Collective Family Decision Making, Temporary Inefficiency and Gender–Specific Transfers*‡

by

Craig Brett

August 1998; Current version: October 1999

Abstract

When behaviour is modelled as the Pareto-efficient outcome of a family decision process, conditions for the existence of a feasible, Pareto improving tax change differ from those that obtain when market behaviour is generated by individuals acting alone. Here, characterisations of Pareto improving tax reforms are presented under various assumptions concerning the ability of a taxation authority to condition demogrants on position within a household. The related issue of temporary inefficiencies is also discussed.

* Some of the work on this paper was done while visiting DELTA in Paris. I am grateful to its director, Roger Guesnerie, for the hospitality he extended to me. I also wish to thank Charles Blackorby, Pierre-Andre Chiappori, David Donaldson, John Weymark, and seminar participants at DELTA for helpful comments and discussions. I bear sole responsibility for any errors that remain. Financial assistance from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

‡ Correspondence to: Craig Brett, Department of Economics, University of Essex, Wivenhoe Park, Colchester, Essex, United Kingdom, CO4 3SQ; email: craig@essex.ac.uk

Keywords: household decision making, tax reform, temporary inefficiency

JEL classifications: D10, D61, H21, H31

November 23, 1999

1. Introduction

In the context of an individual–based model of tax reform, Guesnerie (1977) noticed that under some circumstances the taxation authority may have to choose directions of price change that move the economy inside the production frontier in order to make all individuals better off. However, the Diamond–Mirrlees (1971) result indicates that full optimality requires production efficiency. For this reason the above phenomenon is termed temporary inefficiency. Far from being a theoretical curiosity, temporary inefficiency is of particular importance in cost–benefit analysis. Tirole and Guesnerie (1981) show that, when temporary inefficiencies can be ruled out, the 'fastest' increases in welfare can be attained by changing public policies in such a way to maximise the increase public sector profit at producer prices. Thus, traditional cost–benefit criteria give appropriate guidance. But when temporary inefficiencies exist, this rule selects policy changes that maximise the rate of *decrease* in welfare.

While it is straightforward to check for temporary inefficiencies in particular settings, simple conditions also exist under which they may be rule out *a priori*. Smith (1983) has pointed out that temporary inefficiencies disappear if a poll tax or subsidy is among the instruments available to the planner and if aggregate demand satisfies the Hatta (1977) normality conditions. These conditions may be checked with information on initial producer prices and the income elasticies of aggregate demands.

This study extends the characterisation of Pareto improving directions to a situation in which market behaviour is generated by a family decisions. All families are assumed to have two members. I postulate that each member of a family has preferences over own–consumption of a set of private goods. In line with the collective approach, pioneered by Chiappori (1988 and 1992), I assume that family decisions are Pareto–efficient for the family. This allows intra–family decisions to be described by a sharing rule, that gives the amount of family resources devoted to the consumption of each family member. A good deal of information about the sharing rule can be recovered from family budget data alone. In particular, the derivatives of the sharing rule with respect to prices and individual incomes are identifiable. While this is not enough information to carry out a complete tax reform analysis,[1] it does help describe Pareto improving directions.

The taxation authority is assumed to have at its disposal a full set of linear commodity taxes, including a tax on leisure time, and poll taxes. It cannot, however, effect lump–sum redistributions across families, nor impose the division of resources within families. A simple aggregate production sector is posited, with all pure profit taxed away. For this environment, I derive a characterisation of Pareto–improving directions of reform, paying particular attention to how it is influenced by the types of demogrants available to the planner. Three demogrant structures are compared: poll taxes varying by the demographic characteristic, identical poll taxes for each individual, and redistribution of a fixed total family demogrant between the two family members.

With these characterisations in hand, I present conditions under which temporary inefficiency cannot arise. The planner considered here has potentially two lump–sum transfers available, so we might expect temporary inefficiencies to be ruled out on grounds not unlike those presented by Smith (1983). However, the agents in the present model interact differently than they do in the standard model. In a sense, pairs of agents are forced to behave cooperatively. It is conceivable that this may result in non–standard responses to demogrants. Yet, for some demogrant structures, it is possible to derive restrictions on aggregate demand behaviour and the derivatives of the family sharing rule that ensure the absence of temporary inefficiencies. Moreover, these conditions can be checked using family budget data alone. But temporary inefficiencies remain pervasive when the planner merely redistributes a fixed total family demogrant.

The next section of this paper presents an overview of the model. Characterisation results and a full treatment of temporary inefficiencies follow in Section 3. Some concluding remarks are found in Section 4. Proofs are contained in an appendix.

---

[1] See Brett (1998) for an account of this issue.

2. General Equilibria with Collective Family Decision–Making

Families are assumed to consist of two members, indexed by $i = 1, 2$. They reside in H households, denoted $h = 1, \ldots, H$. Family decisions are assumed to be consistent with the collective model of family behaviour, proposed and analysed by Chiappori (1988 and 1992). The basic consumer unit is the individual, who has preferences over own consumption, x^{ih}, of vectors of n private goods. These preferences are assumed to be represented by a continuous, increasing and strictly quasiconcave function $U^{ih} : I\!R^n_+ \to I\!R$, all i, h. I allow family members to have an initial endowment of goods. For simplicity, it is assumed that all goods are purchased on the market, so that there is no household production process. Consumption decisions are made within the family unit. The exact form of this decision making procedure is not specified. It is merely assumed that consumption decisions are Pareto efficient within the household.

Chiappori (1988) has shown that the assumption of Pareto efficiency implies that the family allocation decision can be summarised by a two–stage process. In the first stage, a sharing rule is decided upon. This sharing rule gives the total value of consumption devoted to goods consumed by individual $1h$, as a function of individual incomes (m^{1h} and m^{2h}) and consumer prices, q. I denote this rule by $\varphi^h(q, m^{1h}, m^{2h})$. Individual $2h$ consumes the remainder of the family budget. It is helpful to denote by μ^{1h}, μ^{2h} the *effective* incomes of family members 1 and 2, respectively. That is,

$$\begin{aligned} \mu^{1h}(q, m^{1h}, m^{2h}) &:= \varphi^h(q, m^{1h}, m^{2h}); \\ \mu^{2h}(q, m^{1h}, m^{2h}) &:= m^{1h} + m^{2h} - \varphi^h(q, m^{1h}, m^{2h}). \end{aligned} \tag{2.1}$$

In the second stage, individuals maximise own–utility, subject to individual budget constraints, which state that the value of consumption must not exceed effective income. The solution functions are denoted by $\hat{x}^{1h}(q, m^{1h}, m^{2h})$ and $\hat{x}^{2h}(q, m^{1h}, m^{2h})$. The within–family aggregate demand for household h is

$$\hat{x}^h(q, m^{1h}, m^{2h}) := \hat{x}^{1h}(q, m^{1h}, m^{2h}) + \hat{x}^{2h}(q, m^{1h}, m^{2h}). \tag{2.2}$$

Levels of individual well–being may then be given by

$$\begin{aligned} u^{1h} &= U^{1h}(\hat{x}^{1h}(q, m^{1h}, m^{2h})) = V^{1h}(q, \mu^{1h}(q, m^{1h}, m^{2h})) \\ u^{2h} &= U^{2h}(\hat{x}^{2h}(q, m^{1h}, m^{2h})) = V^{2h}(q, \mu^{2h}(q, m^{1h}, m^{2h})), \end{aligned} \tag{2.3}$$

where V^{ih} is the indirect utility function dual to U^{ih}.

Families are situated within a competitive environment. There is a production sector consisting of an aggregate firm that maximises profit, given a set of producer prices p. I assume that the solution to the profit maximisation problem defines a differentiable net supply function $y = \eta(p)$, where $\eta : I\!R^n_+ \backslash \{ 0 \} \to I\!R^n$. In addition, I make the following assumption:

Assumption R: $\nabla_p \eta(p)$ *is of rank* $n - 1$.

Homogeneity of the supply function implies that $\nabla_p \eta(p)$ is singular, so that this is a maximal rank assumption. Assumption R rules out the possibility of kinks or ridges in the aggregate production frontier. At any kink or ridge in the production frontier the set of supporting prices is non–unique. Thus, small changes in producer prices may fail to have any effect on production. Assumption R is sufficient to rule out this phenomenon.

The planner can levy a full set of per–unit commodity taxes $t_1, \ldots, t_n$. Consumer prices, taxes and producer prices are related by the identity $q = p + t$. In addition, the government may make transfers to inidviduals. I introduce a demogrant, or common transfer, that does not vary by household. When the within–houshold index i is easily observable (as is the case for, say, gender), I allow the demogrant to vary by i. Denote these transfers R^1 and R^2. In keeping with most literature on the subject, I permit the planner to tax away all profits. Thus, the income of a individual ih is given by

$$m^{ih} = R^i + q\omega^{ih}. \tag{2.4}$$

I am now in a position to describe the equilibria in this economy. First, aggregate demand for goods is given by

$$x(q, R^1, R^2) := \sum_{h=1}^{H} \hat{x}^h(q, q\omega^{1h} + R^1, q\omega^{2h} + R^2). \tag{2.5}$$

The dependence of aggregate demand on endowments is suppressed from the notation. The aggregate endowment is denoted by ω, and is given by $\omega := \sum_{i,h} \omega^{ih}$. An equilibrium for this economy exists at (q, R^1, R^2, p) if

$$x(q, R^1, R^2) \leq \eta(p) + \omega. \tag{2.6}$$

An equilibrium is said to be *tight* if (2.6) holds with equality. If families exhaust their budgets on consumption, Walras' Law guarantees that the government budget is in balance at any tight equilibrium. Assume that the economy is initially in a tight equilibrium at $(\bar{q}, \bar{R}^1, \bar{R}^2, \bar{p})$. In order to avoid boundary problems, I also assume $(\bar{q}, \bar{p}) >> 0$.

A direction of policy change $[dq^\top, dR^1, dR^2, dp^\top]^\top$[2] is said to be *equilibrium preserving* if for initial policy $(\bar{q}, \bar{R}^1, \bar{R}^2, \bar{p})$ it satisfies

$$\nabla_q x(\bar{q}, \bar{R}^1, \bar{R}^2)dq + \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2)dR^1 + \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2)dR^2 \leq \nabla_p \eta(\bar{p})dp, \tag{2.7}$$

where $\nabla_q x(q, R^1, R^2)$ is the Jacobian of aggregate demand with respect to consumer prices. Equilibrium preserving directions of change result in changes in demand that can be met by their associated changes in supply. An alternative representation of feasible directions of policy reform from an initial tight equilibrium, due to Guesnerie (1977, p. 187), is available. Specificially, under Assumption R, for any $[dq^\top, dR^1, dR^2]^\top$ that satisfies

$$\bar{p}^\top \left(\nabla_q x(\bar{q}, \bar{R}^1, \bar{R}^2)dq + \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2)dR^1 + \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2)dR^2\right) \leq 0, \tag{2.8}$$

---

[2] All vectors (both row and column) are enclosed in square brackets.

there exists a direction of producer price change dp such that $[dq^\top, dR^1, dR^2, dp^\top]^\top$ is equilibrium preserving. Condition (2.8) simply describes changes in aggregate demand that stay within the production frontier. It is helpful to rewrite (2.8) as

$$[\Phi_q, \Phi_{R^1}, \Phi_{R^2}] \begin{bmatrix} dq \\ dR^1 \\ dR^2 \end{bmatrix} \geq 0, \tag{2.9}$$

where $\Phi_q := -\bar{p}^\top \nabla_q x(\bar{q}, \bar{R}^1, \bar{R}^2)$ and $\Phi_{R^i} := -\bar{p}^\top \nabla_{R^i} x(\bar{q}, \bar{R}^1, \bar{R}^2)$, $i = 1, 2$.

By substituing (2.4) into (2.3) and differentiating with respect to the government's policy variables — q, R^1, and R^2 — directions of policy change that make every individual of position 1 better off can be shown to satisfy[3]

$$\left[-x^{1h^\top} + \nabla_q^\top \varphi^h + \varphi^h_{m^1}\omega^{1h} + \varphi^h_{m^2}\omega^{2h},\ \varphi^h_{m^1},\ \varphi^h_{m^2}\right] \begin{bmatrix} dq \\ dR^1 \\ dR^2 \end{bmatrix} > 0. \tag{2.10}$$

The intuition behind (2.10) can be demonstrated by considering the equivalent condition

$$x^{1h^\top} dq < \nabla_q^\top \varphi^h dq + (\varphi^h_{m^1}\omega^{1h} + \varphi^h_{m^2}\omega^{2h}) dq + \varphi^h_{m^1} dR^1 + \varphi^h_{m^2} dR^2. \tag{2.11}$$

The left–hand side of the second inequality of (2.11) is the change in the cost of the initial consumption bundle. The right–hand side is the change in effective income brought about by changes in relative prices and demogrants on intra–family allocation. The analogous condition for individuals 2 is

$$\left[-x^{2h^\top} - \nabla_q^\top \varphi^h + (1 - \varphi^h_{m^1})\omega^{1h} + (1 - \varphi^h_{m^2})\omega^{2h},\ 1 - \varphi^h_{m^1},\ 1 - \varphi^h_{m^2}\right] \begin{bmatrix} dq \\ dR^1 \\ dR^2 \end{bmatrix} > 0. \tag{2.12}$$

The interpretation of (2.12) is exactly the same as that of (2.10), keeping in mind that φ^h gives the effective income of person $1h$. Person $2h$ spends the remainder of the family budget.

---

[3] Details of this calculation can be found in Brett (1998).

3. Characterisation Results

Before characterising the feasible strictly Pareto–improving directions of change, some notation is introduced that allows one to consider (2.9), (2.10) and (2.12) jointly. Let

$$\begin{aligned}\Gamma^{1h} &:= \left[-x^{1h^\top} + \nabla_q^\top \varphi^h + \varphi^h_{m^1}\omega^{1h^\top} + \varphi^h_{m^2}\omega^{2h^\top},\, \varphi^h_{m^1},\, \varphi^h_{m^2}\right]^\top;\\ \Gamma^{2h} &:= \left[-x^{2h^\top} - \nabla_q^\top \varphi^h + (1-\varphi^h_{m^1})\omega^{1h^\top} + (1-\varphi^h_{m^2})\omega^{2h^\top},\, 1-\varphi^h_{m^1},\, 1-\varphi^h_{m^2}\right]^\top.\end{aligned} \tag{3.1}$$

Then a direction $\gamma := [dq^\top, dR^1, dR^2]^\top$ is both feasible and Pareto–improving if and only if

$$\begin{aligned}\Gamma^{ih^\top}\gamma > 0,\ h = 1,\ldots,H,\ i = 1,2;\\ \Phi\gamma \geq 0,\end{aligned} \tag{3.2}$$

where $\Phi := [\Phi_q, \Phi_{R^1}, \Phi_{R^2}]^\top$. I make some use of the mathematics of cones in the sequel. Hence, I require the following definition.

Definition: Let $\langle x^i\rangle$ be a collection of vectors in $I\!R^k$. Then the cone generated by $\langle x^i\rangle$, denoted $K(\langle x^i\rangle)$, is defined by:

$$K(\langle x^i\rangle) := \left\{ z \in I\!R^k | z = \sum \lambda^i x^i, \lambda^i \geq 0, \text{ at least one } \lambda^i > 0 \right\}. \tag{3.3}$$

Before stating the characterisation results, I introduce two assumptions.

Assumption A: *There exists a* γ *such that* $\Gamma^{ih^\top}\gamma > 0,\ h = 1,\ldots,H,\ i = 1,2.$

That is, there exists a strictly Pareto–improving direction of policy change, ignoring feasibility constraints. This assumption is generally maintained in the literature (Diewert et al. (1989), Guesnerie (1995)), and is a minimal condition for making the problem interesting.

Assumption B: $\Phi \neq 0$.

A typical component of the vector Φ is the marginal cost, measured at initial producer prices, of meeting the changes in demand induced by a change in the corresponding policy instrument. Assumption B simply states that not all of these marginal costs are zero. This assumption rules out the possibility that all directions of policy reform are tight–equilibrium preserving. If this assumption were violated, the requirement of Assumption A renders the problem uninteresting. The planner could implement the change γ mentioned therein (or any other for that matter) while maintaining tight equilibrium.

3.1. Unrestricted Demogrants

Suppose that the planner can make independent changes in the poll subsidies. Then the following restatement of a theorem of Guesnerie (1977, Proposition 4, pp. 189–190) obtains in the present context.

Proposition 1. *Let Assumptions A and B hold. Then:*

i) $\Phi \in K(\langle -\Gamma^{ih} \rangle)$ *if and only if there exist no feasible strictly Pareto–improving directions of policy change.*

ii) $\Phi \in K(\langle \Gamma^{ih} \rangle)$ *if and only if there exist strictly Pareto–improving directions, all of which are non–tight equilibrium–preserving.*

iii) $\Phi \in K(\langle \Gamma^{ih} \rangle)^C \cap K(\langle -\Gamma^{ih} \rangle)^C$ *if and only if there exist strictly Pareto–improving directions that are tight equilibrium–preserving.*

Although a somewhat technical condition, $\Phi \in K(\langle -\Gamma^{ih} \rangle)$ states simply that the vector Φ can be written as a negative linear combination of the vectors $\langle \Gamma^{ih} \rangle$.

Smith (1983) rules out case (ii) of Proposition 1 by assuming that the Hatta (1977) conditions are satisfied; that is, an increase in a lump–sum transfer leads to a positive change in the cost of (net) demand, evaluated at the original producer prices.[4] This

---

[4] The original Hatta conditions impose this restriction on compensated aggregate demand. In the sequel, I impose similar conditions on uncompensated aggregate demand.

assumption is obviously satisfied when producer and consumer prices coincide, provided that consumers are nonsatiated. The intuition behind this result is clear. Suppose that the planner changes consumer prices in such a way that everyone is better off and the resulting equilibrium is non–tight. Then there exists an excess supply of goods. Suppose now that the planner redistributes this surplus with a lump–sum transfer. Everyone is made still better off, and by the restriction placed on aggregate demand, some of the surplus is consumed.

In the present context, it is not guaranteed that an increase in a specific demogrant makes all consumers better off. Something must be said about sharing within families before such a conclusion can obtain.

Assumption C: $0 < \varphi^h_{m^1} < 1$, $\forall h$ *and* $0 < \varphi^h_{m^2} < 1$, $\forall h$.

That is, additions to the lump–sum grants are 'split' in the usual sense of the word. This property need not hold in general, due to the presence of endowments. As long as the planner may increase the demogrant afforded to each person, Assumption C entails Assumption A of the preceding section because an increase in either demogrant makes all consumers better off.

In what follows I have occasion to use the following assumptions, each of which is in the spirit of Smith's restriction.

Assumption N1: $\bar{p}^\top \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) > 0$.

Assumption N2: $\bar{p}^\top \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2) > 0$.

Like the Hatta conditions, Assumptions N1 and N2 are a form of normality conditions on aggregate demand. It turns out that temporary inefficiencies may be ruled out when either N1 or N2 holds. This is the content of the next proposition.

Proposition 2 *Let Assumptions B and C hold. Then strictly Pareto–improving directions of policy change with temporary inefficiencies cannot arise if either Assumption N1 or N2 holds.*

That only one of N1 and N2 is required to rule out temporary inefficiencies is not surprising. The planner needs but one instrument to redistribute any surplus to individuals.

3.2. Pure Poll Taxes

It may be argued that the planner cannot make lump–sum transfers contingent on the index i. This may be because there is no easily observed characteristic to which it corresponds, or it may be deemed inappropriate to 'discriminate' on the basis of that characteristic. One may also take the view that it is difficult to enact new policies that aim at reducing existing differences in lump–sum payments. Either of these circumstances can be viewed as imposing the restriction $dR^1 = dR^2$ on the directions of policy change available to the planner. It is, therefore, interesting to investigate the conditions under which temporary inefficiencies may arise in this context.

It is necessary to introduce some new notation at this point. Let $\mathbf{0}$ denote the $2N$–dimensional zero vector. Define also the following sets:

$$\hat{\ell} := \Big\{ x \in I\!\!R^{2N+2} | x = \nu \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix}, \nu \in I\!\!R \Big\}, \tag{3.4}$$

$$\hat{K} := K\left(\langle -\Gamma^{ih} \rangle, \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ -1 \\ 1 \end{bmatrix} \right) \backslash \hat{\ell}, \tag{3.5}$$

$$\check{K} := K\left(\langle \Gamma^{ih} \rangle, \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ -1 \\ 1 \end{bmatrix} \right) \backslash \hat{\ell}. \tag{3.6}$$

$\hat{\ell}$ is the negative 45–degree line in the subspace of $I\!\!R^{2N+2}$ spanned by the last two elements of the standard basis. Given Assumption C, the sets $\hat{K}$ and $\check{K}$ are supersets of

$K(\langle -\Gamma^{ih} \rangle)$ and $K(\langle \Gamma^{ih} \rangle)$, respectively. The first $2N$ components of any vector in $\hat{K}$ must be generated as a semi–positive linear combination of the first $2N$ components of vectors $\langle -\Gamma^{ih} \rangle$. Furthermore, the projection of any vector in $\hat{K}$ onto its last two components lies off the negative 45–degree line. $\check{K}$ bears an analogous relation to $K(\langle \Gamma^{ih} \rangle)$. Bearing these notations in mind, the following obtains.

Proposition 3 *Let Assumptions B and C hold. Then the following hold.*

i) $\Phi \in \hat{K}$ *if and only if there are no feasible strictly Pareto–improving directions of policy reform satisfying* $dR^1 = dR^2$.

ii) $\Phi \in \check{K}$ *if and only if there exist strictly Pareto–improving directions of policy change satisfying* $dR^1 = dR^2$*, all of which are necessarily non–tight equilibrium–preserving.*

iii) $\Phi \in \hat{K}^C \cap \check{K}^C$ *if and only if there exists tight equilibrium–preserving directions of policy change satisfying* $dR^1 = dR^2$.

It is interesting to compare Proposition 3 with Proposition 1 when Assumptions B and C (and, hence, A) hold. Because $K(\langle -\Gamma^{ih} \rangle)$ is contained in $\hat{K}$, statement (i) of Proposition 3 indicates that there are fewer values of the vector Φ for which feasible strictly Pareto–improving directions of policy reform exist when poll taxes are restricted. This is hardly surprising. Because $K(\langle \Gamma^{ih} \rangle)$ is contained in $\check{K}$, statement (ii) of Proposition 3 indicates that there are more values of the vector Φ for which temporary inefficiencies arise when the planner operates within the restricted set of poll taxes.

Some insight into circumstances giving rise temporary inefficiencies is afforded by considering a necessary condition for temporary inefficiencies. Suppose $\Phi \in \check{K}$. Then it must be the case that

$$\bar{p}^\top \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) \neq \bar{p}^\top \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2). \tag{3.7}$$

In the presence of Assumption N1 (or N2), it is clear that temporary inefficiencies cannot arise when (3.7) is violated. In that case, the population would act as two sub–populations, each inducing the same change in the value (measured at producer prices) of aggregate demand to changes in the demogrant. The intuition underlying Proposition 2 would apply.

Given that the restricted planner still has some effective means of using poll subsidies, one might suspect that temporary inefficiencies may be easily ruled out. This can be shown by appealing to the following assumption.

Assumption N3: $\bar{p}^{\top}(\nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) + \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2)) > 0$.

Notice that Assumption N3 implies that one of N1 or N2 must hold, but not necessarily both. It is also consistent with condition (3.7). The following proposition may come as no surprise.

Proposition 4 *Let Assumptions B, C and N3 hold. Then strictly Pareto–improving directions of policy reform with temporary inefficiencies cannot arise when* $dR^1 = dR^2$.

The role played by assumption N3 in Proposition 4 is clear. It is sufficient to ensure that any surplus generated by a price change will be 'eaten up' if each poll subsidy is increased by the same amount.

3.3. Intra–Family Redistriubtion

I also wish to investigate the power of a planner who can merely redistribute a fixed lump–sum between the family members. Corollary 1.1 of Brett (1998)[5] indicates that the intuitive argument for the elimination of temporary inefficiencies breaks down, since redistribution of a fixed total alone cannot achieve strict Pareto–improvements. An extreme example of this phenomenon is documented by Apps and Rees (1988): when households act as if they are maximising a price and income independent social welfare

---

[5] This result states that there are no Pareto–improving directions of reform satisfying $dq = 0$ and $dR^1 + dR^2 = 0$.

function, marginal purely redistributive changes in demogrants have no effect at all on intra–family allocations as long as boundary constraints do not bind. However, the collective approach is more general, and allows for behavioural responses to changes in the distribution of a fixed family income. Thus a planner endowed with the power to redistribute within families is not identical to one who has no lump–sum taxation power at all.

In order to state the analogue to Proposition 3 in this context I require some additional notation:

$$\tilde{\ell} := \left\{ (x \in \mathbb{R}^{2N+2} | x = \nu \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix}, \nu \in \mathbb{R} \right\}. \tag{3.8}$$

$$\tilde{K} := K\left(\langle -\Gamma^{ih} \rangle, \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ -1 \\ -1 \end{bmatrix} \right) \backslash \tilde{\ell}. \tag{3.9}$$

$$\bar{K} := K\left(\langle \Gamma^{ih} \rangle, \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ -1 \\ -1 \end{bmatrix} \right) \backslash \tilde{\ell}. \tag{3.10}$$

The line $\tilde{\ell}$ is the 45–degree line in the plane spanned by the last two elements of the standard basis. The sets $\tilde{K}$ and $\bar{K}$ are analogous to $\hat{K}$ and $\check{K}$, respectively, and can be given similar interpretations. Let Φ_Q, Γ^{ih}_Q denote the vectors Φ, Γ^{ih}, respectively, with their last two components deleted.

Proposition 5 *Let Assumptions A, B and C hold. Then the following statements hold.*

- *i) There are no feasible strictly Pareto–improving directions of policy reform that satisfy* $dR^1 + dR^2 = 0$ *if and only if* $\Phi \in \tilde{K}$ *or* $\begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix} \in K(\langle \Gamma^{ih} \rangle)$.
- *ii) There are strictly Pareto–improving directions of policy reform that satisfy* $dR^1 + dR^2 = 0$, *all of which are non–tight equilibrium preserving, if and only if* $\Phi \in \bar{K}$ *and* $\begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix} \notin K(\langle \Gamma^{ih} \rangle)$.

iii) There exist tight equilibrium preserving strictly Pareto–improving directions of policy reform that satisfy $dR^1+dR^2=0$ *if and only if* $\Phi \in \tilde{K}^C \cap \bar{K}^C$ *and* $\begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix} \notin K(\langle \Gamma^{ih} \rangle)$.

A few words of comment on Proposition 5 are in order. First of all, suppose that there are no Pareto–improving directions of change in consumer prices alone, ignoring feasibility. Then, by Motzkin's Theorem, $\mathbf{0} \in K(\langle \Gamma_Q^{ih} \rangle)$.[6] When Assumption C holds, this condition is equivalent to $[\mathbf{0}^\top, 1, 1]^\top \in K(\langle \Gamma^{ih} \rangle)$. Thus, whenever there are no strictly Pareto–improving directions of change in consumer prices alone, adding purely redistributive transfers affords no new Pareto–improving directions of reform. Moreover, $\Phi \in \tilde{K}$ implies $\Phi_Q \in K(\langle -\Gamma^{ih} \rangle)$. Hence, whenever the planner considered in Proposition 5 can find no feasible strictly Pareto–improving directions of policy reform, neither can a planner who does not have the power to use any demogrants. This conclusion should come as no surprise, since $dR^1 + dR^2 = 0$ is satisfied whenever $dR^1 = dR^2 = 0$.

However, purely redistributive transfers can be used to induce demand responses that make feasible some changes in consumer prices that would otherwise be infeasible. In particular, intra–family redistriubitions induce demand responses that lead the economy toward the production frontier whenever $p^\top \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) \neq -p^\top \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2)$. Yet, as the next proposition states, the restricted planner may face the prospect of temporary inefficiencies regardless of the demand responses to lump–sum taxation.

Proposition 6 *Suppose that Assumptions A, B and C are satisfied. If there exists* $\bar{h}$ *such that* $\varphi_{m^1}^{\bar{h}} \neq \varphi_{m^2}^{\bar{h}}$*, then for all distinct* $p^\top \nabla_{R^1} x(q, R^1, R^2)$, $p^\top \nabla_{R^2} x(q, R^1, R^2)$ *there exists* $\Phi_Q \in K(\langle \Gamma_Q^{ih} \rangle)$ *such that* $\begin{bmatrix} \Phi_Q \\ p^\top \nabla_{R^1} x(q, R^1, R^2) \\ p^\top \nabla_{R^2} x(q, R^1, R^2) \end{bmatrix} \in \bar{K}$.

---

[6] Motzkin's Theorem states: for given matricies A, B and C, with A nonvacuous, exactly one of the systems of relations a) and b) below has a solution: a)$\{Ax >> 0, Bx \geq 0, Cx = 0\}$; b) $\{A^\top y_1 + B^\top y_2 + C^\top y_3 = 0, y_1 \geq 0 \text{ (but } y_1 \neq 0), y_2 \geq 0\}$ (Mangasarian (1969, pp. 28–29)).

Proposition 6 indicates that if the effective income of a single individual of type 1 changes due to an intra–family redistriubtion, then temporary inefficiencies are as pervasive as they are when demogrants are completely ruled out. If a price change moves the economy inside the production frontier, a within–family redistribution can be constructed to use the freed resources, but at least one member of family $\bar{h}$ must be made worse off in the second stage of the process.

4. Conclusion

The collective model of family decision making provides a parsimonious representation of behaviour via the household sharing rule. This representation is well suited to the analysis of tax reforms, as it allows the decomposition of changes in welfare into sharing rule effects and more traditional price and income responses. An important sub–class of sharing rule responses are the changes in intra–household allocation owing to changes in the exogenous incomes of family members. When a taxation authority can implement gender–specific demogrants, it has at its disposal a potentially important tool with which to modify family behaviour.

This paper has presented a characterisation of second best optima, from a tax reform perspective, under a variety of limitations on the types of demogrants available. It was shown that some interesting issues in tax reform analysis, including the question of temporary inefficiency, can be resolved with household budget data alone. There is, indeed, no reason to believe that adding a richer description of household behaviour necessarily leads to intractable policy analysis.

APPENDIX

I present here the proof of each proposition of the text.

Proof of Proposition 1:

i) γ is feasible and Pareto–improving if and only if it satisfies (3.2). But (3.2) is satisfied if and only if

$$\nexists \beta^{ih} \geq 0, \text{ not all}\, \beta^{ih} = 0 \text{ and } \lambda \geq 0 \,\text{such that } \sum_{i,h} \beta^{ih} \Gamma^{ih} + \lambda \Phi = 0_{n+2}, \tag{A.1}$$

by Motzkin's transposition theorem. Equivalently, there are no feasible strictly Pareto–improving directions exactly when there are $\beta^{ih} \geq 0, \forall i, \forall h,$ (not all equal zero) and $\lambda \geq 0$ such that

$$\sum_{i,h} \beta^{ih} \Gamma^{ih} + \lambda \Phi = 0_{n+2}. \tag{A.2}$$

Suppose that $\lambda = 0$. Then $\sum_{i,h} \beta^{ih} \Gamma^{ih} = 0$. Then, by Motzkin's Theorem, the top half of (3.2) has no solution. This contradicts Assumption A. Thus, in view of Assumption B, $\lambda > 0$. Hence, I may rearrange (A.2) to give that $\Phi \in K(\langle -\Gamma^{ih} \rangle)$. It is clear that $\Phi \in K(\langle -\Gamma^{ih} \rangle)$ implies the existence of a solution to (A.2).

ii) $\Phi \in K(\langle \Gamma^{ih} \rangle)$ exactly when there exist $\beta^{ih} \geq 0$ satisfying

$$\sum \beta^{ih} \Gamma^{ih} - \Phi = 0. \tag{A.3}$$

Assumption B ensures that at least one β^{ih} is positive. By Motzkin's theorem (with $A = [\Gamma^{11}, \ldots, \Gamma^{2H}]^\top$ and $B = -\Phi$), (A.3) implies the following has no solution

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, \forall h; \quad \Phi^\top \gamma \leq 0. \tag{A.4}$$

In particular, there are no strictly Pareto–improving tight equilibrium–preserving directions of reform. In order for Assumption A to be satisfied, there must exist a γ for which

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, \forall h. \tag{A.5}$$

By (A.4), $\Phi^\top \gamma > 0$ for any such γ. The 'only if' part of Statement ii) follows.

Conversely, let

$$\Gamma^{ih^\top} \gamma > 0, \forall i, \forall h; \quad \Phi^\top \gamma > 0 \tag{A.6}$$

have a solution and let

$$\Gamma^{ih^\top} \gamma > 0, \forall i, \forall h; \quad \Phi^\top \gamma = 0 \tag{A.7}$$

have no solution. Apply Motzkin's Theorem to (A.7) to conclude that there exists $\beta^{ih} \geq 0$ (not all equal zero) and a $\lambda \in I\!\!R$ such that

$$\sum \beta^{ih} \Gamma^{ih} + \lambda \Phi = 0. \tag{A.8}$$

By (A.6) and Proposition 2, $\Phi \notin K(\langle -\Gamma^{ih} \rangle)$. Hence, $\lambda \leq 0$. By Assumption A, $\lambda \neq 0$. (The argument is identical to the one used in the proof of i).) Thus, $\lambda < 0$. Rearranging (A.8) yields $\Phi \in K(\langle \Gamma^{ih} \rangle)$.

iii) This follows from Statements i) and ii). □

Proof of Proposition 2:

Proposition 1 states that temporary inefficiencies occur exactly when there exist $\beta^{ih} \geq 0$ such that $\Phi = \sum_{i,h} \beta^{ih} \Gamma^{ih}$. The last two rows of this equality are:

$$\begin{aligned} -\bar{p}^\top \nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) &= \sum_h \beta^{1h} \varphi^h_{m^1} + \sum_h \beta^{2h} (1 - \varphi^h_{m1}), \\ -\bar{p}^\top \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2) &= \sum_h \beta^{1h} \varphi^h_{m^2} + \sum_h \beta^{2h} (1 - \varphi^h_{m^2}). \end{aligned} \tag{A.9}$$

Assumption C allows me to conclude that the right–hand sides of each equation in (A.9) is nonnegative. N1 precludes the top line of (A.9) from holding. N2 precludes the second.□

Proof of Proposition 3:

i) There are no feasible strictly Pareto–improving directions of change satisfying $dR^1 = dR^2$ if and only if the following has no solution

$$\Gamma^{ih^\top} \gamma > 0, \forall i, h; \quad \Phi^\top \gamma \geq 0; \quad [\mathbf{0}^\top, 1, -1] \gamma = 0. \tag{A.10}$$

But, by Motzkin's theorem, (A.10) has no solution exactly when

$$\sum_{i,h} \beta^{ih}\Gamma^{ih} + \lambda\Phi + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix} = 0;\ \beta^{ih} \geq 0,\ \text{some}\, \beta^{ih} > 0,\ \lambda \geq 0,\ \kappa \in I\!R \tag{A.11}$$

has a solution. Suppose, by way of contradiction, that $\lambda = 0$. Then the last two rows of (A.11) become

$$\begin{aligned} \sum_h \beta^{1h}\varphi^h_{m^1} + \sum_h \beta^{2h}(1 - \varphi^h_{m^1}) + \kappa = 0, \\ \sum_h \beta^{1h}\varphi^h_{m^2} + \sum_h \beta^{2h}(1 - \varphi^h_{m^2}) - \kappa = 0. \end{aligned} \tag{A.12}$$

Now, by Assumption C, the top line of (A.12) implies $\kappa < 0$, whereas the bottom line of (A.12) implies $\kappa > 0$. A contradiction ensues. Therefore, $\lambda > 0$. Rearranging (A.11) yields that $\Phi \in \hat{K}$. It is a matter of straightforward computation to show that $\Phi \in \hat{K}$ implies the existence of a solution to (A.11) .

ii) $\Phi \in \check{K}$ if and only if there exist $\beta^{ih} \geq 0$ (not all zero), and $\kappa \in I\!R$ satisfying

$$\Phi = \sum_{i,h} \beta^{ih}\Gamma^{ih} + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix}. \tag{A.13}$$

Now, by Motzkin's Theorem, it must be the case that

$${\Gamma^{ih}}^{\top}\gamma > 0, \forall i, h; \quad \Phi^{\top}\gamma \leq 0; \quad [\mathbf{0}^{\top}, 1, -1]\gamma = 0 \tag{A.14}$$

has no solution. In particular, there is no solution to

$${\Gamma^{ih}}^{\top}\gamma > 0, \forall i, \forall h; \quad \Phi^{\top}\gamma = 0; \quad [\mathbf{0}^{\top}, 1, -1]\gamma = 0. \tag{A.15}$$

Note that Assumption C implies that there is a solution to

$${\Gamma^{ih}}^{\top}\gamma > 0, \forall i, h; \quad [\mathbf{0}^{\top}, 1, -1]\gamma = 0. \tag{A.16}$$

(Pick $\gamma = [\mathbf{0}^{\top}, 1, 1]^{\top}$; that is, increase both poll subsidies by the same amount.) Hence, there must be a solution to

$${\Gamma^{ih}}^{\top}\gamma > 0, \forall i, \forall h; \quad \Phi^{\top}\gamma > 0; \quad [\mathbf{0}^{\top}, 1, -1]\gamma = 0. \tag{A.17}$$

The 'only if' part of Statement ii) follows.

Conversely, suppose there is a solution to (A.17), but that there is no solution to (A.15). Apply Motzkin's Theorem to (A.15) to conclude that there exists $\beta^{ih} \geq 0$ (not all zero), and $\kappa, \lambda \in I\!R$ satisfying

$$\sum_{i,h} \beta^{ih}\Gamma^{ih} + \lambda\Phi + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ -1 \end{bmatrix} = 0. \tag{A.18}$$

When $\lambda = 0$, the last two rows of (A.18) reduce to (A.12). This violates Assumption C. $\lambda > 0$ implies $\Phi \in \hat{K}$. In view of Statement i), this violates condition (A.17). Thus, $\lambda < 0$. Rearranging (A.18) now yields that $\Phi \in \check{K}$.

iii) This statement follows directly from i) and ii). □

Proof of Proposition 4:

By Proposition 3, temporary inefficiencies can hold only when there exist $\beta^{ih} \geq 0$ (not all equal zero),$\kappa \in I\!R$ satisfying (A.13). Adding the last two lines of (A.13) yields

$$-\bar{p}^\top(\nabla_{R^1} x(\bar{q}, \bar{R}^1, \bar{R}^2) + \nabla_{R^2} x(\bar{q}, \bar{R}^1, \bar{R}^2)) = \sum_h \beta^{1h}(\varphi^h_{m^1} + \varphi^h_{m^2}) + \sum_h \beta^{2h}(2 - \varphi^h_{m^1} - \varphi^h_{m^2}). \tag{A.19}$$

Assumption N3 implies that the left–hand side of (A.19) is negative, whereas Assumption C implies that the right–hand side of (A.19) is nonnegative. A contradiction ensues. □

Proof of Proposition 5:

i) There are no feasible strictly Pareto–improving directions of change satisfying $dR^1 + dR^2 = 0$ exactly when there is no solution to

$$\Gamma^{ih^\top}\gamma > 0, \forall i, h; \quad \Phi^\top \gamma \geq 0; \quad [\mathbf{0}^\top, 1, 1]\gamma = 0. \tag{A.20}$$

By Motzkin's Theorem, (A.20) has no solution if and only if there exist $\beta^{ih} \geq 0$ (some $\beta^{ih} > 0$), $\lambda \geq 0$,$\kappa \in I\!R$ satisfying

$$\sum_{i,h} \beta^{ih}\Gamma^{ih} + \lambda\Phi + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix} = 0. \tag{A.21}$$

When (A.21) holds with $\lambda > 0$, $\Phi \in \tilde{K}$. If $\lambda = 0$, the last line of (A.21) becomes

$$\sum_h \beta^{1h}\varphi^h_{m^2} + \sum_h \beta^{2h}(1 - \varphi^h_{m^2}) + \kappa = 0. \tag{A.22}$$

It follows from Assumption C that $\kappa < 0$. Hence, $[\mathbf{0}^\top, 1, 1]^\top \in K(\langle \Gamma^{ih} \rangle)$. Direct calculation confirms that either $[\mathbf{0}^\top, 1, 1]^\top \in K(\langle \Gamma^{ih} \rangle)$ or $\Phi \in \tilde{K}$ implies the existence of a solution to (A.21).

ii) Let $\Phi \in \bar{K}$ and $[\mathbf{0}^\top, 1, 1]^\top \notin K(\langle \Gamma^{ih} \rangle)$. $\Phi \in \bar{K}$ exactly when there exist $\beta^{ih} \geq 0$ (not all zero) and $\kappa \in I\!R$ such that

$$\Phi = \sum_{i,h} \beta^{ih}\Gamma^{ih} + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix}. \tag{A.23}$$

By Motzkin's Theorem, (A.23) implies that there is no solution to

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, h; \quad \Phi^\top \gamma \leq 0; \quad [\mathbf{0}^\top, 1, 1]\gamma = 0. \tag{A.24}$$

In particular, there is no solution to

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, h; \quad \Phi^\top \gamma = 0; \quad [\mathbf{0}^\top, 1, 1]\gamma = 0. \tag{A.25}$$

$[\mathbf{0}^\top, 1, 1]^\top \notin K(\langle \Gamma^{ih} \rangle)$ implies that there does not exist $\beta^{ih} \geq 0$ (not all zero) and $\kappa < 0$ satisfying

$$\sum_{i,h} \beta^{ih}\Gamma^{ih} + \kappa \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \end{bmatrix} = 0. \tag{A.26}$$

By Assumption A, (A.26) has no solution with semi–positive β^{ih} and $\kappa = 0$. The last row of (A.26) is exactly (A.22), so that Assumption C rules out the possibility of a solution to (A.26) with semi–positive β^{ih} and $\kappa > 0$. Thus, by Motzkin's Theorem, there exists a solution to

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, h; \quad [\mathbf{0}^\top, 1, 1]\gamma = 0. \tag{A.27}$$

Then, by (A.24), there is a solution to

$${\Gamma^{ih}}^\top \gamma > 0, \forall i, h; \quad \Phi^\top \gamma > 0; \quad [\mathbf{0}^\top, 1, 1]\gamma = 0. \tag{A.28}$$

Conversely, let (A.28) have a solution, but let there be no solution to (A.25). Apply Motzkin's Theorem to the first and third components of (A.28) to conclude that $[\mathbf{0}^\top, 1, 1]^\top \notin K(\langle \Gamma^{ih} \rangle)$. Because (A.25) has no solution, Motzkin's Theorem implies the existence of $\beta^{ih} \geq 0$ (not all zero) and real numbers λ and κ satisfying (A.21). In view of (the proof of) Statement i), $\lambda \geq 0$ contradicts (A.28). Thus, (A.21) holds with $\lambda < 0$. Hence, $\Phi \in \bar{K}$.

iii) This statement follows from i) and ii).

iv) Take arbitrary $p^\top \nabla_{R^1} x(q, R^1, R^2) \neq p^\top \nabla_{R^2} x(q, R^1, R^2)$. The existence of an $\bar{h}$ as described in the statement ensures that

$$K\Big(\langle \begin{bmatrix} \varphi^h_{m^1} \\ \varphi^h_{m^2} \end{bmatrix} \rangle, \langle \begin{bmatrix} 1 - \varphi^h_{m^1} \\ 1 - \varphi^h_{m^2} \end{bmatrix} \rangle, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \Big) = I\!R^2, \tag{A.29}$$

since the set of generators contains at least three noncollinear vectors. Then there exist $\beta^{ih} \geq 0$, at least one nonzero and $\kappa \in I\!R$ such that

$$\begin{aligned} -p^\top \nabla_{R^1} x(q, R^1, R^2) &= \sum_h \beta^{1h} \varphi^h_{m^1} + \sum_h \beta^{2h} (1 - \varphi^h_{m^1}) + \kappa \\ -p^\top \nabla_{R^2} x(q, R^1, R^2) &= \sum_h \beta^{1h} \varphi^h_{m^2} + \sum_h \beta^{2h} (1 - \varphi^h_{m^2}) + \kappa \end{aligned} \tag{A.30}$$

Now select $\Phi_Q := \sum_{i,h} \beta^{ih} \Gamma^{ih}$, and the result follows. □

REFERENCES

Apps, P., Rees, R., 1988. Taxation and the Household. Journal of Public Economics 35, 355–369.

Brett, C., 1988. Tax Reform and Collective Family Decision–Making. Journal of Public Economics 70, 425–440.

Chiappori, P.-A., 1988. Rational Household Labor Supply. Econometrica 56, 63–89.

Chiappori, P.-A., 1992. Collective Labor Supply and Welfare. Journal of Political Economy 100, 437–467.

Diamond P., Mirrlees, J., 1971. Optimal Taxation and Public Production I-II. American Economic Review 61, 8–27, 261–268.

Diewert, W. E., Turunen-Red, A., Woodland, A., 1989. Productivity- and Pareto-Improving Changes in Taxes and Tariffs. Review of Economic Studies 56, 199–216.

Guesnerie, R., 1977. On the Direction of Tax Reform. Journal of Public Economics 7, 179–202.

Guesnerie, R., 1995. A Contribution to the Pure Theory of Taxation, Cambridge University Press, Cambridge.

Hatta, T., 1977. A Theory of Piecemeal Policy Recommendations. Review of Economic Studies 44, 1–21.

Mangasarian, O. L., 1969. Nonlinear Programming, McGraw-Hill, New York.

Smith, A., 1983. Tax Reform and Temporary Inefficiencies. Journal of Public Economics 12, 171–189.

Tirole, J., Guesnerie, R., 1981. Tax Reform From a Gradient Projection Viewpoint. Journal of Public Economics 15, 275–293.