CYRIL WELCH

Zeno, Newton and . . .

Preamble

Throughout his *Being and Time* Heidegger speaks as though he were addressing most centrally the question of *Bewegtheit*, of what gets being-there moving: moved down from authenticity into inauthenticity and moved up into full happening; also moved from one historical mode into another — each movement distinct from the movedness of things confronted as on hand, whether apples falling to the ground, planets coursing in their orbits, or people driven by sex or greed or ambition. In his later essay on Aristotle's concept of nature he rightly insists that the ancient question of κίνησις bears on movedness rather than on motion itself. And, as is well known, he argues that the intelligibility of the movedness of things and people rests on the movedness of being-there, an apparently third order.

My own essay here stems from my perplexity regarding how our philosophical tradition has interpreted, variously, both movedness and motion — and from my faith that fresh consideration of these traditional interpretations might place us on the brink of yet another, where Heidegger seems to start.

To our ancient forebears motion appeared much as the blur we today may see on a photo of a moving car taken at a slow shutter speed. Judging from their treatises and dramas, architecture and sculpture, we can deduce that what counted for them was primarily the goal of motion, and then also its impetus. These two — what Aristotle called the telic and the kinetic grounds (final and efficient causes) — appear intelligible: we make sense of something in motion if and only if we anticipate what lies ahead and then also recall what pushes it forward. Apart from these two extremes (i.e., when the draw and the push escape us), what happens in between can only appear as a restless blur.

For millennia, the prime examples of motion were ones we would describe as growth, maturation or development: change of condition as much as (even prior to) change of place. Plants and animals grow from birth to maturity, both in size and capability, then decline; and in many cases there is a craft (anciently, the prime name for human agency) that promotes or thwarts the change. Similarly with humanly devised things (artifacts): bulwarks, wells, and temples not only come into being and pass away, they also decline and require maintenance. But we ourselves remain the prime examples of such development: each from helpless infant to active child, from troublesome adolescent to skillful or ignorant, just or unjust adult — each then fading back naturally, or coming to a violent end. Communities —households and cities —likewise.

Such recognizable motion has stages that we, or at least artisans, will learn to recognize and anticipate. But what counts, what lends intelligibility to each stage, is the final one, the one at which we recognize things as having arrived — first of all at their fulfilment and then, relative to this fulfilment, at their decline. Upon its arrival, the horse and the oak, the ship or the temple, the sage or the city, is at rest, while the stages along the way, forward or backward, *being* only in reference to their respective consummation, *are* not — not yet or not still.

On this understanding of things — of their motion and their rest — Zeno's paradoxes made more intuitive sense to our forebears than they do to us today. We must then exert ourselves especially hard…

We *see with our eyes* an athlete overtaking a tortoise and a projectile hitting a target — despite the conjectured need for each to pass through an infinite number of stages, i.e. ones defined ever freshly relative to the remaining distance. So too we may *hear with our ears* the loud and then the soft of an ambulance's siren at one distance and then at another. So too we may *feel with our hands* a radiator now cold and now hot; or, with the skin on our necks or legs, the air chillier at sunset than at mid-day. These sensations might then lead us to talk of *passing*, *fading*, *warming*, *chilling* as motions. But what we *understand*, with the eye of the soul, is one stage at a time, and especially the final stage, the goal lurking in each of these situations.

Zeno's paradoxes call attention to our intellectual commitments to recognition and anticipation. And to the unintelligibility of the supposition that there *are* infinite stages through which things in motion, changing things, must pass. Unintelligible is motion all by itself, separated from things in *their* beginnings and endings.

And precisely heavenly bodies, which we see neither becoming nor ending, we *understand* when we discover their repetitive paths: these paths remain the same, *are* the meaning of

these bodies in motion, now *understood*, relative to their fixed paths, as at rest.

All this in stark contrast to our own inherited and ingrained understanding of things. For us, careful knowledge (vs. casual acquaintance or mere familiarity) accounts precisely for motion, for change. Newton's physics is science of motion — just as are Machiavelli's statecraft, Adam Smith's political economy, and Darwin's biology; just as are our modern cosmology of solar systems and our modern sociology of multi-cultural conditions. All are sciences of transition between arbitrarily stipulated stages. Sciences of motion *per se*: of a projectile or a planet *as moving* and not only of its trajectory; of change in species and not only differences in the specimens; of development in the wealth of nations and not only of their condition at any one time. Managers of a financial institution or an industrial complex know that their things only *are* in flux, and expend the greater part of their own energies foreseeing multiple ways of responding to possible developments — as do chess masters and military strategists.

Ancient sciences of rest contemplated our individual artisanal participation in natural affairs. Modern sciences of transition allow for collective intervention in the course of human affairs — as when vast teams steer spacecraft to the moon; as when geneticists all over the world cooperate to learn how to manipulate the genetic structure of plants and animals; as when massive bureaucracies navigate national and international ships of state. Sciences of rest bequeath techniques each must remaster. Sciences of motion bequeath technologies that no individual, only a collectivity can master, and always on the fly: education in them inculcates a temperament for learning new versions — as is glaringly obvious today in computer science.

The difference has long been familiar — already early on, in the conflict between Ancients and Moderns, and then in the works of such astute thinkers as Henry Adams (12^{th}-century Unity vs. 20^{th}-century Multiplicity), Henri Bergson and John Dewey.

But how are we to understand *this* difference? Is the change in our intellectual commitment simply continuous progression (as most seem to prefer), or regression (as some might lament)? Or rather an abrupt change from one kind of thinking to another? And, in whichever case, might we today simply dismiss one tradition in favor of the other?

The conflict of the two has largely played itself out since its inauguration by Galileo, Bacon and Descartes. The Platonic-Aristotelian contingent has withdrawn into the ethereal and esoteric realms of nostalgic or scholarly discourse, thereby permitting us to ignore them, our erstwhile competition, and to save our energies for the tasks of expanding our own power, that of the victorious party —just as the collapse of the East Block in 1989 has left capitalism free to ignore the problems highlighted, however ineptly, by communism. Any effort to defend the defeated party engages us in a possibly comic, possibly tyrannical, in any case futile enterprise.

Yet we might take a third, not exactly neutral tact, but also not a belligerent one either: we can reconsider each side on the initial assumption that it formed (in the earlier case) and forms (in the current case) a self-sufficient, internally coherent and, for long whiles, remarkably viable understanding of things at a wide variety of levels, even if not at all the levels endorsed by its adversary. This third tact inaugurates a third understanding, namely that, while the earlier way has died and can never be revived in its earlier form, it drew upon a source vital to our own native understanding of things — and that, while the current understanding is still flourishing in its outgrowth, it has lost its originating vitality and must itself be revived. The task is then to

re-understand *both* sides: to re-inaugurate both, neither defending nor attacking either one.

In my *Contemplative Logic* (1999, amended and expanded into *Logic Ancient and Modern*, 2006) I have undertaken this task in regard to our similarly conflictive traditions of logic. And it could be undertaken again with regard to a number of other mainstays of our tradition. Now, though, let us consider this question of motion (no doubt related to these other questions): Why and how the shift from understanding motion in reference to rest to understanding rest in reference to motion? From intellectual focus on stability transcending flux, to intellectual focus on flux relativizing stability?

Motion implies (even *means*) a difference in condition, posture, quantity, quality or locality over an interval of time. I am ill, and then I am healthy. The gross national product is so-much now and another so-much later or earlier. The Tower of Pisa was erect at one time and leaning at another. A train is at one station at one time and later at another station. — To the extent that these determinations allow of quantification (e.g., health by body temperature or red-corpuscle count), we may establish a numerical ratio of change over elapsed time. A rise in bodily temperature of 2°C over an hour gives one cause to worry, and similarly an increase of 1° over a year in the angle of the Leaning Tower; that the train ride from Luxembourg to Paris used to take four hours, and now takes only two hours, indicates not only some new possibilities for day-trips, but also some remarkable developments in technology.

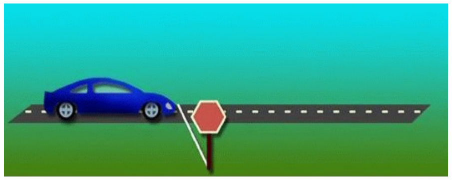
But the ratios obtained from numerical determinations of condition, posture or place, these over intervals of time, do not provide determinations of motion or change at any *one* time, i.e. at a single instant. So far, any ratio pertains to the extremes of a spread — to *stages* — and embraces a blur. If motion *is*, we must be able to determine its being at a single instant, a space-time point. Or rather, we must be able to devise a *method* for determining it. Otherwise, the determinations we make pertain only to a multiplicity of states, each of which is static.

The task is to catch intellectual sight of things in flight — as flying rather than arrested for static determination. We rightly credit Isaac Newton and Gottfried Wilhelm von Leibniz for successfully undertaking this task around the 1670s. But in keeping with the spirit of their undertaking, we can also conceive it as extending over a temporal trajectory with salient points both before and after those fruitful years — back at least as far as Descartes' analytic geometry and forward through Kant's *Critique of Pure Reason* and the formal refinements worked out by a bevy of astute mathematicians in the 19th century.

In short, and precisely in keeping with the spirit of modern physics, biology and social studies, we ourselves may consider the development of the solution rather than only its result as an already full-grown inheritance. And for this purpose I find helpful a cartoon that recently came to my attention.

Here is Zeno out for a drive, enjoying the landscape during the day and perhaps the skyscape during the night, each in its eternity and sameness. Distracted, he passes right through a stop sign at an intersection and gets flagged down by Newton, a recently appointed traffic cop, who claims that he clocked Zeno at 60 kilometers per hour, or 1 km/min., right at that stop sign. Zeno of course denies the charge, and an argument ensues.

Now, Zeno and Newton both have collaborators stationed at that intersection. Zeno's teacher Parmenides produces visible evidence that Zeno's car was standing still at the stop sign — a photo:



But Newton's sidekick Leibniz had taken two photos of the car, one 50 meters prior to the stop sign and another 50 meters beyond it, and with a camera that also recorded the elapsed time: exactly 6 seconds. One-tenth minute to go one-tenth kilometer: surely Zeno had passed the stop sign at 60 kilometers per hour. Zeno, however, rightly replies that Newton and Leibniz are begging the question: they are extrapolating two determinations of stasis and projecting the result back onto that crucial single moment. "Besides," he says (and he here employs the traditional *ad hominem*), "the projection you make unfairly disallows the very real possibility (on your own scheme of things) that I slowed down, stopped, and raced forward for the *average* speed you have determined."

However, foreseeing Zeno's dogged insistence, Leibniz had taken several pairs of photos, and these he now presents: at one meter either side, ten centimeters either side, and one millimeter either side, and the time-intervals reflect the same ratio of traversed distance to elapsed time. But, corresponding to the wonderful technology that allows such determinations, Zeno flaunts an equally wonderful technology allowing his car to slow down, come to a full stop, and race ahead during those ever-decreasing and still blurry gaps, over which Leibniz has established only a average speed. The new policemen are still failing to establish a motion at any *one* time and place, let alone at the crucial moment for which Zeno has visible proof of his stasis.

Motion at a single instant: What can this mean so long as we determine the measure of motion by the ratio of distance over time? What can it even *be*? How can we *think* such instantaneous motion, determine it, assign a predicate to it, certify it?

Newton answers by asking us to imagine the temporal sequence of snapshots going "all the way" in the diminishment of the time-interval — or rather not quite, since that would require us to divide by zero, giving us no ratio at all. So right up to but not onto the line at the intersection. This line, he says, constitutes the *limit* the pairs of photos approach, each still evidencing the ratio of 60 km/hour. And although the pairs never cease being pairs, their unlimited number and unchanging ratio justifies their (and the jury's) concluding that the ratio holds for that single-point limit (instant) as well. In Newton's words:

> . . . by the ultimate ratio of evanescent quantities, i.e. ones that are approaching zero, is to be understood the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish. Those ultimate ratios with which they vanish are not truly the ratios of ultimate quantities, but limits toward which the ratios of quantities decreasing without limit do always converge, and to which they approach nearer than by any given difference but never go beyond, nor in effect attain to until the quantities are diminished *in infinitum*. [Bk I, Sch Lemma XI]

But these words — "the ratio of quantities . . . with which they vanish" at a limit (point) to which they "never . . . attain" until they "are diminished *in infinitum*" — propose an event that, to be understood, would require that we comprehend an infinite number (thus not a number at all) of points in space. So says Zeno. To which Newton and Leibniz respond: while what *you* mean by "comprehending" is the (merely imaginary) act of traversing each stage in thought (analogous to Achilles' parallel act of running with his legs), what *we* mean is the act of watching on the sidelines an intellectual operation that is set in motion and performs itself, much like a computer program will do. To which

Zeno replies: But such intellectual operation is merely imaginary; besides, your computer will have to do its work in a finite interval of time, during which it cannot possibly perform an infinite number of operations; in short, you can only *understand* finitely, one thing, one stage at a time. To which Leibniz and Newton might reply: Just let the two infinities cancel themselves out; Achilles may have an infinite number of marks to pass by before getting to the Tortoise, but then the time-intervals are getting infinitely small. — And so the argument might go on.

Does one account prove the other wrong? Or do they simply assert opposing theses in a hopeless stand-off? Let us look more closely and ask what each is talking about, apart from the conflicting conclusions they engender.

Zeno speaks (1) of someone apparently walking (ever having to traverse a remaining half of the distance to the goal), (2) of Achilles trying to catch up with a moving Tortoise (who has always progressed beyond the point where Achilles arrives), (3) of an arrow flying toward a fixed target (and being always at rest at each point if it is ever to *be* there at all), and (4), in Aristotle's formulation, of "equal bodies moving alongside equal bodies in a stadium, from opposite directions . . . at equal speeds" (from which situation Zeno thinks "it follows that half the time is equal to its double" — *Physics*, VI, 9; the meaning is not immediately clear). In contrast, and in brief, Newton speaks of Quantities approaching a Limit — where the Limit is itself a Quantity.

Zeno talks about *things* in places (only incidentally of time), Newton about things *in motion* (of time, and only incidentally of things). Each deems the other to beg the question.

Parmenides and Zeno, then Plato and Aristotle, focus our attention on what gets things moving: on the strength of Achilles, on Apollo "who strikes from afar," on the glory of athletic games — eventually and for millennia on material, agency, goal and completion — motion all the while being epiphenomenal, i.e. of sensory and therefore of derivative intellectual interest. Newton, some of his predecessor and all his successors, focus our attention on a grid of space and time, introducing eventually the notion of Force (gravitational, centrifugal, magnetic, . . .) to account for "what gets things moving" — motion or change becoming the chief phenomenon of intellectual interest.

Today we can say that each of these two companies has its own "research program" — except that this term reflects again the distinctively modern commitment to change in the volume of collectively accumulated insights, whereas the earlier company, right through Scholasticism and the early Renaissance, understood intellectual development as the return of the individual to the basics: recognition and anticipation of the same, not expansion into the new and different. Thus we today must exert ourselves arduously to slip into the shoes that allowed the earlier company to take Plato's contention in stride: that when we (individually, as philosophers, statesmen, artisans) finally come to know something we are recollecting the matter in its basics and not absorbing information about it provided by others. In contrast, we today immediately appreciate Newton's remarks about his seeing more than others "by standing upon the shoulders of Giants," i.e. by drawing on the insights of his predecessors, and about himself as playing on the seashore "whilst the great ocean of truth lay all undiscovered," i.e. as new truths to be discovered by his successors. Modern intellectual work has its inevitable expression in technological expansion.

In his didactic poem addressed to Zeno, Parmenides tells of being *taken* along the Path, "which bears a knowing man through all haunts," guided by the "daughters of the Sun" to the gates opening out onto the difference between Night and Day, and arriving in the presence of the Goddess who recounts what he

now passes on to Zeno — who must learn "both the untrembling heart of well-rounded Truth and also the opinions of mortals, in which there is nothing to be truly trusted." Essential to the Truth: What *is* is everywhere the same. Thus it makes no *intellectual* difference whether he's in the county jail or at the stop sign, or whether a fine reduces his bank account — these are all matters of conventional concern (opinion).

In the 19th century, John Ruskin echoes Parmenides when he writes about the then-new passion for railway travel:

> The whole system of railroad travelling is addressed to people who, being in a hurry, are therefore, for the time being, miserable. No one would travel in that manner who could help it — who had time to go leisurely over hills and between hedges, instead of through tunnels and between banks . . . The railroad is in all its relations a matter of earnest business, to be got through as soon as possible. It transmutes a man from a traveller into a living parcel. For the time he has parted with the nobler characteristics of his humanity for the sake of a planetary power of locomotion. (*The Seven Lamps of Architecture*, "Beauty").

Toward the end of that piece Ruskin resumes the image. Instead of the huge social investment that has indeed "attained the power of going fast from one place to another," suppose, he says, "that we had employed the same sums in building beautiful houses and churches": for one thing, workers would have been better off (their work satisfying their souls and not just breaking their backs) — "and when all was done, instead of the very doubtful advantage of going fast from place to place, we should have had the certain advantage of increased pleasure in stopping at home."

In Kant's image, but contrary to Kant's advice, Zeno's task is to adjust his thoughts to what is, and not to struggle to adjust things to his thoughts — which entails a struggle against non-being.

But what would it be like to *follow* Kant's advice — to adjust things to our own thoughts? For this is exactly what Newton's account of motion asks us to do! Rather than policing empirical motion, he and his company compute rates of change (here, velocity) from formulas already determining position in space in reference to moments of time. Although the notion was not clarified until long after Newton, we can formulate examples more clearly in terms of a *function*: the position of a car along the road from some point 0 may be conceptualized in advance as a function of the time elapsed from that point. For instance, the position might be a function of the elapsed time squared: $p(t) = t^2$. If distance is measured in kilometers and time in minutes, then at the 5-minute mark the car will be at the 25-kilometer mark.

Such a formula *already* presupposes the whole world of change in time and space, and defines distance in terms of time — dynamically, not statically. It should then come as no surprise that we can, here and in much more complicated cases, calculate directly the velocity at any point along the line — i.e., deduce the velocity-function $V(x)$ corresponding to the position-function, and thereby the velocity of Zeno's car in this second scenario.

We can imagine Newton asking Zeno for a re-run, starting from *one kilometer back* from the stop sign and driving ever faster according to the formula $p(t) = t^2$. And Zeno will comply since, according to his teacher, the Goddess requires him to "learn everything," including the Path "along which mortals wander, knowing nothing, double-minded because perplexity in their bosoms steers their intelligence astray" (exactly the condition of travelers who have become parcels).

First, we may proceed manually, as in our previous example. Recall that velocity is a function of distance traversed over time elapsed. Now take a very short interval of time (Δt), say a nano-minute, n. The distance (Δd) between any one starting point and

the point reached after a nano-minute will be $(x + n)^2 - x^2$. We may then formulate the *average* velocity around any point x:

$$V(x) = \frac{(x+n)^2 - x^2}{n} = \frac{(x^2 + 2xn + n^2) - x^2}{n} = \frac{2xn + n^2}{n} = \frac{2x + n}{1}$$

So, at the stop sign (where $x = 1$), Zeno will be proceeding at two-and-one-billionth kilometers per minute, or a little over 120 km/hour. However, even this "little more" becomes "ever less" as we let Δt shrink from a nano-minute ever closer to zero — even all the way, it seems, since the difference in time no longer appears in the denominator.

The same convenient cancellation of n (of Δt) from the denominator occurs in any normal position-function defined in reference to time. Consider yet another example, a function laden with one or more constants, such as $p(t) = t(10 - 2t)$. In this example, Zeno's rocket-propelled car will be at position 0 both at $t = 0$ and at $t = 5$, and during that interval it first gains and then loses speed. The speed (velocity in km/min) will vary according to the formula:

$$V(x) = \frac{(x+n)[10 - 2(x+n)] - [x(10 - 2x)]}{n} = \cdots = \frac{10 - 4x - 2n}{1}$$

Whereupon we may, once again, let the nano-minute proceed toward its limit of zero. In this example, we note that at $x = 0$ Zeno's rocket-car already has a positive (*forward*) velocity of 10 kilometers per minute, while at the 5-minute mark it has not only returned to its starting point but has a negative (*reverse*) velocity of 10 kilometers per minute: as it traverses that five-minute interval it somewhere, at some moment, reaches a point of rest, which would be the farthest away from the start line. At what point does that *instant* of rest take place — when and where there is no velocity at all? We may read the answer from the velocity-function: when $x = 2.5$ — after two and one-half minutes, at which point it is 12.5 kilometers away. In short, velocity-functions may allow us to determine points of rest (if any) and optimal distances (if any).

So far, time has figured as the variable — the *x* has stood for position as a function of time (i.e., it is defined, variably, by times). Consider now an example where a spatial measurement does this service: the Canadian government has agreed to sell off a relatively small rectangle of land along its Midwest border (the 49th parallel) with the United States — a rectangle of the buyer's choosing, so long as the new portion of the rectangle's perimeter (i.e. the two sides extending up from the border, together with the width of the rectangle) totals ten kilometers. What shape should the buyer choose in order to cut out a maximum area for itself? The formula for the Area, as a function of the length of the side *s*, is $A(s) = s(10 - 2s)$. The Area equals zero both when $s = 0$ and when $s = 5$; furthermore (as we can determine either roughly by hand, or exactly by deriving the rate of the growth and setting it to zero) it grows until $s = 2.5$, after which it declines as it approaches 5. So the greedy buyer will decide on a rectangle measuring 5×2.5, the area of which will be 12.5 square kilometers. All in numerical keeping with our previous example.

But have we entirely succeeded in deriving the rate of change (in magnitude) independently of temporal considerations? In the last example, the variable is indeed "length" rather than time, — restricted by spatial considerations to $0 \leq s \leq 5$. But, looking for the rectangle's rate of growth (first positive, then negative), we allow for all the (infinite) possibilities within that interval. It's as though we were watching, through the lens of our formula, $A(s) = s(10 - 2s)$, the swelling and then the shrinking of the figure. While any determination we make will be spatial (length of a line or area of a figure), we ourselves are following, or rather forming, a progression within which such determinations become possible.

And while we do not measure *this* — our *own* — progression, assigning it any property, we perform it: or, rather, we explicitly perform it to the extent that we are *understanding* the up-front formulations — and not merely applying formulas "tried and true" (as we must do when passing on to more intricate and exacting calculations). And this performative progression deserves to be called temporal — as is only fitting, since the whole intent of Newtonian thinking is to develop a conceptual apparatus allowing us to focus directly on change — on things *as changing* and therefore themselves as essentially temporal. As we begin, so we will remain, and in this case we began by understanding length of a side (and therewith area) as a function of time.

It will be evident, then, that, once we have a position-function defined explicitly or implicitly by points of time, we need no longer imagine snapshots of two distinct positions and a measurement of the time required to traverse the interval. Nor must we worry about maintaining a ratio as the intervals "go to zero": all normally time-defined position-functions will allow us to cancel out the denominator, whereupon we may freely, and without impunity, let the remnant in the numerator vanish The rate of change is decided entirely "in house" and, in keeping with Kant's image, and following his advice, we will expect things "outside" to adjust themselves to our thought: whether machines we have built to travel according to the position-function, or observed events from which we have extrapolated an approximate formula to describe an item's varying position over time.

The same in-house decision allows us to argue in reverse: knowing the speed (deemed illusory by Zeno), we can calculate the distance covered over an interval of time.

Suppose we hear that the Velocity of Zeno's car follows a stated pattern and we want to develop a formula determining the Distance it traverses during any given elapsed Time. Of course, if V is constant we could simply multiply it by T. But how might we calculate D if V varies from moment to moment? One way of approximating D would be to calculate the variable V (in km/min) from the formula at small intervals — say, of 6 seconds. Taking V at the beginning of each interval (throughout which it will vary minusculely), we may determine that, during each of these intervals the car will have traveled approximately $V \times .1$, and we could add these up over the whole interval to obtain the accumulated Distance. And to obtain a more accurate result, we could determine the V at the beginning of every nano-minute, divide it by one billion, and add up the billions of approximate Distances.

Now, as is the wont of modern thought, we can abandon reliance on empirical conditions. Defining (for example) the velocity in reference to time as V(t) = 2t km/min. we let us follow Leibniz's lead by expressing the accumulation from any two points of time a and b as:

$$\int_a^b 2t\,\Delta t \quad \text{letting } \Delta t \text{ go toward } 0$$

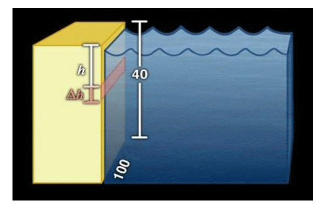
The formula expresses the "addition," the *integration* (∫) of all the products (from time a to time b) of velocity and the time-interval as these intervals get smaller and smaller. So that, if Achilles was running at a describable velocity (whether constant or varying), we could *calculate* the moment at which he would be moving past the Tortoise.

But how can we make use of this summing process of integration? One way consists of finding a position-function that corresponds to the velocity-function, i.e. from which we can derive the velocity function — a formula, then, that gives us the Distance according to the Time. If we succeed in finding this

éminence grise we may simply insert into it each of the chosen times, a and b, and subtract the one from the other to calculate the difference, thereby obtaining the distance traversed during that interval of time. To find such a position-function we must think backwards. And in the example chosen we may recall that the rate of change $2x$ (where x is the variable for time: t) is derivable from the position-function x^2. So the distance traversed starting at a and ending at b is $b^2 - a^2$:

$$\int_a^b 2\,T\Delta t = b^2 - a^2$$

Yet the most impressive use for this procedure is in determining spatialities rather than temporalities. Consider this example: there's a water dam 100 meters wide and 40 meters high and we wish to determine the total pressure exerted by the water on this 4,000 m^2 wall. The hydrostatic pressure around depth h meters on the wall will be (approximately)the product of the height and the weight of water (roughly 305 kilograms / m^3).



The approximate pressure, laterally against the wall at any depth h along the very thin strip of $100\Delta h$ m^2 will then be $100\Delta h \times 305h$ kilos. And if we take Δh to be a nano-meter we can calculate the pressure on each of the 40 billion strips, add them together, and arrive at a very close approximation of the total pressure.

But we may also proceed independently of such minute calculations. The formula for the integration reads:

$$\int_0^{40} 30500h\Delta h$$

Now, just as $2t$ is the rate of change (the instantaneous-velocity function) derived from the total-distance-function t^2, so 30,500h

is the rate of change (the instantaneous-change-of-pressure function) derived from the total-pressure-function $15{,}250h^2$: the first rectangle down the wall, 1 meter by 100 meters, undergoes a total of 15,250 kilos of pressure, the first two such rectangles undergo four times that amount, and so on, in geometric progression up to all 40 meters down. The total pressure exerted by the water on the wall of the dam then comes to $15{,}250 \times (40)^2 =$ 24,400,000 kilograms.

This last inference (from the formula for rate-of-change *back* to a formula for totality) supposes that we can read an apparently static situation as dynamic. Two puzzlements, then: (1) Again, as in calculating the changes in the area of a rectangle as a function of its sides, the change of pressure as a function of depth does not explicitly include any temporal reference. (2) While we may rightly totalize distance covered from the product of velocity over an interval of time, it is not immediately clear that this totalizing works in cases where the rate of change is a function of a spatial point — like the depth of the water.

As for (1): It must be ever borne in mind that the significance of the new mathematics developed by Newton and Leibniz lies precisely in the transposition of statics into dynamics — understanding the determinations we make as themselves changing according to a reconstructed evolution of the circumstances themselves: here, our imagining the water as *rising* and therewith the pressure against the wall as *becoming* ever greater. Although the formulation of the relation between pressure and depth makes no reference to the passage of time (so that we could imagine the water to rise at various speeds), we ourselves — the narrators who actually follow the reasoning through, rather than simply grabbing the result and running off to an application — embody the temporality essential to any apprehension of change. Up front we may only see static situations and extract static

determinations, but out back (transcendentally, Kant would say) we are reconstruing the situation as dynamic: as changing, i.e. as temporal.

Thus any formula denoting a rate of change can fit temporal situations equally well — like those of Zeno's car, the velocity of which might augment at the rate of .30500 kilometers per hour, so that the distance covered after any time t will be $.15250 \times t^2$: after 40 hours the car will have traversed a total of 244 kilometers.

As for (2): The significance of an integration is precisely that it came from an original formula that must be discovered. An integral is a long-hand summation of this original, expressing the addition of infinitesimal products over an interval a to b. And it is always necessary to recur to the original short-hand version when we wish to evaluate the long-hand version with mathematical exactitude rather than empirical approximation. Thus it should come as no surprise that one can demonstrate formally what's called the Fundamental Theorem of Calculus:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $f(x)$ is the rate of change derived from an original (to be discovered) $F(x)$, and dx is Leibniz's (now the conventional) symbol for the difference-approaching-zero.

One more illustration: we may calculate the pressure on the dam wall along a strip one centimeter wide starting 7 meters under water. Over this short difference in depth the pressure will differ slightly. But let's first consider the pressure over the area ($.1\text{ m} \times 100\text{ m} = 1\text{ m}^2$) to be indistinguishably uniform (as though laid out horizontally rather than vertically): we then come to a figure of $(305 \times 7) = 2{,}135.00$ kilos. Then let's calculate similarly the pressure at the deeper side of that centimeter: $(305 \times 7.01) =$ 2,138.08 kilos. The exact figure should then lie between these two. And this figure is provided by the Fundamental Theorem employing the "original" formula (rate of change of pressure as a function of the depth): $[½ \times 305 \times 100 \times 7.01]^2 - [½ \times 305 \times 100 \times 7.0]^2 = 2{,}136.525$ kilos.

The one is the reciprocal of the other: the approximating formula (finite version of the *integral*, as it is called) and the "original" formula (the *anti-derivative*, as it is called) from which the rate of change embedded in the integral may be derived (so that the rate of change is called the *derivative*). This reciprocity, expressed in the Fundamental Theorem of Calculus, works only because the "original" formula expresses distance as a function of time, or pressure as a function of depth. Zeno must already be driving his car at predictable speeds; the pressure against the wall must already be rising in a way predicable from the weight of the water: only from a well defined *integral* can we establish (discover) a well defined *derivative* — or reformulate the integral (the making whole) from the derivative (the rate of change within the whole).

This dependence on uniformity (pre-established harmony of whole and part, Leibniz would say) not only interrelates space and time (newly understood), but leaves room for the introduction of the to-us long familiar, but not at all self-evident understanding of the key element of nature we now call Force. While Zeno may vary his speed erratically, falling bodies do not: the distance they fall is a function of time, and the Force is here mainly that of what we now call gravity (together with any contrary Force, such as that exerted by the air, called friction), and this Force varies regularly with the falling object's distance from the center(s) of gravity. An analogous Force, similarly predictable, is that of magnetism (inherent or electrically induced). Then also there's centripetal / centrifugal force, and all the forces we create by pushing and pulling — and constructing things like dams, the walls of which must be designed and built to push

back against the pressure of the water (a direct function of the Force of gravity). And the water of which, flowing out at the bottom of the wall, may expend this accumulated Force in the turbines that then generate the various forces familiar to us as electrical power.

And so we have the Newtonian principles by which we now investigate nature: nature in motion (essentially changing) — each thing both moved by quantifiable forces and (as a medium of forces exerted upon it) moving yet other things.

So motion *is* — counts among the things that "really are." For each instance of motion (chang*ing* over time) is now intellectually determinable and therefore distinguishable from others. And so it can finally become an essential ingredient in the understanding of things happening around us — whether present things (our earthly economy, the heavenly bodies), things long past in origin (the cosmic formation of minerals, vegetables, or animals), or things in the distant future (problematic developments in our aging populations and in the melting of the ice caps). Motion we no longer put down as the unintelligible, merely sensory blur between cognizable stages of being.

Yet what we now take for granted, as a bequeathed and workable body of knowledge that our educational systems rightly promote as essential to the continuance of our larger systems of production and social well-being, appeared originally as fraught with fundamental difficulties. For, at the time, the then-new up-front manner of understanding things (of investigating nature) brought in its train a number of challenging presuppositions about "things" — about "nature," and also about ourselves as investigators. And although the followers of this new understanding were prepared to move ahead on its basis, others noticed that such presuppositions entail decisions that deserve careful consideration if not emendation — a renewed self-knowledge, including an awareness of the *a priori* implications and limitations built into such investigation.

It was Kant, with his three *Critiques*, who explicated many of the not-always obvious "backroom" presuppositions of the new understanding — largely with a view to justifying it, both defending and adjusting the enterprise as a whole, a century after it had achieved a strong position within the intellectual world.

It was Leibniz, however, who, as an independent inventor of the new calculus, took umbrage at several of the presuppositions that Newton and his entourage advanced and even celebrated: the understanding of Space and Time as intellectually stable constituents of nature, and the understanding of Force as imposed upon things of nature (bodies) — and, interwoven with these two understandings, a vague understanding of Limits as variables "go to zero" or operations "go to infinity."

Motion now appears as "real" because we have learned to predicate it at an instant within Space and Time. But are Space and Time in themselves "real"? We do commonly predicate them: a space can be large or small, full or empty, time we say passes quickly or slowly — but such attributes invariably reflect our own interest in circumstance rather then the grammatical "subjects": we speak metaphorically, it seems.

Motion is always motion of something: it's already an attribute. Essential to the understanding of *a* motion is then an understanding of what gets that thing moving: what pushes or pulls Zeno's car, what applies pressure to the wall of the dam — even who and what is surveying the land. In a Newtonian spirit, we can name the *causes* of motion: the force supplied by an engine of some sort, the force we call gravity, the force we call human agency, along with the tools requisite for performing the actions leading to the resulting configuration. But in what sense

can Force rightly be called "real"? We in fact calculate forces from our own detection of motion, we don't experience them directly the way we may experience a car or a painful sensation in our ears when diving deep into the waters of that dam.

Leibniz especially notes the implication and consequence of Newton's procedure of first establishing the geometry of the bodies investigated (e.g., earth, sun, moon) and then explaining the changes (the motion) in reference to forces (gravity, then also inertia): these forces, he insists, remain essentially extraneous to the things under investigation. Leibniz's alternative, already evident (if only obscurely, he likes to say) in the literature of Plato, Aristotle and Thomas Aquinas, brings us much closer to nature: force as *internal* to each thing — an intrinsic power allowing each thing to act on its own as well as on other things, and to undergo on its own, i.e. in a way essential to its own nature, other things acting on it. A likely example of internal force occurs in the process of nutrition: an organism seems to have forces within itself that require replenishment by nutriment, foodstuffs in turn having forces internal to them. Indeed, the entire study of the life sciences seems to depend on understanding clearly external forces (natural calamities, from forest fires to the impact of huge meteors on the earth) as disturbances of the internal forces of plant and animal life — these latter then count as what's "really real" (ἡ ὄντως οὐσια) in the economy of own understanding, the former counting as "accidental" only.

Newton knew full well that his "mathematical principles of the contemplative understanding of nature" (as he very accurately described his work) confined itself to explicating the manifest (external) motions of bodies. However, at the end of his Preface of 1686 he formulates the "research program" henceforth adopted by all mainstream intellectual work:

> I wish I could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled toward one another and cohere in regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or to some truer method of philosophy.

Since the 20th-century, even biologists have increasingly tended to agree: by now, their method is indissociable from the regulative idea of determining their subject in mechanical ways, i.e. as formed by external forces and therefore manufacturable at will, once the mechanical process has been decoded.

For example: A person is suffering lassitude. Tests reveal a low count of red corpuscles ("tiny bodies") in the person's blood: energy requires a higher ratio of such corpuscles, thus the lassitude is explained. But why the low count? Tests on a specimen of the person's bone-marrow reveal useless elements being produced instead of the desired corpuscles: thus the low count is explained. These bothersome elements are then carefully analyzed to discover a by-now-familiar syndrome indicating a kind of cancer called "myeloma." But how did this dysfunction of the blood-renewing operations of the marrow occur? Some cancers seem to be induced by environmental pollution, some perhaps by a faulty diet, and some by an inherited configuration of the genes (defective, from a standpoint desiring longevity). — Until we arrive at this last stage of questioning, the forces at issue (both of acting and undergoing) belong to the thing itself: such, at any rate, is the way we likely understand them. For the distinction between lassitude and blood count makes sense from

an underlying unity, as does the distinction between the (weakening) operations of the marrow and the (poor) condition of the blood. At the last stage, however, we begin to entertain the possibility of external, accidental rather than necessary forces: clearly if we blame the dysfunction on environmental factors, less clearly if we blame it on inherited genes: here the force would be external not so much in space as in time (faulty transmission via parental semen and egg).

The Newtonian research program insists that we look for external forces accounting for the change in condition from health to illness. It discredits from the outset any search for genuinely internal forces, i.e. one's belonging essentially to the patient's body. For the intent is precisely to develop counter-forces to overcome the decline, to switch the direction of the change, or at least to stall its progress. And these counter-forces will be essentially external — e.g., a variety of chemotherapy. Medicine becomes a form of engineering.

At the outset of his Preface, Newton chooses the term "mechanical" with circumspection on the tradition. Earlier, the word referred to the rough and ready application of principles to materials: "mechanics" viewed the things they were manipulating as *means* toward ends external to themselves. In contrast, "geometrical" referred to the exact study of the principles themselves: "geometers" viewed the things they were contemplating as *culminations*, or ends in themselves (generally considered, in turn, as emanating from an Author). Newton proposes a style of physics that melds these two otherwise contrasting terms into "mechanical philosophy" — a contemplation of bodies, each of which serves as a means (μῆχος) in the whole machine (μηχανή) and has no internality distinctively its own, but only that of the machine itself (generally considered, in turn, as Authored holistically).

Leibniz objects not to the up-front calculations of nature mechanistically construed. He objects to the supposition that such calculations focus our attention on what's really real in nature, ἡ ὄντως οὐσια. And these objections stem not (as so often happens) from a critic unable to participate in such focusing, but rather from a thinker who himself, independently of Newton, invented the method of calculation allowing for it. He then speaks with authority and deserves a careful hearing.

In a manner reminiscent of Kant's later account of the conditions allowing us to know and carefully formulate the course of happenings (phenomena), Leibniz notes that the New Physics presupposes as ultimately real what will, when fully considered, count only as intellectual contrivance for focusing on phenomena. Contrary to Kant, however, he continues to underwrite the aspiration to formulate the underpinnings of the phenomena — *their* basics, *their* sources, *their* causes. Quite consistently, and in keeping with the tradition rooted by Plato and Aristotle and systemized by Thomas Aquinas, he denies to motion the status of reality — confining it (as Parmenides and his pupil confined it) to phenomena "in which there is no true reliance":

> Nam motus (perinde ac tempus) nunquam existit, si rem ad ἀκρίβειαν revoces, quia nunquam totus existit, quanto partes coexistentes non habet. Nihilque adeo in ipso reale est, quam momenteum illud quot in vis ad mutationem nitente constiti debet.
>
> For motion (likewise time) never exists, if one recalls the matter in the most exact [and exacting] manner, since the whole [of motion] never exists, inasmuch as it does not have parts that co-exist [i.e. present all at once]. What's real in motion is nothing but a momentary something that must consist in a force striving for change.

Herein lies the crux: to think this "momentary something" as "force striving for change" — force internal rather than external to the thing that then changes (a thing as moving on its own and then also instrumental in moving other things according to their own forces). The challenge is to think this internal force as "prior to" the external forces calculable in the Newtonian manner — and with the calculus newly devised both by Newton and himself.

What evidence may we cite in favor of internal force as the cause grounding the change, the motion, we otherwise (and "rightly enough") explain in reference to external forces?

The "evidence" intending to justify any metaphysical decision lies ultimately in the phenomenal workings thereby coming to make sense, including our noumenal inheritance. There is a leap of intelligence (noetic insight) in this decision — a leap that takes one simultaneously into this ground and into events "on the ground." No argument (λόγος) can ever force this leap of intelligence upon anyone intent on remaining outside. Talk about principles of piano playing focuses one on the matter itself — but only for one willing to play. So, too, any talk about the evidence for internal forces and any other metaphysical principles — including Newton's (with the difference that our educational system now propagates this noumenal inheritance).

We find in Plato and Aristotle the original evidence for the decision affirming the priority of internal over external force. It is from the tradition they formed, extending through the Scholastic thinkers, that Leibniz draws his own vocabulary for affirming the priority. The original (Platonic and Aristotelian) evidence lies in the contemplation of those who have learned a τέχνη, an ability to work with things to actualize their potentiality — abilities evident in accomplished musicians, physicians, navigators, joiners and even leaders of households and cities.

Such marvelous ability allows and requires the artisan to work with the force, the power, the δύναμις inherent in things, putting *it* to work (ἔργον), thereby manifesting *them* in their own at-work-ness (ἐνέργεια). What counts most as being is this event of things maintained and actualized in their own purpose, i.e. at their optimum (ἐντελέχεια) — a piece of music being played by players "in the zone," a horse in top form racing toward the finish, a privately owned house being put to full use at a hearty family gathering: such things are "most knowable" in the sense that knowing them permits one to know their more common, less-than-optimized versions, to know the elements composing them, and to recognize their many fraudulent look-alikes. Of course, the optimum is most knowable in the sense that it represents what is most essential to know, not what is most easy to know.

From the contemplation of artisanal accomplishment we may, as intellectual "artists," concentrate our attention on the *principles* of things permitting such accomplishment — rather than on accomplishing anything "hands-on." We thereby establish our own discourse (λόγος), one having a strictly "minds-on" focus: Greek ἐπιςτήμη, Latin *scientia*, English "knowledge" (but in the strictly intellectual sense: in contrast to the hands-on as well as mind-on knowledge of flute-players and home-leaders). Essential to knowledge of any sort are the arts of geometry and arithmetic, which then allow not only for such fields as house-building, music and economic theory, but also for fields extending beyond hands-on participation in events of completion: not only astronomy but also the precursors of what we today call ethics, aesthetics, epistemology and even theoretical physics — not to mention the broader developments of mathematics.

Originally, in the works of both Plato and Aristotle, the positive analogue to what Leibniz will later call "force" was internal: the ability to act or undergo, the power (δύναμις) defining the thing and defined by its own purpose (*Sophist*, 247D-E). For the most part, things happen (emerge, grow, improve, and interact with one another) "according to nature" (κατὰ φύσις), i.e. according to in-built purposes. Of course, some things happen "contrary to nature" (παρὰ φύσις: a calf is born with two heads, or a child without all five fingers), but such things are recognizably deviant only in reference to what normally (naturally) happens. In his *Physics* (Book IV) Aristotle says of things happening contrary to nature that they are "forced out of line," i.e. violated externally. To this "force" he gives another name (βία), which could be translated as "violation"; in his *Ethics* (Book III) he says of involuntary doing and undergoing that they derive either from ignorance or from βία — from βία when the origin is external (ἔξωθεν) to the agent, as when a roofer gets blown off the roof by a sudden blast of wind.

Originally, then, force (βία) had a negative ring — the word was used in Attic law to name what we call rape, and in Attic drama to describe the actions of tyrants: violent undertakings, essentially undesirable. Yet sometimes force is inevitable. Aristotle names two related *natural* instances (Book V): we are forced to grow old and to die. And we do need to force some people into line when the preferable route of discussion and persuasion fails: we may force children to undergo dental work. Furthermore, there is one craft that epitomizes the creative employment of force on nature: that of the forger who fires iron or bronze and pounds it fiercely into shapes convenient for human use, such as knives and bowls: Hephaestus provides the divine paradigm of such forceful craft, as also the Cyclopes in archaic theogony (whose force must in turn be subdued by force).

Following these original accounts of what gets things moving (κίνεσις), Leibniz in effect understands it as Aristotle does (*Physics*, 101a10): motion is the fulfillment (ἐντελέχεια considered as happening rather than happened) of something whose nature is to be moved toward its in-built purpose (τέλος). All other perceived motions, like that of car negligently racing past a stop sign or a child being dragged to the dentist, are simulacra, intelligible only in reference to fulfillments κατὰ φύσις (the health of the city or of the child). In Plato's formulation: motion itself *is* not "really" but *is* only participatively (ἡ κίνησις ὄντως οὐκ ὂν ἐστι καὶ ὄν, ἐπείπερ τοῦ ὄντως μετέχει: *Sophist*, 256D).

Yet Leibniz does not let it go at defending the ancients: a modern himself, contributing decisively to the developments of modern science, he insists on tracing the Newtonian understanding of (external) force to the earlier understanding of (internal) power. While endorsing the modern "mechanical" understanding of *phenomena*, he insists that this understanding is rooted in a "deeper" understanding of things themselves, distinct from their (phenomenal) manifestations. In place of the elaborate Aristotelian account of purpose and potentiality, motion and rest, becoming and perishing, Leibniz speaks of *conatus sive nisus*, the in-built "effort or striving" that produces the changes that we all experience and talk about loosely, and that intellectuals then articulate strictly —now with the new calculus.

One way of reading Hesiod's *Theogony* (to which both Plato and Aristotle refer as providing a mythic account of generation and motion) is to trace out the evolution from Eros to Aphrodite. At first, Hesiod sings, where only Emptiness, Earth and Eros appear, generation occurs rather haphazardly and unintelligibly: Earth gives birth to Sky and, among other birthings, Sky indiscriminately fathers, on his own Mother, eighteen Titans, whom he then confines back inside the earth. With the help of the Mother, one of these Titans, Cronus, castrates Sky, putting an end to Sky's wild procreation and indifference toward his progeny. But from Sky's blood soaking into her, Earth gives birth to the Giants, and from Sky's genitals, cast into the Ocean, finally emerges Aphrodite. This Goddess of Love no longer incites to the procreation of monsters, essentially unlike their parents, but rather to the joining of creatures of the same kind to engender creatures of the same kind — humans from humans, educated ones from educated ones, as Aristotle will say. The earlier God of Love, Eros, evidently the *muta persona* in the plethora of procreations prior to Aphrodite, now has competition. That the Goddess gets married off to Hephaestus, the Divine Forger epitomizing the use of force (βία) within the exercise of an art, suggests a tense relation, ever-recurring, between the two.

But why, we may ask, did Newton's (Hephaestus') version override Leibniz's (Aphrodite's)? For we today clearly live in a world intellectualized with external forces: the way the earth now is must be explained from explosions and impacts from elsewhen if not elsewhere, including the excess of CO_2 imposed by modern industry; if I have cancer of some sort, there must have been some element external to my body that caused it (something in the air I breathe or the food I eat); above all, if a person dies or an organism perishes, there must be an external cause (if not an oncoming car or an ingested poison, then something gone wrong in the inherited construct of the body). Indeed, modern research in biology as well as in physics, economics, sociology and the rest, must explain becoming, changing, and perishing in reference to external causes, or it would not count as research. One result of this commitment is that we must postulate entities having no manifest existence: gravity, greed, carcinogens — entities allowing us to explain things as forced "violently," i.e. from outside themselves (as the wind blew the roofer off the roof). Or, alternatively, we postulate the *lack* of some entity to explain, e.g., the criminal disposition that some people manifestly have.

Still, why this predominance of Newtonian science? Of course, we can seek out some external cause for this and other historical developments, and cite landmarks along the way. One general favorite: Descartes' remark that we should develop an intellectual understanding of things allowing us to become "masters and possessors of nature." One of mine, very specific: the remark of a 20th-century biologist (P. L. Kaptiza) that, "until scientists have succeeded in reproducing the contracting of muscle fiber, it is impossible to consider this process understood," and that we must indeed suppose that "these processes can be reproduced artificially and used in practice." — But the remarks of seminal as well as of routine thinkers generally follow as much as engender historical developments: it is wise to regard them as witnesses and often foolish to regard them as causes.

We can simply note that, while Leibniz also bequeathed to posterity the development of calculus so essential to modern science, Newton's bequeathal, unencumbered by recollections of earlier traditions and concomitant commitments, better served the progress of technology, for instance the manufacture of artificial fibers and limbs for medical treatment. — But the question (or at least mine, precisely in the spirit of modern research) is how it came about that progress in such external manipulation became of paramount importance in our relatively small (but ever-expanding) portion of the globe.

Apart from the larger question of why the Ancients gave way to the Moderns, we might well take note of one feature internal to Leibniz's efforts to integrate the earlier into the later, a feature perhaps dooming to failure his efforts in the arena of history: he has entirely forgotten this source of evidence. Plato and Aristotle ask us to behold the efficacy of flute-players, ship-pilots, house-builders, horse-trainers and the like artisans, noting the assumptions driving their vocation — extracting and generalizing these (both for reassessing the standards of leadership and for establishing a third kind of vocation, that of contemplation itself, "above" the making of things and the leading of communities). This source of evidence is barely discernible in Thomas Aquinas's works. Yet Descartes is still formally aware of it, stating it succinctly in the Sixth Part of his *Discourse on Method* of 1637: while the older philosophy, he says, focuses on *les métiers de nos artisans* (the services rendered by our artisans) and thereby remains contemplative (*spéculative*), he himself proposes a philosophy that focuses on the *la force et les actions du feu, de l'eau, de l'air, des astres, des cieux et tous les autres corps qui nous environnent* (on the force and actions of fire, water, air, stars, heavens and all the other bodies that surround us) and thereby becomes practical (*une pratique*). Physics thereby passes from contemplation of the nature at issue in our responsible dealings with her to the study of the nature evident apart from us, where the question is how we can twist it to serve our own interests.

Leibniz's endorsement of earlier metaphysics (abiding forms and entelechies = internal powers to do and undergo) has already lost its original foothold, and his criticism of Newton's absolutizing of external force appears as hardly more than an arbitrary or nostalgic preference. Like Antaeus in this one regard, Leibniz could easily be overthrown, at least by a fully modern Hercules such as Newton, if not by one who succeeds in setting his bulky body back onto the ground.

Any fully philosophical account of what's really real includes an understanding of our own role as interveners. That's the glaring intent of Plato and Aristotle in their accounts of leadership (πρᾶξις). Just as manual craft (ποίησις) requires one to "follow nature while also completing what she is unable to finish," so too does leadership and, finally, contemplation (θεορία): talking, reading and writing bring out what's really real. Such following and completing require knowledge: knowledgeable is one who can follow phenomena by participating in their ground, and thereby bring this ground close to the surface of phenomena. One momentous corollary: we should take as much care in educating (some of) the young to become leaders as we do in training others to exercise manual crafts; for we must all, to be effective, both submit knowledgeably to the internal destinies of things and intervene skillfully in the course of their movement. Including those who exercise the new, the third craft. What gets things moving is primarily their internal destiny and secondarily, within this power, our own compliant intervention.

At the outset of the 17th century, Francis Bacon recites a close parallel: to master nature we must first obey her. But here obedience means close observation: both the collecting of data and the adherence to generalized rules — not participation in the matters at hand but rather participation in a method, a self-contained investigation. Our own role in regard to the things themselves is now manipulation: the conversion of natural powers (forces) to human use — forceful conversion, that of Hephaestus divorced from Aphrodite. Newton's replacement of (internal) power by (external) force has since proved essential to the re-assessment of our own role as interveners — as resuscitated Cyclopes.

The difference is huge, both in what we take to be really real and in how we understand our truest selves, our fullest engagement with reality. But the passage from the one to the other is strangely continuous. After all, Plato and Aristotle sought justification for the three kinds of participative craft in the efficacy of the first, in the marvelous achievements of the manual arts; they both ascribe the efficacy of these arts to their various subliminal intellectual commitments to what's really real. Efficacy then abides as the measure for all craft. And while the crafts of production and leadership may more obviously require participative intervention in the matters at hand, the craft of contemplation seems at first not to require it. Bacon and Descartes, then Leibniz and Newton, argue for a kind of intervention distinctively intellectual, one based on method, an organization of procedures devised to float above things — intervention at a distance, one displaying its own efficacy distinct from that of revealing the inner workings of the first two forms of craft. Such second-order participation then allows intellectuals to work independently for improvements in the mechanics of producing goods and forming communities. Progress in productive efficacy and social organization becomes possible and henceforth even imperative for intellectuals — as proof of the legitimacy of their work, now the mainstay of our global economy.

When, early on, thinkers like Bacon, Descartes and Spinoza wished to repudiate the Aristotelian tradition to make room for modern investigation of circumstance, they went after its very heart: the understanding of purposes. Purpose, these (and others in the 17th and 18th centuries) asserted, belong to human beings only — in the sense that any ascription of purposes to things we handle or investigate are projections of what we want to do with them. This shift in understanding makes sense when we consider tools, which are obviously designed for service in our dealings with other things. One has to strain just a bit to understand organisms as a whole that way: a bee or beehive seems to have its own purpose, its own direction of development, apart from whatever use we may make of them. And we have to strain ourselves much more to understand the purposes of organs such as hearts and livers as simply projected from desire. Witness the vague talk of "natural selection": it supposes a *conatus sive nisus*, a "desire" to survive, if only in progeny, and this in any plant or animal. Whatever the strain, everything we investigate methodologically must appear as a mechanism.

We can only imagine what Plato and Aristotle might have said in reply — could they contemplate the heart of modern investigation. But Leibniz does it for us: he explicitly takes exception to Newton's postulation of what we today call "absolute space" — and, along with this, "absolute time." Newton projects a grid of (up to) four dimensions, with an axis from which we count units of distance and time, starting from their intersection at point 0. Without this grid, we could not quantify change over time or space, differences over points. And we could not imagine, let alone conceptualize those "evanescent quantities, i.e. ones that are approaching zero," in order to obtain the ratios establishing rate of change at a single point. Leibniz legitimately wonders in what sense such space-time (what we now call the Cartesian grid) is real — and in effect says of it what Plato says of κίνεσις, the movedness of things: that it really ("beingly") *is* not, but rather *is* something we construct to enclose phenomena apart from really understanding them. Such real understanding, as distinct from mere collation of phenomena, requires that we penetrate to the inner forces of things (and not rest content with constructing diagrams of space, time and multiple forces, always external).

While Leibniz reverts to the ancient disposition to seek reality beyond space and time, Kant provides the first nuances necessary for a philosophical understanding of this heart of modernity. He starts by noting that space and time are forms of encounter: anything we "really" experience (where "really" now means *phenomenally*), we can always locate within an arena shared by a

multiplicity of things and within a sequence of things appearing one after another. These forms (initial frameworks), always taking precedence over what we encounter, allow for those formal interrelations of space once studied under the name of geometry, those of sequence once studied under the name of arithmetic and, along with the categories of understanding (especially causality), those interrelations now studied under the name of dynamics (Newtonian mechanics of force). Space and time are understood, then, as "transcendental": both as stemming from our own condition and as pertaining to what we encounter.

Hegel nuanced this account of space and time even further: insight into the transcendental dynamism of space and time must be earned from struggling with the contradictions arising when trying to assign reality to the determinations of circumstances essential to experience. Insight must be earned concretely rather than demonstrated abstractly: it consists of becoming self-consciously aware of the human input conditioning both any careful investigation and its results. Restricting the question to the status of scientific determination, serious thinkers in the 20th century have worked hard on instilling this self-consciousness in ways more palatable, at least for anglophones, than that of Hegel in the early 19th century: I think of the continuum from Karl Popper to Thomas Kuhn, Paul Feyerabend and beyond — even to the childish renditions of those who have breezed through the works of these thinkers but, never having engaged seriously in scientific work themselves, blithely declare that all such knowledge is arbitrary construction.

But all such nuancing, including the modifications undertaken by great scientists and mathematicians themselves, leaves the basic engine of Newtonian science in place — both as the replacement of the earlier understanding of internal power and as the basic impetus for on-going investigation. Only the wise will bother to learn the subtleties of self-consciousness; routine scientific work does well enough without it. Illusions arise only when unwise scientists propound the scope of possible developments in their work — since they have no idea of its inherent limitations: its transcendental conditions.

For those who have succeeded both in re-activating the ancient tradition and in activating the modern tradition, it is evident that each "projects" its basic principles. Not arbitrarily, but each in accordance with its own necessity — the necessity inherent in the standpoint: anciently, the standpoint contemplating the workings of hands-on crafts, where the purpose internal to a thing governs the craftsman's knowledge (essentially participative in the things known); modernly, the standpoint contemplating the workings of the hands-free craft of observation (method), where the spatial and temporal framework governs the investigator's knowledge (essentially distanced from the things known, and therefore beckoning subsequent application).

Again, those who have indulged in both may well wonder how the two might be compatible. Certainly, in their anemic forms the two traditions contradict one another, and those who only hear of them may feel obliged to take sides. But I propose another tact ...

Instead of promoting one or the other, let us learn to consider carefully where and how we are prior to our intellectualizing the internal relationships of purpose and the external relationships of space-time. That is, let us learn to consider, i.e. bring into view, where and how we are prior to formalizing powers and forces, prior to explicating the different ways of understanding human intervention, above all prior to the development of that kind of careful knowledge requiring reflexive legitimization. Instead of

seeking roots either beyond or within our ordinary pre-intellectual condition, let us rather seek to expose the roots of our multilayered tradition as they already weave their various strands throughout this prior condition — so that we may give credit to, and marvel at the diversity of the intellectual developments these roots make possible. We may then learn to respect their ultimate compatibility — not by prioritizing one over the other (as Leibniz tried to do) but by prioritizing precisely their, i.e. our own condition.

Among the questions we might then learn to ask are these:

1. How does the geometric understanding of space develop out of our ordinary involvements with things? Already in such involvements things have their places relative to one another. Geometry as an intellectual study somehow formalizes these relations and, especially in its modern developments, leaves the thingly relations out of account. Geometry certainly makes sense apart from thingly relations, but we may still wonder, at least as intellectuals, what transformation of our prior condition allows for such transcendence, and what the status of the results is (especially that of space as distinct from place).
2. How does the arithmetic understanding of sequence develop out of our ordinary involvements with things? And with sequence also magnitude, since the numerically formulated size of something involves a sequence: we start at one end and proceed to the other end, retaining the entire sequence. Ordinary involvements with things are somehow sequential, somehow measured, somehow temporal, without yet being numerical. There are right times and wrong times, as Hesiod informs his brother — referring to the constellations rather than to any timetable. The study of arithmetic formalizes the procedures of beginning and ending (allowing, along with

the force of steam or fuel, ships to depart according to the clock rather than according to the winds and tides). We may wonder, at least as intellectuals, what transformation of our prior condition allows for such transcendence and, again, what the status of the results is (especially that of time as distinct from timing and timeliness).

3. How does projection figure in our ordinary involvements? These already take shape according to visions overlaying the immediate: such visions give direction to our involvement with things. Most prominently, then, intellectuals propose and formalize totalizing visions of our condition: projections of social organization (with issues in mind — Plato's justice, Augustine's salvation, modernity's freedom) and of natural phenomena (with a view to locating their principles of development — anciently, the purposes governing each thing's generation, behavior, and decline; modernly, such Hesiodic items as The Big Bang and Natural Selection). Again, we may wonder, at least as intellectuals, how the modest projections shaping our prior condition might legitimately generate such totalizing projections.

Each of these three questions supposes that intellectual work not only derives from ordinary involvements but also departs from its own legitimizing source. In contrast, one usually supposes that intellectual work reveals the realities more or less obscured by ordinary developments. To intellectuals, this more usual supposition seems inevitable: after all, why do we engage in intellectual studies if not to "get at" what ordinary involvements otherwise fail to detect? There must indeed be things that geometry and arithmetic, and sciences in general, bring to light that otherwise would remain in the dark. But we still might wonder, not only What these are, but also How it is that our

involvements drive some of us to engage in these studies. So here is a fourth question:

4. What gets intellectual work moving? In classical terms: What gets contemplation (θεωρία) going, as distinct from the production, rectification, alteration and mending of things (ποίησις), as well as from the formation and leadership of organizations (πρᾶξις)? This version of the question "What gets things moving" is perhaps the most difficult to retain as a question, since we may only broach it after having been so moved. It's a pardox: only as intellectuals may we ask where and how we are prior to being intellectuals. Yet only by learning to think within this paradox may we genuinely raise the subsidiary question: "What does each of the three ways of being offer that the other two do not?" — so that, relatively speaking, each may indeed be "superior" to the other two.

The classical thinkers from Plato onwards have struggled with the paradox of this fourth question. Among the 20th-century's efforts, I would cite Ludwig Wittgenstein (starting with his *Investigations*), John Dewey (most clearly in his *Art as Experience*), and Martin Heidegger (most obviously his *Being and Time*). But the fact is that readers of the works of these thinkers easily opt out of the struggle and concentrate attention on their answers to subsidiary questions, and in effect reduce their principles to those of the Enlightenment, with its convenient mix of subjectivity and objectivity in every consideration of the question of "motivation." The art of thinking must ever be renewed at its origin.

Learning to highlight intellectually where and how we are prior to intellectual work, we may profitably re-open further questions that have fallen into disuse. For instance, both our classical and our modern traditions build upon the notion of "getting near without arriving": anciently, everything present is intelligible only in reference to a destiny that retains a degree of remoteness —a fulfillment that, allowing us to measure what is incidentally present, constitutes essential presence; modernly, every determination of instantaneous rate of change postulates an approximation to zero that never becomes exact. In either case, we must learn to think what appears to be unthinkable, and to derive from this thought a mass of insights, the marvelous success of which tempts us to forget their original conception.

The ancient version has long fallen into oblivion, but it may help to remember that the modern version is equally bothersome: What can it mean for there to *be* (for us to *think*), "the ultimate ratio of evanescent quantities, i.e. ones that are approaching zero" as "the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish"? This formulation simply relocates Zeno's paradox into our formalism. Newton imagines a continuous temporal movement through a continuous spatial interval. Leibniz imagines a sequence of discrete moments, each introduced arbitrarily closer to the limit than any arbitrarily chosen point: " . . . if any opponent denies the correctness of our theorems [i.e., the determination of an instantaneous rate of change by the derivative of the original function], our calculations show that the error is smaller than any he may assign, since it is in our power to decrease [our determination of] the incomparably small . . . as much as we want" (letter to Varignon, 2 February 1702) — a verbal formulation of the analytic definition symbolically formulated a hundred and fifty years later:

$$\lim_{x \to \infty} f(x) = L$$

means

$$\forall(\varepsilon)\{\varepsilon > 0 \to \exists(V)[N\varepsilon V \,\&\, \forall(x)[x > V \to |f(x) - L| < \varepsilon]]\}$$

where ε stands for "error" (deviation from L) and V for the "value" of x at which ε is noted (N). But as self-questioning intellectuals we may rightly wonder:

5. How do these various notions of "getting close without closing" arise from ordinary involvements with things? For coming in close to things, and keeping our distance from them — seeking and avoiding but also arranging things far and near from one another in "space" (placing) and "time" (sequencing)— already arise in any actual situation, and in ways that are measured long before the employment of geometry and arithmetic, let alone modern calculus. The question is then what sort of modification of ordinary involvement leads to the intellectual versions of this dynamism.

The task is not to explain or justify the intellectual versions, neither is it to diminish their significance. Rather, the task is to rediscover the rich soil out of which these powerful dynamisms grow. And at this level, rather than at Zeno's or at Newton's, to reintroduce the question: What gets things moving?

Posing all these five questions supposes in each case not only that the intellectual development has roots in our pre-intellectual involvements with things but also that its "motivation" is internal to those involvements. What might "internal" here mean? Its opposite, external motivation, seems clearer: from around 1600 many "explanations" were offered to account for why and how it is that linguistic power (rationality) arose (and ever again arises) for human beings. With some remarkable exceptions (Rousseau's and Herder's), the explanations prioritize the practicalities of production and distribution of goods, and draw upon the services of geometry, arithmetic, planning, intellectual work generally and especially the mathematical arranging of things as variably far or near. Such accounts re-enforce the humanism essential to the development of modern industry. But every such positing of external motivation begs the question. It presupposes a particular transcendent concern for the whole, one among others. And it virulently dismisses those others as superstitious shortcuts if not outright blockages to the progress of modern industry. So, in honor of Leibniz we must pose another question:

6. How might the very difference between internal "force" (i.e. power: δύναμις) and external force (βία) arise within our ordinary, our pre-intellectual involvements? The intellectual versions of this difference demarcate our earlier and our later traditions. It would seem, then, that our basic condition allows for both. And that we might, as regenerate intellectuals, bring the allowance into focus.

This question already contains the question of human intervention: How are we to understand our own *varying* response to powers and forces? For, as we understand *them* to be, so we understand our *own* force, our own power, our own being.

Our own being? Herein lies the crux. For each ordinarily *is* not individually but conformitively. The most elementary decisions (to do, to undergo) and judgements (of what's right or wrong) stem from a communal base. A grouping, one "club" set off from others, first dictates the basic predilections and the range of respectable variations. The basic grouping might be a family, a neighborhood, a coterie of friends; then there are groupings of the productive trades, those of political parties, and finally those of academic studies. These various groupings *are* in their power to govern decisions and judgements: without them there would be no community, no production, no action, in short no *co*-operation. And therefore likely no contemplation.

Yet original developments of any sort require individuation: require us (each separately) to stand up alone, to bear witness alone, to decide what to do and what to undergo alone, to judge what's right and what's wrong alone. Plato and Aristotle put it this way (in reference to epistemic groupings): it's one person

who knows, not the many. Within Christianity, truth requires martyrdom: testimony accepting the ultimate consequence. In the name of collective progress, modern philosophy shies away from enunciating such principles, yet it is retrospectively clear that every step forward in the modern sciences was first taken by an individual willing to stand out of the predilections of the group, often to be "martyrized" for doing so. But at least silently dramatic is any intellectual development: a student begins by conforming to inherited expectations and may, if only rarely, learn to appropriate their source — always with the sense of being the first to "really understand" what had hitherto only been comprehended. There are shallow understandings of everything, whether in production, action, or contemplation. And the qualitative difference between the genuine article and the shallow version entails a difference in stance. Even to understand the difference requires individuation. —So we may well ask:

7. What evidence is there, already within ordinary involvements, that suggests the possibility, even the necessity of individuation? Here again the question is whether we may, as intellectuals, detect and highlight the pre-intellectual "seed" of an essential development. And thereby to broaden our intellectual understanding of its possibilities and necessities —its variations of form.

Again, this question bears also on the question of what it means, variously *can* mean, to intervene, to change the course of things.

Yet standing up and standing out — standing alone — does not abandon the grouping: indeed, it first brings the grouping into view. The adolescent version (grudging rejection) most emphatically highlights the basic grouping of family, neighborhood, and outlying "society" (the vague source of expectations not inherently one's own). Creative versions first introduce the concern for the common good — the concern essential to the formation of a community. For only those willing and able to stand up and out and alone are capable of working creatively together, communing, for its own sake. But all this seems clear only to the individual. Indeed intellectuals, and especially academics, often *argue* about the relation between the individual and the community; contrary to the Greek and Roman disposition, the modern disposition must justify the commitment to the community — very subtly in the cases of Kant and Hegel, very utilitarian in the cases of Hobbes, Locke, Mill and their posterity. So what may seem evident to a creative individual deserves reformulation as a question:

8. How does community evidence itself in ("arise from") our ordinary involvement with things, and how does it bear on individuality ?

Again, there are negative as well as positive bearings — as we may easily note in the difference between the first glimmerings of individuality in adolescent opposition to "society" and the fulfilled versions evident in craftsmen, leaders and thinkers who stand up for their service to others and only thereby stand out as individuals.

Full-fledged individuality consists entirely in the way one *comes back to* what one otherwise only *rises above*. One comes back to others, now no longer simply falling in line — somehow in service to them but without pandering to them (as pathetic parents and pathetic teachers may do); somehow in service to the communal good but without relying on the common conception of the good. One returns to an entire situation, antecedently determined and often apparently frozen, even in its uncontrollable chaos (corruption). There are established ways of running a household, a society, a commercial enterprise: these ways are both technical (knowledge-based) and moral (behavior-based). There are, in each case, pre-established understandings

of what the situation is all about: of its purposes (where it should be going), but also of its origins (wherein it is embedded, from which it draws its legitimacy). These ways and understandings define the situation and initially invite conformative participation. They are essentially inherited. The technical and moral ways are bequeathed by example and practice from one generation to another. The overall understandings are bequeathed by oral or written stories that may appear, to outsiders (in epoch or place), as *mere* stories, myths or rationalizations external to the productive and practical bequeathals. But are the archaic Greek stories mere distractions? Similarly, we may wonder about the status of the stories dominating Judeo-Christian literature. And about the power of the stories told by Enlightenment thinkers. Looking around, we also recognize the stories that govern cultures far from our own. Such inheritances seem to get in the way, to be a burden — or to help us along, to liberate us. So there's this question:

9. How do our ordinary involvements within our ordinary circumstances give rise to the extra-ordinary stories we hear and say about overall origins, whether these are dated long ago or far ahead? Initially, they appear to rest on some distant Authority, and our relation to them is one of hearsay. And even if we choose to doubt them we may still wonder why and how they arise. Especially when we note that such doubt usually pertains to one such story — in order to leave room for yet another.

Within this question lies many of the previous questions — regarding time, projection, intellectual work, internal power versus external force. And above all the question of multiplicity: there are *many* conflicting stories already within our own tradition and, supposing they all inform our present condition, we can wonder how to take them in all at once — not which one is “right” (whatever correctness might here mean), but how they “originate” (!!!) in our immediate condition.

And there is a tenth question ... For, all along, we have been assuming not only that there is something we might call “our basic condition” (ordinary involvements prior to the kind of intellectual involvement we are currently performing), but also that we might be able to bring this condition into view intellectually. But this is a supremely paradoxical enterprise. Since ancient times our intellectual discourse has been tailored to the task of determining transcendent features of things — precisely those features at issue in the first nine questions. How can we possibly employ intellectual discourse to bring out, bring into view or into hearing, a condition supposedly prior to intellectual work? And do this for each other — while talking with one another, reading another’s writings, writing for another’s reading? In such discourse, we abandon our immediate condition — lose our innocence, as it were — and the result of any effort will be tainted by our intellectual condition.

So it seems, and so it usually happens. When modern thinkers first turned their thoughts to our “original” condition they immediately talked about sensations (our five senses: abilities to undergo) and extractions (our intellectual faculties: powers to do) — a division of labor essential to intellectual work, and possible only within the position attained (and ever-more nuanced) by laborious intellectual exertion. And when more recent investigators in empirical fields (psychologists, sociologists, ethnologists) turn their eyes to the “basic” conditions presumably as yet untainted, they filter all their findings through lenses inherited from the original modern thinkers or through ones seemingly invented on their own (new categories for supposedly new findings) — in either case contaminating the evidence from the start.

How else? The moment we ask about our original condition our position is intellectual, already “above” the original, already extracted from it. And often proud of it: we celebrate this position as itself an achievement. In the modern tradition, represented now by modern sciences, this position is what’s really real (ἡ ὄντως οὐσια), the place to stand when searching through the ever-shifting sands and waters “below” in search of what *else* might count as real (τὸ ὄν) — their configurations but also, since Leibniz and Newton, their motions.

So let us accept our fate. Our discourse is intellectual, it does not at all represent the sort of talking that goes on within our ordinary involvements — not even the ordinary talk in which intellectuals *also* conduct themselves, already while they are arranging papers and chairs and coffee for an intellectual discussion. For those aware both of the efficacy of ordinary language in ordinary circumstances and of the power of rigorous language in intellectual pursuits, the question may arise most poignantly:

10. Intellectual discourse, this marvelous way of talking about things both individually and together, this strange transformation of talk out of ordinary into extraordinary forms: How does *it* grow out of, and then away from, and then double-back to, our ordinary involvements?

Actually, thinkers from Plato and Aristotle, on through those of the Christian era and as far at least as Rousseau and Mallarmé, consider language to reflect precisely the destiny of our very nature to transcend the ordinary and to lodge ourselves, as much as possible, in the discourse of geometry, arithmetic and all the other domains at issue in the first nine questions, these defining the life of the spirit. Only during the Enlightenment did thinkers begin to consider language as essentially a means devised to enhance co-operation among ourselves in our joint efforts to live out our material lives, with geometry and the rest as especially refined means. Both traditions speak of language (discourse at either the spiritual or the material level) as a tool: as a feature of our nature, in the one case defining our destiny and in the other case extending it. Neither can raise the question of a shared parentage giving each its due.

— All ten questions seek internal gestations — internal powers rather than external forces. But internal to what? To human being, both our ancient and modern traditions say: internal to each individually but, even more, to the species. However, following this scheme, we today, versed in modern social science, run the risk of separating out one exemplar to place under our microscope for closer inspection, or one kind of thing to study statistically — in which case we are no longer contemplating where and how we *all* are, but only where and how "those things" are, to the exclusion our ourselves. What might it mean to contemplate human being inclusive of ourselves?

This last question we can only raise and answer within the raising and answering of it — never in advance, and never really in retrospect either.

One obvious guideline: we *are* first of all as engaged with things — whether setting out coffee and doughnuts for those attending a conference, walking on a sidewalk from a parking lot to a grocery store, grooming a pet or mowing a lawn, changing diapers or dressing for work. In somewhat Aristotelian language, we may say (very intellectually, and already generalizing!) that every such doing and undergoing, gesture and plight, moment and place, distinction and enumeration, interrelates, variably, with others within a dynamic complex we might call, following Heidegger, our being-there with things and people, a being-there already formed (and forming) a world.

The first test of our success in fresh contemplation is whether we can retain the integrity of this complex while also bringing its details to light — rather than merely disassembling it in search of "more real" elements, the listing of which would prove our own intellectual prowess.

Let us then begin…