The text comprises a much-expanded version of *Contemplative Logic*, first published in 1999 (Atcost Press).

Appendices have been added to

- Book Three: “Induction in Arithmetic”
- and Book Four: “Axiomatic deduction”
  - “Axiomatic induction”
  - “Axiomatic set formation”
  - “Probability”

In Book Five, “§2.4 Syntax and truth” has been replaced with
- “§2.4 Thinking and knowing”, and
- “§3. Essay on a fifth interpretation” has been entirely rewritten.

In addition, many errata have been corrected.

March 2009
Other works by Cyril Welch


*Emergence: Baudelaire, Mallarmé, Rimbaud* (State College: Bald Eagle Press, 1974). Co-authored with Liliane Welch

*Address: Rimbaud, Mallarmé, Butor* (Victoria: Sono Nis Press, 1979) Co-authored with Liliane Welch

*The Art of Artworks* (Victoria: Sono Nis Press, 1982)

*Linguistic Responsibility* (Victoria: Sono Nis Press, 1988)

*Contemplative Logic* (Sackville: Atcost Press, 1999)


*Dante: Three Meditations* (Sackville: Atcost Press, 2007)

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**Logic Ancient and Modern**

Cyril Welch

Atcost Press

2006
The first three of the five “books” appeared earlier in *Contemplative Logic* (1999), except that I have added an appendix to Book Three (induction in arithmetic). In Book Four I have relocated the section on Zermelo’s theory of set formation to a much-expanded appendix, along with appendices on Russell’s theory of deduction, mathematical induction (with meta-theorems), and probability (with an axiomatic account). Book Five has been entirely revamped starting at the last section on Kantian axiomatics and continuing to the end. In addition, and throughout, I have corrected typographical errors and made some stylistic improvements. This new text is 186 pages longer than the 1999 text.
Preface

This work comprises my lectures on logic, reworked annually over the last three decades of the twentieth century. The lectures themselves would take place over four semesters: the first on syllogism and paralogism, the second on induction and ontological axiomatics, the third on natural deduction (Gentzen’s), and the fourth on formal axiomatics (Russell’s, Zermelo’s and others).

Throughout, the point of departure remains that of Greek logic as it first appeared in Plato’s dialectic and then received organized treatment in Aristotle’s works. The intent is to rehearse the shift of concern that led into modern logic, specifically the logic associated with the mathematical sciences and modern institutions generally.

Initially (around 1970) the motivation for this study lay in a peculiar revulsion. Fresh out of graduate school in 1964, I was soon asked to teach a standard course in logic. Still in thrall to the contemplative reading, writing, and talking engendered by the great works of our tradition from Plato through Kant and Heidegger, I found the “thinking,” the “reasoning” structured by popular textbooks incompatible with the careful thinking and reasoning at play in such contemplation.

Of course, there are stunted versions of everything — of logic, of marriage, of jazz. The moment one realizes that one has been caught up in such a version, one may proceed variously: one may acquiesce in it, abandon the field entirely, supplement it somehow (active acquiescence), or strive to recover its source and thereby restore the enterprise itself. I have aspired to this last.

At first, I assumed that the decisive battle would take place between the popular (stunted) version and the original (philosophical) version. However, I gradually learned that this facile discrepancy conceals a deep-seated conflict between two possibly very healthy versions, the ancient and the modern.*

Since writing this work, I have studied Oswald Spengler’s The Decline of the West — which, I can now see, set many of the themes governing the intellectual concerns of the 20th Century. He characterizes the difference between ancient and modern religions and arts (visual, musical, literary) as that between Apollo and Faust (with the “magical soul” wedged between them). Similarly, the two originary traditions of logic might be called the Apollonian and the Faustian (with the Scholastic wedged between them).
Roughly from the mid-nineteenth to the mid-twentieth century, the conflict seemed to stem from questions of convenience. Traditional logic clearly could not keep pace with the rapid developments in modern mathematics and natural science: it presupposes a universality of reference that modern investigation was abandoning for a universality of notation. Yet already this one question of convenience points to differences of an ontological order. Again very roughly: the old logic is subject-intensional and the new logic is object-extensional; the one invites transcendent, the other transcendental grounding.

And the ontological differences correspond to phenomenological differences masking a transformative battle over another question: the venue of rationality. For the original logic finds and investigates its evidence in artisanal and political exigencies, while its modern progeny finds its evidence in the exigencies of its own investigation and its own organizational imperatives. Do we enact rationality by entering into things, finding ourselves answerable primarily to their exigencies? Or rather by holding ourselves aloof from things, finding ourselves answerable primarily to our own exigencies? In the original venue, the question bore on the relation between intuitive and discursive reason.

Historically, the conflict seems to be settled: our entire technological civilization rests on the infrastructure secured by the victorious party. Both the successes and the failures of our sciences, our governments, our educational institutions remind us daily of our dependence on the infrastructure of discursive reason. Yet the original battle still rages—pressed into the underground, as it were, where it then resurfaces in various forms of neurosis.

I have undertaken to bring the battle back into the open. And while its plot, like that of Homer’s *Iliad*, can be summarized in a few lines, its significance emerges only in the myriad episodes. Thus the length.

The story of the battle is best told from the inside—as enacted. Yet the danger here is that we may merely take the occasion to re-tell our own story, the modern version. To counteract this danger, I tell the story primarily from inside the Trojan camp, the Platonic and Aristotelian side defeated on the plain of the strictly historical conflict. On this approach, it remains constantly imperative to re-enact, freshly, the modern side as well.

In Plato’s sense of the distinction, the style of the present study is largely mimetic. For a purely recitative style serves the construction of outside stories told with unquestioning confidence in the narration of one’s own age. Such an approach (historical rather than philosophic) relieves us of the claims at work in our tradition, obscures their power, and thereby reduces them to curiosities.

Modernity has lost its naïve verve and has become, despite its name, yet another tradition weighing on us. Awareness of this development has engendered the genealogical style, often rightly (if immaturely) associated with Nietzsche. Yet this style, too, places us at a crossroads. Down one road travel those who wish to free themselves from the confines. Down the other road travel those who aspire to re-appropriate the confines themselves, their source. The first appeals to adolescents—who will quickly flee back, like homing pigeons. The second appeals to those who know that they must slowly make a home wherever they are.*

* * *

Modernity has a special claim upon our understanding of rationality: our immediate tradition aspires to establish techniques of calculation and observation enabling us to discern or devise structures for academia, for government, and for industrial production and commercial distribution—all of which is often summarized under the single rubric of information. Even when foundering, the abiding aspiration and its past achievements continue to provide the framework to which we awake each morning, and within which we will work out any alternative.

It has become fashionable, in some circles, to call attention to modernity’s less savory effects (basically: managerial imperatives at odds with the communal spirit and technological abuse of the natural environment), and to seek relief in a cultural relativism (basically: to each his or her own story of community and of environment). Yet when

* Martin Heidegger remarked already in *Being and Time* (1927) that it was of no consequence whether his own account was new, that what mattered was whether it was old enough to appreciate and appropriate the possibilities disclosed by our ancestors. Hans-Georg Gadamer develops this thought (“fusion of horizons”) in his *Truth and Method* (1962). Similarly, Gianni Vattimo remarks in his *End of Modernity* (1984): “Plato’s work cannot be rethought today in terms of whether the doctrine of ideas is true or untrue, but only in terms of trying to recollect the clearing, or the originary historical opening, within which anything like the doctrine of ideas is able to appear.”
distancing ourselves prematurely from our immediate tradition, without any reconsideration of the stakes, we rush headlong into alternatives that can only be negative and thereby vacuous.

The vacuity stems from the thoughtless and historically erroneous assumption that modernity adequately represents rationality itself, whereupon the two must appear on trial together. Yet a careful consideration of our original western tradition reveals that modernity has rather hijacked rationality. The task, then, is to take it back.*

From the logic of Plato and Aristotle we may also have to distance ourselves. Yet theirs was a logic that embedded rationality integrally within our immediate dealings with circumstances, namely within craft (τεχνη), where sensation and intellection, intuition and discourse, find both their differentiation and their roots. Contemplation of this unity of reason can provide a perspective through which to discern and appreciate modern rationality as representing only a fragment, albeit all the more (humanly) powerful for having suspended the claims of its parentage.

Most novel about my own work is perhaps the sustained effort to slip into the shoes of a thinking that precedes even what we call traditional, really scholastic logic. And only afterwards take the measure of modern logic. Obviously, I here reverse the usual procedure.†

The Introduction lays out the Four Interpretations that have formed our tradition over the last two and one-half millennia. Of these four, only two receive extensive attention throughout the rest of the work. Although sketchy, the account on these pages sets the stage for understanding inherited forms of logic as illustrating interpretations of human being, and especially of human rationality.

Book One on syllogism and Book Two on paralogism eke out the basics of Aristotle’s account of categories, of assertions, and of sound and unsound inference — always contrasting the earlier accounts with subsequent developments. These contrasts intend to bring the original achievements into sharper focus: most importantly, their venue.

Book Three presents induction in the format of the New Stories advanced by Plato and Aristotle. On their own accounts, Socrates was the first to practice this form of induction, clearly within the venue of the Athenian respect for the arts, including the art of leadership. Induction of this original sort belongs to the art of dialectic: person-to-person maturation of insight entailing a transfiguration. Again, the contrast with Baconian induction highlights the significance of the original version.

Book Four gives full reign to mathematical logic. Novel perhaps is the recurrent detection of the Stoic ontology silently at work throughout the development of the modern interpretation of rationality. Apart from this detection and the incumbent contrasts with Plato and Aristotle, my account simply rehearses the achievements of modern logic during the first third of the twentieth century, with considerable attention to formal axiomatics.

Book Five re-enters three versions of ontological axiomatics: Aristotelian, Kantian, and post-modern. Each raises the question of the status of the Axiom of Non-Contradiction: being vs. non-being. This question repeatedly forces itself into earlier considerations — in ways, I hope, that will have already rendered these final considerations imperative.

During the decades in which I was writing and re-writing these pages I profited immensely from the intellectual companionship of others: above all that of Liliane Welch, whose literary expertise and poetic gifts would draw me back into the concretions from which logical abstractions grow, and then also that of my many alert students, whose natural desire for clarity would lead me to see the need for both stylistic and substantive corrections. I am immensely indebted to both.

Cyril Welch
September 2006
Introduction: Differences

Our nature is to be rational. In the Greek formulation: our condition is one of λόγος, we “have” logos. Logic is the study of our nature as thinking and talking, as accounting for ourselves and our circumstances. Our rationality is not a tool we simply use, much less a property that we simply enjoy. It is a potential we must develop. And part of the development is to learn what it means to be rational, to think and talk logically.

§1. Thoughtfulness

To think: What does this mean? What all does it mean — so that we can consider it as naming our nature? Instead of searching immediately for one meaning of thinking, let us savor the multiple ways we in fact manifest our thinking nature.

(1) To think is to infer. Whatever my vocation, I am constantly applying rules to current events in order to foresee them. I think ahead, make adjustments now so that things will turn out later as my vocation requires. However, foreseeing the future manifests only one form of inference. As a horticulturist, I also infer from the color of a plant's leaves that the plant has lacked sufficient water: I think ahead into the past. As a doctor I infer from the outbreak on a patient's skin that the patient's liver is diseased: I think ahead to another location. And we all learn in school to draw inferences somewhat in abstraction from time and place: if I have so much money in my pocket and if I spend so much of
it, I shall have so much left; if I drive such-and-such a distance averaging so much speed, the trip will take so long; if physics requires theoretical talents and engineering practical talents, and if my talents lie more in theory than in practice, I will find physics easier than engineering. In any event, to be human means to live by rules which we then apply to cases. Or: to think is to make judgements taking us beyond what is given outright, whether the beyond be understood in reference to time, to place, or to formal relations.

(2) To think is to criticize. Human being seems essentially attuned to the possibility not only that things might, indeed incessantly do fall short, but also that people themselves might and incessantly do go wrong. Criticism of events reflects the clash of circumstances with one's own rules, a frustration of human will. Criticism of others reflects a clash of their performance with one's own expectations. Critical thinking then naturally fulfills itself in self-criticism, the assessment of one's own performance within circumstances. Thus the practice of any vocation requires one to think critically in all three ways: to note how events diverge from the norm, to question whether others have responded appropriately to events, and to wonder whether one's own way of approaching situations does justice to them. In one word, criticism brings presuppositions into the open for review and possible revision: even the most elementary criticism—mere disgruntlement with the way things are going — evidences a dim awareness of a presupposition, namely the standard against which events appear in negative relief. The more one develops a vocation, the more one reviews the presuppositions themselves — as a doctor knows that “having cholesterol” may mean many different things, as a mechanic knows that the failure of an engine to run may reflect many different malfunctions, as a chemist may wonder whether impurities in the samples could have affected the results obtained by analysis. Especially as we advance in our work, we become increasingly critical in one, two, or all three ways: for the interplay between presuppositions (one's own rules) and the affairs of the moment allows for every kind or error, deception, and even self-deception. One tends, then, to become more thoughtful.

(3) To think is to search. Not simply to search out conclusions based on one's rules or presuppositions, and not simply to detect the presuppositions already in force, but to discover hitherto untried, unknown principles. Being human, we carry a burden: we have to learn. And to think is to accept this burden. Of course, learning in some instances requires one merely to adapt one's behavior to socially defined manners, often merely to familiarize oneself with the sayings of others — two rather thoughtless accomplishments. However, thoughtful learning of principles brings one first of all into an intimacy with one's circumstances: as a doctor, in learning the principles of medicine, becomes intimate with human bodily being; as an automobile mechanic, in learning the principles of combustion and the like, becomes intimate with engines; as a physicist or chemist or biologist, in learning the principles of nature, becomes intimate with the materials and equipment in his laboratory; as a lawyer, in learning the principles of law, becomes intimate with the workings of society. Learning principles requires one to think backwards, to discover implications underlying our dealings with the human body, with engines, with chemical reactions, with questions of justice. At the same time, however, principles work in the opposite direction: once learned, they invite application, and in application they become presuppositions for inferences, and as presuppositions they may replace the original intimacy and become blinding rather than lenses. So learning is a continuous affair insofar as it is a genuine affair at all, and a thinking person appears as one who is always learning rather than as one who has already learned.

(4) As a special case, but one which defines the specific interest of logic: to think is to ponder what it means to think! Granted that our thinking nature leads us to infer, to criticize, to search — that we fulfill our nature by developing the capacities of inference, criticism, and research — what do these performances mean? What do they require of us? In any vocation, one can be struck by the fragility of inferences, of criticisms, of discoveries. We think incessantly, but very often to no avail: the inferences we draw, the criticisms we level, and the principles we discover easily prove unwarranted. So much so that we sometimes hear advice to avoid thinking, to take things simply as they come. Yet such advice perhaps intends rather to suggest that the development of the capacity to infer, to criticize, and to learn requires one not only to approach situations more actively, more cuttingly, more deeply, but also to realize the tentative status of any such approach. A thoughtful person in any vocation appears as one who knows that his expertise cannot truly pre-empt what actually arises in encounter. Thus in common parlance we speak of a thoughtful person, a thinking person, as considerate: one who
takes the side of what or whom is encountered — inferring, criticizing, discovering but at the same time acknowledging that his thinking must incessantly measure itself within the encounter and not simply against his own standards.

Most obviously, logic has evolved in western civilization as a school discipline, namely as a systematic study of inference, criticism, and discovery — under such names as deduction (syllogism), seduction (paralogism), and induction (neologism). Over the centuries logicians have developed so many techniques and called for so many distinctions that teachers and students alike can easily devote all their academic energies to familiarizing themselves with the results and illustrating their efficacy in solving problems arising in their own house. So much has logic become a self-contained challenge that the discipline has begun to disassociate itself from its original home, namely philosophy — just as many other academic disciplines have done, e.g. psychology and sociology. As soon as a field of questions becomes laden with an abundance of apparently fixed answers, the pedagogical exigencies, the daily tasks of teachers and students, invite all concerned to concentrate on the fixities and to forget the original questions. Yet only these questions — which, in the case of logic, all lead back to the question, What does it mean to think? — really justify the answers. More: only the questions bring into relief human thinking itself.

In any case, logic becomes a contemplative affair only when we broach the questions of inference, criticism, and discovery explicitly as questions. That is: when we approach these three kinds of thinking as dimensions of our condition, as manifestations of our own being conditioned by λογος, rather than as separate bundles of techniques and distinctions simply to be mastered. Contemplative logic then has its home in philosophy itself. Or, rather, it makes us at home in human nature.

§2. Human being

One can reap at least one advantage from the long and variable history of logic. For we can see now, in retrospect, that each established version stems from and therefore promotes an interpretation of human nature — of what it means to think and of where thinking leads us: our destiny. And we can see that these interpretations have guided human efforts and accomplishments throughout the ages for which we have historical records: they have not lain to one side of the road as idle commentaries on passing events. In a reflective mood, and one taking us briefly beyond the specific questions arising in logical study, we can discern in western literature at least four essentially contrasting interpretations of human being.

(1) In the works of Homer and Pindar, Aeschylus and Sophocles, perhaps also the comedies of Aristophanes and some fragments of the pre-Socratic philosophers, we come upon a unified vision of human being in the image of the hero, the responsible warrior. In these works, what happens to human beings appears totally erratic: it follows from the will of the gods or demi-gods. The hero acknowledges his position as essentially vulnerable, takes a stand in the face of variable and ultimately fatal circumstances, and thereby shows others that such a stand is possible. Such heroism receives its sole reward in the recognition of one's courage, a reward bestowed by others. The two together, the display of courage and the bestowal of honor, also appear as the foundation of the early πολις, the formation and preservation of clan-based communities and, later, the more broadly based cities. If the hero is here a thinker, his thinking consists largely in recollecting his pending death and his continuous destiny to stand up to hostile circumstances so that his name can stand as an inspiring memory for those who follow. In one word, the earliest distinctively western interpretation of human being calls us the mortal ones.

(2) In the works of Plato and Aristotle, as well as of their successors, we come upon another, to us more familiar vision of human potentiality: man is here the maker, most obviously the artisan but then also the politician-orator as maker of the πολις in eulogistic, deliberative, and forensic speech. Here the central task of man becomes that of seeing the end: seeing the purpose and thereby foreseeing the completion of things. According to Plutarch, Solon had already encouraged the development of crafts and made it a crime not to be a maker in some sense. Such making, be it that of the potter, the horse trainer, the doctor, or the politician, requires a split relation with circumstance: a relation to the givens (the starting points, the materials) and a relation to standards (the ending points, the forms). The performance of a craft requires one to stand in the middle, to mediate with the hand or the tongue between these two modes of being. Of course, sophists and philosophers debated the status of beginning, middle, and end. But all agreed, in contrast to
the Homeric tradition, that human beings fulfil themselves in creative mediation of some sort — in taking responsibility for seeing the end and doing something to bring about what is foreseen. In one Latin word, the second distinctively western interpretation of human being calls us the rational ones.

(3) In the opening chapter of Paul's First Letter to the Corinthians, in Augustine's Confessions, and of course throughout much medieval literature, we find the Christian vision of human being: God alone can fulfil our nature. Every pretension to wisdom in the making of things, to comprehension of things as a whole, to status in society — in short, every strictly human accomplishment — is foolishness when taken to answer, even in part, the question of human fulfillment (salvation, redemption). Ultimately, we can at most allow God, by way of Christ, to fulfil our being: our central task, if we can still call it a task, is to acknowledge our dependency upon our Creator and upon mediation by His Son. Yet we are in fact vocationally engaged: bound within split relations, desire to comprehend, and societal status. Moreover, we are also free: responsible for making something of the gift of our being. Theologians may debate the conflict between our debt to Caesar and our debt to God, and that between our recognition of free choice and our acknowledgement of God's grace. In any case, a Christian falls into heresy, e.g. the Pelagian, if he dreams of standing on his own or of creating his circumstances. In one word, the Christian interpretation names our being as created, and calls us creatures.

(4) In the literature of modernity human potentiality reappears as knowledge. Yet the knowledge here differs in some essential respects from that of the late Greeks. Instead of taking the craftsman and the orator, intellectuals took themselves as the model. Bacon and Descartes commenced this new vision, Leibniz and Newton culminated it, but only Kant worked out its implications for human being generally. An intellectual's knowledge is not one of mediating between matter and form, as it seems to be in the case of a potter, a horse trainer, a doctor, or a statesman. Rather, an intellectual devises systems in interaction with observations. Modern philosophers debate the exact workings of the interplay between the two (concepts and data). In any case, however, knowing now means predicting, controlling, changing: adapting upcoming affairs to one's own system (rather than “partly completing what nature is unable to finish, and partly imitating her,” as Aristotle understands art and knowledge in his Physics, 199 a 16). The prime example of the knower is here the modern scientist. But this prototype spawns others: the industrialist, the merchant, the modern politician. Eventually it sets the standard for all human vocations. In one word, the modern interpretation of human being calls us operators.

These four interpretations have completed themselves; they define in large part the scope of our western heritage. None simply describes human nature, each rather states a standard of fulfillment. Each overlaid a predecessor (the Homeric seems to have overlaid the Minoan cult of Mother Earth) and was overlaid by a successor, so that our heritage does not consist of four neatly stacked piles. And each has it own lightweight version, always more evident and therefore confusing the study of our heritage: corresponding to the Homeric hero we have the braggart, the macho, the movie star, the TV personality; corresponding to the Athenian maker and orator we have the hobbyist and the demagogue; corresponding to the Christian we have those who simply submit themselves to doctrines of some church; and corresponding to the operator we have the bureaucrat. For the heavyweight versions we must turn to the source literature, e.g. to the works of Aeschylus, Plato, Augustine, and Kant, works which we can only understand if we re-enact them.

With the distinction between heavyweight and lightweight, we can sense yet a fifth interpretation of human being and human thinking. Its lightweight version we may easily name: the consumer. In the twentieth century we find for the first time an interpretation explicitly and widely prescribing what earlier interpretations had described in order to proscribe: human being consists essentially of desires, the chief one being the desire for longevity, and human fulfillment consists in clearly recognizing and actively satisfying these desires — rather than transforming them. Most obviously, western political and economic systems proceed on the basis of consumerism; they are basically demand-oriented, even if leaders sometimes lay emphasis on the supply side. More subtly, mainstream intellectuals generally promote the processes of consumption; they find their work respected and supported if (and, one fears, only if) it serves society, i.e. the processes of production, communication, and protection surrounding the life of consumption.

Is there a heavyweight version of the fifth interpretation? In the nineteenth century thinkers like Hegel, Marx, and Nietzsche, Emerson,
Thoreau, and Whitman, and in the early twentieth century still others like Henry Adams and Henri Bergson, John Dewey and Martin Heidegger, explicitly acknowledge the danger of our historical position now that the Fourth Interpretation has simply installed itself, and they explicitly proffer some first steps toward meeting this danger. Similarly, scientists like Niels Bohr, Werner Heisenberg, and C. F. von Weizsäcker have argued that science itself can only proceed profitably under a new interpretation of what scientific work means. And on a popular plane the environmentalists urge us to develop styles of life preserving rather than simply consuming circumstances. If none of these alternative interpretations has yet installed itself definitely, we might console ourselves with the thought that a genuinely live interpretation always demands further thought, further development, and therefore never appears as simply established. Indeed, the peculiar vantage point of the twentieth century — an age so obviously riding on top of a manifold of established interpretations of human being — suggests that the moment an interpretation installs itself definitely it loses its status as a live force and transmutes itself into a sometimes inspiring, more often shackling memory.

§3. Logic

As hitherto known in the West, logic represents a central development only in the Second and Fourth Interpretations of human being. For only in the late Greek and then again in the early Modern ages does human being appear as intellectually responsible. The Homeric heroes display responsibility to the utmost, but their responsibility requires not knowledge but rather a stand within circumstances in the face of death. A Pauline or Augustinian Christian displays only a grounded responsibility, a pre-intellectual submission to the Source of all. And the present-day consumer (a Kantian heteronomous man, a Nietzschean last man) displays for the first time the articulation and recommendation of essential irresponsibility: a concern for familiarity (called “knowledge”) solely for recognizing, prolonging, and satisfying immediate desires. The vision of human being as essentially conditioned by λογος, one for which we are ourselves responsible, is not only distinctively western, but strictly derived from two of the four interpretations firmly comprising our heritage.

On the late Greek interpretation, we are all basically makers. As a house-builder I foresee the house to be built; this foresight penetrates the otherwise obscure purposes of nature, penetrates the various needs of the human body, the services rendered by various materials, and the joints and joinings necessary for “completing what nature is unable to finish.” As a polis-builder I foresee the community to be preserved, the groupings to be formed. Here my foresight penetrates the very obscure purposes of human being, of human beings in all their variability, penetrates also the effects of structural arrangements on human cooperation. Similarly with a horse trainer, a potter, a farmer, a sailor: all stand at the crossroads of the given and the foreseen, the individual and the universal. To be human is to decide the one with the other, to embody the one in the other: to judge, to subsume the individual under the universal. Judgement manifests itself not only in recognition, but also in reparation, not only in naming things as ones of kinds, but also in helping things become their kinds. At this crossroads arise the necessity of inference, the possibility of error, and our destiny to mediate. As logicians, we analyze these modes of λογος to display intricacies, difficulties, and implications.

On the early modern interpretation, we are all basically operators. The investigator, observer, calculator now serves as the model, the prototype of the thinking human being. In the passage of givens one aspires to detect patterns, to design patterns: patterns of events. Instead of subsuming givens under universals, variable instances under fixed forms, the modern investigator distinguishes and relates eventual data (in some sense fixed at a time and a place and so beyond appeal) and human constructions (in some sense open to modifications in the course of time and change of place). The thinking at issue in modern investigation requires above all a well formulated and well regulated home base from which one ventures out into fresh events. The modern investigator then also stands at the crossroads. However, this stand allows for an apparently neat division of labor: on the one hand, the investigator must keep cleaning up his own house; on the other hand, he must face extra-domestic events. Inference (within the household), error (at the doorstep), and discovery (beyond the confines) here take elaborate forms. And the logician must be prepared to analyze these modes of λογος to display their intricacies, difficulties, and implications.
Are there then two logics? One can answer this question variously, according to what one means by “logic.” On the one hand, we may take the word to name strictly the manipulation of extracted forms: formulation. While such formulation would then presumably represent the thinking which human beings more or less do, we would, as logicians, abstract from any contextual considerations, e.g. those evoked to distinguish the various interpretations of human being. Ignoring the difference between the two logic-generating visions of the human condition, we would find logic to be one and the same throughout history, all differences appearing evolutionary and otherwise incidental. The obvious advantage of this approach is that the study of logic here requires no careful reconsideration of the human condition at all. One disadvantage is that logic then comes to mean mainly a study of formal inference and becomes increasingly a mathematical concern.

On the other hand, we may take “logic” to name more broadly the contemplation of emerging form: of formulation as reflecting our basic condition as human beings. And since our basic condition remains questionable — has in fact received four or five very different interpretations in the West alone — contemplation here leads us to appreciate differences (unless, of course, we believe that we have already penetrated to the single basic condition of human thinking). Certainly we today can distinguish between the historical assumptions and ambitions giving rise to logic in the late Greek and again in the early modern eras, and we can appreciate the diverging directions each “logic” took. Yet the historical differences here only serve to remind one of the necessity to devise and assess logical formulations in constant reference to some context of assumptions and ambitions. The obvious advantage of this approach is that we can proceed in our studies along two lines at once, the logical and the philosophical. One disadvantage of the contemplative approach is that logic becomes a study requiring us periodically to journey back into the penumbral underworld of the human condition where the supposedly solid forms in the upperworld appear as mere shades: logic, after all, purports to be a study of fixed form, and now (on the contemplative approach) the fixity appears as indebted to a source and therefore as tenuous after all.

Meanwhile, we engage in the discipline called “logic” only if we do in fact extract forms of thinking which at least appear to have a life of their own. The formal and the contemplative approaches differ mainly on the question whether one must retain a sense of the generation. And, indeed, why should we worry about the source? Constant reference back to the source and generation of form impedes the forward flow. Pedagogical considerations inveigh against such reference, especially nowadays when so much detail must be learned.

One could justify the slower approach by arguing that some decisions about form require a prior understanding of the subtle conditions of its generation; that the historical evolution of form — whether in ancient or in modern times — in fact depended upon a sensitivity to origins of generation. And we might suspect that any future development of logical insight will require a return to the source once again.

While accepting the basic tenor of all the arguments in favor of the slower approach, I propose yet another. It seems to me now, at the outset of the twenty-first century, that both versions of logic, as they are established within our heritage, suffer from a confusion of two antithetical interests: in each case a substantial and a vested, an original and a subsequent interest. A contemplative approach, by eliciting the substantial interest in both versions, might liberate the study of logic from the vested interests, with the result that both versions can receive full attention. For the vested interests set the proponents of the one against the proponents of the other. In brief:

Greek logic analyzes inference, perversion and discovery to reveal the universal at issue in all thinking. No matter what my vocation, I think my way through given presentations in constant reference (whether wholesome or not) to their respective kinds: I deal with horses according to my understanding of the horse-ness each represents, and so on. It seems, then, only a slight step from this analysis of human responsibility, with its linguistic and intellectual commitments for all human arts and sciences, to a doctrine of the way things already are prior to and independently of any human engagement with them: to a description of the universe. And this step was in fact taken sometime during the Third Interpretation. On the Aristotelian version, the fixity of universals (of species) remains a hovering anticipation within each art or science, something making sense only as it is being realized, actualized. On the Church version, the fixity of all universals in creation is a haunting recollection which pulls one back out of one's art or science and into the Source of all beings: it renders one's own unaided effort to help beings
be, to “complete what nature is unable to finish,” ultimately peripheral to human being, even blasphemous if taken centrally. In keeping with the Third Interpretation of human being, and of all beings, as creaturely (created by God), the Church introduced a vested interest into Greek logic: See, said the Fathers of the Church, even our thinking and talking requires an acknowledgement of universals, fixed species, and therefore of God’s creation and of God as the creator. In pedagogical and political situations this vested interest can become blatant: teachers can reproach their pupils for not recognizing fixity, and rulers can claim to represent the one way of government.

Modern logic analyzes inference, discovery, and error to reveal and formulate the rules at issue in each: as an investigator I look for recurring patterns in events. I first make sure I can observe an array and I then develop a system of my own which allows me both to re-enforce my observation and to play a game of prediction. Again, it seems only a slight step from this analysis of human thinking to the doctrine that the purpose of human thinking is to twist all events into the satisfaction of human desires: domination. And this step has in fact been taken in our modern consumer society. We today assume that intellectual work must somehow contribute to the processes of getting things to behave the way we want them to behave. Our liberal society owes whatever stability it has to the accomplishments of physics, biology, computer science: How could modern democracy work without modern engineering, agribusiness, mass propaganda, and data processors? And, since modern logic lies at the basis of all these developments, we find the study of it once again weighed down by vested interests.

The two vested interests not only clash with one another, often driving intellectuals to pursue one version at the expense of the other. They also drive many away from the study of logic altogether. Certainly, modernity was born partly out of a weariness with the Church’s vested interest in fixity of species in the pedagogical and political domains. And today the vested interest in fixity of law leads many who recognize the essential poverty of consumerism to reject the logic which consumerism has adopted and corrupted—to reject all thinking about thinking, and thereby unwittingly to hand all the power of logic over to the latest lightweight interpretation of human being.

Perhaps a contemplative approach to logic can liberate it from the vested interests which distort and ultimately discredit it. Such a liberation would require that we replant logical formulations in the soil of their original generation. Perhaps, too, in overcoming the clashes of the vested interests and allowing for a retrospective unity of logical study, a contemplative approach might eventually generate an interpretation of thinking and of human being prospectively appropriate for our time: a heavyweight version of the Fifth Interpretation.

§4. Basic questions

Ancient logic developed in an effort to enhance participation first of all, not directly description. That is: to enhance the realization that each being one encounters is but a part of its destiny, and that we ourselves can make sense of what we encounter only by taking part in that destiny.

Modern logic developed in an effort to enhance construction first of all, not directly domination. That is: to enhance the realization that each moment of encounter in space and time requires a prior organization on our part, and that we can, even must withdraw from what we encounter, resetting its terms.

Participation entails an intimacy with the transcendence of what we encounter. Construction entails a distance from what we encounter, a transcendence backwards into our own domain of response. Borrowing Kant's distinction, I would call the one logic transcendent, the other transcendental. Both engage us in transcendence—engage us in contemplation of the transcendence marking the human condition.

The most obvious wonder of our thinking, of our logical condition, is that we routinely extract and occasionally assess a way of thinking in abstraction from what we are thinking. In the special language of our discipline, we say that thinking can be valid or invalid already at a distance from the content of our thinking. The question of validity we can then keep distinct, at least for a while, from the questions of accuracy or truth.

Anyone who has carefully filled out an income-tax form with nary an error in the calculations, only to find that the tax authorities disagree with the final figure, knows that thinking can proceed well in itself and yet fail to reflect or meet its full purpose. Similarly, Copernicus' reasoning was valid and his conclusion even correct when he argued that the other planets could not revolve around the earth since their distances from the earth varied over time: however, he was assuming that any
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A candidate for the central position would have to hold each of the planets at a constant distance from itself, i.e. his argument assumed that planets must move in perfect circles. The argument is valid, i.e. the premisses lead to the conclusion, and still today we consider the conclusion to be accurate: but one of the assumed premisses we now consider wrong.

Anyone who has correctly assessed the individual costs of a trip but has added them up incorrectly, knows that thinking can proceed poorly in itself. Similarly, I may think myself an accomplished poet because all accomplished poets write strange things and I write strange things, too: here the two premisses may be correct, but the conclusion does not follow, the argument is invalid—even if the conclusion happens to be correct as well.

Very formally stated, thinking appears as a movement (inference) from premisses (basically two at a time: A and B) to a conclusion (C). One question is whether C does indeed follow from A and B. Another question is whether A, B, and C, taken separately, deserve our endorsement. Each question points out a dimension of human thinking, a direction in which we measure our thinking. The one represents the role of the distance, the other suggests the role of the intimacy engendered in or by thinking. In the study of logic one learns to separate these two even though, or precisely because, in actual thinking the two go together.

The question remains, of course, how the two dimensions relate to one another, rightly or wrongly. For the longest time, philosophers understood intellectual work to consist largely in chains of reasoning in which distance seemed to play the chief role. For instance, in his treatise On Monarchy, written at the beginning of the fourteenth century, Dante could say: “Any truth which is not itself a principle is demonstrated as following from the truth of some principle” (I, 2): and the treatise proceeds to speak of human affairs in a long line of deductions not unlike those in Euclid’s book on geometry. On this view, the world becomes intelligible only as we extract ourselves from the immediacy of our condition and install ourselves in pure reason. Yet what about the principles, the premisses, giving rise to the concluded truths?

Obviously, thinking at its best is not only valid but also to the point: not only will its conclusions follow from its premisses, but the premisses will reflect a close relation to what the thinking is about. An argument (a λόγος) embodying success in both ways deserves a special name: Aristotle called it demonstrative (it shows what it is talking about, in the talking), we today tend to call it sound. Evidently, an undemonstrative or an unsound argument is one that fails in one or both of the two ways.

The crucial question is: From what must inference begin in order to be not only valid but also “to the point”? Here our traditions diverge widely. In brief:

In early Greek literature we find no logic, no accounts of inference, but only mythic, testimonials of origins. For instance, in the Iliad (11, 484 ff.) we read the invocation:

Tell me now, Muses who have homes on Olympus,
For you are goddesses, are present, and know all things.
Whereas we hear [only] rumor and know nothing....

Once we begin talking to one another, whether casually (in conversation) or carefully (in song), we find ourselves at a remove, moving in hearsay. Only divine intervention can bestow legitimacy on such helpless talk: we have to be told. Truth here depends upon accepting our speaking selves as media of the divine. However, talking is only one manifestation of our being: acting is yet another. If we turn to the testimonials of such acting (which run the risk of being mere rumor), we see what it means to act from origins: as exemplified in Achilles and Odysseus, it means to strive for victory but, as in the speaking of the poet, it means to do so with the help of the gods; the Greater Ajax made a profound mistake in claiming that he needed no such help (Sophocles’ Ajax, 765), as did the Lesser Ajax in claiming victory for himself (Odyssey, IV, 500 ff.). Still, one must strive (Pindar’s 4th Isthmian, 30), and one’s task is to be versatile, many-sided, as the epithets of Odysseus indicate: much-enduring, much-attentive, much-contriving, much-seeing. The two go together: in settling back into divine origins one must respond to all that comes.

In later Greek literature we find the first logical consideration of the question of the origin (intimacy) of reason (distance). In his Republic (509D ff.) Plato has Socrates illustrating the question graphically: one kind of knowledge leads downward from principles, the other upwards toward the principles allowing the downward movement. It is this “allowing” that remains puzzling, since we have to start already with the principles which become evident only later. Paradoxically, no amount of “solid reasoning” leads one toward origins: one either starts there or
fails. We shall have to address ourselves to this paradox in the sequel. We easily fail to begin. For instance, we read in Aristotle’s *Nicomachean Ethics* (1144 a31):

> Practical syllogisms are conditioned by a principle \[\alpha\rho\chi\eta\] taking the form “Since the purpose, or the best thing is, ...” (whatever, it makes no difference). But this purpose does not appear to one who is not himself good. Perversion cuts one off from, even falsifies the practical principle. Hence it is clear that one cannot be intelligent in practical affairs without being good.

Being good means, on the Second Interpretation, being in touch with things as they are, promoting what is good for them, their own purposes. Not only downright failure (perversion), but also more everyday conditions prevent this contact and destroy the significance of reasoning which may be formally valid by itself. Thus Aristotle goes on to say (1147 a18):

> Those undergoing sleep, drink, or madness speak demonstrations and verses of Empedocles, and beginning students reel off speeches without knowing what they are talking about: growing together is necessary, and this takes time.

In these comments we can read a faint remembrance of Homer: for all their love of speech, Greek philosophers remained acutely aware that speech *may* only amount to rumor. The best speech, truth in speech, requires an origination evident perhaps only in, but never guaranteed by speech itself.

In Christian literature we find the same Greek logic of formal inference, but a different logic of principles— if “logic” it can properly be called. For instance, at the end of the second century Tertullian could say that one believes anything fundamental partly because it is “absurd” (*ineptum*) or “beyond possibility” (*impossibile*): formulation of truths which are simply apt or possible are so because they follow from something else, fit in with human reasoning which we must judge according to our own standards. Two centuries later we find the same thought in Augustine, who could say that anything fundamental must be a gift of God based on no human merit. And toward the end of the Christian era Dante says that belief in principles is a condition for, never a result of an argument; in the middle of an arduous process of reasoning we read, “... if he is not of the faith, he will have no interest in the proof” (*On Monarchy*, II, 11). The best speech then never intends to give a reader or a listener something he has not already been in touch with himself. Rather than transmitting anything to us, genuine speech draws out our own knowledge, makes it explicit: it reminds us of, engages us in a recollection of our basic condition—an achievement requiring that we already be “of the faith.”

In modern philosophers we find the logic more familiar to those engaged in modern science. Most obviously, the forms of speech called hypothetical (“if this then that”) rather than those called categorical (“this is that”) gradually assume the status of paradigms. More subtly though, a new logic of principles evolves, a logic which is only today becoming clear. In answer to the question, “Where must discourse begin to be not only valid but also to the point?” modern philosophers have wavered between appealing to rational and appealing to empirical foundations: between reliance on self-sufficient reasoning and reliance on self-controlled observing. Do I best begin with the conditions for thinking our things out all by myself, or by assembling facts about my circumstances? Much philosophical debate from the time of Francis Bacon (say, 1600) down to the present century (say, 1950) has consisted in a seesaw of these options (epitomized, say, in the difference between Leibniz and Locke). However, thinkers such as Bacon at the beginning, Einstein at the end, and Kant in the middle, have always insisted on the interdependence of reason and experience in any actual knowledge—often drawing quaintly on Aristotle for precedent. Until very recently, i.e. before science became a business like any other, modern thinkers at either end of the seesaw generally agreed on two principles, both essentially negative: first, reject authority (the reminder of tradition); and, second, don’t confuse reason and perception (keep form and content separate). Instead, then, of pondering the ways thinking owes its operations to (finds its justification in) truth, modern logicians ponder the relation between validity of reasoning and accuracy of observing. Our thinking appears “to the point” only when we succeed at performing these two tasks somehow in tandem.

Truth: so obviously a prevailing issue in our actual condition, yet so devious an issue the moment we aim for it explicitly. In ancient literature, the question of validity (in syllogism, paralogism, and
neologism) sets the stage for the question of truth. In modern literature, the question of accuracy (in formulation and observation) sets the stage. The actual dramas take the form of battles over principles. In the Second Interpretation the battle rages between the Homeric and the Platonic views of the human condition, and in the Fourth Interpretation the battle lines are drawn between the Ancients and the Moderns: between the Statics of reality represented by the Church (which in turn draws upon Aristotle) and the Kinetics of reality represented by Science (once called new: Bacon's *Novum Organum* in 1620, Vico's *Scienza Nuova* in 1725). However, in the rise of both the Second and the Fourth Interpretations truth appears as *adequation*: our thinking becomes true when it becomes equal to, adjusted to, lined up with, what we are thinking about or dealing with, whether this latter be static or kinetic, at rest or in motion. As adequation, truth then depends on some sort of conversion on our part: either a turning of ourselves toward what faces us, or a turning of what faces us toward ourselves. During the First and the Third Interpretations there seems to have been considerable doubt about whether we could, all by ourselves, perform this act, effect truth. And nowadays, at the Dusk of the Fourth Interpretation, truth as any kind of adequation has once again become suspect—not only in the eyes of the active promoters of technology (who believe only in validity and accuracy) and the passive followers of consumerism, but also in the eyes of many who are thinking their way toward a heavyweight version of the Fifth Interpretation: these take adequation not as an answer to, but as itself setting the stage for, the question of truth.

§5. Language

One might be tempted to suppose that language merely expresses in sensuous form a super-sensuous domain of thinking. Several common experiences fortify this temptation: the frustration we feel when trying to formulate in words what we have on our minds, and the frustration we experience when hearing or reading the words of others without understanding what they mean; also, the recognition that someone (asleep, drunk, insane, or simply very young) is presently repeating formulations which at one time made sense but now appear hollow, and the recognition that we ourselves, in concentrating on formulating our sentences well, may miss the things themselves. However, if we erect into a principle this difference between linguistic manifestation and actual thinking, we institute an estrangement which we will soon regret.

Rather than postulating as fundamental a dualism of inner thought and outer manifestation to account for the difference between successful and unsuccessful language, we can, to retain the integrity of language as naming our condition, postulate instead a dualism of artful and inert language: language in the *making* vs. language already *made*.

Especially as intellectuals, we live and move and have our being in fixed forms of discourse: the sayings of our forebears provide the established arena of our work, an arena which becomes increasingly fixed as we struggle to talk with one another. After a period of initiation in which we struggle to speak and to listen, to write and to read intelligibly, we can easily slip into patterns of discourse apparently impeccable but really only repeated from the past and geared essentially to establishing ourselves in the social context of our work: to talking with our teachers, our peers, our children and our pupils. We then have a vested interest in the conventional forms and run the risk of identifying them with the integrity of intellectual work and devoting our intellectual hours simply to preserving them. For we will discover that the ongoing affairs of life utterly fail to fit the moulds, whereupon we eventually reject one in favor of the other, most likely in alternation (and go on to erect into a principle the opposition between the inner life of the spirit and the outer life of ordinary affairs). Language then appears paradoxically both as the essential medium of thinking and as inessential in its obvious manifestations, since only the spirit “underlying” the letter gives the latter its momentary meaning.

The alternative is to refuse to give to language already made the primacy, let alone the exclusive title of language. We may then insist that genuine language *is* only as a live response to new situations, and that the accumulated responses surrounding us in the fixed print of books and daily talk of academia represent only the simulacrum of language—just as it is often said that the painting of an apple or of a bed derives from but does not provide either nourishment or repose. Initially we must have faith that such simulacra do indeed stem from, and therefore can mutely hearken to the real thing. For on this alternative our task is to reanimate the original relation—not to reject language already made, but to return it to the status of live response. And this return can only be
The distinction sketched out in these last paragraphs has occupied the attention of philosophers from the beginning, but especially of late (including myself: \textit{Linguistic Responsibility} [Victoria, B.C.: 1988]). Plato recurrently insists that monologue falls short of genuine language — so that \textit{written} works should fall immediately under suspicion (\textit{Phaedrus}, 274B ff.). Aristotle distinguishes between inquiry directed toward the sayings of others (or even of one’s own) and inquiry directed “toward the matter itself” (\textit{On the Heavens}, 294b 7-13). Heidegger’s distinction between “idle talk” and “discourse” (\textit{Gerede} and \textit{Rede}: re-talk and talk) in \textit{Being and Time} is the likely source of Husserl’s discussion of the “temptations of language,” viz. of falling into forms of social intercourse devoid of originary evidence (“The Origin of Geometry,” in \textit{The Crisis of the European Sciences}); and also of Merleau-Ponty’s distinction between “speech spoken” and “speech speaking” (\textit{Phenomenology of Perception}, Chapter 6), as well as of H. G. Gadamer’s elaboration of the principle that “we understand [a speech, a text, a person] differently if we understand [it, him, her] at all” (\textit{Wahrheit und Methode}, II, I, c). Quite independently of Heidegger, Wittgenstein has addressed the same question in the first pages of his \textit{Logical Investigations} (and also of his \textit{Blue Book}): the meaning, the “life,” of spoken and written signs lies in their “use,” i.e. our entering into the discourse and putting language to use in response to live situations; he argues that most seemingly fundamental problems in intellectual work stem from falling back on words in the hope they will do the work \textit{for} us, whereas we must learn to do our work \textit{in} them.

As logicians we hardly enjoy immunity to the temptations of hollow language. On the contrary. Since we both \textit{start} with formulations by way of examples, and \textit{aim} for formulations satisfying one or another requirement, we easily find ourselves stuck in them as already made. Direct concern for the workings of λόγος augments rather than diminishes the opportunity for betrayal. One principle is then especially worth bearing in mind: exactitude of formulation only \textit{results} from originary response (truth), it never paves the way \textit{toward} anything fundamental. Just as football players, coaches, and even fans often inspect videotaped reruns of games, all the while practicing themselves in a make-believe manner, so in logical study we often inspect ready-made language, printed examples, to detect and rehearse the issues governing our involvement in them. Yet in such logical make-believe we must constantly transform the language already made, we must relive ours: we must take upon ourselves the original relation. Thus any adequate response to old sayings automatically engenders new sayings.*

Quite independently of Heidegger, Wittgenstein has addressed the same question in the first pages of his \textit{Logical Investigations} (and also of his \textit{Blue Book}): the meaning, the “life,” of spoken and written signs lies in their “use,” i.e. our entering into the discourse and putting language to use in response to live situations; he argues that most seemingly fundamental problems in intellectual work stem from falling back on words in the hope they will do the work \textit{for} us, whereas we must learn to do our work \textit{in} them.

The distinction between language already made and language in the making hounds all intellectual life. In logic we institute it at the outset by distinguishing \textit{proposals} from other forms of discourse, most obviously from the wide variety of utterances (from grunts of approval and disapproval, to the issuing of commands and the posing of questions), but also from terminal judgements (dogmatic assertions, proclamations, announcements). To propose something, I must bring something forward for consideration: not only must I struggle to make something clear, I must also maintain its suspense, suspend judgement about it. The language of a proposal must be both incisive and open. How rare it is, then, that our language actually takes the form of a proposal! Normally, I repeat rather than propose: “utterance” names the common event most broadly. And when listening or reading I more or less imitate rather than consider. To overcome such passivity, it seems at first that one must become assertive, simply declare to the world one’s decision: “judgement” names this very special event. Yet a proposal is neither simply an utterance, nor has it come to the fixity of a judgement. Rather, a proposal requires that we move in the realm of possibility: that we approach something, keeping \textit{it} rather than our own responses in view, and suspending fixed and final judgement until all the possibilities have been aired. In a word, by concentrating us on proposals, logic teaches us to \textit{discuss}: “dialectic” is the traditional name for the art of dialogue, the art every intellectual must learn, and an art in which speaking and listening, reading and writing, take place in perfect, although ever-moving consonance with thinking.

In assuming for us the status of proposals, language loses in innocence what it gains in thoughtfulness. For the language of proposals places us in a tense position where we must constantly resist both the tug of mere utterance and the tug of final judgement. In logical terms, we must resist the temptation to assume that the language in which we are moving simply repeats \textit{fact} or somehow embodies \textit{necessity}: each of
these modalities must be earned during a prior sojourn within possibility. In ancient times the language of the legislature and the courtroom (deliberative and forensic speech) exemplified the need to start with proposals and only gradually to discover facts and to decide upon necessity. Nowadays, the language of scientific investigation serves the same purpose. Yet even in discussing a day's program with another I obviously name not so much the way things are as the way they might be, and in diagnosing some malfunction a doctor or a repairman must review the wide variety of possibly relevant symptoms in correlation with possible causes. Only within such discussion or review will some determinations assume the status of facts: we agree on the amount of money we have available for the day, just as the doctor registers as a fact that the patient has a temperature and the mechanic that the battery is low. And after much, perhaps a lifetime's consideration, one judges definitively that mortality necessarily belongs to human nature, or that the energy delivered by a system necessarily derives solely from the energy put into it. Yet we learn also to return all statements of fact or necessity to statements of possibility, i.e. to rejuvenate them as proposals: for only in the form of proposals do statements allow us to think about them and to discuss them with others.

Still, the logic of the Second and the logic of the Fourth Interpretation differ remarkably in the ways they ask us to understand the relation between the modalities evident within language and the realities articulated by language. For the ancients, proposals tune us into reality, whereas routine utterances take us along as in a dream, and proclaimed judgements run the risk of erecting elaborate viewpoints which obstruct the view: since reality here requires us to embed ourselves in the potentiality of things and to participate in their actualization, speech attuned to reality itself must begin in the mode of possibility. For the moderns, however, proposals simply articulate the arena in which we must make further determinations: here we touch upon the source of the modern disposition to believe in the accumulation (rather than the recollection) of knowledge, and in the unsatisfactory status of proposals (since these must pass quickly into definite judgements in order to make room for fresh proposals).

To illustrate the difference, we can consider the proposal which lies at the heart of logic itself: that "to be human is to have λόγος," that "man is the rational animal." For Plato and Aristotle, down to Thomas Aquinas, this definition of human being names primarily our destiny: philosophers discussed it in the mode of possibility, as reflecting human potentiality, doing so for the sake of bringing out its necessity, our own actuality. Anciently, the definition made no claim about how human beings in fact, i.e. already and manifestly, are. In contrast, for Descartes and Locke, and more obviously for modern empirical studies of human being, the definition becomes increasingly suspect: modern thinkers eschew the potentiality/actualization distinction on which the definition anciently depends; yet neither does the definition name a distinctive fact about human beings. The common view today is that "having λόγος" is simply an accomplishment of some people, and even then an accomplishment which varies in form from person to person: it is a variable fact.

And the difference on the question of the relation between modality and reality raises the crucial question about the status of logic itself: In devising techniques for logical formulation, are we developing (consummating), or are we abstracting (idealizing) our linguistic being, our language? At the beginning of the Second Interpretation, philosophers generally believed that ordinary language is a kind of leftover of genuine language, and that logical analysis restores the original, or rather exercises us in the source of our linguistic being. At the end of the Fourth Interpretation philosophers came to believe that "ordinary language is all right," as Wittgenstein says in The Blue Book, and that the "ideal languages" we construct only serve special purposes, ultimately those of ordinary language itself.

One of the overall tasks of a contemplative study of logic is to accentuate, occasionally even to hyperbolize the differences in logical interpretation. For we thereby sharpen the questions to focus on the matter itself.
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Truth and Validity

The Workings of Syllogism

Once some things have been said in a conversation or in an investigation, others seem to follow: liking them or not, we infer them. Whether one set of words legitimately generates another, whether the second set follows rightly from and not just incidentally upon the first, whether we infer properly, is one question: the question of validity. On what basis the following or the inferring takes places is another: the question of truth.

Long before one learns to contemplate the logical questions, one learns to perform numerical inferences. Having determined how many people will attend a social function and how much wine each will likely drink, we can figure out in advance how much wine to order. One learns the rules for such inference without questioning the basis of their legitimacy. Only in the more advanced questions of mathematics does one worry about the principles of validity and truth in these operations. Not only do practical questions press in upon one more urgently than do the theoretical difficulties, but the legitimacy of the mathematical operations themselves appears initially as self-evident—perhaps because one learns them when still very young. Interestingly, precisely this self-evidence, or apparent self-evidence, has provided, in the West, a standard for what one would like to achieve in the domain of general discourse, i.e. in situations articulated by words rather than by numbers.

One obvious context of word-inference is the courtroom. In legal matters, unlike in casual conversation or even in curious investigation, a definite question must be decided within a finite interval of time: the judge or the jury must draw a conclusion within the channels of pre-
established alternatives. Moreover, a well defined collision of interests leads the opposing parties to present the ingredients of the discourse in definite formulations. In such debates (discourses under formal stress), the participants explicitly focus not only on the separate formulations but also on the way they fit together to generate a final judgement.

In Albert Camus’ *The Stranger* the prosecuting attorney successfully persuades the jury to condemn Meursault as a criminal. On what grounds? The novelist shows the following argument slowly shaping up in the courtroom:

- Criminals have no respect for social norms.
- Meursault has no respect for social norms.
- So: Meursault is a criminal.

No doubt the first premiss is correct. And the story shows Meursault precisely in the light of the second premiss: he smoked during the funeral of his mother, etc. However, the two premisses together still do not generate the conclusion. Although the words of the two premises mention the same thing (lack of respect for social norms), they themselves do not interlock to produce the judgement in fact pronounced by the jury. In this example, as in much idle conversation, the discussion simply lumps together various sayings from which the participants generate, for other and unstated reasons, their own conclusion. Here, then, the syllogism is a sham.

In the famous Scopes trial (Tennessee, 1925), both the prosecution and the defense agreed on the *de facto* question, namely that Scopes had indeed taught the theory of evolution to his pupils (as set forth by the biology textbook prescribed by the State!). Scopes had simply allowed himself to be the object of a test case for the then-recently passed Butler Act which read, in part, that “it shall be unlawful ... to teach any theory that denies the story of the Divine Creation of man as taught in the Bible, and to teach instead that man has descended from the lower order of animals.” The debate focussed on the *other* question: Could the admitted fact (Scopes’ actions in the classroom) clearly interlock with the formulated law to generate a conviction? The answer to this question depended on a clear meaning of the law itself, which was in effect being put on trial (the *de jure* question). For instance, what does it mean to deny the story of Divine Creation? As one witness (a biologist) testified; “Evolution is not a power or a force; it is a process, a method. God is a power, a force; he necessarily uses processes and methods.”

In the 17th-century debates about the role of human freedom in the reception of divine grace, both the Church and the Jansenists agreed on the *de jure* question, the premiss stating the principle at issue: anyone who ascribed to the five points of doctrine alleged extracted from Jansen’s book *Augustinus* (1640) would be guilty of heresy. But the Jansenists (among whom were Pascal and the authors of the *Port-Royal Logic*) argued that neither they themselves nor Jansen’s book in fact endorsed those five principles. The debates raged for a century or so, despite the condemnation of Jansenism by Rome in 1642 and 1653.

These three examples illustrate a cardinal point: a syllogism may contain only two briefly stated premisses and a brief conclusion, yet the actual discourse may run on for hours or even years. The syllogism, then, namely the condensed concatenation of words most obviously locating the imputed generation of a conclusion from premisses, is an extract: something like a country map making distinctions, showing relations, and tracing out ways of getting from one place to another. It allows us to distinguish carefully between the proposed conclusion and the proposed reasons, to discuss each reason separately and to decide whether they do, taken together, generate the conclusion. Like a map, however, a syllogism draws its significance from our own involvement in the matter from which the extraction is made. We would then commit a serious mistake if we were to assume that either a map or a syllogism could convey us from one place to another, apart from any motive power, any involvement of our own; soldiers scouting foreign territory discover that they must reconver the extracted relations, formulated so neatly on paper back in camp, into concrete recognition of the circumstances themselves.

Scientific investigations illustrate the need for direct contact. Imagine an expert biologist and his untrained assistant out on a field trip in a desert. At one point the biologist says: “There must be water here.” “Why?” asks the assistant, since the whole landscape appears dry as a bone. “Because there are plants here, and plants require water,” the first replies. Rightly understood, these two reasons map out questions for the assistant: if not, he may look for a pool of water or assume a significant yearly rainfall, whereas the biologist might include under “there is water here” the possibility of significant amounts of dew at night, or minute quantities of moisture seeping up from below. Moreover, what counts as
a plant? With his trained eye, the biologist may perceive a plant where his companion sees only a strangely colored rock.

The analogy of syllogism with mathematical proof may mislead one. In a Euclidian proof, for instance, one might discern a discourse in which every sentence serves as a link in a long chain of reasoning. However, in the debates of a courtroom or the investigations of nature, syllogisms interrelate various judgments and thereby map out a field of questions rather than a sequence of answers. Does the conclusion follow from the premises? Does each of the premises hold? What do the various terms within the proposals mean? A syllogism allows us to divide the question, to consider the details, to contemplate. Taken as a result of contemplation, it allows us to reconvert prejudices (judgments already settled and therefore replacing direct consideration) back into insights (judgments presently enacted and therefore in contact with the matter).

Syllogisms need not even lie on the surface of discourse. Indeed, they usually rise to the surface only in the heat of a controversy. Just as a painter may draw lines of perspective on his canvass only to paint them over, so a writer usually allows the syllogistic structure of his treatise to sink well below the surface of his prose. One of the many pleasures of careful reading consists in gradually discovering the background structure and the way it lends powerful but invisible support to the foreground of observation.

Still, some styles of writing and speaking bring syllogisms fairly close to the surface, so that reading and listening must immediately account for them. For instance, we read in Balzac's *Old Goriot*:

> Like all narrow-minded persons, Madame Vauquer was not accustomed to consider why events happened; she concentrated her attention on the events themselves.

The seemingly innocuous comparison (“like...”) really catalyzes two premises: “all narrow-minded persons pass over the reasons for things” and “Madame Vauquer is a narrow-minded person.” Thus the main commentary appears as a conclusion calling attention to an instance of a general consideration. Similarly, whenever we read a reason (for instance, in the common word “because”) the author is likely alerting us to a syllogism. Consider the verses from Dylan Thomas' famous poem:

> Though wise men at their end know dark is right,  
> Because their words had forked no lightening they  
> Do not go gentle into that good night.

Most obviously, we are asked to contemplate the suggestion that wise people are incapable of forking lightening and that therefore they, too, remain uneasy at the end of life. However, there is also the further suggestion (premiss) that the inability to fork lightening (to ensure clarity in human doings?) leaves one essentially uneasy. In any case, an entire syllogism implicitly complements the initial thought expressed in the opening subordinate clause: “wise men at their end know dark is right.”

Some forms of irony also draw a syllogism almost to the surface, leaving the rest to the imagination. During a news conference (October 26, 1973), a CBS journalist asked Richard Nixon, then President of the United States, whether he was angry with the media. Nixon replied: “Don't get me wrong. I'm not angry with you. You can only be angry with those you respect.” Here the conclusion (“I'm not angry with the media”) is justified by one reason (“one can only be angry with those one respects”), but the other reason is conspicuous as well—by its absence, as we say. In this instance, notice, the listener not only concentrates on the unstated element, but does so while completing the syllogism by a kind of reverse inference: in order to infer the stated conclusion, one must infer backwards to the missing premiss (“I have no respect for you”) necessary for the forward inference.

One might assume that the best, or the most honest discourse would expressly state all its ingredients. While some exercises in logic and mathematics do exact such thoroughness and thereby sensitize one to its possibility, effective speaking and listening, reading and writing, in fact require that we develop the ability to think with only some of the necessary ingredients expressly stated: the remaining ones must, from a speaker's or writer's standpoint, be left to the participation of others—must, from the listener's or reader's standpoint, be supplied from one's own experience, imagination, or insight. When the speaker or writer tries to do all the work, the result is rather boring for others.

Some people are especially talented in remarking immediately (whether rightly or wrongly) a full array of ingredients when confronted with only one or two. This talent is called wit. Aristotle defines it as the ability to hit upon reasons for the obvious “without any noticeable interval of time”: one sees someone talking with a rich man, and remarks
that the one is angling for a loan from the other; or one hears that certain people are friends and remarks that they have a common enemy. These and other examples from Aristotle show syllogism at work in the perception of events. Marcel Proust, on the other hand, defines wit as the ability to say something which, in a nutshell, points out and calls into question the syllogisms (prejudices) currently in vogue: everyone assumed that the Duchess Oriane de Guermantes would be attending the fancy-dress ball of the new Greek Minister, but when asked what she would wear she replied, “Why, nothing at all” and this single comment, coming from a person of her standing, “at once set every tongue wagging, as revealing Oriane’s opinion as to the true social status of the new Greek Minister and the proper attitude to adopt toward him.” This example from Proust shows syllogism at work in social consciousness.*

Except in textbooks and in comedies such as Ionesco’s *Rhinoceros*, people do not actually speak or write in syllogisms. Like maps, syllogisms structure the whole of a discourse in a way allowing us to examine the parts separately but also to discern whether these parts do indeed join together to generate the conclusion. The map-like character becomes evident in Aristotle’s account: here, the conclusion appears already at the start of any discussion, and the participants must discover and examine premises. Often, the conclusion is simply an acknowledged fact: the moon always has its bright side facing the sun, a poor man and a rich man are talking with one another, the Roman Empire collapsed, the economy of the Canadian Maritimes is chronically depressed. The premises then supply the reasons, the causes, the explanations: they locate the principles which simultaneously propose some insight into reality and allow us to understand the fact as an instance of that reality;

* Aristotle defines wit (αγχινοια) as a form of syllogism in his *Posterior Analytics*, Book I, Chapter 34. In his *Nicomachean Ethics* (1142 b 5) he expressly contrasts this way of “having one’s mind instantaneously close to things” with the ability to speak and to think gradually toward reality, as is necessary in deliberations with other people (the usual location of syllogism). Proust defines wit (esprit) as well as the perversion of it (mere cuteness) and the lack of it (gullibility) in *Remembrance of Things Past* (Random House, New York, 1981, Volume 2, pp. 477-97). The difference between the two definitions reflects the difference between ancient and modern concerns generally. On either definition, be it noticed, wit likely terminates rather than inaugurates discussion, however much it may spice the exchange of commentaries.

for example, the moon depends upon the sun for its brightness, the Canadian Confederation discouraged local incentive in the Maritimes. In such instances, syllogism is *causal*: some fact is assumed, and the syllogism proposes a whole account in which the conclusion states the fact and the premises articulate the principle (cause). For Aristotle, causal syllogisms are the ones of full human interest, reflecting as they do both the exigencies of any concrete affair, the structure of any concrete inquiry, and the wonders generating philosophical contemplation.

Still, once we have extracted a syllogism we can ask of it, as a surveyor may ask of a map, whether the proposed relations among the parts “add up.” No longer looking at the substance, no longer stalking and contemplating the terrain itself, we recoil into our previous determinations to calculate whether the premises formally generate the conclusion: here, we treat the syllogism as *formal* rather than causal — exactly as we would a mathematical problem (e.g., how many bottles of wine to order). Indeed, many affairs appear as formal anyway, as whenever we apply established rules or principles to upcoming events: once the Nazis made their policies of racism clear, anyone member of an unfavored race could anticipate his fate, conclude it in advance, if he were to fall into their hands — quite formally. While full-blooded arguments are generally causal in their original intent, formal considerations occupy our attention when we come to the question of validity.

Besides causal and formal moods of argument, we may record a third: *exhortive*. A mother argues exhortively when urging her child to do or not to do something (to pursue the study of medicine, or not to date so-and-so) on the grounds that such-and-such is good or bad for the child. Grammatically put, the conclusion of such an argument contains some modal verb like “should” to express the exhortation (or, stronger, the imperative): “You are one who should pursue medicine, because you are one talented for that vocation (or disposed to enjoy the resulting social status), and anyone so talented (or so disposed) should pursue medicine.” Similarly, we read in *Matthew* (5:19-21):

Lay not up for yourselves treasures upon earth, where moth and rust doth corrupt, and where thieves break in and steal. But lay up for yourselves treasures in heaven, where neither moth nor rust doth corrupt, and where thieves do
not break in nor steal: for where your treasure is, there your heart be also.

The conclusion assumes the form of an imperative: "Do not..." (or, more formally, "You are well advised not ..., You are one who should not..."). Yet there are reasons given for this conclusion, and we could construct syllogisms from the prose (whereupon we would have to interpret the metaphors as well). Again, then, we could conceivably consider the argument in a purely formal manner. However, in doing so we would abandon the mood of the argument itself, since its intent is mainly exhortive.

Many, perhaps most concrete arguments display at least a modicum of all three moods: causal, formal, and exhortive. For an argument is rather vacuous if it does not help us understand the causes of what is formulated in the conclusion, and it is flippant as well if it does not leave us with a sense of what is important for us to endorse. Yet if there is no claim that the conclusion follows formally from the premises — once all are duly formulated — we have no argument at all. As logicians we must extract the formal structure, thereby treating arguments provisionally as though they were only formal. However, the extraction locates the challenge; the actual decision of formal validity or invalidity usually becomes a very simple matter once we complete the extraction. Indeed, it is possible to list and memorize the formal relations and techniques for deciding validity and invalidity — much as we all learned to recite multiplication tables when still very young. Yet the logical interest in validity lies elsewhere: in the consideration of the conditions for obtaining the well formulated syllogisms allowing for formal decision.

§1. Categories

Under what general conditions can one set of words lead to another set? Nowadays, the search for technological systems has led to the effort to interpret the workings of inference so formally that an electronic device can compute them. In Book Four we shall return to consider the very intricate working of this interpretation. In the meanwhile, we shall concentrate on the interpretation which reigned with variations for about 2000 years — from the time of Plato and Aristotle down to sometime in the 16th or 17th century: let us call this the classical interpretation, according it one name even though internal disputes in both the Second and the Third Interpretations of the human condition led to many diverging developments which only now, after the establishment of the modern interpretation (the Fourth Interpretation of the human condition), appear unified in general significance.

The principle of the classical interpretation is this: words follow from one another legitimately only when they all flow more basically from the emergence of what we are talking about. On those occasions when our discourses display genuine coherence they do so because we have succeeded in focussing on the subject matter under discussion: the subject matter guarantees the coherence, not the mechanisms we have devised in view of the subject matter. For instance, the argument concluding that water must be present in a given stretch of desert, not because one perceives the water but because one perceives plants and because plants require water for their existence: this argument appears valid on the classical interpretation because it represents a full experience of plants — because it flows from or points to such full experience. Indeed, on the classical view language itself (λόγος) owes its origin to such emergence, or to our own innermost destiny to focus on beings as emerging: on plants if we are botanists or farmers, on the community if we are leaders, on horses if we are horsemen, on rocks if we are rock climbers, on human beings if we are physicians or philosophers or poets.

It may take a while to get used to the classical principle. In effect, its states that truth occurs as we succeed in focussing on things, as we participate in their emergence, and that error occurs otherwise (whether as casual oversight, faulty presumption, or intentional avoidance). It sounds too simple! Especially to one confronted with the need to master the ever-growing mathematical complications and accumulated determinations of a modern science. Yet the supposed simplicity of the classical principle requires the most drastic readjustment of thinking on the part of the modern mind.

Proposals must take categorial form — both to embody the classical principle of truth and to satisfy subsequent formal requirements for determining validity. That is, the subject of the proposal must name a being, and the predicate must say something about this being, something

* In his Movement of Animals (7), Aristotle illustrates the exhortive mood of argument: e.g. “all humans ought to walk, I am a human, therefore I ought to walk.” The examples here proffered explain overt doings as resulting from inner thoughts (silent arguments all one's own).
which allows the being to emerge for consideration and also allows us to focus on this emergence. For instance, if we want to think and talk about a horse, we name the horse and then say something about its size, its color, the way it compares with other horses, where it is, when it is the way we describe it, the pose it has assumed as a result of having done or undergone things, the condition it is in for doing or undergoing things, what it is in fact doing, and what it is in fact undergoing. Instead of a horse, we might take as the subject of our discourse a certain person: “horse” and “human being” are the two examples Aristotle cites when listing the ten “ways of saying something” (categories) in his work called *Categories* (Chapter 4):

1. the being (substance: man, horse)
2. how much is it? (quantity: two feet long, three feet wide)
3. what is it like? (quality: white, grammatically expert)
4. how does it compare? (relation: half, double, greater)
5. where is it? (place: in the agora, in the lyceum)
6. when is it? (time: yesterday, last year)
7. how is it poised? (posture: lying, sitting)
8. how is it prepared? (condition: shod, armed)
9. what is it doing? (action: cutting, burning)
10. what is it undergoing? (affection: getting cut, burned).

Following the Latin tradition, the English names for the categories (substance, quantity, quality, etc.) abandon the force of the original Greek: Aristotle's is first of all a list of questions we can ask; elsewhere, even the first category appears as a question: what is the being? In any case, the list offers a scheme for undoing the blurriness which ordinarily prevails in letter-writing, in casual conversation, and even in academic discussion. Once we do focus, once what we are talking about does emerge, the scheme itself cedes to the event, and wonder becomes the appropriate response: thus our admiration for a detective who, like Sherlock Holmes, gets disparate predicates to focus on one thing; for a scientist who, accompanying us in the desert, sees and helps us to see what is there; or for the writer who, strangely, makes present to us in his work people and environments which do not exist in our ordinary lives.

You might well ask: What counts as a being and therefore as a legitimate subject of discourse and thought? A horse, a man, a house, a ship, a university—all things we can care for, work on, destroy, ignore. But also horses, people, houses, ships, universities—not as collections but as kinds. To know horses may mean not merely to be acquainted with a number of horses, but to know what it means for any horse to be a horse, so that one can judge or tend any given horse. Most intellectual discussions (especially books and lectures) speak not only about individuals but also about species: usually, they cite individual cases only as examples of the kind of being under study. Indeed, in answer to the question, What counts as a being? we might answer provisionally: a being is what can be approached and known either as an individual or as a species. This provisional answer conceals a paradox of great interest to western philosophy. On the one hand, I know a person, a horse, a university, only as I come into contact with it and treat it as an individual: Aristotle likes to say that the doctor cures not man, but Socrates or Callias, you or me. On the other hand, I know any one individual being only if I know what it means to be that kind, what it means for a horse to be a horse, a human being to be human, a university to be a university: a doctor can treat Socrates of Callias, you or me, only because he has studied “human anatomy” and the like—the species man.

In general, we judge how good an individual being is by measuring it against the idea we have of its kind—of how it is supposed to be. And we try to improve or change it accordingly. Indeed, we even identify each given being as one of a kind, prior to making explicit judgments about it: the barest act of recognition (itself already a judgement) presupposes the difference between the individual and the species (that’s an elk moving through the bushes, and not another hunter). This difference has generated philosophical discussion for more than two thousand years.

Focused discourse has as its ultimate subject either a being as singular or a being as species (Aristotle says: a being in either the primary sense or the secondary sense of the word). The other nine categories name auxiliary ways in which we, in our talk, may bring forward some being for consideration—ways in which they variously emerge for us.* Yet the ultimate, the logical subject may not correspond

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* In Greek, to “categorize” means: to bring something to the agora to public attention). The anglicized Latin translation, to “predicate,” means: to speak something out before others (to proclaim). Only much later does “category” come to mean simply a classification and “predicate” a part of a sentence.
to the grammatical subject evident in prose formulations; we may have to extract it. Consider a sentence from Stendhal's *Scarlet and Black*: “Speech has been given to man to hide his thoughts.” Traditional grammar tells us that “speech” is the subject and the rest the predicate. But is “speech” a being? Does the word name a singular being or a kind of being? Not really. Speech is an *activity* of human beings. The aphorism claims to say something about us, about human beings, about man: it belongs to our nature to speak and listen, read and write, but in doing so we aim primarily (according to Stendhal) to conceal rather than to reveal our thoughts (judgments, intentions, aspirations, desires, or whatever). If we fail to spot the logical subject, we might imagine that “speech” is something we can examine all by itself, as though it were itself a being. On the other hand, once we do spot the logical subject we have something concrete to examine (the people around us, and ourselves): we have a measure for judging whether the aphorism is correct or not: Do human beings move within language primarily as a means of concealing their thoughts? We may judge the aphorism to be correct or incorrect, but the positioning of ourselves in the face of the subject already places us in truth. On the other hand, the failure to focus places us in untruth, in error, in falsehood (e.g., the insistence upon imagining a little package of reality called “speech” and investigating it apart from examining the “human being” anchoring the activity — and wondering whether it is given to us for the purpose stated).

Singular beings seem to stand most easily as subjects of predication. In any case, the list of ten categories applies to singular examples most readily: “Napoleon is a short well-spoken man of the early 18th century who was politically more effective than Bismarck in unifying Europe.” Yet we often predicate a species-being as well: “Oaks are deciduous trees with alternate leaves”—where “trees” names a broader kind of secondary being (a genus), “deciduous” indicates something oaks do, or are prepared to do, and “alternate leaved” states a posture. However, the predication of a species presents difficulties, since individuals of the same species vary considerably in detail: “Human beings normally walk upright, have either white, yellow, black or reddish-brown skin, most always learn to speak with their mouths and listen with their ears, on average have a height between five and six feet, usually live to an age of about seventy but sometimes to an age of a hundred or more; and the female of this species is generally smaller than the male.” We have to strain to say something correct about an entire species of being, and we rightly hesitate. Indeed, “science” names the effort to discover and to formulate predications which hold of species rather than simply of individuals. A scientist, then, must pay careful attention to the difference between singular being and species-being, and must keep those predications which vary among individuals separate from those which do not (from those which finally and accurately apply to the species and therefore to all the individuals of that species).

Besides individuals and species, we seem to talk about each of the other nine categories as well. In mathematical calculations we predicate numbers: the subject seems to be quantity. Painters may talk about colors rather than any being that is colored: about qualities. Teachers may talk about literacy rather than about children able to read and write: about conditions. And most commonly we talk about activities without stating explicitly what or who is performing them. For example, in Stendhal’s work we read: “All competent reasoning gives offense.” It seems clear, however, that Stendhal here asks us to consider something about *people* who engage in the activity of carefully reasoned speech. And we may wonder generally whether in taking quantity, quality, condition, activity and the like as grammatical subjects we are not relying in some way upon an original logical subject falling under the first category. In any case, many such proposals not only lend themselves to reconstrual (what is the novelist talking about, if not human beings?), but allow us to assess them only if we in fact do reconstrue them as talking really about some being or other. Yet the wonder is legitimate. For most modern and post-modern science asks us to talk and think in the vocabulary of quantity and relation, action and affection — in marked contrast to our older intellectual traditions, including that of ancient science, which assume and often argue that talking ultimately, and thoughtful talking always, zeroes in on beings by way of the other categories and not primarily or genuinely on determinations falling under these others.”

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* For Aristotle’s own discussion of the basis of predication, see his *Categories*, especially Chapter 2, and his *Metaphysics*, especially Book VII. Both accounts draw distinctions and raise questions dependent on a concern for what counts as a being, and both employ “separability” as a criterion for deciding the question: a being is that which, although an individual, evidences a “separable” species which *also* counts as a being, whereas other predicates
There is yet one more common subject of discourse, the one most familiar, perhaps, to academics: we talk not only about beings (in the primary and the secondary senses), not only about quantities, relations, and the rest, we also talk about what others (Plato, Aristotle, Balzac, Stendhal, our teachers and our friends) say. “Tolstoi says that human beings cannot be both idle and at ease”: What are we to make of this? Are we to store the entire proposal away as saying something about Tolstoi, claiming that we now know something about what this man wrote? One can do precisely that—for the sake of impressing others, e.g. guests at a party or teachers who evaluate one’s talk, or a general audience expecting a learned speech. Yet one might also “look through” Tolstoi’s words, along with the elaboration in War and Peace (VII, 1), taking them as an occasion to think back upon idleness and peace of mind as postures or conditions and then forward to human being itself—to ourselves and to our fellows as emerging in their postures and conditions and as founding or not the words uttered by Tolstoi. Especially when listening to others speak or when reading what others have written, we struggle to understand. But what does “understanding” here mean? That we can follow, record, reproduce, possibly paraphrase and interrelate the flow of words we hear or read? Or that we move through the flow, meet the subject (being) under consideration? On the first view, the words appear to flow from their author and, in a way, to belong to, point toward the author: the chief question will be whether we have got the words right, and then possibly whether we like them. On the second view, the words appear to flow from the subject (being) itself, its emergence, and the speaker or writer appears as a medium of the flow, a witness to the emergence: the chief question will be whether we ourselves are attuned to the subject, and then possibly whether the given words are appropriate to the attunement. The speech of academia generally wavers between these two ways of talking about the sayings of others.

(Those cited as examples of categories 2 through 10 on the list) are not in the strict sense (even if in some senses) “separable” and therefore not in the fullest sense intelligible. However, the Categories takes its cues more from the question of proper speech, while the Metaphysics relates the question of being (which, for us philosophers, is “primarily and solely ours to contemplate”) directly to the questions of wisdom and of truth.

Philosophy has its origin in wonder, as Plato first said.* And in considering the workings of categorial discourse we already see at least three levels of wonder. First, we talk about, think about, come to understand beings both as individuals and as ones of kinds: Is it not strange that I understand my spouse, my dog, my lawn, the soil and the rock by attending not only to these in their individual givenness but also by grasping something of what it means to be a spouse and a human being, what dogs in general are and need, what grass as a species is and needs, what soil and rock in their various forms can do and undergo? Second, we talk about, think about, come to understand beings not simply but rather indirectly, by considering them as being something else as well—as being of a certain size, as being colored in a certain manner, as comparing with other beings in a certain way, and so on: Is it not strange that I cannot rest with beings all by themselves but must constantly shift my attention to features about them? And, third, we talk about, think about, and try to understand discourses, books and speeches of others (scientific, artistic, or philosophic): Is it not strange that we attend to these rather than to reality itself? In all three cases, the wonder is not only that we seem to take a remove from what we are talking about and trying to understand, a detour instead of a direct route, but also that the remove can result in an incisive return, a greater intimacy with the primary beings which we have abandoned when aiming for their species, listing their features, or perusing speeches about them. All these efforts may either produce a smoke screen distracting us from the subject, or provide a powerful push driving us deeper into the subject. As human beings we live largely within a never-ending battle between these two powers of categorial discourse.

Following Aristotle, we may distinguish two kinds of wonder. At first, wonder arises as a curiosity in the face of a perplexity, an impasse (aporia) which we desire to overcome: we fail to understand something, and we pause to ponder the difficulty. Indeed, in much academic work we simply pose and overcome problems. But once we have passed through such impasses, wonder comes to mean something else: we discover that genuine difficulties root in the emergence (genesis) of things, and we pause to ponder this genesis. Similarly with the study of

*Aristotle, Theaetetus, 155D: “Wondering is certainly the whole feeling of the philosopher, for there is no other origin of philosophy than this one.” Aristotle elaborates the thought in his Metaphysics, 982 b 12.
categorial discourse and the syllogism built upon it: at first one notes difficulties, but in the end one contemplates not these but their roots. Academic work runs the risk of remaining with the first kind, thereby displacing rather than enhancing the wonder. Aristotle, however, urges his students to examine carefully all forms of life, “excluding none” (Parts of Animals, I, 5):

For while in some things there is nothing delightful for sensation, yet in contemplation their productive nature provides extraordinary pleasures for those who can concentrate on causes, i.e. are by nature philosophic. Finding joy in the contemplation of images, since we then also contemplate the productive art of painting or sculpting, it would be illogical, even absurd, if we did not take even more delight in contemplating the things of nature, whose causes we can also fathom. Thus it is most important not to be vexed, as children are, when examining living things that are not noble. For within each and every natural thing there is something of a wonder.

On this account, not the detection and resolution of difficulties, but the contemplation of nature and art as productive (demiurgic) preserves the wonder of the philosophic life: the contemplation of nature-made and man-made things as recalling the causes by which they come into being (emerge) rather than simply lie around us.

§2. Affirmation and negation

We notice and think and say not only how things are, but also how they are not. Or: we deny as well as affirm predicates of a subject. I say of another, “He doesn’t smoke” or “She’s not bothered by people smoking.” A teacher says of a student, “He can’t write properly” or “She’s not sufficiently prepared.” A doctor may say of a wounded soldier, “He’s no longer alive,” and a captain may say of a ship, “She’s unfit for sailing.” In all such cases the denials intend to bring some being into focus, to testify to its emergence, but they do so in the mode of non-being, negation, rather than simply in the mode of being. A being comes forward, but as lacking, as deprived of something.

Linguistic forms do not always make the negation explicit. Some camouflage it, either lightly or heavily. In “I never smoke” the “never” obviously collapses “not ever,” “not at any time.” The prefix “un-” in “the ship’s unfit for sailing” collapses “not fit” (although the prefix in “He behaves irrationally” may not be reducible to “not,” since the whole word has come to mean something positive, like “wildly,” “passionately,” “willfully”). But what about the doctor’s judgement that someone is dead? Grammatically understood, the sentence is affirmative, and there is no explicit sign of a negation. Yet the judgement seems to say not how something is but how it is not: not alive (not capable of recovering from its present state). More superficially, euphemisms like “leaves much to be desired” and “has reached his level” also conceal denials under grammatical affirmations. On the other hand, some grammatical negations conceal logical affirmations. A locution like “not only...but also...” means, logically, “both...and...”; and litotes like “not unreasonably,” “not a few,” “not a bad guy,” and “no rare occurrence” amount to double negatives, i.e. logical affirmations.

But apart from the discrepancies between the grammatical shape and the logical meaning of negations, difficulties arise in determining the logical meaning itself.

First of all, in direct perception. I can easily notice that someone smokes: I see or smell him smoking, either here and now or on a variety of occasions. But what does it mean to notice that someone does not smoke? Do I recall other people smoking (or the same person smoking) on other occasions, compare the present case with the recollected ones? If so, I still do not notice that someone is not smoking. I notice that I am not noticing him smoking now (or in general) — on the basis of my having in fact noticed someone smoking on other occasions. Or consider the stronger sense of the proposal (not being a smoker at all, over time): How can I perceive that the fellow does not ever smoke? By tailing him, and repeating the first kind of observation over and over again? But the person may give me the slip, just for a minute — and light up! A more likely context: a witness in a courtroom may be driven in cross-examination to reduce the force of his testimony from “the accused does not smoke” to “I have not, to the best of my recollection, ever noticed the accused smoking”—whereupon the witness may find himself pressed to explain his own experience of smoking serving as the standard of comparison. In any case, negation here begs for a change of subject: the witness does not negatively predicate the accused, but rather himself, and does even this on the basis of an affirmative predication, his own positive experiences.
Then also by way of accumulated evidence. Affirmative judgments draw upon positive experiences: a teacher judges that a pupil understands a given exercise because the pupil in fact talks, writes, calculates in certain ways; an engineer judges that a beam will hold a given structure by referring to the relation between its strength and the stresses and strains exerted upon it. But a teacher also judges that a certain pupil does not understand a given lesson, and an engineer that a beam will not hold. On what grounds? Again, probably the pupil’s measured performance, the beam’s characteristics relative to the stresses and strains exerted upon it: in each case, the “not” expresses a deviation from the norm. Let us assume that one has measured correctly, and that the norm in each case actually works (i.e., fitting the norm indicates dependable performance in future tests). But can we also assume that deviation from the norm (and from all its known variants) indicates lack of competence, lack of future performance? This further assumption seems presumptuous. In a more humble mood, then, the teacher and the engineer might formulate their respective judgments: “Until now, I have not discovered the measurements I have been looking for.” Again, such a reformulation restricts negation to one’s own performance, and leaves the question open.

Do we then necessarily err, innocently or presumptuously, in denying predicates of subjects? Should we by rights confine ourselves to affirmative predications of what we face? In ordinary situations there is much to be said for a positive attitude, for concentrating on what we can affirm and for waiting out the lacks we experience rather than transforming them into rigid denials. And at the early dawn of the Second Interpretation (perhaps really the dusk of the First), Parmenides in fact argues that insight, as distinct from opinion, is only possible as directed toward what is and remains forever barred from what is not—a thought pregnant with consequences, not the least of which is a criterion for distinguishing wisdom from foolishness.

So powerful are the considerations apparently discrediting our habit of denying predicates of beings, that Plato offers (in his Sophist and elsewhere) long and arduous, to most readers today seemingly abstract arguments to justify such judgments. And in the course of the justification his works re-interpret the possibility as well as the destiny of human insight. In brief:

As responsibly and effectively involved in knowing and transforming my environment (as engaged in an “art,” e.g. that of leading a community or weaving cloth from scratch, of raising horses, captaining a ship, or tending the sick), I am essentially required to judge given instances (an action or a fabric, a horse, a ship, or a patient) in relation to what it is supposed to be (the “emergence of the measure”): I identify the being in question, and in doing so I judge it affirmatively as approximating to what it needs to be. But in doing so I also judge it negatively, as falling short. Only as responsibly and effectively involved with something, and precisely when so involved, am I in a position to affirm things knowingly, i.e. fully and genuinely—not as they simply appear to me according to my likes and dislikes (to how they suit my own desires), but as they are in themselves (an action proper to the community, a garment good for protecting the body from the wear and tear inflicted by use in the environment, and similarly a good horse, a good ship, and a good body). So involved with things, I necessarily distinguish any given instance of them from its highest possibility: I know it does not itself reach the perfection it approximates; it can, in principle, supersede itself or be superseded by other instances. More simply put, the given being does not exhaust its kind, its species: it is not its species, but only a passing instance of the species. Or: in every positively experienced and therefore affirmatively predicable being there is also something negative and therefore deniable: the “not” belongs to every given “is” (and, as later thinkers insisted, every determination of how things are is also a negation of other possible ways of being).

Still, the “not” does not belong to a being the way a quality, a quantity, a relation, etc., may belong to it: as something we perceive or infer in some way. Rather, it belongs to the entire complex of our dealing with things, and therefore to ourselves as well as to things. As practicing an art of some sort, we fly over beings in their givenness, perceive given beings accurately in their qualities, quantities, relations, etc., only because and to the extent that we fly high above and in a sense away from them in their factual givenness and glide rather in the aether of their normative being: how they might or should be (the imagery of Plato’s Phaedrus). Such flight (both as positive and as negative: as a flying and as a fleeing) suggests the truth possible and destined for us, as well as the way our own position in truth governs our subsequent ability to predicate beings accurately. In the language of later philosophers, our condition is essentially one of transcendence, and this
condition accounts simultaneously for our destiny to truth and our logical enwrapment in negation. *

Once we understand our condition in the face of beings as one of transcendence, we can and must distinguish between two kinds of negation: the profound recognition that a being is deprived of what by nature belongs to it, and the simple registration (easily mistaken) that a certain feature of a being happens to be absent from it. For instance, we recognize that a blind person lacks what belongs to him or her, and assume that the blindness resulted from a genetic defect, an accident, the wear and tear of old age, or some act of cruelty. On the other hand, when a human being is not smoking, not as tall as his father, not white-skinned, not lying down, not at home, not on time, we simply register what we take to be the absence of a possible predicate, and we do well to formulate our judgement as pertaining more to ourselves, i.e. our own inability (perhaps failure) to discover the feature in question. In general, recognizing a privation involves us in one kind of negation and denial, one rooted firmly in our condition as “artisans,” whereas registering an absence (really: not registering a presence) involves us in another kind. It may well be that the latter kind would never occur to us, never become explicitly formulated in judgments, if we were not bound up in the first kind: we would, as presumably both plants and animals do, respond only to presences, liking or disliking them; not missing anything in the beings themselves, we would not likely be able to register what they themselves happen to miss.

* During the Third Interpretation the understanding of transcendence broke away from the Greek setting of art and introduced a thought analogous to, but different from the earlier one: that all givens are cursed (see Romans, 8:21, and Genesis, 4:10-11). Yet here there is no positive responsibility for beings; while we read passing suggestions that the curse may be removed and that heaven should come down to earth, one's obvious responsibility is to heaven itself. At the dawn of the Fourth Interpretation, Descartes says (Meditations, IV): “I am a sort of mean between God and nothingness.” However, he understands both extremes as justifying the Interpretation of the truth of the human condition as lying in the perfect human control of circumstances. Finally, in his Being and Time (1927), Heidegger re-interprets the nothingness at issue in all previous Interpretations, independently of the standards of individual efficacy, divine destiny, or secular progress: all human effort is essentially “held out beyond beings, into nothingness.”
sustained effort to learn, exercise, and develop an art. Or perhaps several arts, corresponding to the various occasions of life (as a woodsman, a parent, a reader and a writer). For philosophy, too, is an art one may develop on occasion, the art of drawing out and making thematic the issues lurking silently within other modes of human contact, and in the lack of such contact. We, for instance, are presently engaged in specifying what counts as essential in well ordered and pointed discussion.

For a general sense of what might count as essential according to the Second Interpretation, we might note the examples of privation cited by Aristotle himself. The most frequent example is that of eyesight: to recognize that a person is blind is really to recognize that he is deprived of something natural to him; it would follow, then, that when a poet like Homer portrays a hero as seeing things the portrayal brings out the hero (along with ourselves) in his (and our) essence, at least partly. Other examples: health is essential and disease a privation, beauty is natural and ugliness a privation, and so with strength vs. weakness, hearing vs. deafness, waking vs. sleep. Some examples likely run counter to current scientific conceptions: light, we read, is natural, whereas darkness is the privation of light, warmth is natural and cold the privation of warmth; knowledge is natural to us and ignorance a privation (ignorance as a failure to heed, not as a lack of information); and “matter” is, in one important sense, the most emphatic embodiment of privation (as when the life is taken from a friend and we are left with the corpse, as when a building collapses and we face the rubble). Interestingly, still today we talk metaphorically of a cold person, a dark person, and the remains of a person as though these attributions did signal privations, just as we speak of someone being blind, deaf, or asleep to suggest that by nature he or she should be heedful: it seems that we naturally agree with Aristotle!

* For Aristotle’s general comments on privation, see his Categories (10) and Metaphysics (IV, 2); examples are cited in On Sleep (1), Sense and Sensibilia (3), On the Heavens (1, 3), On Generation and Corruption (1, 3), and Topics (9). Poets have traditionally complicated the predication of “seeing” by asking what it means. Thus both blind singers (Demodokos in the Odyssey) and blind prophets (Teiresias in Sophocles’ Oedipus) appear as “seers”: they see what those of us with eyes fail to see. And Shakespeare’s King Lear may be heard or read as a sustained reflection on seeing and being seen, along with metaphorical and literal privation thereof; see Stanley Cavell’s excellent commentary, “The Avoidance of Love: A Reading of King Lear,” in Must We Mean What We Say? (New York, 1969).
can appreciate the more complicated version proposed in Aristotle's *Topics* (I, 5), where we read of four kinds of predication:

1. A complete statement of essence is a definition. Here we have a topic in itself, or rather the ultimate topic of classical logic. At this point in our study we are working on the definition of human being, taking as our initial definition the verbal formulation: “living being conditioned by λογος” (truncatingly translated, via the Latin, as “rational animal”). The general requirements for definition, along with the prerequisite developments of insight, comprise the final topic of Book Three.

2. A predicate which belongs only to the species, but without stating the full essence of the being, is a property. Aristotle’s immediate example of a property is “grammatical” as applied to human being: only human beings can be such, but we can be human without having learned the proper forms of our native tongue. In general, a property is a possible predicate which derives, as a possibility, from the very nature of the species to which the individual belongs: being expert in grammatical form follows as a possibility from our essential destiny to make sense out of our circumstances in speech (λογος), so that any being said to be grammatical is also understood to be human. In a sense, then, properties are also “essential” determinations of things — i.e., derivatively so. Interestingly, Greek literature generally construes the predicates “being awake” and “being beautiful,” and often “seeing,” “hearing,” and “waking” as properties of human beings, possibilities deriving from and calling attention to human being — and applicable to other beings only by extension (metaphorically, one might say, although the conventional view now presumes that the special and in-depth meanings of such terms must derive from supposedly easy and obvious ones).

3. A predicate which simply locates the place to look for the essence, but neither the essence itself nor any property, is a genus. For instance, “Algae are plants, not animals” suggests the place to begin understanding algae, as “Universities are man-made institutions, not nature-made growths” suggests another place to begin understanding something: in fact, both are beginnings of definitions. This sort of “rough” location allows us to draw an initial bead on the being in question. We are tempted, of course, to consider “genus” as simply the older word for our modern “classification.” However, the Greek word refers not to a formal set but to a condition of genesis, generation, genealogy, and even generosity: it names a source, an origin. A “species” embodies and reflects the “genus” — as a son or daughter grows up and reflects his or her family, carries on its name, its function, and its honor. Thus, a university’s being, generically, a social institution reminds us of a feature of its being which we must carry through in our understanding of it as, specifically, devoted to research and education. That human beings are not only conditioned specifically by λογος but also start out by being animate (animal, and not just vegetable and mineral) means that we become ourselves not only by actualizing our logical capacities but also by embodying, in our λογος, one way (among others) of being alive, being lively, being animate. A genus then has a crucial role in pointing up the destiny of a being — even if it does not yet specify the destiny. This role becomes all the more evident in Descartes’ subsequent and very modern rejection of “genus” when defining what it means to be human: on the grounds that the traditional species-genus definition would confuse his inquiry by requiring him to proceed in two different directions at once (What is animal? What is rational?), he insists on analyzing his thinking nature (his rationality) first, and independently of all else. No wonder, then, when he returns to the more “remote” question of his bodily nature (his animality) he discovers not an origin to be realized but an opposition, a stand-off between his thinking nature and his corporeal nature.*

4. Whereas the predications of a property or a genus (as bringing a subject into relief) amount to partial statements of essence, the predication of an accident somehow leaves the being of a subject untouched, leaves us out of touch with it. But how so — untouched — if the predicate still says something about the being? Aristotle answers: in the sense that the feature may or may not belong to the being — as is

* For Aristotle’s definition of man as “the only animal having λογος,” see his *Politics*, 1253 a 10. For Descartes’ decisively modern rejection of this definition (i.e., the insistence upon discussing the “specification” of rationality independently of the “generalization” of animality), see the second of his *Meditations*. At the end of the 19th century Nietzsche is still very Greek in one important way when he claims that wisdom and animality will have to come together (again?) in post-modern times (*Unschuld des Werdens* II [ed. by A. Bäumler; Stuttgart, 1978], §923). Descartes’ comments on rationality mark the historical watershed dividing logic as the modern effort to strengthen our solitary condition as standing off against nature, from logic as the original effort to specify our generic condition as rooted in nature.
usually the case with the quality of being white, the posture of being
seated, the relations obtained by comparing two things to see which is
more preferable, which more pleasant. Such predicates certainly leave the
species untouched: in thinking deeply about human being or about
education we do not care about the color of the skin or of the buildings.
But in understanding an individual human being or in constructing an
individual building on a university campus, we do care, and rightly so,
about such accidents: a white man among blacks will have a life largely
determined by his difference in color, as a building on a campus exerts
a special force by its color; in both cases, one who knows (a novelist or
an anthropologist, an architect or a designer) will take such differences
very seriously. As Aristotle says, in a given situation at a given time —
i.e., with regard to an individual — an accident becomes a property. Thus
we to a theater director the posture of an actor at a given moment in a
stage play appears “essential” — as the color of a patient's skin may tell
the doctor something important about the patient's condition.

However great the difficulties may be in deciding what is missing as
against what is absent, what is essential as against what is incidental, we
enter into the truth when we acknowledge the basic difference and apply
ourselves to discern and promote the one rather than the other extreme.
Once inside the battle of the difference, we may make determinations
about things with gusto, as it were, and with direction — and concern
ourselves for the accuracy of such determinations inasmuch as such
accuracy can serve the discernment of the difference and the promotion
of its positive pole. Truth, we can then see, precedes accuracy, names a
condition for accuracy to make sense. On the other hand, the drive for
accuracy all by itself most likely leads one to highlight the incidental, the
superficial features of things (since accidents lend themselves more readily
to determination than do genera, properties, and essences) and thereby
lands one in untruth, something much worse (on the classical view)
than any inaccuracy.

§3. Full and partial focus

The primary subject of focussed discourse is ultimately an
individual: I want to understand my friend, my house, my university, the
community in which I work; the doctor must understand and treat each
patient, the horseman must understand and train each horse, and so on.
But how can one achieve the full focus needed in each case? How can
we “see” the being whole? Evidently, not by collecting the variety of its
incidental features, neither those of an individual nor those of many
individuals. Rather, by heeding its essential destiny, what it (e.g., the
horse) needs to be, what it should be. The secondary subject of a
focussed discourse is then the wholeness itself, the species as what
specifies the fullness possible for each individual being of a kind. What
is a human being? What is a friend? What is a university, a community,
a house, a horse?

A singular subject is logically expressed as “This...” a deictic
expression referring us to something actually given, an individual
requiring attention in its own right here and now or, derivatively, then
and there in the past (“My father, now dead, used to ...”) or in the future
(“My daughter, when she grows up, will ...”). On the other hand, a
species-subject is logically expressed as “All ...,” a semantic expression
referring us to a universality not given as an individual is, but still
requiring attention as the fullness of being possible for the various
individuals of the species. However, these logical expressions simply
standardize a wide variety of ways we think and talk. Instead of saying
“This friend ...” I more likely say “My friend ...” just as I commonly
identify a house or a horse already with a possessive adjective. Or I
employ a proper name: Lincoln, the Eiffel Tower, The Cedars (meaning
my summer cabin), or Nugget (meaning a horse). And to express
universality we rarely say “All friends are ...” or “All houses are ...,” but
more likely “A friend is one always ready to help” or “A house must
basically provide shelter from the elements” or “To be a friend is to be
ever ready to help,” and so on. Also: “Every doctor must ...,” “Any
university should ...” Grammatical expressions for logical meanings vary
considerably from tongue to tongue: for instance, in English we
sometimes express universality by simply dropping the article, as in “Man
is a rational animal” and “Society demands conformity from its
members,” and sometimes by using the plural form, again without an
article, as in “Algae are plants” and “Horses need rest, too”; but in
German and French one often employs the singular form with a definite
article. So long as we are thinking and talking in context such variations
pose no insurmountable difficulties.

But we also think and talk about beings partially. Not just about the
parts of a being (the nose of a person or the leaves of oak trees, blood or
sap: we shall return later to the topic of parts and wholes in this sense).
Rather, in talking about the species, we often take into account what applies only to some individuals exemplifying it: “Some human beings are wakeful and some not, some are lethargic and some not, some can read and write and some not”—properties, in these cases. “Some people are snub-nosed and some not, some tables are varnished and some are painted”—more likely accidents. The partial focus, the incompleteness of the subject, is logically expressed as “Some ....” an expression commonly employed in English to denote the limitation imposed on our talk, a limitation on the range of the subject and not just on the adequacy of the predication (all predicates, except those of complete definition, fall short of specifying the being talked about). Still, we do vary the expression of the logical meaning by saying “People sometimes have snub-noses” or “Tables occasionally have only three legs” (the subjects here are species rather than individuals, and the temporal adverbs reduce the focus); “People can be brown-skinned” or “It is possible for clovers to have four leaves” (the reference to possibility here reduces the focus to particularity). And sometimes we use the plural form: “People are out on the street tonight” or “Horses come up in Greek literature frequently.”

It seems, then, that there are three ways we can zero in on a subject: on a “this” (an individual), on “all” (a species of a being, a universal) or on “some” (particulars). And it seems reasonable to ask of someone who storms in to announce “Women are cheaters” whether he has in mind an individual woman (the plural simply lending emphasis), woman-kind (the plural naming the unitary species), or some women. Similarly, the master woodsman likely notices that the apprentice confuses what applies to all and what applies only to some trees, and thereby achieves a very poor focus on any individual tree (whether the poverty follows from the confusion or causes it).

* In the 13th century, Aquinas explicates the legitimacy of an affirmative proposition precisely as the inclusion of its predicate in the essence of its subject (Summa Theologica, I, 2). Leibniz still writes in the 17th century: “And so, in a universal affirmative proposition, it is obvious that the predicate is contained in the subject considered by itself. But if the proposition is particular affirmative, then the predicate is not contained in the notion of the subject considered by itself, but in the notion of the subject with something extra added; that is, the predicate is contained in a special case of the subject.” Leibniz, Philosophical Essays, Hackett Publishing Co., 1989, p. 11.

But difficulties arise. The original philosophic reasons for dividing focussed judgments into singular, universal, and particular stem from an ontological interpretation: in focussing fully on an individual I both rely upon and aspire to realize what it, he, or she is— as distinct from the various “hows” named by the other nine categories. However, the threefold division tempts us to suppose a numerical interpretation of the differences: on the scale of natural numbers, “this” seems to stand for one, “all” for a negatively understood infinity, and “some” for a floating number between one and infinity. Classical thinkers already raised questions about the ontological interpretation, and modern thinkers have subsequently objected to the numerical interpretation.

“O Plato,” one Antisthenes remarked, “I see a horse, but horse-ness I do not see.” Here is the classically stated objection in a nutshell. What is the status of a species, and of its name? We certainly talk about horses in ways implying that we relate not just to this or that horse, but also to what it means to be a horse. Perhaps, though, the so-called species or form or what-ness of a horse is only a name and not some-thing begging for consideration in its own right: if so, then back to this or that horse! Only fools would get wrapped up in questions about the nature of horses in general, i.e. as a so-called species, and one who really understood horses would also know that even particular judgments merely record features of individual horses experienced or anticipated.

To this objection Plato and Aristotle recurrently reply that we do in fact look for horse-ness, need to do so, and occasionally succeed when doing so—only when we ourselves become master horse trainers, master horse doctors, or master horse riders, and never when simply wandering out to the fields or to the barn to look at them. We must learn to see, learn to look for the essential, develop the “eye of the soul.” And such development on our part requires that we give ourselves to what we only subsequently see: to see horses fully we must give ourselves fully to them, i.e. vocationally.*

* About giving ourselves fully: In Book Three we shall consider the classical principle that we ultimately identify a being in truth (in its fullness) by identifying with it. The need for such identification, as well as the modern disposition to deny its legitimacy, comes through very powerfully in Peter Shaffer’s stage play Equus (1973): about a boy’s obsession with horses in conflict with his desire for membership in society as well. — Antisthenes’ remark is recorded in Simplicius’ commentary on Aristotle’s Categories (Kalbfleisch’s
We find in the literature a number of other rejoinders as well ("scare" arguments one and all): (1) without developed commitment to universals, all the manual arts will collapse, and the city with them (for artisanship depends on concern for kinds, and not just for individuals, and the city depends on the arts); (2) without firm commitment to principles public debate will lose its coherence and the city will be destroyed (since the syllogism providing the needed coherence requires at least one universal premiss, as we shall see); and (3) philosophy makes no sense except as a search for universal conditions and destinies for human kind (forsake universal conditions and you have nothing to do as an intellectual).

Modern authors such as John Stuart Mill (but David Hume and others earlier on) simply assume that Antisthenes was right and that the **numerical** interpretation of the three foci must lie at the basis of categorial discourse. But then the limitations of such discourse become glaringly evident: the "all" must mean something like "all that one has seen in the past and, consequently, expects in the future," since none but God could survey non-finite, in principle uncountable instances of any being. However, the restrictive clauses reduce the initial universality to a factual particularity, and particularity itself to a numerical consideration of individuals.* Would it not be more honest, more helpful, more accurate, to list the horses one has seen? And to admit that the application of the proposal to foreseen, i.e. as yet unseen horses is merely a psychological habit? For then, at least, one avoids prejudicing, prejudging, individuals, and one can remain open rather than becoming closed in one's habits. (Such is the advice of David Hume, for instance.)

In any case, the syllogism henceforth simply structures our disposition to record and to project: "All horses I have ever seen have been gentle, so I shall approach this horse, too, on the assumption (perhaps mistaken!) that it will be gentle like the rest." (Such is the interpretation of John Stuart Mill, for instance.) But once one abandons the classical interpretation (species-orientation) and "psychologizes" its logic as Hume, Mill, and others do within the development of the Fourth Interpretation, one must re-orient the affair of thinking entirely to preserve its integrity, as thinkers at the end of the Fourth Interpretation do (the topic of Book Four).

Apart from the external objections raised by the likes of Antisthenes and John Stuart Mill and touching the relation between "all" and "this", internal questions also plague the relation between "all" and "some." What is the logical force of "few," of "many," of "most"? In the proposal "Most people wear clothes in public," we likely hear an accident (if we think simply of how people, including children and primitives, happen to appear) or possibly a property (if we think of the distinctively human ability to rearrange, in openly variable ways, the environment—an ability deriving from λόγος). In either case we would not find objectionable the more cautious logical formulation "Some people are dressed in public," even if this reformulation bypasses the meaning of "most" as "more than half." But sometimes the numerical consideration determines the outcome of reasoning: from "Most Canadians are in favor of allowing smoking in public places" and "Most Canadians are non-smokers" it follows mathematically that "Some non-smokers are still in favor of public tolerance of smoking" (whereas two "some" premisses would not generate any conclusion at all, as we shall soon see).

In non-essential predication of a subject we rightly hesitate to affirm the "all." Prudently, we learn to soften many initially universal proposals into particular ones (and to ask or state "how many"). But the softening of essential predications illustrates something we have to learn about beings themselves: the difference between the norm and the fact, the destiny of the subject and its success (or failure) in living up to its destiny. Consider the definition of ourselves as "the rational animal": whatever we choose or learn to mean by "rational" ("logical"), we will not find all human beings living up to that meaning all the time. And so we soften the claim to "Some human beings are rational" or, more loosely, "Human beings are (can be) rational, are sometimes rational." But do the harder and the softer versions oppose one another absolutely? Do we have to choose between the two? Interestingly, the Greek formulation allows for both by employing the category of condition in the definition: "Man is conditioned by λόγος," has rationality as something he must enact; that we "have to be" rational is universal, and whether we in fact are so at given moments is another, an abiding question. In general, the "specific" part of a definition names (as we shall later see)

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* Contrast Aristotle: "One takes no proposal to speak about what you have seen as number, what you have seen as rectilinear, but rather about all" (Posterior Analytics, 71 b 4).

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edition, p. 208), and the image of the "eye of the soul" occurs in Aristotle's *Nicomachean Ethics*, 1144 a 30, and Plato's *Republic*, 533D.
a norm often contrary to fact, whereas the “generic” part names the absolutely inevitable basis (take away our animal-ness, our animation, and we cease altogether to be).

Aristotle seems to have been the first to name, if not to draw the lesson: in working truly with primary beings (individuals), we face, understand, and elicit them in their potentiality. We understand a fully natural being as striving to actualize its potentiality, a man-made being (an artifact) we understand as actualized by human agency (an art). As a teacher I understand my students as developing, as a gardener I understand my flowers as growing; and because I realize how precarious such development or growth is I aid their actualization, rightly fear their stagnation and demise. On the other hand, as a house-builder I understand each house as something made and then maintained — as arising and persevering only through human agency. In either case, whether as self-generating or man-generated, individual beings arise for us as potentially what they are — not as completed, and therefore not as assured. The difference between the logically expressed “all” and “some” in our talk reflects the conflict in which each being participates and in which we, too, participate inasmuch as we engage with these beings artfully and knowingly: the difference highlights the temporality of our own condition as we are dealing with the potentiality of each being.

To the question, What is an individual being? we can now offer one more answer: as a potentiality, an individual being is essentially in movement; and, deprived of the conditions of movement (logical challenges for human beings, water and sun for flowers, maintenance work for houses and automobiles) the being degenerates, falls ever more short of what it is meant to be, and eventually ceases to be. This understanding of beings entails an understanding of ourselves, more exactly the conditions for our residing in truth. A teacher, for instance, is in the truth when acknowledging and aiding students as “moving” — and in untruth when insisting that they already fall into fixed conditions and postures; a leader of people is in truth when eliciting their movement — and in untruth when presuming them; and a house-builder (or homeowner) and a gardener (or a farmer) are in truth when acknowledging the man-made and nature-made beings they deal with as being what they are only in continual movement induced, or continuous movement aided, by (often their own) human agency — and in untruth when taking their houses or fields for granted. Beings are as kinetic and dynamic (these two Greek words are traditionally translated as “moving” and “potential”), and we are true to them only when we acknowledge them as such and participate in their becoming: an acknowledgement and a participation only possible, according to our classical traditions, in recognition of the fullness of being (species) hanging over or lurking within each being.

Modernity has instituted a challenging twist on antiquity’s view of the dynamism to be acknowledged in every given being. Aristotle and his long line of successors (as well as Plato before him) interpret movement as arising from something like a magnetism, namely the proximity to a completeness attracting individual beings, governing their development, and defining our understanding of them. Modern thinkers, on the other hand, start at the other end: Descartes and Newton devise a theory by which non-human beings (bodies) move not by a pull from ahead but by a push from behind — like clockworks. Lamarck and Darwin devise a theory by which plants and animals evolve from the hardships of material conditions into ever-changing species. Finally, novelists like Balzac, doctors like Freud, and philosophers like Nietzsche interpret human being with a view not to human fulfillment but to human failure. In Aristotle’s terms, modernity concentrates on privations in a sustained effort to appreciate the dynamics of beings — except that these privations are now understood independently of the perfections against which they count as privations: privations come first not only in time, but in reality itself, and the way they happen to be overcome is not determined in advance (all species evolve, i.e. change). For instance, Nietzsche’s proposition that man is the sick (rather than the rational) animal calls attention to where we in fact begin (rather than to where our being culminates). In reversing the traditional assumption, Nietzsche also claims that our factual beginnings (historical and physiological givens)

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* Time, Aristotle tells us in his *Physics* (221 a 32 ff.), we are to understand as spelling destruction rather than generation of a being: time marks what happens to a being, apart from naturally or humanly induced actualization of its potentiality. It follows that we do not understand things in their fullness by appealing to the category of time. Rather, Aristotle often sums up the categories of posture and condition, action and affection, under the simple heading of movement (which he in turn defines as the fulfilling of a potentiality: *Physics*, 202 b 25). For a discussion of this summing up, see Abraham Edel’s “Aristotle’s Categories and the Nature of Categorical Theory,” *Review of Metaphysics*, XXIX, 1 (1975), pp. 45 ff.
locate our definiteness while, by contrast, our “health” remains indefinite, essentially and forever open: man is the sick animal because he is the as yet undetermined animal. In general, readers of 19th- and 20th-century literature must accustom themselves to the proposition that the recognition of sickness can form the basis for understanding and enacting human freedom, creativity, and responsibility.*

If, then, we cannot understand the “all” as recalling the species of individuals, and the “some” as predicating something less than such perfection, it is no coincidence that routine academic work today (most obviously in the social sciences) looks for universality at the “lower” end of the dynamics: all societies suppress their constituents, all people are greedy, all people have trouble liberating their sex drives from the initial affections provided by their parents, all animals aim primarily to survive. And much the same disposition to articulate universality at the level of failure is evident in novels, plays, paintings and other contemporary art works. Does this new point of departure (or “origin,” Aristotle would want to say, except that he contrasts these with privations) indicate a morbid disposition, a fascination with failure, a failure of Western Civilization itself? Not necessarily. When insisting that “man is the as yet undetermined animal,” modernity at its best confines the “all” to the undeveloped beginnings of the human condition so that we can reserve singular proposals for reflecting the (open-ended) development of the individual. As both Plato and Nietzsche insist, any actual instance of human individuality occurs as the “rare exception”—in contrast with “the many.” While “all people are sickly,” it is also possible that “Some people (i.e., individuals) become healthy.” If you or I are to become healthy, we will have to take our “essential” sickness upon ourselves—as something (as the chief and abiding thing) to overcome.

The difference between the ancient and the modern justification of universality raises the question of modalities with special, for us decidedly historical poignancy. Aristotle says (Prior Analytics, I, 2) that “every proposal is of what simply pertains to the subject, of what necessarily pertains to the subject, or of what possibly pertains to the subject”; and over two thousand years later Kant repeats this threefold division of modality (Critique of Pure Reason, A74, B100). But what a tremendous difference in meaning between the two interpretations! Both remind us of a basic responsibility we have when speaking and listening, writing and reading: we must bear constantly in mind for each proposal that it is indeed a proposal, namely a road toward something, and one of our tasks is to signal how well we find ourselves travelling on that road. As logicians, we do not have to determine what in actual experience falls under each of the three headings. Yet as artisans, statesmen, poets, or observers of nature (“physicists,” in the original sense) we do find it relevant to rank our proposals one way or the other; and as intellectuals talking with one another we find it especially relevant to structure our discussions along these lines (often to recycle sayings throughout all three modalities). Must a house have such-and-such a foundation, or do houses simply have it, or is such a foundation merely one option among others (for this one house, or for its kind)? Must a community stem from friendship among its members and banish stand-offs among them, or does it simply arise that way, or is it only one possibility among others? Must Oedipus leave Corinth and go to Thebes, or ...? Must plants have water or ...? Must human beings actualize λογος, or ...? At moments when answers to such questions come forth, they do so with a stated, implied, or perhaps confused certification—as necessarily so, simply so, or possibly so.

Of the three modalities, necessity locates both a kind of preference and a source of difficulty. Especially in decision-making, but also in coming to understand most anything, we want to know what is necessary and, as intellectuals, we would like to formulate or entertain statements which can carry the full force of necessity. Yet there are evidently different senses of the word: (1) necessary but not sufficient, namely material conditions (food is necessary for life: we must feed or graze our horses), (2) hypothetically necessary in the attainment of goals (if we are going to raise horses we will have to build a barn), (3) bitterly necessary (I must shoot my horse: she’s broken a leg), and (4) logically or mathematically necessary (the premises necessitate the conclusion, the three inside angles of any triangle necessarily add up to 180°).

Now, although each of the four kinds of necessity suggest differing human responses, Aristotle derives them all from yet a fifth: the nature of a being, what is necessary by its nature, what cannot by nature be otherwise. Only by focussing on this fifth kind of necessity can we ultimately make sense out of the other kinds. Only by understanding the

* The proposal “Man is the sick animal” (i.e., the only one!) is found in Nietzsche’s Genealogy of Morals (III, §13); the proposal “Man is the as yet undetermined animal” is found in his Beyond Good and Evil (§62).
initiate proposals for consideration. Recalling the need for a being as logical subject, our own need to affirm or deny predicates of the being, and to zero in on the subject as either singular, universal, or particular, we know that we must recurrently decide which of six forms we are trying out, whatever the subject (S) and predicate (P):

This S is P  All S are P  Some S are P
This S is not P  No S are P  Some S are not P.

Not having undertaken the journey, one will find this six-fold scheme imposing arbitrary restraints: it becomes a nuisance rather than a guide in thoughtful speaking and listening, reading and writing. Once understood, however, it helps us to break the surface of discourse to discover what, exactly, it (I, you, a book ...) is talking about. Consider again Stendhal’s aphorism, “Speech is given to man to hide his thoughts.” Which of the six forms fits the proposal, discloses what it is talking about? Grammatically, the aphorism is singular and affirmative — and the first of the six forms seems to fit. But is “speech” a being? Does it make sense to consider an individual being called “speech”? Well, no (we might say), so the aphorism must ask us to consider speech in general, “all speech is such and such.” But is “all speech” given to man? What would that mean (perfect speech, all languages...)? Well, no... the point is that “man” is given speech to hide his thoughts. And here we have it: the aphorism talks about mankind, “all human beings,” while “speech” and “hiding thoughts” (a condition and an action) intend to focus our attention (rightly or wrongly, we can now wonder) on a being which can concretely present itself to us for our consideration (whereas we will search the ends of the earth in vain for the predicates by themselves, floating free of any being). Breaking the grammatical surface and searching for logical form, we resist the temptation to reify predicates and to wash our hands of the proposal itself (here, the Platonic-Aristotelian definition of ourselves, redirected by a Stendhalic-Nietzschean amendment). Breaking and searching in this apparently trivial way, we already position ourselves in the truth.

In general, the effort to reshape the proposals of ordinary talk forces to the surface the decisions necessary in fully meaningful speech; necessary if we ourselves can fully mean what we say and hear, and do not rest content with word-filled air. Notice, however, that we need not reshape ordinary talk itself. For the most part, the reshaping proceeds in silence, as is most obvious in reading and hearing others, but also evident...
in speaking and writing, where we rightly give full rein to grammatical techniques of emphasis, tone, image, mood, and interrelation (in short: to style)—if only to remind others and ourselves that the truth of speech requires recurrent repositioning of ourselves in it. Nothing violates truth more than the insistence upon talking already in logical forms, since then we deliberately confuse the two levels and wash our hands of the need to attend carefully to the difference, i.e. to the subject itself.

For an example of the seductive power of grammatical form, and of our own logical responsibility to crack the surface, consider the ambiguity of such frequently heard formulations as “All Canadians are not in favor of legislation against smoking”: tonal emphasis on “all” and a neutral tone on “not” actually gives us to understand that “Some Canadians are not in favor...” Some S are not P. Here the “not” slips in late during the speaking and hearing, and one would have better said (even grammatically) “Not all Canadians...” On the other hand, we also hear such formulations as “All Canadians are not allowed to serve as Presidents of the United States”—where context and tone give us to understand that all are excluded, “No Canadians are allowed...”: No S are P. Thus we see that the grammatical shape “All ... not ...” may yield very different logical meanings, depending on stylistic features such as tone and context. Despite its visual similarity with the 3rd and the 6th forms on the list, then, this shape cannot properly serve as a logical form.

We shall soon find it possible and even relevant to manipulate proposals in a somewhat mechanical fashion once we have reshaped them to fit one or another of the six forms. Yet it will remain helpful and even necessary to keep in mind how we have packed those little words “is” and “are” with meaning. In our logical sense, these little words do not entail identity of subject and predicate. In ordinary talk they often do in fact serve to state some sort of identity, e.g. (1) when expressing equations (“two and two is four”), (2) when translating (“horse is our word for what the Germans call Pferd”), (3) when paraphrasing (“to say he is limited is simply to say that he is stupid”), (4) when stating what is called numerical identity (“Mark Twain is really Samuel Clemens” or “the morning star is the same as the evening star”), and (5) when talking about names (“the name of that construction over there is the Eiffel Tower” or “John Diefenbaker is the name of the man who served as Canada’s Prime Minister from 1957 to 1963”). Such uses of the verb “to be” pose their own difficulties, and have given rise in modern times to logical considerations of some importance. However, the categorial sense of the verb means something else: that the subject emerges both as itself and as something else, becomes itself in light of the predicate, or emerges for attention as withdrawing from the light of the predicate. The tension of this emergence and withdrawal lurks in the underworld of both the difference between affirming and denying and the difference between individuality and universality. Categorically understood, the verb “to be” locates us in that tension. For if the reduction of all verbs to “is” and “are” can enrich rather than impoverish discourse, the dynamism discussed under the headings of “privation” and “potentiality,” “movement” and “necessity” must lurk within, or under, this one verb.

And precisely because the categorial sense of “being” carries such a pack of meanings, proposals can significantly remain in the grammatical tense and mood called the “present indicative.” Our ordinary talk rightly roams through various other grammatical tenses (several futures, several pasts, even several presents) and moods (subjunctives and, especially in English, modal verbs such as might, may, can, should). Yet it is generally possible, if at times grammatically awkward, to reshape these variations into logical form: “Julius Caesar is the Roman emperor who did X” or “All human beings are beings who will have to die” or “This house is a construction that can serve as a public restaurant, might collapse at any time, should be torn down.” Such is one traditional grammatical view of all verbs other than “to be”: they imply a present indicative of the basic verb, into which other meanings then enter, including often an “adjective” (some sort of doing or undergoing), a temporal inflection (other than present being), and mood (condition or posture—originally of the subject under discussion, more recently of the speaker’s own attitude toward the proposal). From the logical point of view, however, such grammatical reshaping appears justified only so long as our talk intends to bring the nature of the subject forward for consideration: Julius Caesar’s deeds as having present being for us,

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*In his discussions of syllogisms Aristotle uses the grammatical expression “P belongs to S” rather than “S is/are P”: grammatically, the logical predicate (P) appears as the subject! Moreover, the verb translated as “belonging” reeks with meaning: ὑπάρχειν suggests something like “lying at the basis of” and retains the question of origin (ὅφρη) which surfaces later in Aristotle’s discussion of axioms: How can one thing (P) belong to, lie at the heart of, pertain to the inner being of another thing (S)?*
although deriving somehow from our past; mortality as having present significance for us, although fulfilled only in the future; a house as having a basic present structure, although one permitting alternative developments, various human responses to its basic presence.

However, the moment we abandon the supposition that our talk must ultimately derive its significance from the subject-being it may elicit (the supposition that beings form the needed focus of our dealings and so of our speech), the verb “to be” loses all its claims to primacy, and especially the primacy of the present tense: henceforth, as in modern scientific talk, temporal qualifications become absolutely essential to the integrity of any proposal; indeed, an intellectual discipline appears to some moderns as legitimate only to the extent that it can formulate its findings in systematic reference to temporal qualifications. At such-and-such time Julius Caesar did and suffered this and that, the patient will likely die, there was or will be an earthquake, the victim was or will be shot, the economy was or will be performing well. And henceforth beings need not undergird grammatical subjects: at such-and-such times and places, running strengthens the heart, love makes creative activity possible, logic helps us think straight, riding horses costs money. In all these latter-day examples, we would destroy the significance of the proposals if we were to drop the temporal (and spatial) qualifications: for the proposals are context-bound, not being-bound, and time assumes central significance in defining the context of talk (along with space, although in our present age scientists and mathematicians tend to rank time above space in their formulations). Context-bound proposals will come under our logical scrutiny only much later, but we shall soon see that some contextual qualifications can be moulded into categorial discourse after all (when we employ “parameters”).

From an aloof (formal) standpoint, perhaps the greatest significance of the six propositional forms lies in their interrelations. In considering a proposal about an individual being, we most concretely move within a single tension: Is my friend dying, or not? Is this building safe, or not? This clear opposition we call contradiction: one or the other must, in the end, be; we must, in the end, testify to one or the other. And to deny the one is to assert the other: it is not so that my friend is dying, i.e. my friend is not dying; it is not so that this building is not safe, i.e. this building is safe.

But the interrelations become more complicated when we consider proposals having for their subjects kinds rather than individuals. Aristotle discusses these in detail, but only centuries later did logicians provide a heuristic diagram mapping them out in what is now called the Square of Opposition:

\[
\begin{array}{c|c|c|c}
 All S are P & No S are P \\
 Some S are P & Some S are not P \\
\end{array}
\]

Here it becomes evident that the absolutely incompatible pairs are those kitty-corner to one another. These we call contradictories, and together they propose the same forced options as do the opposing possibilities of a singular proposition: to deny one is to affirm the other, and to affirm one is to deny the other — as any example you choose will make intuitively evident. Now, the top pair also displays an opposition — more vehemently than do the diagonal pairs, you might insist. But some examples of S and P (“houses” and “two-story”: any being predicated with either a property or an accident) make it intuitively evident that both these top-line forms may rightly occasion denials (neither version being correct). On the other hand, it seems evident that we cannot endorse both in any given example (at least one version must be incorrect). The opposition of this pair, not forcing us to endorse at least one of the two, is not that of contradictories, but that of contraries.

What, now, about the bottom pair? (These two have no name useful outside the study of logic, but textbooks often call them “sub-contraries.”) In contrast to the top pair, we might affirm both in any given example of S and P: Some people are literate and some not. Significantly, we must learn that affirming one does not itself imply that we must affirm the other, although context and tonal emphasis often does imply it. When I tell a colleague that “Some students of mine are having difficulty” I may

* See “Apuleius and the Square of Opposition,” by David Londey and Carmen Johanson (Phronesis [1984], Vol. 29, No. 2). The authors highlight the important differences between the ancient and the modern views of the diagram.
mean that I have confirmed such difficulty in some cases, and still leave open whether others are or are not having difficulty; and in general a scientist will talk about some S being P without at all intending to exclude the possibility that all S are P. However, if we deny one member of the pair we seem committed to affirming the other: for to deny that “Some dogs are not capable of being house-trained” we must affirm that “All dogs are capable...,” and from there it seems that we must also be able to drop from the top to the bottom, and affirm that “Some dogs are capable....”

Yet we here have a telling question: How can we — can we legitimately — drop vertically down, affirming the particular (the “subaltern,” as logicians sometimes call it) on the grounds that we affirm the universal version above it? On the Second Interpretation, we can. On the Fourth Interpretation, we cannot — or not without extra care. The different reasonings behind the differing judgments force to the surface the issues that each Interpretation usually only presumes. Moreover, we shall soon see that arguments sometimes depend for their validity on whether or not we do in fact endorse the “drop.” Why, then, do opinions differ?

On the classical view, proposals about “all” of anything derive their meaning and their legitimacy from a focus on the species (or kind) of being, and claim to bring out the nature (or essence) of every such being. But proposals about “some” of anything also derive from a focus on the species but claim only to spot something “permitted” for the being in question. Both the universal and the particular talk about the being as a species; the difference between the two consists in the emphasis of the particular version on the permissible (or the possible) and the release of any claim to bring forward the essential (or the necessary). To be sure, it sounds strange to a master woodsman when his apprentice talks about some oaks losing their leaves every fall, but the wrongness lies in its incompleteness rather than its incorrectness — and would become genuinely wrong only if the apprentice insisted on the restriction. The same sort of strangeness often occurs in deliberate understatement, and may even make us smile (“It can be annoying to break both your legs” or “It does help to have a roof that stays on your house”). In short, the “drop” signals a relinquishment of a wisdom which, under some circumstances, may be crucially important and may even represent an earlier stage of knowledge; but it is hardly wrong in itself.

Now, on the modern view, proposals about “all” of anything cannot derive their legitimacy from species or kinds, natures or essences, of beings. Early on, Hume and Mill experimented with the thought that the “all” merely sums up past experience (and acquired habits of expectation), and amounts to “All I have ever experienced.” Eventually, however, it became necessary to admit that the “all” is not bound to incidental experiences in this way, since the strict reasoning of modern science makes no sense if given a merely psychological foundation. Rather, it came to mean: “all by definition.” Thus, for purposes of communal investigation and enterprise, we define a class, we agree among ourselves to talk about “human-like figures who live forever” or “people over ninety years of age accompanied by their parents” or “organisms capable of living on the surface of Mars” or “teachers who satisfy themselves, their students, and the economic needs of society” or “office buildings over a mile high.” The classes defined in each case (perhaps at greater length) are matters of convention (convenience: how we agree to talk among ourselves), not matters of nature (insight: what we have discovered about the inner destiny of what we face). The classes themselves, whether appearing as subjects or predicates in a proposal, do suggest a kind of universality, but one that does not allow us to “drop” to the particular. For we have, as moderns, divided our talk precisely here. Universal proposals state only what we mean among ourselves. And particular proposals talk about what we have in fact found to be the case: “such-and-such person, aged 92, accompanied by mother of age 119 and father of age 123” or “such-and-such building in New York, 1.27 miles high.” Once we construe a universal as having a conventional rather than a natural base, the particular shifts from “a partial view of a species-being” to “recorded examples, one or more, meeting the criteria defining the class.” The “lower” version makes not a weaker claim, it claims something else entirely.

We may also formulate the difference between the classical and the modern interpretations of the Square of Opposition as a difference in the understanding of existence. On the classical view, every fully meaningful proposal talks about (occurs in our talking as we are talking about) beings that concretely arise, have arisen, or will arise: that are, that exist. The

* In Ancient Greek, “is” and “are” served both to state existence (as a presence of a being) and to connect (as a copula) a predicate with a subject. The relation between these two services is subtle, ad one can wonder whether there
universal catches the existence in its innermost destiny (nature, essence), whereas the particular catches it only in flight: having the first allows one to “drop” to the second. On the modern view, however, a universal talks about sets (classes) and a particular talks about findings rendering a proposed set non-empty. Having defined a set I might still decide that (or leave open whether) I can find no member for it: it may be that there are no “organisms capable of living on the surface of Mars” or no “people ninety years of age or older accompanied by their parents” (the point of a sign I once saw posted over the cash register at a cafeteria: “Credit extended only to those ninety years old ...”). If we now mean by “existence” something “findable” (as distinct from “conceivable”), the particular requires not less, but more than the universal: we can hardly “drop” from the conceivable to the findable without apology.

Returning from the general (the universal and the particular) to the singular forms, we may again raise the question about existence and essence. Traditionally, a proposal like “Homer is a poet” presents us with both: even if Homer, conceived of as a person eating and drinking, sleeping and passing the time of day with friends, does not exist, Homer-as-poet does exist, does have a decided presence through his work. Or: I tell you that “My friend is dying” and thereby refer you both to an existing being and to his being a friend of mine — and therefore to a kind of being called “friend” (a natural species having an essence). In general, every singular proposal implies or states an identification of the individual talked about, and every identification refers us to the universality of the being — a referral occurring in a purer form when I introduce a house or a person to another: “This is a house” or “This is a friend.” Of course, in telling you that “My friend is dying” I refer you to yet a third claim: he is dying (whether this predicate functions as an action or an affection, a condition or a posture, depends both on how we understand human beings to die and how my friend happens to be dying). A singular proposal is then loaded to the gunwales. And the load becomes especially evident in denials: “It is not so that my friend is dying” may mean (1) that, though he is my friend, he is not dying (he is healthy); or (2) that, though I am willing to call him my friend, he no longer exists (he is dead and gone); or (3) that he is no friend of mine (no matter whether he exists or is dying) — all depending on the context and the tone of the sentence.

Both in ordinary and in logical talk we bandy singular sentences about, as though they were easy to come by. But how often are we in a position to consider a singular proposal? Not very often. We seldom succeed in focussing on an individual being. For to focus in this way, and subsequently to propose what emerges at the point of focus, requires that, before we testify to the predicate (dying, or whatever), we face the individual both as individual and as its kind: a primary being in its secondary being. In Aristotle’s language, we actually know a being only when focussing on and participating in its actualization, the completion of its potentiality. Individuality becomes manifest to us only as totality (whereas universality manifests unity and particularly plurality). And the totality of my friend, my university, my house, or my horse is indeed difficult to come by.

So difficult, in fact, that Aristotle understood the most common form of error to lie in knowing perfectly well the universal (“all about” friendship, education, architecture) but failing to recognize an instance of it (this is the “mistake,” ἀμφιβολία, he says a tragic hero commits) or failing to actualize instances of it (the anguish of apprentices in any art). In Book Two on Paralogism (λογος in which we shirk our task) we shall consider the multiple forms of this failure, and some of the dramas engendered in efforts to overcome it. And in Book Three on Neologism (the renewal of λογος in which we learn to do better) we shall consider how genuine learning both starts and ends with individual beings — starts by the (rare) intrusion of one into the vagaries of universals and particulars, and ends by our (rarer) attunement to totalities (to the recurrent convergence of unity and plurality, as later philosophers sometimes put it: for example, a truly learned physician recurrently tunes in to each patient both as one of a unified kind and as having various symptoms and needing various treatments all his or her own).

Despite the factual difficulties we have in focussing upon individuals and considering singular proposals, we shall soon discover that in the formal workings of the syllogism we need not account for the two singular propositional forms as distinct from the two universal forms. That is, for the formal purpose of determining the validity of an argument, we can and shall treat singulars as universals. Why? We can, because singular propositions behave exactly like universal ones in their

are two different meanings here or two parts of one deeper meaning. For a scholarly defense of the second possibility, see Mohan Matthew’s “Greek Ontology and the ‘Is’ of Truth” (Phronesis [1983], Vol. 28, No. 2).
interrelations within a syllogism; and we shall, because treating them as universals reduces the number of relevantly nameable forms from six to four — to the four appearing in the Square of Opposition and, on the classical interpretation, requiring a species in the subject position. And perhaps also because syllogisms ultimately structure discussions underway toward actualized knowledge, i.e. toward foci on individuals, so that individuals (and the singular proposals focussing us on them) most properly stand “outside” argument, i.e. as its basis and goal.

Meanwhile, let us not forget the popular view that individuals are easy to come by, and singular proposals easy to initiate and consider. It might seem that we need only look to see, or listen to hear individuals, and then open our mouths or manipulate our pens to give utterance to the individual perceived. Yet philosophers, painters, and poets have decried the illusion of such ease: unless we do something with it, really work with it, at least open ourselves to it, and then learn to be with it, a being (our house, our friend, our institution) passes us by as it is in itself and as individual, and we register only its traces as it affects us or as it fits into our dealings: it remains little more than a name of a kind of thing, and our chief reaction is to ignore it, at most to like or dislike it, pursue or avoid it. But the only proof that philosophers, painters, or poets have to offer is their own work in which beings somehow declare themselves to us as passers-by, and which thereby offer us a second chance.

In Book Four on Formalism we shall return to the modern interpretation of singularity. But a quick preview now: Agreeing with the classical interpretation that singulars are difficult to come by, the modern mind differs as to the reason. Not a looming totality, but a perceived given, constitutes an individual. A given (a datum), however, is the result of a careful analysis of events, and is distinct from and yet related to the theory engendering and the interpretation employing it. Such individuals, construed as empirical data, are then collected into a plurality to form a particular proposition (this S is P, and so is this one and that one... so Some S are P). Such particulars stand necessarily in eternal contrast with the conceptual unities (conventional rather than natural universals) articulated by theories. Here then, individuals (in Greek: atoms) spawn particular propositions; whereas, on the classical view, individuals (beings as wholes) spawn universals. Still, on the modern interpretation singular propositions also behave as universal ones in the workings of the syllogism — because a proposal about a “this” talks about “all” of it — the “it” here being a numerical identity rather than an embodied species.

The modern division between “conceivables” and “findables” (as a replacement for the relation between “essence” and “existence”) was a long time in the making; it started already during the last gasps of the Third Interpretation, and it has climax ed only recently. But it came to something of a watershed in The Port-Royal Logic (originally, La Logique, ou l'art de penser, published anonymously by Arnauld and Nicole in 1662). In this textbook, written by Churchmen enflamed by Descartes' work, we find the insinuated assumption that thinking comes first, actual encounter second: that we can separate off and develop a straight thinking which can wait for subsequent application. Proposals about “all” of anything — the stuff of reasoning and so of logic — now refer us primarily not to beings but rather to our ideas. In general, acts of speech now appear as expressions of our inner thoughts. Formally put, the “terms” (S and P) of a proposition have two neatly divided functions: they have “comprehension” (later called “intension”: we would ordinarily say “meaning,” but as something strictly “inside” us), and they have “extension” (or, as we would ordinarily say, “application” to individuals happening to exemplify the meaning). The analysis here follows the Cartesian divorce of thought from reality, a divorce crucial to the development of the Fourth Interpretation. Logic thereby becomes a study of our own isolated thinking rather than a study of our way of being taken up into reality."

Perhaps Hegel's Science of Logic (1812) offers the last all-out and influential attempt to stem the tide of modernity, to save the unity of thinking and being, the marriage of our own being with the being of our circumstances, and the study of logic as the study of reality (i.e., of our

* Two contemporary results of this divorce: (1) the word “idea” shifts its meaning from naming what we must learn to face to naming something we somehow already have (the popular meaning today); (2) “language” appears as evolving on its own, and intellectuals speculate on the origin of language in time as though it were one handy possession among others for expressing our thoughts, rather than already at work in completing our encounter with things (rather than expressing beings, so to speak). Noam Chomsky's Cartesian Linguistics (1966) celebrates these developments as highlighting our creativity; for all the fervor and apparent novelty, however, Chomsky's work re-asserts and entrenches the basic tenets of the Fourth Interpretation.
One can read Marx's *Capital* (1867), and *German Ideology* (1846), as first discerning one historical significance of modernity: the destruction of nature-based-ness (*Naturwüchsigkeit*) and the introduction of society-based-ness, and thus the transformation of science itself, which “big industry separates from labor and presses into the service of capital” (*Capital*, XIV, §5); “big industry engenders world-history ... and subsumes natural science under capital” (*German Ideology*). Hegel and Marx were the first to blow the whistle on the naïve assumption that modern science simply studies nature.

Hegel's logic supersedes the Port-Royal logic in the realization that meanings (thoughts, intentions, ideas) are dead left-overs when residing simply in us; all such “inners” are alive (are what they really are) only when dynamically functional in some facing of “outers,” and therefore in conjunction with something faced — with “extensions.” Or, rather, “intentions” emerge for what they are only within our outreaching toward our circumstances. In the abstract, Hegel's claim here corresponds to Aristotle's account of actual vs. potential knowledge, but the removal of any definite basis in what is faced corresponds to, even lends a helping hand to, the modern insistence on a strictly human development. In Hegel's work, as in the work of moderns generally, “actualization” is ultimately societal, i.e. communal activity, and it takes place not as we assume our place in Nature but as we assume a role in History. Oddly, then, Hegel's work confirms rather than challenges modernity.

§5. The 256 syllogistic forms

A syllogism, Aristotle likes to say, is a λογος in which, certain things being posited, something else necessarily follows from them (see the first pages of his *Prior Analytics* or *Topics*). As is customary on the

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Second Interpretation, the successful version engenders the name, so that only the successful version deserves the name. If we were to retain the Greek spirit, then, we would have to say of a sequence of proposals in which the conclusion does not follow from the premisses, not that it is an invalid syllogism but that it is no syllogism at all: “here we fail to have a syllogism,” Aristotle says of such a λογος.

However, to avoid frustrating our modern disposition to grant a name to an example and only then to ask whether and how successful the example is, let us accord the name syllogistic form to the structure of any argument displaying even the pretense to generate a conclusion from two premisses. And we shall compensate for this initial permissiveness by insisting on a number of formal requirements in the actual transcription of such arguments: (1) that there be exactly two premisses and only one conclusion, (2) that all three proposals take one of the four propositional forms (employing the universal forms to represent singular proposals), (3) that there be three and only three “terms” (nameable subjects and predicates), and (4) that the two premisses be so ordered on top of the conclusion that the premiss containing the subject term of the conclusion be placed directly above it (this premiss is called the “minor” one) and the premiss containing the predicate term of the conclusion (the “major” premiss) be placed on the very top. Failing to meet one or more of these formal requirements, an argument has not yet assumed syllogistic form at all, and we shall not even try to determine whether it is valid or not.

How does a syllogistic form differ from a plain syllogism? As Aristotle says, a syllogism is a special kind of λογος: it is an account which generates a conclusion. Concretely, a syllogism arises in a discussion. For example, one person remarks that people seem to be uneasy in and about their lives, someone wonders why that should be so, and finally a reason emerges: “People are uneasy in life because they owe whatever substance they have to circumstances beyond their control.” Here we have a syllogism, an argument. And from it we may extract a form, a transcription which both satisfies the intent of the argument, and meets the requirements stated by the four rules.

\[
\text{All M are P} \\
\text{All S are M} \\
\therefore \text{All S are P}
\]
where S = human beings, P = uneasy, and M = beings dependent on circumstances. To transcribe the original syllogism into syllogistic form, we must drive out into the open all the elements of the talk: this is the first challenge and the first reward of formal transcription. And once transcribed the argument itself (the “content”) may momentarily fall out of consideration: we may calculate and decide by form alone the relation between the premisses and their conclusion. In commercial accounting the procedure is much the same: one first determines the exact values of assets and liabilities, debits and credits, and then adds and subtracts by form alone.

Most actual arguments fall into this scheme: S is P because S is M; the reason for S being the way it is (namely, P) lies in its being something else as well (namely, M = the middle term, what Aristotle calls the “cause” of S’s being P). All such arguments we may transcribe into the same syllogistic form as in the first example. Often, of course, the subject term of an argument (defined as the subject term of the conclusion, and recurring in the minor premiss) reads “This S...” (although we have agreed to transcribe this type of proposal into universal form for the sake of determining validity). But we often reason with “Some S...” as well: “Some birch trees die prematurely because their roots fail to adjust to fluctuations in the water table.” And in this case we extract a different syllogistic form:

$$\begin{align*}
\text{All M are P} \\
\text{Some S are M} \\
\therefore \text{Some S are P}
\end{align*}$$

where S = birch trees, P = unusually short-lived, and M = beings very fussy about their water table.

Now, these two first-figure affirmative forms do seem to represent our natural reasoning. At least they in fact structure much of our ordinary talk. Moreover, we learn to be careful about introducing the first reason (the minor premiss) in this scheme, since others might hold us responsible for the major premiss completing the argument (and ask us to recant the minor if we cannot endorse the major). And, finally, these two forms appear intuitively valid: they simply display “in large” the workings of every encounter “in small.” Just as I identify every individual by its kind, so I learn to subsume every proposal (judgement, thought, belief) under a principle. Or: these two forms of argument simply display, in reference to a middle term (M), the guarantee that S and P are indeed connected (that we are not mistaken in believing that all people are uneasy, that some birch trees are dying prematurely — that uneasy people and dying birch trees are not just mistakes or mere exceptions).

Backimg off now from syllogisms themselves and looking simply at these two syllogistic forms, we notice that there must be 254 more such forms. For in each of the three proposals of an argument form there are four possible propositional forms:

- All S are P called an A
- No S are P called an E
- Some S are P called an I
- Some S are not P called an O.

Right here, then, we have 64 possible moods, of which the AAA and the AII are only two. Furthermore, although both our examples have illustrated the same locations of the middle term (M) in their premisses, there are a total of four such “figures”:

- 1st figure: All M are P, P M, M P, S M
- 2nd figure: S M, M S, P M, P S
- 3rd figure: M S, S M, P P, P P
- 4th figure: S S, M P, P S, P P

with the result that we have 256 differing moods and figures. As we shall see, only a handful of these are valid and most of the rest never reflect the form of any natural argument. Yet several invalid forms do in fact structure arguments which come close enough to our natural reasoning patterns to tempt people time and again. And a few of these 256 argument forms raise questions about natural reasoning, so that arguments in these forms will be judged valid or invalid according to how we answer these questions.

Tradition has ranked the figures according to how well they reflect natural reasoning when valid. The first figure is most natural both in the sense that we ordinarily follow the scheme “S is P because S is M” and

*Ernst Kapp (Greek Foundations of Traditional Logic, New York, 1942 & 1967, pp. 70-74) discusses the importance of recognizing that, for Aristotle, syllogisms most concretely arise as causal rather than as formal. Modern logicians often miss this location of syllogism and therefore misinterpret, or at least over-interpret Aristotle’s frequent formulation of syllogistic forms as conditionals; see I. M. Bochen’s A History of Formal Logic (1956, 1961), p. 9, and J. Łukasiewicz’ Aristotle’s Syllogistic (1951, 1957), p. 2.
in the more philosophic sense that we naturally want to conclude how things are — how an S is singularly or universally — and the first figure is the only one allowing an affirmative universal (or singular) conclusion. On the same grounds, the second figure is less natural: as we shall see, it allows only for negative conclusions, and we cannot live (on the Second Interpretation), cannot even think (according to Parmenides) by concentrating on how things are not. The third figure is even less natural, since it will allow us to draw only particular conclusions, and we need universal ones to satisfy our condition as knowing beings (and at least some universal ones even to syllogize).

The fourth figure (which allows conclusions in the form of E, I, or O) was formally introduced into logical consideration only hundreds of years after Aristotle. It has occasioned much controversy. Obviously a formal possibility once we back off from given arguments and confine our view to their formal structures, this figure nonetheless seemed to escape notice throughout the heyday of the Second Interpretation and struck later philosophers as highly unnatural, somehow not genuine — even as late as Kant in the heyday of the Fourth Interpretation. In hindsight today (when philosophers routinely accept this figure) we can ponder the “difference of opinion” as forcing once again into the forefront the basic principles of antiquity and modernity — principles otherwise merely presumed in each case and justified only in their respective fruitful employments.*

On the classical view, the study of the syllogism is still a study of λόγος itself. And the study of λόγος is, precisely, the study of the human condition, of our nature as manifested in artful and speechful dealings with beings. Every logical consideration, then, takes us back to concrete situations — at the very least, back to “eristic,” the “gymnastic” of debates, exercises in preparation for concrete dealings such as litigations in a courtroom, deliberations in an assembly, or pure commendation (eulogy, poetry, philosophy). In a sense, then, and despite the often abstract appearance of ancient treatises on the subject, logic remained for centuries a study of arguments (syllogisms): one did not back away from them to view their formal structures only — anymore

than one could back away from horses to study their essence (horse-ness) only. The ancients assume that the study of form evolves within the study of “content” (reality). As a result, the question whether some formal arrangement devised by logicians reflects how human beings naturally reason remains central to the whole enterprise of logic.

In modern times, the study of logic has become primarily a study of inference — of the workings of inference discernible in syllogistic forms (whereupon it soon became evident that the syllogism as we have defined it represents only one type of inference, and a rather limited one at that). The inference here under study has no natural home: in determining structures and their validity, we have nowhere else to look, we appeal neither to dealings in concrete situations nor to workings of the human brain. For the whole enterprise stems precisely from the divorce of straight thinking from every such home — of mind from matter (Descartes), of “comprehension” from “extension” (Arnauld and Nicole), thinking from being (Hegel, although his work consists in an effort to annul the divorce). In more recent times, intellectuals often feel obliged to divide every complex question, prior to debating it, into questions of logical meaning and questions of empirical fact. As a result, the question whether something so obviously logical like a form of inference is “natural” must now appear as a question whether it is “empirical” — and the question itself must be dismissed as confusing the two realms.*

The modern verdict does appear cleaner and safer than the ancient one. For it embodies the principle of self-sufficiency of logical thought: we need not appeal to any standards outside our own subject matter — to anything we ourselves have not in some way produced. Yet the modern, hard and fast distinction between syllogistic form and empirical argument requires something more, not just less, than appears necessary to Aristotle and his tradition. For we must expend very special effort, and exercise

* Such is the approach one finds in The Development of Logic (Oxford, 1962), by W. and M. Kneale: “Much energy has been spent in disputes about the existence of a separate fourth figure, but it is not at all clear what the disputants wish to prove or disprove. Sometimes they talk as though the question were about the way men naturally argue. If this is the problem, the proper way to solve it is to hold a large scale empirical survey” (p. 101). See also the section on “The Doctrine of the Syllogism” (pp. 67 ff.), which carefully expositions the detail of Aristotle’s peculiar way of structuring syllogisms, and distinguishes this way from later ways.
great care, in the transcription out of ordinary (empirical) into formal (logical) language. Since we cannot handle an argument in its natural setting, we must reconstrue it entirely. And this transcription must proceed on its own recognizance, i.e. we must simply leap into the ordinary, take what we can, and leap back into the formal: and nothing guarantees that the formal result will do justice to the original thought. The result may be clean and safe, but the venture obtaining it can be rather messy and dangerous.

Transcription becomes especially challenging and even revelatory when we take upon ourselves the requirement that the subject of an argument (and all subject terms) must bring a being forward for consideration. For the grammatical subjects of many naturally occurring arguments do not name beings at all, but rather actions or affections, times or places, perhaps conditions or postures, quantities (in mathematics) or qualities (the noble, the useful, the pleasant). For instance, consider Vico's famous argument in *The New Science* (1725), §331:

> ...a truth beyond all question: that the world of civil society has certainly been made by men, and that its principles are therefore to be found within the modifications of our own human mind. Whoever reflects on this cannot but marvel that philosophers should have bent all their energies to the study of nature which, since God made it, He alone knows; and that they should have neglected the study of the world of nations, or the civil world, which, since men have made it, men could come to know.

The conventional paraphrase of this argument (the first absolute defense of what we now call the social sciences) runs: “We can only know what we (and our likes) have made, therefore all genuine science has society for its subject matter.” We can paraphrase it yet again to obtain a complete argument approximating syllogistic form:

All human products are societal in essence.

All genuine science studies human products.

∴ All genuine science studies society.

Pinching our eyes to blur our vision, we might see an AAA-1 here. However, even apart from the slight variations in the “terms” (certainly in the verbs), we may notice that the “subject” of the minor premiss and the conclusion (”genuine science”) is not a being, but an action (knowing), while the “subject” of the major premiss (human products) is indeed a being (although highly generic: “artful” beings, Aristotle says — as opposed to “natural” beings). To obtain the three terms necessary for syllogistic form, to get rid of the verb “study” for the sake of “are,” and to clarify what the being at issue is, we need to acknowledge that “science” is shorthand for “activities of human beings in knowing something” (S = AHK). These activities, the conclusion proposes, are “activities focussing on social institutions” (P = AFS) — because they are “activities in which human beings explain human products” (M = AEP):

All AEP are AFS

All AHK are AEP

∴ All AHK are AFS

This transcription introduces a category (action) occurring in each of the three terms and hanging over the being named in the subject and middle terms (ourselves as knowers, makers, citizens). The category of action serves as a parameter, a common measure grammatically heading up each term and requiring us to consider the being at issue already under a category other than being. Grammatically, we even seem to have abandoned the principle that only a being can serve as a proper subject of proposals. But grammatical appearance is not the same as logical meaning. And, logically, a parameter such as “activities” and any other “inseparable” category (including “being” in the secondary sense!) remains anchored in, and meaningful only in providing a framework for, the being (here: ourselves) named “inside” the parameter.

Time and place also serve especially well as parameters in the process of transcription, and allow us to account for our natural disposition to reason about beings so measured: “Wherever water tables vary considerably you will find birch trees dying prematurely” = “All places where... are places where...”. “We should not pride ourselves when receiving good fortune” = “No times we receive good fortune are times we rightly pride ourselves.” “Horses may kick out if you walk behind them without warning” = “Some times horses get approached from behind without warning are times they kick out.” “Christmas is a sad season of the year” = “All times people officially celebrate the birth of Christ are times they become melancholy.” “Autumn is beautiful in the Maritimes” = “All times in the Maritimes when trees and other forms of life are on the whole receding from their proper activities ARE times
they appear beautiful to us.” — In such cases, parameters allow us both to create and to analyze names: while we have a grammatically single name for “horses” we do not have a ready-made single one for “horses when approached from behind without warning” (the being under two categories, time and affection — to which a third, action, is added in the predicate of the proposal); and while “autumn” is grammatically singular it begs for analysis so that we can discover under its grammatically temporal reference the logically named being (here, “nature” but considered both at a time and a place and as failing to engage in an activity). On the classical view, naming ultimately stems from beings themselves, i.e. we come to name things because beings have declared themselves to us, partially or wholly, for what they are: names are natural when working well, but their work is also our work, and if we allow them to fall into disuse, i.e. to stop working well, they are merely conventional (thus the fascinating debate in Plato’s Cratylus). The modern demand for syllogistic form, and the consequent need for parameters, can help one appreciate the human responsibility required for the careful naming of beings — despite the modern tendency to view names as essentially conventional rather than natural (as simply man-made rather than arising in our facing of nature).

By employing parameters in careful transcription, we can sometimes pinpoint the crux of an important debate — the exact point at which opinions diverge. For instance, in his Summa Theologica (1265), Thomas Aquinas disputes the argument: “...mercy is a kind of sorrow...there is no sorrow in God, and therefore there is no mercy in Him” (Question 21, Article 3). In weak fulfillment of the requirements for syllogistic form, we could easily transcribe the argument, rightly employing the category of affection as the parameter:

All feelings of mercy are feelings of sorrow
No feelings of God are feelings of sorrow.
∴ No feelings of God are feelings of mercy.

However, the major premise fails to name any being. We must ask (as Thomas Aquinas does): Whose feelings of sorrow? A wolf’s, as he spares another wolf acknowledging defeat? A human being’s, one saddened by the plight of others and intent upon relieving it? Or God’s, who created all beings and holds them totally in his power? If we satisfy the strong requirement of syllogistic form by completing the predicate term of the argument to read “feelings of mercy had by human beings,” we can endorse all three proposals of the argument — but end up with a trivial conclusion. If, on the other hand, we complete the predicate term to read “feelings of mercy had by God” we may well reject the major premise: it may be, as Thomas Aquinas argues, that a human being is brought to the action of helping others (mercy) in response to being affected (sorrowed) by their plight, but God can engage in the action without being driven to it (without “affection” in the categorial sense of “passivity” and “being overpowered”). Until such time as we discover the options and decide upon one of them, we easily slip from one to the other, affirming the major premise implicitly completed by adding “...had by human beings” and affirming the conclusion implicitly completed by “...had by God” — whereupon we really have four terms in the syllogism and fail to have a syllogistic form at all.

Many actual arguments display yet one more peculiarity: they most immediately appear as sequences having three or more premisses for generating one conclusion. For example: Since (1) all things of nature are ephemeral, and since (2) all human beings are things of nature, and since (3) we are human beings, it follows that we are indeed “creatures of a day” — as we are called in Pindar’s 8th Pythian Ode. It seems like one argument, yet when transcribing it into syllogistic form we must treat it as two interlocking ones: (1) and (2) generating an unstated conclusion (“All human beings are ephemeral”) which then combines with (3) to generate the final conclusion. We may call the original sequence a sorites, a “piled up” syllogism. Such sorites (“pile-ups”) work most clearly as sequences of affirmative universals, zeroing in from wider (generic) beings down to species or individuals. But they also work with negative proposals, as in Vico’s fuller argument: our knowledge cannot rightly be directed to nature because (working up the sequence) (3) it is always directed to man-made things, and (2) directedness toward nature is reserved for God and (1) divine knowledge is not really concerned with man-made products. With a parameter:

1. No activities of God knowing are directed to human products
2. All activities directed to nature are activities of God knowing
   (∴ No activities directed to nature are directed to human products)
3. All activities of human knowing are directed to human products
   ∴ No activities of humans knowing are ones directed to nature.
Transcribed in this way, the sorites makes explicit a basic tenet of the Fourth Interpretation. The conclusion, for instance, states outright a proposal contrary to the Aristotelian principle that “all activities of man directed to human products are activities directed to nature” (“an art partly completes what nature is unable to finish and partly imitates her” — Physics, 199 a 15). Perhaps, too, the first major premiss states a proposal contrary to the Third Interpretation (“all the hairs on your head are counted” — Matthew, 10:30). In any case, careful consideration of the meaning and employment of the third premiss reveals very concretely the divorce of the human from the natural that has increasingly defined, albeit often quite silently, our own understanding of what it means to know: to refashion our environment.

§6. Validity and invalidity

We could, if we so wished, construct a booklet comprising exactly 256 blank pages, one for each of the syllogistic forms. And on each page we could, as time went on, inscribe several proofs of its validity or invalidity — i.e., of the validity or invalidity of the arguments occurring in (transcribed into) that form. We would then have the satisfaction of a “complete induction,” a review of all the possible entries in the parade of argument forms, and a considered judgement (several times over) of every one. Following Aristotle, we would discover 14 valid forms; following the Medieval Schoolmen, 19); following Leibniz, we would discover 24; and following the precepts of recent set theory, we would discover only 15 (rejecting natural naming and accepting only conventional names). Let us consider how we could follow all these paths, discerning in each both the mechanical component allowing for formal decision and the semantic component justifying the mechanisms and reflecting the Interpretation at issue. Let us start out on the path followed in the heyday of the Fourth Interpretation.

§6.1 The path of 24

We take two 1st-figure forms as reflecting naturally valid arguments: the AAA-1 and the AII-1 (Barbara and Darii: in the Middle Ages logicians christened arguments with proper names, the vowels of which named the mood’). But why naturally valid? Aristotle had answered earlier by interpreting all arguments as stating relations of belonging. An AAA-1 argument then runs: P belongs to each and every M, and M belongs to each and every S, so P belongs to each and every S. The relation of “belonging” here appears sequential, linear, transitive: P is passed down, through M, to S — wholly (as in an AAA-1) or partially (as in an AII-1). As already noted, we can then understand the two 1st-figure forms as “blow-ups” of the same “subsumption” (belonging) mysteriously but effectively operative in the tiniest categorial proposal of any S as a P.

We can ask, then, which of the other 254 forms can be “reduced” to one of the other of these two — which ones display the same evaluation of relations as (which are “equivalent” to) a Barbara or a Darii. Consider, for instance an argument in the 3rd figure, an IAI-3:

\[
\text{Some M are P} \\
\text{All M are S} \\
\therefore \text{Some S are P}
\]

Hypothesizing in advance that it is valid, we can transform it, without (we hope) violating the claims of its various proposals, into an AII-1. For if we know that “Some Snakes are Poisonous,” do we not also know, does not our knowing simultaneously mean, that “Some Poisonous things are Snakes”? It seems that, in principle, we can recast any particular affirmative proposal into either form — by conversion: that “Some S are P” is intuitively equivalent to “Some P are S.” It follows that any argument taking the form of IAI-3 may be recast as:

\[
\text{All M are S} \\
\text{Some P are M} \\
\therefore \text{Some P are S}
\]

See the Kneales' Development of Logic, pp. 72 f. and 233 f. for an explanation of the proper names ascribed to argument forms during the Middle Ages; in his Galen and the Syllogism (Pittsburgh, 1966, p. 31) Nicholas Rescher lists those of the fourth figure also. I here name all twenty-four in medieval fashion. First figure: Barbara, Celarent, Darii, and Ferio. Second figure: Cesare, Camestres, Festino, and Barbaro. Third figure: Darapti, Felapton, Disamis, Datisi, Bocardo, and Ferison. Fourth figure (non-Aristotelian): Bramatip, Camenes, Dismaris, Fesapo, and Fresison. Weak forms (corresponding to their parents' names): Barbari, Celaont, Cesaro, Camestros, and Camenos.
where we have converted the two particular affirmatives and restacked the premisses to reflect the change in subject term of the argument (the real S is now designated above as “P”). We have then proved the validity of this one form by leading it back to its naturally valid form: the method called reduction (see Prior Analytics, I, 45).

While we cannot rightly convert either an A or an O proposition (try converting “All dogs are mammals” or “Some people are not Italians”), proposals in the E form do appear convertible: “No Saints are Perverts” seems equivalent to “No Perverts are Saints.” However, conversion of this negative proposal leaves us with a negative form and therefore never yields, by itself, a Barbara or a Darii. We need another mechanical technique to show, for instance, that an EAE-1 is valid.

Besides converting proposals in the I and E forms, we can obvert proposals in any of the four (in fact, any of the six) forms. We need only (1) change the “quality” of the proposal, whether from negative to affirmative or from affirmative to negative, and (2) replace the predicate by its opposite: Nietzsche's proposal that “All humans are sick” is equivalent, it seems, to “No humans are well” (if “sick” and “well” are fully opposite, i.e., if not being one means being the other); and the common recognition that “Some movies are hopelessly bores” is equivalent to “Some movies are not going to excite anyone” (again, supposing that “hopelessly boring” and “somehow exciting” are fully opposite). With this method of obversion, then, we can easily transform the EAE-1:

\[
\text{No M are P} \\
\text{All S are M} \\
\therefore \text{No S are P}
\]

into an equivalent AAA-1 (lead it back onto its natural, i.e., “perfect” form) — by obverting both the major premiss and the conclusion:

\[
\text{All M are non-P} \\
\text{All S are M} \\
\therefore \text{All S are non-P}
\]

where the “non-“ formally reflects the need to decide what constitutes the genuine opposite of the term. (In one's zeal to obvert one must also remember that each obversion introduces a new term; otherwise one easily slips into arguments having more than three terms — which disqualifies them from being syllogistic forms and thus from being candidates in our booklet of 256 pages; above, both instances of the

With these two techniques of conversion and obversion we could successfully reduce 13 of the remaining 254 argument forms to obtain a total of 15 valid ones (note that our failure to reduce the rest only says something about us, not about the argument forms themselves: we need another technique to prove arguments invalid). However, having passed by the AAI-3 when proceeding through our booklet, we might have wondered. Returning to it now, we notice that we cannot, without ado, convert either of the premisses; that obverting them gets us nowhere; and that, besides, the combination of two universal premisses and a particular conclusion does not appear on our list of (two) naturally valid forms. However, if “All philosophers are wise” and “All philosophers are crazy,” does it not follow that “Some crazy people are wise”? And (to illustrate, finally, how a singular proposal behaves even mechanically as a universal rather than a particular), if “Socrates is a philosopher” and “Socrates is hated by his fellow citizens,” does it now follow of necessity, that “Some philosophers are hated by their fellow citizens”? In any case, logicians in all but recent times have judged this argument form to be valid. To show it to be so, we need only notice that anyone who knowingly proposes an argument with two premisses in the form

\[
\begin{align*}
\text{All M are P} \\
\text{All M are S}
\end{align*}
\]

also, and of necessity, knows and proposes that

\[
\begin{align*}
\text{All M are P} \\
\text{Some S are M}
\end{align*}
\]

because the minor premiss “All M are S” includes “Some M are S,” which is equivalent to “Some S are M”; and then the conclusion follows of necessity in an AII-1. And since the EAO-3 (Felapton) and the EAO-4 (Fesapo) are equivalent (by conversion and obversion) to the AAI-1, we can reduce them all by subalternation to AII-1, and thereby add three more to our list of valid argument forms. This third technique simply requires that we choose the universal premiss whose subaltern works to yield eventually the AII-1.

\*

Of course, we cannot rightly convert a singular proposal, as would seem to be necessary in arguments where M is, say, Socrates. Aristotle insists that a primary being arises for us precisely as what generates the need for
But once we allow subalternation we can add five more forms to our list. For there happen to be five valid combinations of universal premisses generating universal conclusions; and, for the same reason at issue in deciding the validity of the AAI-3, we can decide that their “subaltern moods” are valid as well (e.g., if we have a Barbara, and thereby affirm the conclusion in the form of “All S are P,” we also affirm the “weaker” version, the AAI-1 — Barbari — without having to dicker with the premisses at all).

So much, then, for the mechanics. Now for the semantics: What happens to the meanings of proposals when we convert them? When we obvert them? When we settle for their subalterns? To consider these questions we must return to what proposals mean — and to examples of proposals rather than to their abstracted forms: after all, the four (or even all six) forms have the formal meanings we grant them on the basis of our understanding what the examples concretely mean.

Converting “Some snakes are poisonous.” I must take “poisonous (animals)” as my new subject. I now concentrate on these (a genus) as they come forward in the form of the new predicate (a species: snakes). I have changed the focus (intent) of the proposal, even if I have not changed the scope (extent) — as I would indeed do if I were to convert “All dogs are mammals” to obtain “All mammals are dogs” — also a species-genus relationship. Still, I seem obliged to accept the change in focus if called upon to do so, since to object to (deny) the converse “Some poisonous animals are snakes” would, by contradiction, amount to asserting that “No poisonous animals are snakes” which seems to be downright incompatible with the original. In any case, this particular change in focus seems to do no irreparable harm, and it does have its uses, e.g. it allows us to judge the AII-3 form to be valid.

But consider what happens when we convert “No humans are perfect beings”: we obtain “No perfect beings are humans.” In the original version, I consider a proposal in which humans are brought forward, and I can ask whether “perfect” belongs to them. In the new, the converted form, I must consider “perfect beings” and ask whether they can take on the predicate “human.” But perhaps the very reason why humans are not predication, and therefore can never really serve as a predicate itself. Thus the proof of validity in this form was called “exposition” (the laying out). See Aristotle’s *Prior Analytics*, 25 a 15 to 28 b 21, and the Appendix on the reconstrual of singulars.

But we can get around these difficulties of conversion by declaring that proposals in the form “No S are P” require of us positive knowledge of what is covered by both terms: require, for example, that we know both “human beings” and “perfect beings,” and then judge the two to be incompatible. To know something in this sense would then require knowing both what it means for something to be S, what it means for something to be P — and to know these meanings by focussing on the beings themselves. It would require, in summary, that we know the essences of existences — and that essences not be considered by themselves but as of existing individuals. Satisfied that these conditions of knowledge are met (or that it is rightly proposed that we meet them), we can convert a proposal in the E form and bring forward for consideration what was named originally as a predicate. Yet we will then have to decide, in any formally given case of a proposal taking the E form, whether it is really a proposal at all: whether it (i.e., those initiating or considering it) can fulfil the conditions of knowledge.

Another quandary: obverting “All human beings are rational.” I must first exercise some care in naming the opposite of “rational.” Especially in this case, it might be best to suspend the decision and to say simply “No human beings are non-rational”; for “irrational” tends to mean something like “erratic,” in any case a mode of actual behavior, whereas the original “rational” means (at least to a philosopher) something like “capable of reason” (a potentiality). But what does it mean to consider a proposal like “No human beings are non-rational”? More generally stated, what does it mean to bring forward a being for consideration, trying then to decide whether it is “non-X” — is “anything except X”? Surely, the natural meaning is expressed by the positive perfect is that perfection is impossible for any being. In this case, I can hardly bring “perfect beings” forward for consideration. And some such “bringing forward” is a condition for truth — a condition which, if unfulfilled, leaves us with (and in) a world of “mere proposals,” i.e. of gab. The technique of subalternation then aggravates the matter: while from the original we obtain “Some humans are not perfect beings” (which may reflect a tentative appraisal of one’s own experience), from the converse we obtain “Some perfect beings are not human” (which, though supposedly a tentative appraisal of experience, actually claims to reflect experience of perfect beings — as well, of course, as the decision that they happened not to be human).
predicate, and thus in the original rather than the obverted form. And converting the obverse makes matters even worse: What does it mean to consider a proposal like “No non-rational beings are human beings”? How can we, to fulfill a necessary condition for truth, bring forward “non-rational beings” to decide, for ourselves, whether they deserve the predication of “human” or not? A “non-X” is not a being: the phrase does not allow any delimitation of a being to serve as a subject for subsequent predication. And neither does it arise as a subject in production, action, or contemplation: as artisan, statesman, or philosopher I do recognize “non-X” as an issue, but I do so first by devoting myself to the being itself and then by acknowledging its privations.

Although obversion is highly suspect inasmuch as it introduces negatively delimited (i.e., undelimitable) terms and thus deprives us of our natural ability to consider and decide proposals in reference to reality itself, we can at least appreciate the technique as a grammatical device for eliminating such undelimitables in the process of transcribing arguments into syllogistic forms. Consider, for example, the proposal “When he's sick he's unapproachable”: we might transcribe it as “All times he's sick are times he's not approachable.” But what does it mean to consider whether “times he's not approachable” applies to the subject? It would make more sense to consider whether the positive version of the predicate applies to the subject. For to ask whether “No times he's sick are times he is approachable” drives the negativity out into the open between the two terms instead of burying it within a term. Similarly with any grammatically positive but logically negative term like unapproachable, unlimited, irrational, perhaps also sick, boring, stupid, and (debatably) independent, used up, evil.

Finally: subalternating a universal proposal like “No times he's sick are times he's approachable,” I assume that there are “times he's sick.” Although we moderns might ask whether a (conventional) term applies to existing circumstances (past, present, or future), our ancient forebears assumed rather that a (natural) term stems from and thereby draws out our circumstances. Descending from the universal to a particular version requires only that we drop from concentrating on the complex essence-of-existence to noting the simpler and easier existence. However, the ancient assumption then serves as a condition for any genuine proposal taking the A or the E form: before judging it to be a proposal at all, we would have to decide whether it stems from and draws out the existence subsumed under the essence.

In an effort to clarify the question of the “existential import of propositions” (as logicians call it), we may distinguish between a name (of a species) and a description (of a class): a name is genuinely a name when it (our utterance of it) issues from and draws out a being as including both essence and existence, whereas a description may (in it we may) restrict a named being under other categories (as we do in employing parameters: a very un-Greek procedure). We would then deceive others with, or be ourselves deceived by proposals naming something that does not exist. But, in the description of a class, deception occurs only if we fail to ask the supplementary question: Have there been times when he's been sick? Are there customers over ninety years of age who bring their parents? Will there be another coming of the Messiah? Still, the distinction raises a number of questions of a logical nature: Are there species (and therefore names) which are not merely humanly constructed descriptions? Can we describe a class without some reference to existence (to a “he” — even if there are no times he's sick)? Furthermore, it seems obvious that much powerful discourse names individuals who do not exist in the seemingly evident way my house and my town, my wife and my neighbor, exist: historic and poetic literature may name ancient Rome or ancient Troy, Socrates or Odysseus, Jesus or Dido, the statue of Zeus at Olympia or the Island of Circes. We can explain such literary power reductively by claiming that historic names of individuals have reference to past existences which we take on faith from testimony left over to us, and that poetic names are really pseudonyms for references intentionally concealed in the recitation for the sake of affecting us with the predicates ascribed to the subject.

Plato’s “Doctrine of Forms” agrees that essences (forms, species, universals) are always of something, see R. C. Cross’ “Logos and Forms in Plato” (Mind, 1954, especially pp. 446-450).

* The principle of subalternation is stated in Aristotle's *Topics*, 119a 36: “if P belongs to all S, it belongs to some; and if it belongs to no S it does not belong to some.”
Or we can contemplate such literary power inductively by asking whether historic names might not stem from and draw out our undercover selves, our own destiny as bequeathed to but forgotten by us, and whether poetic names might not be alethonyms for ourselves and our homes, our lovers and our gods as the subjects of the predicates otherwise only floating through the pages of discourse.

For the pure pleasure of mechanical manipulation we may certainly convert, obvert, and subalternate — and, in a purely formal way, approve of all 24 syllogistic forms. And the quandaries these techniques plunge us into do not discredit any actual argument in these forms: quite the contrary, the questions about these techniques intend to highlight the wonderful credibility of actual arguments — by pinpointing the assumptions (or presumptions) at work when we transcribe them and assess the resulting forms. Still, the requirements evidently conditioning the conversion of proposals, limiting the employment of obversion, and complicating the use of subalternation, generally drive logicians either back to the original Path of 14 or forward to the contemporary Path of 15.

§ 6.2 The path of 14

In three short chapters of his Prior Analytics (I, 4-6), Aristotle takes the four valid moods of the 1st figure (besides Barbara and Darii, also Celarent and Ferio) as “perfect,” reduces eight more moods of the 2nd and 3rd figures to one or the other of the first four, and reduces two more by showing in each case the impossibility of affirming the contradictory of the conclusion while still affirming both the premises (a technique later called “reduction per impossibile”). The mechanics of these procedures appear simpler when recast into those we have already reviewed, but in the process of recasting them we lose the semantics distinguishing Aristotle’s procedure from later ones. Let us then concentrate on the differences and thereby on the semantics.

First, all four valid moods of the first figure appear as perfect, intuitively valid, and therefore as paradigmatic for deciding the validity of other moods and figures. Why? Because they all show transitive relations of belonging or not belonging. For instance, Ferio (EIO-1): “P belongs to no M, but M does belong to some S, so P does not belong to every S.” Whereas the verb “to be” wavers in colloquial tongues between identity and classification, and suggests categorical predication only when we force it a bit, the verb “to belong” takes us down a line of “having” and “not having” from P through M to S. Of course, we have to concentrate (think) in order to travel down the four lines justifying these four forms, and for that reason we usually prefer mechanical techniques. However, such concentration on belonging (pertaining to origination: 
\[\deltaι\alphaρχειν\]) expresses a principle of categorial logic: that truth only occurs as we zero in on beings as they are (and as they are... not).

And why only three figures? Because Aristotle never draws up a visual panorama of figures, and does not even have a fixed number of syllogistic forms (as we have in our booklet of 256 pages): for him there are only 14 syllogisms. In a syllogism, he says, one end (predicate, or major) is brought into a belonging (or not-belonging) relation with another end (subject, or minor) via a middle term — either in a perfectly transitive way (as in the first figure: linearly) or in an imperfectly transitive way. These imperfect ways are only two: M might belong to both S and P (the second figure), and thereby pry S and P apart (generating only negative conclusions) or both S and P might belong to M (the third figure), and thereby relate S and P partially (generating only particular conclusions). Thus, for Aristotle, there is no fourth figure because these three semantic arrangements exhaust the possibilities — once we ask only how P can belong to S rather than asking how the three terms can appear visually in a sequence of three propositions. Again, the adequacy of three (rather than four) figures reflects the logical primacy of belonging over purely formal (visual) schemes.

Two of the third-figure forms (Darapti and Felapton), and two fourth-figure forms (Bramatip and Fesapo), are proven not by sub-alternation, but by a procedure later called “conversion by limitation”: from All S are P one obtains directly Some P are S. Thus the proof of Darapti reads in full: “if P belongs to every M, and S belongs to every M also, then, since S belonging to every M means, in addition, that M belongs to some S, we have a syllogism concluding that P belongs to some S.” We today may understand this procedure as a round-about way of subalternation and conversion, but Aristotle understands it as an inference based on the meaning of 
\[\deltaι\alphaρχειν\]: if I belong totally to my wife, in some sense my wife belongs to me; if rationality belongs to every person, then personhood belongs to some instances of rationality. Aristotle has no interest in the weak forms (Barbari, Celaront, ... or even
our fourth-figure Fesapo) because his procedures highlight belongings in the concrete and not simply extractions abstractly possible.

Two forms, the AOO-2 and OAO-3 (Baroco and Bocardo) defy reduction to the Darii—if, that is to say, we forbid ourselves, as Aristotle forbade us, to employ the procedure of obversion. Aristotle follows Parmenides when he insists that we cannot think about, or even bring up for discussion a “non-X,” rather than as a subject or as a predicate (Prior Analytics, I, 46); rather, our task, already abundantly difficult, is to focus on what is (the recurrent argument in Aristotle’s Metaphysics and elsewhere). In these two cases we must employ another technique: we induce an impossible result from supposing we could deny the conclusion. Formally: we suppose the contradictory of the conclusion, combine this with one of the premises, and draw a conclusion (using Barbara, Celarent, Darii, or Ferio) that contradicts the other premiss. For instance, Baroco reads:

\[
\begin{align*}
\text{All } P & \text{ are } M \\
\text{Some } S & \text{ are not } M \\
\therefore \text{ Some } S & \text{ are not } P
\end{align*}
\]

If this form were not valid, All S are P would be compatible with the two premises. But from the original major premiss and the supposed possibility, i.e. from

\[
\begin{align*}
\text{All } P & \text{ are } M \text{ (original major premiss)} \\
\text{and } \text{All } S & \text{ are } P \text{ (supposed possibility)}
\end{align*}
\]

we may derive, by Barbara:

\[
\therefore \text{ All } S \text{ are } M,
\]

which contradicts the original minor premiss. Some S are not M. We have then shown that denying the original conclusion leads to an impossible condition, namely the affirmation of contradictory proposals. Thus it appears necessary to reject the supposition that led us down this path, namely the supposition that All S can be P. And rejecting this we must accept its contradictory, Some S are not P. And this is what we set out to prove."

* Leibniz, who abhorred conversion as a violation of focus, recommended that we employ the method of reduction per impossibile for all proofs. Interestingly, this technique lands us in contrariety rather than contradiction when applied to Darapti, Bramatip, Felapton, and Fesapo: the question then becomes whether we can accept contraries.

---

If we ask Aristotle how we can know that a supposed syllogism is really not one (that a given syllogistic form in our booklet of 256 is not valid), we discover once again an appeal to semantic belonging which also provides a mechanical technique. He asks us first of all to concentrate on all the possible relations of belonging among the three terms of the premises (not among the three propositional forms in the argument form). Within such concentration we may establish, for a given arrangement of terms, where belonging and its opposite deliver a conclusion; and in each case of a successful delivery we have a syllogism. In the course of such concentration, however, we must still decide when to abort, and in the process of this decision we may detect a mechanical technique. For instance, consider an argument in the second figure beginning with EO (Prior Analytics, 27 b 14):

\[
M \text{ belongs to no } P \text{ and does not belong to some } S; \text{ it is then still possible for } P \text{ to belong, whether wholly or not at all, to } S; \text{ terms showing the non-belonging [of } P \text{ to } S \text{] are:} \\
P = \text{ black, } M = \text{ snow, } S = \text{ living.}
\]

What is distinctive about Aristotle’s analysis here is that it asks us to consider whether the modality of “P belongs to S” is merely that of possibility rather than that of necessity: in the example cited, contrary belongings remain possible, therefore no belonging is necessary, therefore there is no syllogism, therefore any syllogistic form in the second figure beginning with EO is invalid. And because no belonging is necessary we can then find an example to illustrate the nonsense generable with such premises:

\[
\begin{align*}
\text{No black things are snow} \\
\text{Some living things are not snow}
\end{align*}
\]

which, taken together, are quite compatible with “All living things are black” (to illustrate the nonsense of the EOA-2) and with “No living things are black” (the same for the EOE-2) — contrary conclusions themselves incompatible with one another.

Out of Aristotle’s procedure we may extract a neat mechanism: for any argument form which has resisted our efforts to reduce it to one of the paradigms of validity, we may try to devise an example (three terms) in its form, an example which must satisfy two requirements: (1) that the resulting premises both be reasonable (accurate by normal standards of experience) and (2) that the conclusion be ridiculous (by the same
standards). This technique came to be called “reduction to absurdity” or “refutation by analogy.” It bypasses the semantic arguments otherwise necessary (and the concomitant ontology), but it still presupposes that we can naturally recognize obviously correct proposals and absurdly incorrect ones.

Working through our booklet, armed with our new technique, we shall find some forms very easy to refute by analogy. For instance, consider an argument in the AEE-1:

All dogs are mammals
No cats are dogs
∴ No cats are mammals.

And an argument in the IAI-1:

Some mammals are dogs
All cats are mammals
∴ Some cats are dogs

Others, however, seem to require a willingness on our part to accept the principle of subalternation, as the following argument in the EOO-2:

No mammals are snakes
Some (!) humans are not snakes
∴ Some humans are not mammals.

Indeed, we must be careful not to judge the subaltern of a correct universal as absurd. Consider the following in the OAO-1 form:

Some fruits are not oranges
All apples are fruits
∴ Some apples are not oranges.

While to prove a statement factually wrong we must claim that current knowledge of the world is not only correct but exhaustively so, a self-contradiction is logically wrong and therefore relieves us of the empirical burden: it is a predication in which the predicate undoes the subject (rather than bringing the subject out as being or not being something).

The negation of a self-contradiction is necessarily a tautology (the predicate says the same as the subject, reasserts it at least in part). Tautologies prove to be useful in several ways. Consider, for instance, an argument in the AOO-3 form:

All dogs are mammals
Some dogs are not bulldogs
∴ Some bulldogs are not mammals.

To obtain a self-contradiction, we must first introduce a tautology and then combine it with the major premiss above:

All dogs are mammals (original major premiss)
All bulldogs are dogs (tautology)
∴ All bulldogs are mammals (by Barbara).

Now we have in our “system” both an O-form proposal and its contradictory A-form. This condition is already one of contradiction (the “self” being our self), but to obtain the requisite contradiction within a proposal we need only put the two contradictory proposals together:

All bulldogs are mammals (most recent conclusion)
Some bulldogs are not mammals (first conclusion)
∴ Some bulldogs are not bulldogs (by Baroco).

* See, e.g., A. Heyting’s *Intuitionism* (Amsterdam, 1956 & 1966), pp. 17-18: “in intuitionistic mathematics, only falsity ‘de jure’ can play a part; the introduction of ‘de facto’ falsity would conflict with the principle of constructivity.”
To obtain a self-contradiction in a mood concluding an I-form proposal, we must employ a very non-Aristotelian procedure: \( P \) must be the complement of \( S \). For instance, an argument in the III-1 form:

\[
\begin{align*}
\text{Some furry beings are non-dogs} \\
\text{Some dogs are furry beings}
\end{align*}
\]

\[\therefore \text{Some dogs are non-dogs.}\]

And in the case of forms concluding universal-form proposals, we must be prepared to consider self-contrariety as a kind of self-contradiction. For instance, in the EAE-3:

\[
\begin{align*}
\text{No collies are bulldogs} \\
\text{All collies are dogs}
\end{align*}
\]

\[\therefore \text{No dogs are bulldogs.}\]

As empirical, this last conclusion could, conceivably, be wrong. A singular \( S \) leads more obviously to a logically self-contradictory conclusion; e.g., for the AEE-1:

\[
\begin{align*}
\text{All collies are dogs} \\
\text{My bulldog is not a collie}
\end{align*}
\]

\[\therefore \text{My bulldog is not a dog}\]

And the fifteen invalid forms concluding a universal affirmative will require us to devise a conclusion in the strange form of all \( S \) are non-\( X \), where the \( X \) contains at least a part of \( S \); e.g. All bulldogs are non-dogs. And once again to accept self-contrariety as a kind of self-contradiction.

Finally, a form that has raised, and continues to raise eyebrows. As an example of Felapton we have:

\[
\begin{align*}
\text{No lizards are mammals} \\
\text{All lizards are reptiles}
\end{align*}
\]

\[\therefore \text{Some reptiles are not mammals.}\]

We want to conclude the E-form proposal, but the EAE-3 has already been proved invalid above. In this case, only a partially correct

\[\text{Interestingly, while the subaltern of this conclusion (Some dogs are not bulldogs) is not a self-contradiction, the subaltern of its converse (Some bulldogs are not dogs) is. For this reason, some logicians have wished to declare that E-form proposals must not be assumed to have existential import — unlike A-form ones (affirmation requiring an existential focal point). Such is the wish of Lewis Carroll in his Symbolic Logic of 1896. Others, like Leibniz, have wished to ban conversion.}\]
4. any term having such a universal belonging relation in the
conclusion have it also where the term appears in the
premisses.

The first and second rules (stated by Aristotle himself) touch upon the
primacy of affirmation. (1) If we have only negative premisses we can
only pry things apart and cannot bring P to bear on S even negatively.
Mephistopheles, the spirit of negation, cannot really argue, he can only
refute and seduce. In trying to argue with negation only, we commit a
fallacy we may call “total negation.” (2) But even if we do have an
affirmative premiss, the moment we introduce a negative proposal,
whether as the remaining premiss or as the conclusion, the other must
also be negative: we can neither obtain a negative from two affirmatives,
nor generate an affirmative with a negative premiss. In trying to argue
in either of these forbidden ways, we commit a fallacy which we may
call “unbalanced negation.”

The other two rules appear only in the Middle Ages and in the
course of contemplating a slightly different question: To what extent
does the belonging relation within a proposal bring a term to bear on the
individuals it covers? In singular proposals the predicate simply belongs
to or stays away from the subject and, moreover, to the entire “extent” of
the subject (because, as the Scholastics began to say, there is only one
individual to which the predicate can extend). In general proposals (the
other four) the belonging relations are more complicated. To make use
of Rules 3 and 4 we must become clear about these relations.

When we propose something about S in the universal affirmative, we
call to account all individuals under S: we must take responsibility for
the full extent of what the subject names — for each and every S.
Abstractly stated (i.e., eliminating reference to our own position within
the activity of speech): the A proposition fully distributes its subject
term.

When we propose something about S in the universal negative form,
we likewise call to account all individuals named by either term. Here, P
calls up only part of S and, rather underdemocratically (or perhaps stingily)
distributes its goods to this part (some individuals) only — or at least promises nothing more for the
moment. And the convertibility of such proposals again suggests that if
we consider S to belong to P, only part of P is the beneficiary. Our
commitment in such proposals extends only to parts (some instances) of
what each of the terms denotes. Abstractly stated: the I proposition does
not fully distribute either its subject or its predicate term.

When we propose something about S in the particular affirmative
form, we have it easy: we do not take total responsibility for individuals
named by either term. Here, P calls up only part of S and, rather
underdemocratically (or perhaps stingily) distributes its goods to this part
(some individuals) only — or at least promises nothing more for the
moment. And the convertibility of such proposals again suggests that if
we consider S to belong to P, only part of P is the beneficiary. Our
commitment in such proposals extends only to parts (some instances) of
what each of the terms denotes. Abstractly stated: the I proposition does
not fully distribute either its subject or its predicate term.

When we propose something about S in the particular negative
form, we deny that P belongs to some parts (individuals) denoted by S:
we take responsibility for withholding all P from some examples of S.
Or: for a portion of S, we refuse to distribute any of the goods named
by P. There is, then, a universality of non-belonging here claimed in our
understanding of P. Graphically: I must know all manners of smoking,
all possible manifestations and meanings of the term, to justify my claim
that “Some people are not smokers.” And the nonsense engendered when
converting O propositions (if not obvious in “Some smokers are not
people,” then try “Some Italians are not people”) suggests the asymmetry
in claims. Abstractly stated, then: the O proposition fully distributes
only its predicate term.

Following the terminology developed in the late Middle Ages, we
can read “fully distributed” for “universal belonging relation” in Rules 3
and 4: (3) in a valid argument the middle term must be fully distributed
at least once, and (4) any term fully distributed in the conclusion must
also be so in the premiss containing it. We may call the violation of
Rule 3 the “fallacy of undistributed middle” and the violation of Rule 4
the “fallacy of undistributed major (or minor) term.”

The semantic justification of the four rules raises questions about the
“agency” of distribution: Do propositions, do terms, or do we (as
speakers or listeners, readers or writers) do the distributing, the covering,
the knowing? And also about the meaning of “under”: What does it
mean to speak of individuals “under” or “denoted by” a term? Apart
from such questions, however, the rules mechanically work: even if one remains unconvinced about the semantics of the rules, one knows immediately whether to try proving the form valid or to try refuting it with an analogy. However, be it noted that the rules apply to all 256 syllogistic forms, and require us to endorse all 24, i.e. also the 9 depending on “conversion by limitation” or “absurdity by self-contrariety.”

For the special purposes of set theory, mathematical logicians have introduced a fifth rule: a valid syllogism may not have two universal premisses and a particular conclusion; and the violation of this rule they call the “fallacy of existential import.” The reason for introducing this rule is that universal-form proposals appear in formalistic work as based on conventional rather than natural names, so that they have no necessary relation to their subalterns. The only remarkable effect of the rule is that it eliminates from our list three forms that have intuitive credibility and logical interest as well: Darapti and Felapton in the third figure, and Fesapo in the fourth figure, function with an overload on the middle term (Bramatip, in the fourth figure, becomes Barbari upon conversion of the conclusion, and the five weak forms have no logical promise whatsoever).

§6.3 The path of 15

Aristotle analyzes syllogism as one kind of activity specifically belonging to us as λογος-conditioned. But he does so with a view to a still more specific activity of which we are capable: the activity of showing or discovering fully — ultimately of showing or discovering beings in their nature. This kind of complete showing Aristotle calls “demonstration.” Everything he says about syllogizing then serves both as part of a larger effort to understand our logical nature (thus all the examples drawn from public deliberation and schoolroom eristic) and as preparation for the narrower effort to pinpoint the basic issues of thinking (thus all the terminology drawn from ontological considerations). Ultimately, the philosophical purpose of logical study is to understand how these inside workings of demonstration, the ways we show or discover things, bring out the origins at work in our reasoning (origins embodied in the premisses, never in the conclusion!).

But we may wish to stop short with syllogism itself and not travel on to demonstration — either because we do not believe that the showing of basics must or even can occur in syllogizing, or because we have enough trouble already with syllogizing, or simply because we find our pleasure in syllogizing at a distance from beings (the pleasure Plato and Aristotle detect in geometricians and arithmeticians). Once demonstration (full showing) falls out of consideration, the study of logic becomes strictly “formal” i.e. severed from “content.” Such is the modern tendency, one that brings in its train a wholly different conception of λογος, our linguistic capabilities. Now that we are at the crossroads, and before traveling further down the original path, let us take a brief excursion down the modern path which we shall explore more fully in Book Four.

In his Letters to a German Princess, the Swiss mathematician Leonard Euler wishes to explain in pictures the various interrelations of the terms in propositions.” Instead of worrying about S “coming forward as” P, or about P “belonging to the heart of” S, simply imagine four a syllogism in which knowledge occurs” (Posterior Analytics, 71 b 18: so that the premisses themselves must be “true, primary, non-mediated, meaningful to us and preceding as well as grounding the conclusion”). More: the “beginnings,” the premisses of any demonstrative syllogism, must also be necessary, inexorably and already so, in order to justify the derived necessity of the conclusion (ibid., I, 6). “Of the figures, the first is best for knowledge” (ibid, 79 a 17: for in this figure we most naturally syllogize and come to contemplate both how things come about and what they are). Of course, all this leaves open the question how we tune ourselves into the requisite beginnings (ibid., 99 b 17: the topic of Book Three). Yet you need not wait for examples of demonstration: you only need to consider some of those I have already offered in the process of illustrating categorial transcription of arguments and proposals; for in each case we position ourselves in the face of things — in the truth — whether or not we rest content with the given formulation. “While neither syllogism nor demonstration can come up with what a thing is, this [a beginning] comes out [gets revealed] in the course of syllogism, i.e. demonstration” (ibid., 93 b 16).

See the letters dated 7 February through 3 March, 1761 (pages 153-80 of the 1843 edition: Lettres à une Princesse d'Allemagne, Paris); I have, however, modified the way of diagramming particulars. While clearly insisting on the spatial and visual significance of his method, Euler continues to endorse 19 forms (all but the five weak ones).

* “Demonstration is a kind of syllogism” (Prior Analytics, 25 b 30: therefore we must begin our lessons with syllogism in general). “By demonstration I mean
different arrangements of two corrals each; and to visualize the basic logical relations possible for the two terms (e.g., humans and mortals) imagine gathering everything called “human” into one corral and everything called “mortal” into the other — allowing, however, for dual membership. The A proposition you may imagine as claiming that the corral of “humans” will have to be located completely inside the corral of “mortals”:

```
       Mortals
        ⊆
       Humans
```

whereas the E proposition you may imagine as claiming that the two corrals have nothing to do with one another:

```
Humans       Mortals
```

Then you can imagine the two particular propositions as visually requiring some overlapping — but also, in these cases, some indication of where the “human” is that you are talking about. The I proposition looks like:

```
Humans       x
             ⊆
             Mortals
```

with a “x” in the intersection to show that there is at least one “horse” in that sub-corrals, i.e. in both corrals. And the O proposition looks like:

```
Humans       x
             ⊈
             Mortals
```

where the “x” indicates at least one “horse” forbidden to graze in the sub-corrals.

In these pictorial versions of the four general propositions we must play upon our intuition of space to visualize “corrals” (classes, sets) which have “horses” (members) in them. For the two universal forms we must not indicate membership but notice only that if there are any “horses” in the corral for humans then they are also included within (in the case of the affirmative) or excluded from (in the case of the negative) the corral of mortals. However, in the case of the particular forms, we must indicate membership even to distinguish between the affirmative and the negative. Thus, already in reconstruing categorial discourse into class relations we have revalued the propositional forms: the particular says more, not less than the universal. Also, we have undone the primacy, even the meaning of the subject: whereas Aristotle insists that the subject be a being and the predicate a way in which the being may evolve, the mathematical reconstrual leaves us with terms indistinguishable on these grounds, each naming a description which may or may not fit givens. Universals are indifferent to whether anything is given to fit, and particulars insist only that there be something to fit.

To show the relation, if any, between two premisses and their proposed conclusion, we may say to our princess: You need only superimpose the two pictures and ask whether you can read the conclusion in the result; if you can, then the conclusion follows “of necessity”; and if not, then not. For instance, the AAA-1:

```
All M are P
All S are M
∴ All S are P
```

where P = the largest circle and M = the middle circle — which then contains S = the smallest circle. You can now see that any horse grazing in S will also graze in P. And so we finally have living (i.e., pictorial) proof of this otherwise unproven form — a form only assumed, for millennia, to be intuitively valid. A little practice with pen and paper will allow you to see that the conclusions are readable in superimposed pictures of the premisses in all 15 syllogistic forms, i.e. those of the 24 which do not commit the “fallacy of existential import.” For instance, the OAO-3 looks like:

```
Some M are not P
All M are S
∴ Some S are not P
```

where the “x” illustrating “Some M are not P” is also included in S and excluded from P. Interestingly, if you go through all 15 forms you will notice that there are only six distinct diagrams — one for each of Aristotle’s four paradigms, one for the OAO-3 and one for the AOO-2
Syllogism: Truth and Validity

(Bocardo and Baroco — the two Aristotle proves valid per impossibile). In a sense, then, there are only six distinctly valid forms; the other nine look the same as one or the other of Aristotle’s basic four, and therefore are the same — mathematically.

You may see that the premisses necessitate the conclusion in each of the 15 valid forms. You can see it with the eyes in your head, we shall tell our princess, and you see it on the paper in front of you: reasoning has taken on a visual manifestation, and you see that it is right. As teachers and students we may all rejoice in such seeing, since it gives us confidence in our decisions and allows us to show our decisions to others. Yet as philosophers we cannot forbear noticing that the seeing here has bypassed the kind of seeing which Aristotle and his tradition assume belongs to our nature rather than to our eyes: it does not at all focus the “eye of the soul” on “beings in their Being.” And on the development of this focus depends both theoretical demonstration and effective practice. And without it we have what Aristotle calls reasoning, talking, or thinking (λόγος), but no intelligence, intellection, or insight (νοῦς).∗

Reconstrued and shown visually, syllogizing no longer takes place in moods and figures. We may continue to line up premisses and conclusions in the traditional way, but simply for the sake of our booklet of 256 pages. For the purpose of showing the reasoning, we need only the two premisses in each case, and the proposed conclusion to look for in the superimposition. Indeed, if there is need for any order at all, we would place the universal premiss first in those arguments containing a particular premiss: as our princess will soon discover, we must here draw the universal picture first (so that when drawing the particular we will know where to put the “x”).

And as subjects and predicates move from logic into grammar, the ontological distinctions like privation vs. absence, necessity vs. possibility, potentiality vs. actuality, all become superfluous or get conscripted into some use devoid of any reference to reality. For their

∗ Cf. Aristotle’s Nicomachean Ethics, 1096 b 29: “as sight is in the body, so νοῦς is in the soul.” As already noted, Plato and Aristotle both aim to develop the “eye of the soul.” One may read Kant’s three Critiques as the first monumental effort to demonstrate, taking modern science as a paradigm, that νοῦς (which Kant translates and criticizes as “intellectual intuition”) does not allow of self-certification even though it is necessary.

Validity and Invalidity: Path of 15

basis, namely the mystery of belonging, now appears as a mere metaphor of class membership, which is better expressed in spatial relations.

Following Euler, our princess will find singular propositions a bit embarrassing. In Aristotle’s account, singulars lurk as the often silent but still essential partners. Now, however, a singular enters the pictures either as a class with only one member or as an “x” with a proper name, so that an AEE-2 about Socrates might look like

All P are M
S is not M
∴ S is not P

where we allow Socrates either to hog an entire corral or to graze anywhere outside M — in either case he is visually excluded from the inner Pasture.

An obvious disadvantage of Euler diagrams is that they become messy when we try to picture premisses which will evidently not allow us to read the proposed conclusion — i.e., when testing syllogistic forms which prove to be invalid. While accurately representing the superimposed premisses, we must often resort to broken lines showing possibilities (and thereby the non-necessity of reading the conclusion — without, however, suggesting any other necessity). Consider, for instance, the AEE-1:

All M are P
No S are M
∴ No S are P

where we first draw one premiss (here, the minor: showing the exclusion of S and M) and then the other (here, the major): but then we notice that there is nothing in the diagramming of P either to prevent or to necessitate its stealing over into S. If we had allowed our template to move a bit to the right we might have drawn the circle for P neatly away from S; or a move to the left would have firmly ensconced at least a segment, maybe even the whole of S within P; and our template would have become the adjudicator! A broken excursus saves the day, but we would obviously have to alert our princess to such dangers: and only some sort of extra-visual attention on our part will avert disaster.

In his Symbolic Logic of 1881 the English mathematician John Venn presented another technique for showing the reasoning embedded in
arguments, a technique not only much neater visually but also more suited semantically for the development of set theory. Instead of relying on three operations (exclusion, inclusion, and indication of existence) we can proceed with only two: negating and positing (or, exclusion and indication). Starting out every time with two overlapping circles (sets), we can diagram each of the four general propositions as follows:

<table>
<thead>
<tr>
<th>All S are P</th>
<th>No S are P</th>
<th>Some S are P</th>
<th>Some S are not P</th>
</tr>
</thead>
</table>

where each shading represents an empty sub-set and each “x” indicates a non-empty sub-set. Both universals appear as negating existence, both particulars as positing existence: no mean re-assessment of the ancient distinction, which assumed that we must first distinguish between affirming and denying, and only then (derivatively) between whole and partial predication. And in a sense both the particulars require us to think negatively: to avoid naming the existence posited, we speak of sub-sets as non-empty (admittedly a double negation).

Certainly one of the chief advantages of Venn diagrams is that they allow us to begin testing any argument form with the same schema of three circles:

\[ \begin{array}{c}
S \\
M \\
P
\end{array} \]

where the order of the two top circles represents (in western languages) our visual direction of reading the relation between S and P. Given any syllogistic form, we need only map each of the premisses, negating and positing S and P in relation to M. For instance, the All-I:

\[ \begin{array}{c}
S \\
M \\
P
\end{array} \]

where shading out all M which is not P represents the major premiss (M which is not P is empty, MP = 0) and installing an “x” in the intersection of S and M (where space remains) represents the minor (M which is S is non-empty, MS ≠ 0). Both premisses are now readable in the diagram, and I need only ask: Can I read the conclusion? Relying solely on the eyes in my head, I must answer: Yes. And I have shown this paradigmatic argument form to be valid.

For displaying invalidity, let us consider the AEE-1:

\[ \begin{array}{c}
S \\
M \\
P
\end{array} \]

which shows both premisses in cooperation but also that there may be some members in the remaining sub-set of S and P: quite literally, then, the diagrammed premisses leave the proposed conclusion open; they do not necessitate the conclusion, therefore we must write “invalid” on this page of our booklet. In general: an invalid argument concluding an A or an E will leave some space open which would have to appear closed (shaded out) to require our eyes to read the conclusion; and an invalid argument concluding an I or an O will leave the “x” (if any) straddling two sub-sets and likewise leaving the conclusion undecided. E.g., the AOO-I:

\[ \begin{array}{c}
S \\
M \\
P
\end{array} \]
where we must be a bit careful when inscribing the minor premiss: our pen must sweep over the entire sub-set of \( S \) which is not \( M \) and place the “\( x \)” on any intersecting line we find there (to show that this horse may graze on both sides of the fence).

Construed as basically negative in import (because behaving like universals), singular proposals pose no special problems in set-theory: as mathematicians we can simply say that an argument about “my wife” requires a class of one member only, so that “My wife is beautiful” reads: the sub-set composed of things (i.e., of the one thing) called “my wife” and not called “beautiful” is empty (\( WB = 0 \)).

\[
\text{My Wife} \quad \text{Beauties}
\]

However, whether as philosophers or as linguists, we notice that the semantics of the original proposal does not, as does the set-theory interpretation, leave the question open whether the sub-set \( WB \) is empty, but rather insists (in any actual occurrence) that the sub-set is not empty (in which case two acts seem to be required: negating and positing) and that the existence indicated is the only one (as the set has in fact been defined) so that the “\( x \)” is identical to my wife. To avoid the multiplication of acts here, we could inscribe the singular “\( W \)” (= my wife) from the start as occupying the single set of “beautiful things”:

\[
W \quad \text{Beauties}
\]

However, in postponing the question of “existential import,” the set-theory version does allow us to consider proper names of individuals as possibly fictitious: in the mathematical spirit, we can first figure out the relations and then ask the supplementary question whether the term applies to anything. From an historian’s standpoint, for instance, it is certainly conceivable that, in some sense, Homer’s Achilles has no existence — just as Homer and even Napoleon may have none. Moreover, in courtrooms one may legitimately ask whether a proper name really intends to indicate existence and, if so, how (the question of “legal persons” and “limited responsibility”). Many letters you receive from large businesses appear with a “personal” signature indicating no existing person but rather a function within the organization. And Thomas More wrote a book about a being which, the title (\textit{Utopia}) assures us, exists nowhere — following Plato who wrote about the nature of a city, which can only be “in \( \lambda \delta \gamma \gamma \zeta \), nowhere seen on earth” (\textit{Republic}, 592). If “existence” means anything like “readily available as a datum, independent of anything we do to actualize it,” then only during the Fourth Interpretation do intellectuals worry much about it. For during the earlier Interpretations “existence” was bound up with the dynamics of actualization, in which we must first learn to act (\( \pi \rho \delta \zeta \zeta \), “practice,” precedes and conditions our discovery of existence, Plato says in that passage just quoted).

Liberating reasoning from the traditional claims of subjects, predicates, and figures, set-theory also liberates us from the fixation on two premisses. We can, for instance introduce \textit{three} premisses to handle actual arguments appearing initially in the form of AAI-3:

\[
\begin{align*}
\text{All } M & \text{ are } P \\
\text{All } M & \text{ are } S \\
\text{All } GT & \text{ are } P \\
\text{R} & \text{ are } P \\
\end{align*}
\]

\[
\vdash \text{Some } S \text{ are } P
\]

where diagramming the first two premisses leaves empty space shared by both \( S \) and \( P \), a space then declared non-empty by the third premiss. For the “subaltern moods” we need “\( S \) is a non-empty set” and for the AAI-4 “\( P \) is a non-empty set” to obtain readably valid arguments.

And we can even conclude, from three universal premisses, that a set is empty. Consider, for instance, the argument: “All good teachers are people who devote at least 10 hours each day to preparing for their classes, but they must also devote 10 hours every day to their own research; so, since no one who devotes 10 hours to preparation can devote another 10 hours to research, there are no good teachers!”

\[
\begin{align*}
\text{All GT} & \text{ are } P \\
\text{All GT} & \text{ are } R \\
\text{No } P & \text{ are } R \\
\end{align*}
\]

\[
\vdash \text{GT is an empty set}
\]

And from this example we can understand the practical necessity of deciding that “contraries” are \textit{formally} compatible. Given simply the
names of two sets, S and P, we can find ourselves legitimately and even necessarily affirming both “All S are P” and “No S are P”:

\[
\begin{array}{c}
\text{All S are P} \\
\text{No S are P} \\
\text{S = 0}
\end{array}
\]

More fully stated: (1) once we have decided that S (e.g., “good teachers”) is an empty set (that S = 0), we necessarily affirm both contraries at once (whatever P is brought into contact with S), for only the two together will assure that there is no room for any “x” (i.e., prevent the affirmation of either contradictory); on the other hand, (2) so long as we have not decided the “existential import” of S (i.e., so long as we acknowledge that both “S = 0” and “S ≠ 0” are possible), it is possible to affirm either the A or the E and even possible to affirm both at once. Of course, whether and how we learn or decide that S is empty or not is another question entirely; but (3) once we have in fact decided that a given S is not empty, the older Aristotelian interpretation of contrariety holds: it is impossible to affirm both the A and the E.

Meanwhile, we can see why it is absolutely necessary, once we embark on the modern project, to construe universal propositions themselves (apart from supplementary considerations) as semantically negative in import — as negating sub-sets rather than positing existence of any kind. For in the modern project terms or names, classes or sets make sense primarily as descriptions: they are conventional rather than natural. And although we may be able to legislate these descriptions we cannot, in the end, legislate existence, i.e. declare by fiat that things exist to fit our descriptions: we must decide existence separately.

Although the obvious purpose of John Venn’s diagramming technique is to develop a mechanical decision procedure, the diagrams already suggest and ultimately require us to envision the three sets against a background set — called the “universal set”:

\[
\begin{array}{c}
\text{U} \\
\text{S} \\
\text{P} \\
\text{M}
\end{array}
\]
to “items of private property” — a grammatical joke, as Wittgenstein calls it (Investigations, §111).

There are other illustrations of the usefulness of thinking backgrounds. For instance, we are told as children that we cannot add 3 apples and 4 oranges. Yet both apples and oranges emerge, as Plato and Aristotle say, from a genus (e.g., fruit) or, as John Venn and Lewis Carroll say, from an incidental universal set (edible things, spherical things, juicy things). Against whatever background we choose, we may now count 7 of them.

We might also ask the background set to carry the burden of the Aristotelian insistence that every legitimately formed proposal display a being in the subject position. Stendhal’s “All competent reasoning gives offense” and our own “No syllogisms having two negative premises are valid” make sense to us without verbal reference to a being because both stated subjects are performances (category of doing) against the background of “human beings.” Indeed, the universal set of much significant discourse is precisely “human being,” and for this reason we need not keep mentioning it. Similarly, at a horse race we need simply remark, “That one is really good”: until otherwise announced, the universe of discourse is obviously “horses racing,” and so we need not complete the subject with “horse” and the predicate with “at racing.”

Some of the puzzle of empty sets might dissolve once we recognize that sets require a shared background. A conventional name such as “people who are over twenty feet tall” may refer to nothing, in which case universal proposals in which this name occurs in the subject position will have no existential import. Accepting the necessity of a universe of discourse in any case, we can propose that conventional names, being compounds, are sub-sets, and that each genuinely basic component, drawn from a background set covering them all, necessarily refers to, stems from, things in fact encountered (existent: here, people and things twenty feet in height). Yet we might still wonder about fictional names — names of people like Little Red Riding Hood and Hamlet, and of places like More’s Utopia and Bacon’s Atlantis.

And riddles such as the Mad Hatter’s “Why are ravens like writing desks?” illustrate the way we play tricks on universal sets: things that produce flat notes, things associated with tails/tales, things on which Edgar Allen Poe wrote, things whose names begin with the sound “ruh” (r, wr). The grammatical joke in these cases consists in formulating a universal set that defies existence: the individual words supposedly naming the background set for the sets “ravens” and “writing desks” are homonyms or homophones, so that each “answer” shifts attention from existence to the (ambivalent) formulation itself — whereupon we may as well answer the riddle by shifting the scene altogether, replying, “Because there is a ‘b’ in ‘both’.”

Interestingly, Aristotle and set-theorists alike agree that dealing with (understanding, forming) even one term (idea or set) requires a background, a universal set. In Book Three we shall consider how a “real definition” creates a focus on something by drawing the “something” out of a genus. And in Book Four we shall note the mathematical necessity of construing a set always as part of a larger set (Zermelo’s Axiom of Separation).

Logical study aims predominantly at exactitude in the middle-ground: at methods of “exactizing” formulations and inferences. As it turns out, however, all exactitude in the middle-ground depends on, draws upon, refers us to a background that we never fully exactize: for to do so we must bring elements up into the middle-ground having, once again, a background. Furthermore, exactitude in the middle-ground issues from operations we ourselves perform: stretching the metaphor, we might say that these performances lie in the foreground (our position). And this foreground, too, defies any effort to exactize it in abstraction from the moment of performance. Although we might perform exactly, the exactitude remains adverbial: we may be very exact in formulating a rule, but we cannot formalize a rule for applying the rule without leaving ourselves with yet another rule requiring once again an open (free) judgement on our part. The study of logic becomes and remains of philosophic interest when we find ourselves mediating (embodying, enacting) the interplay of back- and foreground on the middle-ground.

Thanks to notes by Martin Gardiner in his Annotated Alice (1960, Penguin Books 1970). We shall return to the theme of homonymity in Book Two; Aristotle also interprets the event as one of shifting attention from πράγματα, the proper focus of λογοι, to words themselves.

On the limitations of formalizing rules, see Kant’s Critique of Pure Reason, A134/B175 and Russell’s Principia Mathematica, Summary of 2.
§7. Conventional vs. natural naming

In one obvious and important sense, all Paths agree on the results (whether we number them 24, 14, or 15): on each we can handle all the results once we understand in the mode of possibility such logical concerns as the syllogistic figures, the negating of terms, and the positing of existence. Interestingly, too, both extremes (the method of ontology and the method of diagrams) leave us with only six basic argument forms.

Yet to obtain their respective results, the Paths obviously diverge in some very important sense: at one extreme we are asked to understand the whole process as one in which human beings reason about natural names naming natural beings, and thereby bring to light (show, demonstrate, but in premises and not in conclusions) the nature of things, beings in their Being. At the other extreme, we are asked to understand the whole process as one in which we, as logicians, devise ways in which we can reason with conventional names and thereby bring to light (show, demonstrate) our own reasoning as an activity which suspends our relation with beings—in which we “show our stuff,” our own being.

How did this difference evolve? Why the change in, the reversal of direction?—If we pose these questions in a strictly modern frame of mind, we likely direct them all to the ancient Path, and wonder how anybody could have believed in species, in belonging, in privation, in actualization, and the like. And no matter how open-minded we are, no matter how interested we become, we will learn nothing about our present condition, i.e. the Fourth Interpretation. If, on the other hand, we can reverse our position for a while, pondering these questions in an antiquarian frame of mind, we can direct them forward: How could modernity ever branch off from the earlier Interpretations? How could our forefathers ever come to deny that our condition is nature-based (naturwüchsig, as Marx acknowledged in celebrating the development of modern organized industry and insisting upon our destiny to become community-based)? How can we today believe that we can think independently of the dynamism of nature, returning to it only later as a more or less important afterthought?

It is easy enough to trace the development backwards from modern textbooks to Venn to Boole to Euler to Leibniz to Descartes—i.e., down through all modernity. Some have even traced it back to Plato and beyond, discerning in the interstices of the ancient passion for understanding “principles” the first seeds of dissension finally ending in the divorce of λογος and φυσις, human articulation and the nature of things. But we can still ask what drove modern intellectuals to file for the final divorce. And if we listen to the first petitioners (Bacon and Descartes, for example) the grounds soon become clear, at least in outline: their immediate predecessors, the Scholastics (the Schoolmen, the professors plying their trade in the name of the Church and, usually, of Aristotle), concentrated their attention on the subtleties of linguistic forms in judgement and argument rather than on the ways we relate to, are already bound within nature herself. The moderns initiated a new science, a new logic, not to get away from nature but to get back to her. The divorce in the intellectual domain followed both from a weariness with school philosophy and from a love affair out of school. As the moderns soon discovered, however, the paramour firmly resisted the advances of her suitors, with the result that the adventure terminated in a loneliness unprecedented in all recorded history.

In the Scholastic concern for the “properties of terms” we can detect the seeds of the ontological dissonance which soon revealed the grounds for divorce. For instance, the property of “distribution” for justifying the rules for valid syllogism: this property of S, P, and M requires us to ask whether the term’s position in a categorical proposition leads us to think the term as referring to all or only to some of the individuals it names. However, this form of the question reverses the original direction. Originally, Plato and Aristotle ask: How do proposals bring beings out in their being? How do predicates belong to beings? How does an S come out as P? How can P enter into a belonging relationship with S? This original direction starts with beings and asks how they arrive in speech, how they give rise to λογος. Looking to examples of speech, we

* One can also read Henry Adams' two great works, Mont-Saint-Michel and Chartres (1905) and The Education of Henry Adams (1918) as efforts to understand modern “multiplicity” from the pre-modern standpoint of “unity”
here ask not how terms apply to things but how beings themselves generate (and lead us to notice) various belongings; and in looking at this belonging relationship we ask not how each term separately refers to individuals, but how the belonging of P to S (mediated or not) stems from primary beings revealing themselves as and for what they are. Although the Scholastic direction of inquiry no doubt reflected an authentic wonder at the power of λόγος to orchestrate the emergence of beings and our own clarity about them — although very pious and ancient in origin, the new questions silently locate us and our language on one side (in the library of a university or a monastery) and reality or nature on the other (the outside, as intellectuals have increasingly said). From then on, and most remarkably in the dissidents of modernity, the question of distribution becomes one of whether one's thoughts and words refer to anything at all — no longer whether distribution is full or partial, but whether something exists corresponding to the given linguistic term (and, if so, what form it can take and what name it deserves). Starting not with beings but with names, we are henceforth plagued by questions of reference, and we must eventually admit that names have no natural, but only conventional power and warranty.

To appreciate the extent to which the Schoolmen began to dote on names we may turn to an early 13th-century textbook on logic, William of Shyreswood's *Introductiones in Logicam*, which distinguishes nine modes of predication.* We are asked, first, to distinguish between two basic varieties of belonging (in Latin: *suppositio*, “under-placing”): the “material” and the “formal.” In saying (1) “Man is monosyllable” we take the predicate as belonging to the *material sound* of the subject; in saying (2) “Man is a noun” we take the predicate as belonging to the *material sound* of the subject; in saying (2) “Man is a noun” we take the predicate as belonging to the *material sound* of the subject. Obviously, only *formal* belonging puts us in the vicinity of categorial discourse. However, in saying (3) “Man is a species” or (4) “Man is the noblest of species” or (5) “Men fall victim to hay fever in Rome as well as in Athens” we already have three kinds of “simple formal belonging”: the “manerial” (we identify a name as signifying a species, but we say nothing that would help anyone understand what is signified), the “repetitive” (we repeat the species and then say something about *it*, and then also about any individual but only inasmuch as he represents the species), and the “vague” (the subject is not touched in its species but only in its accidents: *some* people having hay fever are in Rome, *some* in Athens — the subject is not fully distributed). And now, on the home stretch, we can distinguish four kinds of “directed formal belonging” (“personal belonging,” the Latin reads): in saying (6) “This man is running” we say something determinate; in saying (7) “Every man is somehow or other going to die” we take the predicate not as determining anything specific about the individuals under discussion (it remains “merely confused” and does not tell us much); in saying (8) “Every man is mortal,” and meaning this as a statement about human nature, we take the predicate as saying something definite about the species and therefore about all individuals covered by the species (the predicate is “distributively confused” — not “messy” but “blurred” in reference to individuals); and in saying (9) “Every man fails to do at least one thing he has set out to do in life” we take the predicate to belong confusedly and distributively to the subject, but no definite meaning we give to the predicate will fit all individuals named by the subject (the predicate becomes “immobile” as soon as it becomes definite: *some* people fail to fulfil their desire for riches, *some* their desire for adventure, *some* for love).

Aristotle, you recall, belabors only four modes of predication: definition, genus, property, and accident. And *his* four start with the most essential and derive the other three by analysis of the essential. Aristotle also illustrates the meaning of “material” by referring to the letters of the alphabet (and even to *premises* as “material” for argument: *Physics*, 195 a 15); but on his account the “material” of anything is either its aspect when destroyed (letters and proposals deprived of their function...
in human response are then only materials, just as the corpse of a man is material, a cut-down tree is material) or its aspect when entering into production (for the farmer soil, wind, sun, and rain water are materials; for the potter clay, air, fire and fetched water are materials). In any case, there is here no sense in looking to the materials for understanding any kind of belonging: one must look to the purpose as evident only in the actualization of potentiality: just as a severed hand is a hand in name only, so a name severed from its (our) response to beings in their being is a name in name only.

In silently assuming names as severed, the Scholastics certainly aid the shift from natural to conventional naming. From this assumption it is just a small step for Descartes to insist upon a split between thinking (λόγος as our activity) and being (φύσις as what we must face and fathom); this split immediately engenders, first, a transformation of “thinking” into an activity independent of our supposedly “generic” nature as “animal” (living within circumstances), and, second, a transformation of “nature” into geometric interrelations (“extended substance,” Descartes says). And from here it is no step at all for Arnauld and Nicole (Churchmen, no less) to write in their textbook, subsequently named after their Abbey, that “meaning” (intension) is one thing, “application” (extension) another. For a while (say, 1600-1900) the two domains appear synchronized by a Third Factor of timeless belonging: thinking clearly and distinctly, we fathom God's eternal order (the laws of nature), whereupon we simultaneously understand what we in fact face (the events or givens or data of nature). But what if the Third Factor drops out of the picture? Nothing then guarantees that thoughts by themselves, no matter how clear and distinct, rightly apply to any reality; rather, they may serve more to insulate us from reality. In any case, thoughts (and names) must henceforth win their legitimacy in direct battle with the space-time grid of actual experience — where conventional naming appears urgent and natural naming impossible.

Notice, however, that an historical sketch only illustrates, does not answer the question. We might still wonder how the Scholastics, and all modernity afterwards, could reverse the direction of naming. But even this question assumes that the original direction was indeed natural rather than simply naive or foolish. Even to contemplate the significance of the reversal we must somehow convince ourselves that the original has, or at least had, its own substantial justification. In an antiquarian mood, then, let us ask: How could our ancient forebears assume that names (and therefore terms in an argument, and so also validity) were natural rather than conventional? How could this assumption arise and work? What, in fact, does it mean?

As antiquarian philosophers we must first ask: What counts, for Plato and Aristotle, as basic evidence? The answer is that of the Second Interpretation: man the maker, homo faber. Making (producing, creating) here appears not only to philosophers, but also to poets and statesmen already in the age of Pericles, as naming the basic human activity and therefore the basis of the human condition (our lurking destiny); and it has remained a cornerstone of western civilization despite all the subsequent alterations effected by the Third and Fourth Interpretations. Yet this determination of human being entails a special responsibility on our part: we only are, we only actualize our condition, by learning and exercising an art (a τεχνη). And an art, unlike the technology of the Fourth Interpretation, requires that we artisans “complete what nature is unable to finish” and that we (as potters or farmers or shipmasters or statesmen) do this finishing work by following nature.* Thus making things requires of us, in learning and exercising an art, to participate in the hidden nature of things, and to acknowledge this participation and this hiddenness: it requires that everything made for human use be primarily nature- rather than community-based: natural before being artificial.

And names as well. Except that we in fact (“in time”: in growing up, and in helping others to grow up) start with names already made by or predecessors: by our parents and teachers and friends, our oral if not written traditions. We hear and talk about horses and pots and cities long before we understand these things (thus we learn our mother tongue easily, whereas our desire to understand names slows us down in learning a foreign language). We come to understand only very few things — namely, those things which we learn to finish by following nature. Thus, already in learning an art we must overcome conventional meanings for the sake of natural meanings: we must learn to speak out of the hidden ways of nature rather than the familiar ways of society. And one whose art is essentially one of talking (most obviously: the statesman, the poet, and the philosopher, but also any genuine parent or teacher) must

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* δλος τε ἡ τέχνη τὰ μὲν ἐπιτελεῖ ἡ δὲ φύσις ἀδύνατον ἀπεργάσθαι. τὰ δὲ μεμείται. (Physics, 199 a 15)
expressly work on regrounding names: must assume that names originally arise from someone’s insight into hidden ways of nature, must recognize that they now appear to others only in their conventional meanings, and must lead others back to nature (including their own) where names will come back to life. Moreover, just as practice (πρᾶξις; creation and preservation of communities) takes its evidence from artisanal human nature (follows and finishes it), so also theory (θεωρία, contemplation) elicits the natural conditions of production (ποίησις) and action (πρᾶξις) — for those who wonder. In contemplation, then, names appear explicitly naturally — perhaps “for the first time,” since makers and actors work with such names rather than on them.∗

Here we have a clue, a possible answer to the riddle: whatever the reasons (again, a question!), the fact seems clear that sometime in the Middle Ages intellectual work abandons the evidence of artisanal human nature, ceases to draw this evidence out, and retires to the study: contemplation then directs itself to created human nature (as is only appropriate during the Third Interpretation) and draws out the evidence of divine λόγος. And this monastic contemplation necessarily asks, not how names arise in the learning and exercising of an art, but how human beings can relate divinely given but humanly corrupted names to creatures. Such is the complaint levelled by Bacon and Descartes: their predecessors had dwelled too much in their own minds and in their cloistered studies. Yet the moderns, in leaving the study to read the book of nature, take their first encounter with the fresh air and soil, dew and sunlight, as the evidence: they take not artisanal, not created, but

∗ For Plato’s suggestion that our initial condition leaves us with names whose “functions” we have forgotten (or with personages whose “deeds” we have forgotten), see his Critias, 109D ff. (plus the “recollection” discussed in his Phaedo and the imagery of “forgetting” discussed in his Laws, III); that λόγος owes its origin to insights, see his Sophist, 259E; that “no man of sense will entrust his soul to names” as they first appear, see his Cratylus, 440C. For Aristotle’s suggestion that in public discussion we start with the conventional meanings of names, see his Topics, 148 b 20; that a name lands us at first in a conventional universality which must be broken, in the course of λόγος, to reflect nature (just as “a child begins by calling all men father and all women mother, and makes distinctions later on”), see his Physics, 184 b 10; that knowledge is actualized by bringing the universal (which risks being merely conventional) down to individuals, i.e. beings primary in and by nature, see his Metaphysics, 1087 a 11.

observing human nature as the basis of human articulation (as befits the Fourth Interpretation). And here names can only serve to record and transmit observations: they must be merely conventional — precisely to distinguish the computations of the study from first and subsequent encounters with nature.∗

The Path clearly divides on the question of contemplation — on the question of where and how we stand in contemplating the nature of things. Pythagoras’ reply to a ruler’s question about the meaning of “philosopher” serves as a kind of Rorschach test you can apply to yourself or others to determine the Interpretation most instinctively espoused. Pythagoras says that to him:

the life of man seems like a fair (mercatus) at which are celebrated the great games of Greece, with all their ado. For here some strive with trained bodies for the glory and the nobility of the crown, others are drawn to seek wealth in buying and selling, while there is also a certain type, and these are the noblest, who seek neither applause nor wealth, but come for the sake of seeing and eagerly perceiving what is done, and in what way. So we too, as though having left some city for a fair with much celebration, proceed to our present life out of some other life and nature, some serving glory and some serving money, but

∗ In his lengthy work La Connaissance de l’individuel et la logique du réalisme (Neuchâtel, 1975), J.-Claude Piguet broaches the same fundamental question. He argues that the original λόγος, in which things and their names were somehow united, has progressively, in the course of western intellectual development, broken up into three orders: the order of things as we can perceive them, the order of ideas which (since the Middle Ages) can exist independently of things (as objects of our own thinking), and the order of names by which we articulate the ideas of things. Given this historical development, Piguet argues, we must now learn to undertake a “semantic reversal”: we must learn to consider neither the order of ideas nor the order of names as signifying (setting the standards of meaning for) reality, but rather to let reality itself give sense to these two other orders. Thus, at our present stage of history, we must consider names as neither simply natural nor as simply conventional. As we shall see in Book Five, logical considerations themselves lead to some such conclusion. However, without a developed sensitivity to the original Greek understanding of λόγος one likely fails to fathom the contemporary exigency for “semantic reversal.”
If you prefer the life of competition, scoff at the money-makers, and look at the spectators only as bestowing the honors, you endorse the First Interpretation, will worry about deserving a name, and appreciate Homer's use of ἐιδος (striking form of a person). If you understand the life of contemplation to be one of recollecting the nature of things as dimly evident already in your “previous life” of learning and exercising an art (and, negatively, in the temptations to settle for money instead), you endorse the Second Interpretation, will worry about how beings give rise to names, and appreciate Plato's use of ἐιδος (the hidden form of beings at issue in the performance of any art). If you understand the life of contemplation to be one of service to the Creator of beings rather than to glory, to money, or to natural bases of human effort, you endorse the Third Interpretation, will worry about the meaning of divine names in the original λόγος, and will appreciate the later Latin translations of ἐιδος (eternal species, or divinely created forms of nature). And if you understand the life of contemplation to be a sideline activity that in no essential way attends to “previous lives” but rather constructs a life in the observation of things passing under review, you endorse the Fourth Interpretation, will devise systems of conventional names for organizing (interrelating and predicting) observations, and will appreciate the modern use of ἐιδος (from Descartes and Locke through Hume and Kant translated as “idea,” and meaning something lodged inside and referring, possibly, to something outside).*

And it may be that many, even at our universities, will endorse none of the Four, but prefer to mull around in some crowd, hawking popcorn or hotdogs: and these will laugh at naming, declaring it to be a totally arbitrary affair and names to be simply tokens of exchange (whereupon}

some few will take pains to invest them carefully for themselves and to accumulate tremendous power over the crowd). Yet intellectuals who shrink from such vulgarity (the political and social consumerism generated in and by the Twilight of the Fourth Interpretation) must still decide where they might go — what it can mean, in Pythagoras' simile, to sit in the grandstand of life or, in another image from Cicero's work (II, 25) “in peace, at home, in our easy chairs.” Are we to back into one or the other of the Four? But then, unless we become very clear what it means to be the “noblest” (maxime ingenium) or the “best” (liberalissimum), we likely fail to find a definite seat, and wander from one to the other, confusing both the games being played and our own status in contemplation. Ultimately, the question might well be: How can we move forward into a Fifth Interpretation? Such a move might, however, require that we first understand each of the first Four.

* See Cicero's Tusculan Discussions, v. 3. Cicero himself, although addressing a Roman audience given to glory in speech and war but eschewing artisanal contact with things, still seems to endorse the Second Interpretation, especially when he goes on to praise Socrates as the “first to call philosophy down from the heavens and to locate her in cities and in homes.”
Appendix: The Reconstrual of Singulars

Cf. pp. 107-110: In Aristotle's account, singulars lurk as the often silent but still essential partners. Now, however, a singular enters the pictures either as a class with only one member or as an “x” with a proper name....

The following passage from Joannis Duns Scoti, *Opera Omnia*, Vol. II (ed. by Wadding in 1639, reissued by Vivès: Paris, 1891) illustrates the Scholastic struggle with singularity that led to the modern reconstrual of each singular as a class with only one member.

Scholars agree that this one work is not authored by the John Duns Scotus (d. 1308) we know as the author of other works at the time.

For the translation I am grateful to James C. Morrison and Thomas Goud.

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**On Aristotle’s *Prior Analytics***

**Question 11:** Whether an expository syllogism holds by virtue of its form.

1. It is argued that this is not so, because this inference is not valid: *Socrates’ coat is white, Socrates’ coat is black; therefore black is white.* Its premisses are true and the conclusion is false, yet the syllogism is expository since the middle [term] is a discrete term.

   Similarly it does not follow that: *This man is good, this man is bad,* [these] different [propositions] having been demonstrated; *therefore good is bad.*

   Secondly, because it does not follow that: *This man or ass is an ass, this man or ass is a man; therefore a man is an ass.*

   Thirdly, because it does not follow that: *This man is an animal, this man is not every man; therefore not every man is an animal.*

   Fourthly, because it does not follow that: *This man is an animal, this man is no ass; therefore no ass is an animal.*

   Likewise, it does not follow that: *Sequana [the river Seine] is 100 years old, Sequana is this water; therefore this water is 100 years old,* which [conclusion] is false because perhaps [the water] had been produced by rainfall not long ago, or by springs.

   Again, it does not follow that: *This divine Essence is the Father, this divine Essence is the Son; therefore the Son is the Father.*

   Likewise in negative syllogisms it does not follow that: *This man is an animal, this man is not an ass; therefore an ass is not an animal.*

2. The opposite is argued by Aristotle in Chapter 2, which seems to prove the conversion of a universal negative [proposition] by the second rule.* Similarly, syllogisms of the third figure are proved by an

* In Aristotle's work we read: “If A belongs to no B, so also B belongs to no A. For if B did belong to some A — say, to a C — it will not be true that A belongs to no B; for the C is one of the Bs.” Or: C (a singular) is A (by definition) and C is also B (B belongs, by hypothesis, to it), therefore Some B is A (by Darapti, and contrary to the original claim that A belongs to no B). The “second rule” is Scotus’, not Aristotle’s.
expository syllogism, and hold by virtue of the rule: *If two things are the same as a third thing they are the same as one another.*

3. In this Question it will be seen from what terms an expository syllogism comes about and what it is, [and] similarly that it holds by virtue of its form, both in affirmative and in negative [moods].

It must be noted that an expository syllogism is that whose middle [term] is a discrete term. But it must also be noted that a term is discrete in three ways. One [way is this]: it signifies one thing only, so that it is contrary to its own signification to signify any one of [a group of] different things without a new imposition or demonstration.

In another way a term is called discrete which denotes* or signifies a singular thing when this nevertheless signifies any one of [a group of] different things which are the same as that singular thing. E.g., the term, *This Divine Essence* is called a discrete term because it signifies only one singular thing which is nevertheless the same as any one of [a group of] different things. And a term is called discrete in a third way, as the term *Sequana*, or similar [terms], which according to grammarians is a discrete term by the fact that it is a singular term, yet has the peculiar quality that nevertheless this term signifies successively any one of [a group of] different things without a new imposition; because now it signifies water which is now, and later it will signify water which later will be completely different from this, yet these waters have or had or will have some union with one another on account of the continual succession of the parts among themselves.

4. Now, according to this account, some solve the question by supposing that a term called discrete in the first way is properly discrete, and the others less properly, hence they say that syllogisms made from terms discrete in the first way are syllogisms good by virtue of their form, but if they come about from other terms they are not really [good], just as before the opposite was argued of the divine Essence and Sequana.

It is argued against that [proposal] because then it would follow that an expository syllogism would not be a formal inference. This consequence [i.e., result] is false for two [reasons]. First because the conversion of a universal negative [proposition] is proved by an expository syllogism, which conversion is a formal inference† [as remarked already in §2]. Second, because by it are proved syllogisms of the third figure [e.g., Darapti and Felapton] which are formal inferences: but if an expository syllogism were not a formal inference, more evident inferences would be proved by a less evident principle. The inference is [supposedly] proved on the grounds that an expository syllogism does not hold by virtue of the arrangement or form of the propositions and terms but precisely by virtue of the terms signifying one thing or another, i.e. by virtue of the [subject-] matter, as is evident from the preceding Question.

5. And so one must say otherwise, [i.e.,] that an expository syllogism holds by virtue of the form in [the case of] all terms, even though the premises are subject to the rule of *What is said of all or of none,*‡ so that a discrete term is distributed by means of the mediating phrase which is, e.g.: *Socrates runs* must be resolved into *Everything which is Socrates runs,* and then the expository syllogism is a formal inference.‡

6. But note that he to whom it is certain that no thing is the same as several things would posit the expository syllogism [as] a formal inference apart from the rule of *What is said of all or of none,* so that for

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* The Latin translated as “denotes” is *supponit*: perhaps “has as a support.”

† The Latin for “inference” is *consequentia*; “conclusion” is reserved for *conclusio*, and “consequence” for *consequens*.

‡ The Latin here is *per dici de omni, vel de nullo*. These two principles appeared to many Scholastics as summing up the inner workings of syllogism. Aristotle says (*Prior Analytics*, 24 b 27): “That one thing is wholly in another is the same as the one [the latter] being said [categorized, predicated] of another [the first]. We mean by ‘being said of all’ that nothing can be found of which the term cannot be said. In the same way we mean ‘being said of none’.” This passage relates the two different ways Aristotle speaks of the relation of S and P; e.g., “P belongs to all S” = “All S is P.” Yet already in appealing to these rules Scotus leans toward a class interpretation. In Question 7, he had already discussed these rules; in §5, he claims that syllogism rests upon “subordination under a distributed term;” i.e. the subsumption of S under M (and M under P— in Barbara).

§ Here we witness the reconstrual of singulars. Already in Question 7, §6, Scotus interprets “God is the Father” (two singulars!) as “Everything which is God is the Father.”
him this inference would be formal: *This man is Socrates; this man runs; therefore Socrates runs,* without any of the premisses being distributed negatively or affirmatively.

And please note that I say, *to whom it is certain,* and not to whom it is evident, for *certitude* applies in more [cases] than evidence. For *certitude* belongs to any part of some proposition, and can exist with respect to error just as with respect to truth, but nothing is *evident* unless true. And this way of preserving the expository syllogism as a formal inference was posited by Averroes, the Commentator on that book.

7. Now, in the special case of the affirmative expository syllogism, I say that in any of these figures a good affirmative expository syllogism can occur. An example of the first figure is: *Socrates is musical, white is Socrates; therefore white is musical.* Likewise, in [the case of] divine things it follows that, *This divine Essence is the Father, the Son is this divine Essence; therefore the Son is the Father,* [it being] posited that the premisses are subject to the rule of *What is said of all,* and hence the major [premiss] is false, just as the conclusion, whence this is false: *All which is this divine Essence is the Father,* because the Son is something which is this divine Essence and yet the Son is not the Father. Likewise, *Sequana* is an example of those terms, and in this way: *Sequana is a hundred years old, this water is Sequana, therefore this water is 100 years old.* Because if the major [term] is distributed, it ought to be distributed thus: *Everything which is Sequana or was Sequana is 100 years old,* and this major [premiss] is false. Similarly, it is possible that this be said by admitting *This water is 100 years old* in just the same as way as *Sequana is 100 years old,* because in both cases there is a similar [kind of] identity, namely, according to the continuous succession of parts.

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* Note that Scutus' abandonment of the singular allows him to place individuals in the predicate position, something Aristotle forbid. Note also that the “is” here is identificational, not categorial.

† Modernity extended this principle of understanding identity as a “continuous succession of parts” to the question of the identity of any being: a necessary step in the process of abandoning “predicates belonging to individuals” in favor of “individuals belonging to classes.”

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Appendix

An example of the second figure, as *White is this man, musical is this man, therefore musical is white,* is reduced to the first figure by conversion of the major [premiss].

An example of the third figure, by arguing as follows: *This man is white, this man is grammatical, therefore grammatical is white,* is reduced to the first figure by conversion of the minor [premiss].

8. Then, about the negative expository syllogism, I say that the negative expository syllogism is valid in the first figure by virtue of its form, given the negative major [premiss].

Someone might deny this, because it does not follow that: *This Father is not the Son, this essence is the Father; therefore this essence is not the Son.*

I respond that the inference is formal, and just as the conclusion is false, so likewise the major, if it is subject to the rule of *Said of none,* is false, whence the major [premiss]. *Nothing which is the Father is the Son,* is false, because the divine essence is the Father which is the Son.

9. Second, I say that a negative syllogism is not valid in the first figure if the minor [premiss] were negative, and the cause [of this] is that if the minor were negative and the major affirmative, then the [one] extreme term, the major, is not distributed in the premisses; indeed it stands determinately, yet confusedly, and distributively in the conclusion, but by form a distributed never follows from a non-distributed. For example, it does not follow that: *This man is an animal, a certain ass is not this man; therefore a certain ass is not an animal,* indeed, it is the case that every ass is an animal.

10. Third, I say that such a syllogism is perfectly fine for concluding indirectly. † E.g., *This man is an animal, a certain ass is not this man; therefore a certain animal is not an ass.* And the same can be said about the second and third figure.

11. As for the reasons [against the plausible objections listed in §1]. To the first I say that this complex term, *Socrates' coat,* which is the middle

† Again, notice “essence” cannot, in Aristotelian logic, stand as a subject of a categorial proposal about beings. To say, “The essence of man lies in his rationality” is to say: “All human beings are rational.”

‡ Concluding “indirectly” seems to mean concluding “with regard to an unnamed singular”—as in the modern employment of “a certain *x.*”
[term] of the above-mentioned syllogism, is not a discrete term, but rather it is a common term,* and so [the whole] is not an expository syllogism.

To the second I say that it is not an expository syllogism, because the discrete term which is there the middle [term], does not signify [anything unique], as is demonstrated too in each premiss, and so the middle [term] is taken equivocally in the major and in the minor on account of the difference of the demonstration.

To the third I say that it must be concluded: Therefore something which is not every man is an animal.

To the fourth I say that this must be inferred: Therefore something which is not a ass is an animal.

To the fifth [I say that] the inference is denied, nevertheless none of those inferences would have to be admitted if it [here: each conclusion] were taken distributively, and then the inference would be valid.†

Other things too were addressed in the question. And thus we have addressed the point at issue.

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* Scotus interprets “Socrates’ coat” (Socratis cappa) as a general term: Socrates has many coats, each one occasioning a different “imposition.” Similarly in the example that follows in §1: “this man” Scotus interprets as an expression used on different occasions for different people. He is introducing the temporal aspect of predication — as P. F. Strawson does in his analysis of “I am over six feet tall” and “I am under six feet tall” in his Introduction to Logical Theory (London, 1952 & 1966, p. 4).

† That is, the conclusions about Sequana and the Divine Essence do not follow from the acceptable meaning of the premisses (as reinterpreted with omne quod est). Yet the arguments could be made valid by rewriting the premisses, rendering them unacceptable. In either case, these two arguments are not sound. (Scotus does not analyze the very last argument in §1: a conclusion such as “Some asses are not animals” simply does not follow.)
Truth and Perversion
The Workings of Paralogism

We go wrong in many obvious ways. We add up figures incorrectly when drawing up accounts, we eat badly and gradually lose our health, we behave poorly in disrespecting those we later learn deserved our respect, we pursue lines of study or sport unsuited for our interests or talents. Such oversights may stem from carelessness. Some we may easily correct. Others may lead to disasters.

We also make wrong judgements. We uproot the shoots of garden flowers thinking they are weeds, and leave the shoots of weeds thinking they are flowers: here we mis-assess the species of individuals. We overwater our plants and kill them, we underestimate the conductivity of the paint on the handle of a screwdriver and get a nasty shock: here we misconstrue the properties of a species. These errors, too, may stem from haste and lead to disaster, but they are basically errors of knowledge.

And we also go wrong when out of tune with reality. Dazed by a blow on the head, by a painful stomach illness, by drunkenness, even by great sorrow or great joy, we fail to bring our knowledge to bear on the situation of the moment, go wrong in a way akin to, but different from both simple oversight and wayward judgement.

The going wrong of logical interest is a special variety of the general failure to attune: we do not face reality, we back off from situations which we must otherwise (elsewhere, elsewhen) admit are ours to face. Here we go wrong because we latch on to something which we recognize, at least in contemplation, to pave not a way in, but a way out.
In classical terms — still in the Second, as drawing upon the First Interpretation — our problem of error comes down to cowardice. Conditioned by λόγος, we actualize ourselves in the activity of speaking and listening — in handling situations discursively. But to handle, we must face. And in facing given situations we can, we might, come to know the individually encountered beings for what they are, their “species”: my house, my plants, my children, and my horse in their house-ness, plant-ness, child-ness, or horse-ness. But the activity goes together with, and always depends on, a receptivity on our part (categorically speaking: an undergoing). The most rudimentary error consists in the effort simply to act, i.e. to bypass the full requirement of opening oneself to what acts on one: here, one latches on to a universality and misses the individuality of beings. One thereby rubs out the given features of what one undergoes: one fails to complete it by following it. The basic failure is a failure in one’s art as a house-builder or home-maker, parent or gardener, horse-trainer or King of Thebes.*

Knowledge, we read in Plato and Aristotle, requires courage — a strange suggestion if we assume that knowledge is chiefly illustrated in having information from books or even being familiar with many kinds of things. But the suggestion is not so strange if we consider that knowledge genuinely stems from and ultimately returns to our dealings with individual beings: that even information gathered and remembered, familiarity with the properties of species, and the ability to speak and listen, read and write fluently, stem from and return to the actualization of our condition, i.e. a process of “completing (by following) beings.” For knowledge then requires that we fully take upon ourselves (in our activity) what is given to us (in our passivity). So long as our dealings run smoothly, i.e. take the form of routine, our doings and undergoings appear wedded to one another. On the other hand, when a being offers resistance, when we can no longer handle it routinely, the difference between ourselves and circumstance becomes evident — our customary responses and the customary appearance of things lose their apparent self-evidence. The price we likely pay for an earned wedding is a troublesome marriage. But then the question arises: Can I genuinely act? For it is possible that I merely re-act, do something in response but only to avoid rather than to face the trouble.

In simply re-acting we go wrong in ways which become thematic both in comedy and in tragedy. In comedy the reactions appear stylized and harmless: the lack of attunement to givens actually appears funny to us on the sidelines — perhaps because in reading a play like Aristophanes’ The Clouds or viewing a TV series like All in the Family we can fool ourselves into thinking that proper attunement would come easy to us. In tragedy, the reaction appears individual and powerful, the lack of attunement terrifying — perhaps because we recognize that the passage from reaction to action, e.g. in Sophocles’ Oedipus or Shakespeare’s Hamlet, requires the ultimate response most arousing fear.

Of course, the “goings wrong” warranting special logical attention are those in which the lack of attunement to what one undergoes gives rise to, or at least surfaces in the (stunted) activity of λόγος itself — warped language, whether warped in the speaking or warped in the listening. As logicians, we can formulate a number of such warped manifestations — formulate them in a manner revealing the way we deviate from the full actualization of our condition: our paralogism. In formulating them we aim to understand them: both the kinds of paralogism and actual instances of them. Such understanding requires us recurrently to reconsider the original condition they only stuntedly actualize. As philosophers, then, we must do what composers of comedies and tragedies do — but only insofar as is necessary to anchor whatever fallacy we happen to be formulating. A preliminary question, then:

Why do we go wrong? Why do we not straightaway actualize our λόγος-bound nature, our native condition and potentiality? A question ages old — literally: a Homeric hero can ascribe his failures to the whimsical intervention of some god; a Christian can associate his sin with his creatureliness, with his original departure ... (to conclusions) beyond what the facts warrant, a jumping expressing his natural will to establish his dominion over

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* Aristotle analyzes voluntary and involuntary error in his Nicomachean Ethics, Books Three and Seven. In his Poetics he remarks that the tragic hero commits a “big error” (1453 a 10): the Greek word is διαμέρισσα, analyzed elsewhere by Aristotle as the kind of mistake which occurs from lack of attunement to the individual even though one understands the universal. In what follows, I take διαμέρισσα to be one of the philosophical issues underlying paralogism in public discourse.
circumstances; and our contemporary interest in psychoanalysis stems from a dissatisfaction with the extent of the modern panacea for error: whereas this panacea (provisional abstention and rational attention) covers mistakes committed on the stage before us, many misunderstandings might well stem from self-incurred oblivion of our personal origins, suppression of infantile experiences, and repression of natural desires — resulting in a blurriness out on the open stage of life.

But there is a simpler reason — less imaginative, perhaps, but also less prone than any other to fantasy: we have to act. In action, we move out from principles toward conclusions, and the principles are necessarily left behind, at least for a while, and might long remain unconsidered, whereupon we ourselves remain in the dark. Yet these principles are precisely the grounds of our possible attunement with circumstances — to the very beings which we must approach in our action: "there's the rub," as Plato and Aristotle understand it, and Hamlet in their wake.

In its widest sense, action (πραξις) is the actualization of desire in a circumstance, an actualization displacing contemplation of the grounds for such action. I know that all humans need exercise and I straightway engage in a rigorous program of calisthenics and jogging — without pausing to consider my own special condition as a human being: here, I slip past the minor premiss, perhaps at some cost to my health.* Obviously, we must rush ahead, act quickly, on many occasions — with or without having considered both premisses. And, just as obviously, we are bound to make mistakes.

In its narrower, more philosophic sense, action (as opposed to production, ποιησις, as well as to contemplation, θεωρια) is the kind of doing in which we participate in the creation or preservation of communal affairs — ultimately, of some community as a whole (family, team, university, city). And such action is inseparable from decisions jointly made. Action is a special kind of doing, a doing in which decisions not only yield passing effects but institute abiding policies. Indeed, on the Greek view, a human being only counts as free to the extent that he or she can speak and listen effectively in the ongoing formation of public affairs.

* The example is from Aristotle's piece On Motion in Living Things, 701 a 26, where we also read that action (πραξις) is the actualization of desire.

In the narrower sense, action also locates the primary dramas of paralogism. For in public speaking and listening we most obviously rush forward to conclusions. In courts of law we must worry about who is guilty (in criminal cases) or who is responsible (in civil cases) — and how much. In legislative bodies we must worry about formulating decisions (whether laws or alliances) and foreseeing their effects. On a variety of occasions we must publicly praise or blame personages, either to influence decisions in litigation or legislation, or simply to encourage others to adopt or reject certain standards of conduct. In all these cases, we scurry on to something else: the public syllogism scurries us along. There is no obvious justification for turning around to contemplate all the premisses. And so we easily fall into distortions — either with intention (as when speakers push for conclusions on grounds they themselves know to be spurious) or through inattention (as when listeners simply wish to actualize their desires and pass over the ways for doing so).

In short: when speaking and listening in public action, we work within the purview of persuasion. Or: syllogism here takes the form of exhortation — we drive toward a decision and the reasons simply add power to the drive. It should come as no surprise that we often borrow power wherever we can get it, forgetting its exact nature and implications. Indeed, the surprise, the wonder, comes rather when we turn back to recall, to contemplate, the full power of the principles forgotten by us when engaging in action.

For a salient contrast with public exhortation, consider those rare occasions of intimate conversation when we pause to ponder all the premisses of an argument. Here, we have withdrawn for a while. Such withdrawal is possible only if we have previously engaged in action and currently have leisure time (or at least take the time). In conversation we then probe one another. Each recollects what he or she can, and in the conflict of memories and hunches we spark distinctions, discover what matters, what goes unnoticed in any rush to meet circumstances. Such is the contemplative life: the stakes lie in learning rather than simply persuading, we engage in dialectic rather than in rhetoric. The logical topic is induction rather than deduction.

But there is no safety in private conversation, either. Indeed, with friends we run even a greater risk of becoming smug, losing ourselves in a flow of big words. Perhaps out of frustration with the resulting gab, the empty talk, some people inevitably evolve into "wise guys" who play
with words rather than with reality, who “syllogize not with a view to the emergence of the matter itself, but with a view to names” (Topics, i, 18). And so long as we are indeed lost in words these wise guys will play with us as well—paralogically, to be sure, but only because we ourselves are committing the most basic paralogism: the failure to attune ourselves to reality within words, the attempt to solve questions by words, to get something from words.

Following Aristotle, we will eventually distinguish between fallacies “resting with talk” and fallacies “lying outside talk” (within λέξις and beyond λέξις): between what logicians now call verbal as against material fallacies. Yet all fallacies of logical interest occur in language (λόγος). Thus, we can, in contemplating them, locate the error very exactly (and therefore name them properly) by reconstructing the argument in which they occur: by supplying the missing premiss while carefully transcribing the entire argument. For in this way we bring out all the matters deserving contemplation, whether legitimately assumed in action or illegitimately serving to effect decisions while tuning us out of reality.∗

§1. Enthymeme

An argument in which we rush ahead to a conclusion, not raising all the questions and not discussing all the premisses, is an Enthymeme: a syllogism in which we rely on the general spirit (θυμός) already operative within (ἐν) the company for which we are speaking. Oversimplifying: an enthymeme is an argument in which we must supply an otherwise missing premiss—in public debate, almost always the major (a first-order enthymeme), but otherwise possibly the minor (a second-order enthymeme). We can even discern a third possibility: we might state or hear two premisses only, with the conclusion left up to the imagination (“The law says you must be at least 18 to drink, and you are less than 18…”).

But on what can we, on what do we rely? Evidently, on common preconceptions, on prejudices, on what people already believe, on what appears evident to people: on opinion. Aristotle says: on “what all people, most people, or wise people take to be so”—whether rightly or not. The immediate moral is clear: to speak effectively, one must keep in tune with the “climate of opinion”—no mean achievement in itself. But what form do such opinions take? Once detected and transcribed to complete the syllogism, the opinion takes the form of a proposal—usually the major premiss. But on what do such proposals touch? Evidently, something contributing to the making of decisions in action—in litigation, legislation, or general praise and blame. Aristotle says: they touch on probabilities and maxims, signs and examples.

§1.1 Probabilities

What counts as a probability? The Greek word (related to “icon,” a “likeness”) suggests “something very like the truth.” A probable argument in court might be: “The accused must have hated the murdered man, because he envied the man’s success”; here, the probable principle that the prosecutor himself leaves undiscussed when in court, but that we might like to discuss in friendly conversation, is: “Envious people also hate those whom they envy”—likely do so. Or: “These two youngsters are not lovers because they never show affection for one another in public”—on the likely but undiscussed principle that “Lovers show affection.” Such principles sound reasonable, reflect propensities, do point toward something essential in our condition (here, about love and hate: see Prior Analytics, 70 a 5). But they do not refer to any essence, let alone dwell on it; and they do not necessarily hold in any given instance.

∗ For Aristotle’s account of the life of contemplation as against the life of action (reminiscent of Pythagoras’ distinction indicated on the last pages of Book One), see his Ethics (i, 5, and x, 9 ff.), his Politics (vii, 2), and his Metaphysics (ii, 1; vi, 1). Nearly the whole of Aristotle’s discussion of paralogism asks us to contemplate action. The purpose of such contemplation is not only to become familiar with and adept at speech in action (λόγος in πράξις), but to extract what does remain the same, the issues and exigencies which do not vary and which are forever the same. In this sense, contemplation is profoundly wedded to action.

∗ “The probable (τὸ ἐλκόδε) pertains to what often comes about (τὸ πολὺ γνωστὲν)—not absolutely, as some say, but only to what permits of being otherwise” (Rhetoric, 1357 a 35). Until modern times, “probable” meant “well probed, well attested” by trustworthy people for daily dealings. Such “probity” was the most that could be expected in temporal affairs. The opposite of “probable” was “demonstrable,” where demonstration pertains only to super-sensible matters. See Ian Hacking’s The Emergence of Probability (Cambridge, 1975).
In his *Rhetoric* (II, 23) Aristotle provides numerous examples of enthymematic arguments based on probabilities. Let us review some of them, transcribing them into nearly syllogistic form.

Arguing from contraries: “Our economic troubles will come to an end by lowering interest rates, because our troubles came about by raising them”—with the unstated premiss that things brought about by one procedure will be eliminated by simply reversing the procedure: significantly possible but not absolutely necessary. Or: “You must admit that mortals are poor judges of true sayings because you know that they are poor judges of deceitful sayings”—with the unstated premiss that “All people good at spotting true proposals are good at spotting false ones”: generally correct but sometimes incorrect.

Arguing from relation: “I had a right to kill the man because he well deserved death”—on the likely principle that anyone deserving death is rightly put to death (but: by anyone?). Here we contemplate the workings of tragedy (Orestes killed his mother, who may have deserved it, having killed her husband. Orestes’ father: but should Orestes have done it?). Also the quandaries of courtroom battles (Jane Stafford kills her extremely cruel, sadistic, bestial, destructive husband: Was she acting in self-defense, or should she, could she have turned the case over to the police?). Here we have a “probability” which lends itself to intimate discussion — both for understanding the principles of society and for deciding the case at hand.

Arguing from more or less: “I could hardly be expected to know the dynamite would explode, because even the explosive experts had failed to predict it”—on the probable assumption that “Those who know more are more responsible than those who know less.” Or: “This fellow deserves no special pity because others have suffered more than he and have complained less than he”—on the unstated grounds that recognition of a greater suffering allows us to scoff at a lesser. Or: “Paris had the right to kill Achilles, because Achilles had the right to kill Patroklos” or “We must be doing no wrong in killing terrorists, because they claimed they did no wrong in killing innocent civilians”—on the assumption that justice consists in responding in kind (an assumption eventually challenged by Jesus and latter-day pacifists, e.g. Quakers and opponents of capital punishment).

Arguing from a consideration of time: “You wanted to marry me before I had fallen into poverty, therefore you should want to marry me now”—on the assumption that the difference in one’s fortunes does not touch the basic reason for marrying, e.g. beautiful character (the same afterwards as before). Similarly in law courts and the like: we may argue for consistency in the application of principles over time.

Arguing from the speaker back to himself: you say “The accused bad-mouthed his colleagues,” but your accusation is nonsense since “You obviously hold first prize in bad-mouthing your own colleagues”—on the likely assumption that an accusation holds only if the accuser himself is free of it. Aristotle pauses to approve of the principle: “In general, it is absurd to reproach others for doing what you yourself do or would do, or to urge others to do what you neither do nor would do” (1398 a 13).

Arguing from definition: “I must believe in the gods (in divinities), since I believe in the godly (in the divine)”—on the assumption that “All people who believe that some things are holy are in effect acknowledging the existence of gods as the sources of such things” (on the definition of the action of those who believe in divine things). Such is the argument Socrates presents in answer to the charge of atheism (in Plato’s *Apology*). Aristotle and others (today, Martin Heidegger) have contemplated the major premiss of this probable argument, and argued that one must begin by finding and affirming things as divine; that beginning the other way around (in gods to justify holding some things holy or divine) leads one into an unholy commercialism, mere trade-off (emporium) — as in the case of Euthyphro in Plato’s dialogue carrying his name.

Arguing from good or bad results: “You should not pursue your education, since you will arouse the envy of your family”—on the unstated assumption that you wish at all costs to avoid such a (bad) result. Or: “You should pursue your education, because you will then discover what you can actually do in life”—on the assumption that you desire this (good) result. There are always good and bad results following from any action, but some arguments on this score cut both ways immediately: “You should not speak up in public because you will be hated (by your fellows if you speak justly, by the gods if you speak unjustly)”; however: “I should speak up in public, because I will be loved (by the gods if I speak justly, by my fellows if I speak unjustly).” Mark Twain remarked: “Always do right. This will gratify some people, and astonish the rest.”

Arguing from the difference between the openly avowed and the secretly wished: You say “War is now the most noble course of action,”
but this is nonsense, since you obviously mean that war is profitable for your arms factory. Aristotle says that this form of contradicting someone else's proposal is the “weightiest of those upsetting appearances.” It is built upon the melancholy philosophical awareness that, as a rule, “one openly praises justice and nobility but secretly wishes advantage for oneself.” On occasion, we must be prepared to counter the charge—or, at the very least, we must avoid creating situations in which there might even appear to be a conflict of interest.

And arguing from the purposes of action: “The accused must have done it, because he could do it easily and profitably, with no reason to think he would be caught”—on the likely principle that people do or forgo things primarily for the purpose of maximizing profits. Or the argument of the Chief Magistrate in Anton Chekhov’s “The Head Gardener’s Story” (1894): “I swear he is not guilty. That anyone would dare kill our friend the doctor, I do not concede. Man cannot sink so low.” For the (murdered?) doctor had been something of a saint, and no one, not even the accused (who so clearly seemed to be implicated) could possibly have had any motive for the crime: the judge upholds the saintliness of the dead doctor and the goodness of human nature generally, perhaps at the cost of letting a scoundrel go.

Probable principles may even surface explicitly in public discourse, providing only that they allow for succinct, impressive, and memorable formulation. Such are the maxims (wise sayings, gnomes) which can give direction to much debate. In his Rhetoric (II, 21) Aristotle defines a maxim as a proposal pertaining not to individuals but to wholes, and not to beings as wholes but to actions as wholes. He distinguishes four kinds of enthymemes based on such proposals:

Those in which the maxim has already gained assent. For example: We had better forbid smoking in the classroom because we now know that passive smoking also endangers one's health and, although the new rule will deprive active smokers of their private pleasure, “Health is the best thing of all.” Proverbs serve especially well as guides requiring no further discussion: The project deserves further study—“Look before you leap.” It is time to take a stand on the matter—“Between two stools one sits on the ground.” Let us compromise on this question—“Better to bend than to break.”

Those in which the maxim likely gains assent immediately even if it is not current: the fellow is not to be trusted to participate in the most intimate discussions of management, since he has on occasion been more loyal to his family than to his job—and “He who is not always loyal is not really loyal.” Such invented principles can win immediate assent, if only they are formulated symmetrically and rhythmically to recall common experiences.

Those in which the maxim needs and also receives a justification: We should not provide special education for the gifted, because we would then make some people wiser than others, and “No man should cultivate himself beyond his fellows' reach”—because, appearing vain to others, one then arouses hostile jealousy. Such maxims occur mid-way in a sorites, both justifying a conclusion and justified by a further enthymeme (here: by the probable principle that in social decisions we must avoid dissension among the constituents).

And, finally, those in which the maxim “is enthymematic but not part of any enthymeme”: being Christians, we must have pity for the man; being mortal, we must not cherish the wrath reserved for immortals; like all narrow-minded persons, he never considers why people do what they do; my child, you must listen to the advice of your elders; O Lord, I place my fate in your hands. In all such cases, the opening phrases imply reasons for the remaining proposal, we could reformulate them into general principles guiding actions (you are a child, all children must listen to their elders, etc.).

As the use of maxims shows, an enthymeme is such, not because a formulation is missing, but because the discussion of it is missing—because one leans on it, whether mentioned or not. Indeed, we generally delight in hearing explicitly stated the universal principles which cover and thereby justify our experiences—so long as these principles are succinctly formulated and permit us to pass quickly on. For example, since most parents have trouble with their children, many will gladly hear it said that “In sorrow you shall bring forth children” (Genesis, 3:16). Or one whose neighbor's children tramp down the flowers in one's garden might like to hear W. C. Fields' comment that “Anyone who hates children and dogs can't be all bad.”

As Aristotle remarks, we judge the character of others precisely by the maxims they cite, since these express the basic choices one has made in life. Accordingly, it is sometimes appropriate to challenge popular sayings, arguing, e.g., that we should provide the children with improved recreational facilities since, contrary to Fields' famous saying, it is our job...
to make children (and even dogs) lovable. Or: we should fight to the bitter end—disagreeing with the saying that “It is better to run away and live to fight another day,” since one cannot truly live a single day without honor.

If we contemplate the otherwise unconsidered, undiscussed principles of public discussion, we immediately notice that they do not necessarily hold for all cases: that they are only probable, tentatively functional, not absolutely certain in any given application. Yet we might also ask whether we can ever know for sure that a person is guilty as charged, that a proposed policy will prove beneficial, that we ourselves are acting to create something and not simply reacting to avoid something. We may prefer to deal with certainties, in which case we had better study mathematics. But in matters of application, of action as well as of production, we must accept the slipperiness of our own condition. As Aristotle says (Rhetoric, 1402 b 32), “it behooves us to decide matters (of application) not from necessities alone, but from probabilities as well”: we therefore fall into a salient paralogism whenever we wash our hands of a matter simply on the grounds that any judgement will fail to have the status of necessity. And perhaps even philosophical contemplation consists not in extracting certainties all by themselves, but rather in turning toward and puzzling over the uncertainties of action. For contemplation consists in pondering potentialities, what can or might be: we may detect and appreciate what must be only as we ourselves actualize what we acknowledge to be potential. On the Second Interpretation there are no substantial self-contained certainties possible for humankind; such become of interest only during the Fourth Interpretation—when “theory” and “practice” become entirely separate affairs.

§1.2 Signs

What counts as a sign? The Greek word (related to our “semantic”) suggests a “familiar indication”—as doctors look to symptoms as signs. “He must be ill, since he has a high fever” or “She must be with child, since she has milk”: the first assumes that people who are hot in a certain way are also ill, the second that pregnancy at some time necessarily precedes a woman’s ability to nurse. Given our own state of knowledge, we assume that some signs (here: fever and milk) serve as necessary middle terms: that the formulation of the minor premiss suffices for drawing the conclusion, since “everyone” knows the unstated major premiss.

There are also non-necessary signs. “She must be pregnant because she looks haggard” or “He must be a thief (or an adulterer) because he disrespects social norms”: formulating the requisite major premises we obtain either (invalid) 2nd-figure arguments with (probably) correct assumptions or (valid) 1st-figure arguments with (definitely) wrong assumptions; the trial of Meursault in Camus’ The Stranger operates on this slippage (see Book One, Prelude). Similarly, 3rd-figure enthymemes invoke non-necessary signs: “Wise people must be concerned for the good of the city—because Socrates was concerned for the good of the city, and we know that he was wise”: but the premises only justify concluding that some do.

A sign, then, is something directly encountered (in sensation, in αἴσθησις) that we know or believe points to something else (formulated in the conclusion). We take things to be signs all the time: bubbling water on the stove as a sign that the water is hot, squeaking doors as a sign of dry hinges, a first beard or a first menstruation as a sign of puberty, a formation of clouds as a sign of pending rain. And the learning of any craft consists largely in learning to recognize presented signs as indicative of how things were or how things will be—and also indicative of the ways we ourselves can and must “complete what nature is unable to finish” by following up her signs. In either case (in simply getting on in life and in exercising one’s special craft) one lives largely by moving from the visible either back to the invisible (to understand what has happened) or forward to the invisible (to help make the future).

But the legitimacy of our sign-reading depends on the legitimacy of the often unstated major premiss. It may be that “All females with milk are females with progeny”—so that “having milk” can serve as a necessary sign. But it hardly seems the case that “All women looking haggard are pregnant” or that “All people scoffing at social norms are thieves”—although the converses may hold, at least as probabilities: “looking haggard” or “scoffing at social norms” are inconclusive signs. Yet we tend to be rather careless in these matters. Because the ground outside is always drenched immediately after heavy rainfalls, we assume that drenched ground now is a sure sign that it has recently rained (whereas there are other ways the ground can get drenched); because we learn that a huge national debt is a sign of troubles to come, we assume
that our troubles now are due to a huge debt (whereas there are other sources of economic troubles). As Aristotle puts it, our frequent errors in interpreting signs arise because we rely on a (wrong-headed) converse of what “everyone” does in fact know — we convert the enthymematic major premiss.

One reason we wrongly interpret signs becomes clear when we note carefully the difference between the order of discovery and the order of being. For instance, “This woman has a protruding belly, therefore she is pregnant” represents a temporal and sensuous order of discovery, whereas “This woman is pregnant, therefore she has a protruding belly” represents an eternal and intelligible order of being. The first exemplifies an inference based on a sign, and the syllogism answers the question “Why do you believe...?” and the second exemplifies an inference based on an understanding of female physiology and answers the question “What is the cause of...?” But the one major premiss is the converse of the other.

Many signs seem to be primarily “aesthetic”: from something presently seen or heard, smelled, tasted or touched to something no longer or not yet sensed— but still at some time sensed. And there are many questions we can and do level at given examples, wondering how reliable they are: what counts as knowledge obviously changes in the course of time. Indeed, David Hume has argued that all so-called signs are merely customary associations, on a par with “drenched ground indicates recent rain.” And modern psychologists generally agree with Hume, understanding human behavior with signs as gratuitously conditioned (a matter of incidental repetition, even if seasoned occasionally with a pinch of creative imagination). Yet for Plato and Aristotle, even for most mature thinkers of the Fourth Interpretation (e.g., Kant), the interpretation of signs conditions the very possibility of experience.

But some signs, often the most important ones in decision-making, are only half aesthetic: I take the frown on my lover’s face as a sign of unhappiness about what has happened and of trouble to come; we take the easy gait or the hesitant gesture of a person as signs of confidence or its opposite. The tone of voice tells us, we think, about the disposition of a person — as might also neatness or sloppiness of dress or of immediate environment (bedroom or desk). Acting in theater and film consists largely in embodying such signs; and we follow the drama accordingly. In all these instances aesthetic (somatic) signs send us toward psychic dispositions. In living and conversing with intimates we interpret such signs incessantly, and learn that others interpret us in the same way. In a job interview we must strain to discern something of the soul of the applicant, since his future contribution depends on his inner nature as much as on his outer credentials.” In legal cases, especially in anglophonic law, we must strain to discern the intentions of the accused — even, in the case of British common law, the intentions of the original law-makers, the “signs” of which often lie in the casual conversations reported in newspapers.

Aristotle devotes the closing paragraphs of his Prior Analytics to a discussion of signs as of the body to the soul. Hegel likewise devotes several pages of his Phenomenology of Mind to the same question (“physiognomy,” in both Greek and German). Our obvious quandary is this: we must carefully discern, from manifest bodily phenomena, inner intentions, dispositions, abilities, potentialities; yet signs here are never necessary, we constantly learn that there are exceptions, discover that we ourselves and others often dissemble. While we must interpret, only time will tell. As Aristotle puts it, such signs depend upon a convertibility of the major premiss, difficult if not impossible to effect correctly: ultimately, only choice, as manifest either in word or in deed, reveals character (the soul). As Hegel puts it, the actuality of such signs lies solely in the event of the inner becoming outer — it lies in action, so that we make a fundamental mistake if we assume that we can formulate a law, know for sure how the soul is by virtue of how the body is, know in advance of the action.

Above all, arguments drawing upon signs require that one learn to distinguish between essential and accidental predications: “having milk” may belong essentially to motherhood, while “looking haggard” may belong only accidentally to mothers. Indeed, the day-to-day function of signs as middle terms provides a phenomenal field rich in ontological

* In his discussion of shame (Rhetoric, II, 6), Aristotle cites a series of examples of our natural sensibility to signs of bad character: avarice, obsequiousness, etc. The painter Robert Henri reminds his students that their task is to draw an eyebrow as a sign of “sadness, boredom, surprise, dignity ... the force or direction in the action of looking” — even though, taken in itself, it is merely “a series of small hairs growing out of the skin” (The Art Spirit [New York, 1923 and 1960], pp. 24-25).
questions. For instance: What is the relation between a being and its predicates? All categories other than the category of being serve as signs for identifying beings: we try to identify honey by its color (but then take bile for honey, as Aristotle picturesquely says), gold by its glitter (but then take iron sulfide, fool's gold, for the real thing), aspirin by its place in a bottle on a shelf (but then take a mis-bottled or misplaced poison for aspirin), an intimate by his touch or smell rather than his voice (but then take Jacob for Esau, as Isaac did) or by her color rather than her voice (but then take a wolf for our mother, as the Seven Little Goats did). In answer to the question how we genuinely identify beings, Plato and Aristotle argue that something both counts as a being and gets correctly identified as the being it is according to its dynamism: according to what it can and does do, can and does undergo, in a subtle interweaving of condition and posture, effecting and resisting (see Plato's *Sophist*, 247D-E, and most of Aristotle's *Metaphysics*). In the end, then, signs have their proper, their fullest meaning in the actualizing of potentiality, in the exercising of our own participative role as lending a helping hand or a helping word to the emergence of what is (to be) identified.

Even individual beings serve as signs—as signs of what it means to be the kind of being the individual exemplifies. From “Socrates is wise” and “Socrates is concerned for the well-being of the community”—from these proposals as premisses—we only obtain (in the form of an AAI-3) “Some wise people are concerned for the well-being of the community.” However, if we take one of the proposals as the conclusion (e.g., “Socrates is concerned...”) and come to believe that the reason for his concern is expressed in the other proposal (“Socrates is wise”) we rely on a major premiss “All wise people are concerned...”. If this rhetorical argument turns out to be wrong, its wrongness will consist in wrongly treating one predicate as a sign of the other, as the middle term of the AAA-1. Yet, whether rightly or wrongly in any given case, we necessarily engage in some such procedure both in the process of growing up and in the mastery of any art. For in praising and blaming others, in taking them as heroes or villains, we set standards for ourselves, standards which we then imitate to the exclusion of others: so-and-so (my mother or my father, an athlete, a statesman, or a TV star) is really great because he or she has such-and-such properties (qualities, quantities, relations, places, times, doings, undergoings, conditions, postures).

Similarly, in taking some practitioner of an art as our master, his exact doings as worthy of imitation, we gradually become intimate with wood (if the art be that of carpentry) or with the human body (if the art be that of medicine): this joint obviously holds, but it holds because the master has prepared and glued the wood in such-and-such a way; this man has obviously recovered from his disease, but recovered because the doctor treated him in such-and-such a way. In all these cases we may wish to say not “because” but more simply “and”: the cautious, the sober version. But then we ourselves are stuck with particular conclusions (in the AAI-3), and for all our caution and sobriety we learn nothing.

As a special, slightly more complicated case of signs, we can take the use of examples (paradigms of action). For instance, to argue that “Waging war in Nicaragua will prove disastrous,” we can cite the example of another war: “The Vietnam war was disastrous.” Why? Because “The Vietnam war required us to pit open organized force against underground indigenous resistance.” The employment of the parallel example silently introduces the principle that “Wars of this type are disastrous for us”—a principle which then serves well in the present debate: we need only draw out the parallel, i.e. introduce a minor premiss to the effect that “Waging war in Nicaragua will require us to pit... .”

In his *Rhetoric* (II, 20) Aristotle distinguishes three kinds of example-giving: we may recall affairs already come about, we may draw comparisons with things generally in force, or we may devise stories of our own. In arguing against military involvement in Nicaragua by referring to what happened in Vietnam, we draw upon past events. In arguing, as Socrates recurrently did, that we should no more choose our rulers or judges by lot than we would designate by lot who shall be a doctor, a carpenter, a horsetrainer, or a helmsman, we drive home again and again the same major premiss: only those with proven competence in the performance of a given art (including statesmanship) deserve our trust. The third variety, akin to the second in requiring ingenuity (both require a “making” on our part) is illustrated by the fables of Aesop or La Fontaine, as well as the folk tales gathered by the Grimm brothers: an animal loses something (food, tail, life) because it listened to flattery, or whatever; Little Red Riding Hood gets into trouble because she did not obey her mother, trusted a stranger, or whatever.
The three kinds of example-giving correspond neatly with the three kinds of literature: historical, philosophical, and poetical (Poetics, 9). Machiavelli emphatically illustrates and defends the first kind: one who wishes to understand human affairs must carefully examine what has already happened and extract from these actual examples the principles of action—rather than make up tales to illustrate what should happen: “He who abandons what is done for what ought to be done will bring about his own ruin rather than his preservation” (The Prince, 15). Aristotle himself illustrates and defends the second kind: the philosophic disposition consists precisely in the ability and the desire to detect similarities among diverse instances (Rhetoric, 1394 a 5 and 1412 a 12; cf. Poetics, 1459 a 5); however, examples of philosophic interest appear already as universalized: one looks not to a given helmsman or a given carpenter to see how he comes to exercise his craft, one rather recalls helmsmen in general and carpenters in general. Leonardo da Vinci follows this path of universalized examples when he writes in his Notebooks: “Iron rusts from disuse, stagnant water loses its purity, and in cold weather becomes frozen: even so does inaction sap the vigor of the mind.”

The third kind of example-giving, the poetical, deserves separate comment. A mother may recite an Aesop-like fable to exhort her child to perform or to avoid certain actions, but she then relies on established tales. A poet, on the other hand, more or less constructs the tale—if not entirely from scratch, at least in showing the exact manner in which given people respond to given circumstances. And such free construction immediately brings into enthymematic play some principles of response. In Boris Pasternak’s novel, for instance, Dr. Zhivago appears over and over again as one who flees from his public role during the Russian Revolution into the private life of his own love affairs (“release me from the cast,” we read in one of the poems Pasternak ascribes to his Doctor): moreover, he appears as withdrawing into private life because he is one who genuinely cares about the well-being of his fellows. Precisely by concentrating on the responses of this one character, the novel enforces the general principle: “Anyone who cares for humanity will have to withdraw from the social movements of the time.” No wonder, then, that the successors of the Russian Revolution condemned the novel in the 1950s. Wishing to defend freedom of expression, we in western democracies may claim that a novel simply portrays a series of predications to affect the reader: Dr. Zhivago does and undergoes this and then that and then this, etc., and readers merely conclude that some people who do or undergo X also do and undergo Y (touching, perhaps, but otherwise inconsequential). However, do we not then trivialize literature to save it? If, on the other hand, we defend censorship on the grounds that literature portrays causal relationships, arguments in the enthymematic AAA-1 form, and that the principles it engenders and enforces are therefore true or false (that “poets are the unacknowledged legislators of the world,” as Shelley said), do we not terrorize literature, insist that each new example simply re-introduce the established principle of the day?

Many readers and listeners feel more comfortable with examples—and recoil from all else as “abstract.” Yet, oddly, examples engender and hold our interest precisely because they mean something, because they somehow suggest universals: they appear concrete precisely because we detect in them what applies to further examples, perhaps our own. True, life comes in examples only—in occasions. But life also keeps coming: we must respond ever afresh to new examples. We take pleasure in one example partly because from it we learn something helping us to respond to others, both past ones which puzzled us and future ones which will often put us to the test. As Aristotle says (Problems, XVIII, 3), we delight in examples because in them we detect similarities: in both life and in literature, examples are signs of universals.

A major form of error consists in imposing some significance upon, rather than detecting it within a given example. Perhaps we wrongly impose on Nicaragua what we rightly detect in Vietnam; perhaps Socrates wrongly imposes on examples of civic responsibility what he rightly detects in examples of manual crafts; perhaps Dr. Zhivago does not genuinely represent full concern for humankind. Yet can we fully understand each instance all by itself? We seem driven to see in given instances something universal, something applicable to other instances. Even the mantic arts have a certain appeal: the claim to see in the layout of cards, entrails, tea leaves, birds, or stars the invisible realities in store for human beings. Plato and Aristotle try to curb this drive; they argue, in effect, that from the flight of birds one can learn mainly about birds (their physiology, their relations to one another and to the wind and the weather) and not about the outcome of battles among human beings. Jesus, too, warns us not to seek for signs as assurances of divine presence.
(Luke, 11:29), presumably because our destiny is to witness the divine, to stand within in and to be a sign for it.

As readers or listeners we of course find it much easier to appreciate the examples provided by others than to devise examples of our own—to become writers or speakers. Indeed, intellectuals generally prefer to enunciate general judgments—to speak and write “abstractly.” To formulate examples ourselves requires the development of an art—the art of portraying individuals (primary beings) while both focussing on the details (predications of the individuals) and paving the way for application to other individuals: one must learn to treat individuals as signs of their own universality, must learn to allow the universality to emerge on its own from our concentration on individuality. To this kind of learning we shall return in Book Three.

§1.3 Responding

Just as one normally takes four years to complete a college education, so there are four different ways of responding to public arguments. The freshman desires to fall in line, to swim comfortably within the beliefs of “all people, most people, or wise people”—and seldom bothers to distinguish among these three sources. The sophomore delights in spotting and criticizing the underrunning beliefs, and drives the undercurrent to the surface where it loses its force. The junior spots and analyzes the undercurrent in order to swim cleverly within it, guiding its direction in small ways so that it carries him closer to what he personally wants. And the senior: What does he or she do to graduate, to commence? At least this: become a doer and a speaker.

Modern textbooks in logic speak to their readers as passing from freshmen to sophomores, i.e. from being gullible to being skeptical. Plato and Aristotle, in contrast, speak to us as passing from juniors to seniors, i.e. from being manipulatively cynical to being creatively philosophical. To comprehend the full gamut of involvements, one must nowadays take special care to appreciate the fourth one: for only then does one discover the privative nature of the other three. And one can then also understand the modern tendency to ignore the Platonic and Aristotelian interpretation of our λογος-bound destiny.

Where must we learn to speak and listen? Aristotle answers: in and for (1) deliberative assemblies, (2) judicial proceedings, (3) commemorative occasions (public praising and blaming), (4) sophistical disputations, and (5) philosophical discussions. The first three modes of discourse (traditionally called deliberative, forensic, and epideictic) serve action, the decision-making processes which, one hopes, contribute to the preservation of communities. And the remaining two serve contemplation, the formulation of interconnected proposals which, one hopes, contribute to a focus on the basic realities at stake for humankind.

In all five locations persuasion is the issue: what formulation, what proposal, the participants ultimately affirm, finally trust. One does not make policies in assemblies or judgments in courtrooms the way one makes tables and chairs in a workshop: one must come to believe in one formulation as against its contradictory, and one’s belief contributes to a decision affecting others as well. When praising and blaming we especially set the standards of human conduct and accomplishment. Genuine persuasion may seem to vanish as an issue in sophistical disputation (“eristic” it was at one time called, “contentious argument” we might say today). Yet this exercise was much lauded in ancient times, even by Aristotle (Topics, I, 2), namely as a preparation for entering into the other arenas of life where speech locates the issues urgently demanding affirmation or rejection. Involvement in the fifth, in careful and concerned and intimate dialogue (philosophy) was long considered the necessary counterbalance to the danger of remaining in the arena of merely contentious debate: for only in such dialogue do we meet head-on the question of what we can and must ultimately trust—namely, beings in their being rather than any of their other predicates (e.g., the fulfillment of human nature within communities rather than any isolated proposed action, posture, place, or whatever).

But to enter into any of the five locations of discourse, or at least to consummate our speaking and listening therein, we must (on the Greek view) be free: politically (we must be recognized as having a voice in affairs), economically (we must be able to take time off from earning our bread), and spiritually (we must be able to take our stand in what we do for, rather than in what we get from our circumstances). Our freedom in any of these three senses is always tentative, never guaranteed. Our constant task, evident in our linguistic life, is to pass from mere reaction to action, from merely squabbling and squirming in an effort to protect our established interests to fully standing within our established condition.
in an effort to face and uphold the realities streaming in upon us. Unfreedom begins in a withdrawal into a defensive position.

At the beginning of his *Rhetoric* (1355 a 22 ff.) Aristotle justifies the study of enthymeme and paralogism explicitly in reference to our need to become seniors; here one even detects an element of heroism borrowed from the First Interpretation.* The study is useful, he says, “because the true and the just being by nature stronger than their opposites, we ourselves are necessarily the weaker if decisions come about inappropriately.” All five arenas of speech are analogous to the arenas at athletic games: the participants enter as competitors and those who lose have no one to blame but themselves; it would be absurd, Aristotle says, not to be ashamed of incompetence in defending oneself in speech-activities if (as Aristotle takes for granted) we are ashamed of not being able to defend ourselves in body-activities — since our own being lies more in our speech than it does in our body. Once we take upon ourselves the art of speaking and listening, we will find ourselves able, if not to win at least to promote whatever victory is possible (a distinction Aristotle insists upon: an art guarantees not success but only the promotion of success). For just as in sports the superior athlete performs the best, so in speech we learn that “the true and the better are always more amenable to proper syllogizing and to persuading.”

The probabilities and signs saturating public discourse may indeed mislead us, as both sophomores and juniors well know. However, the senior, knowing that such premisses are indispensable in action, takes on the task of reorienting them in the service of truth.

But what is all this talk about “truth”? Having difficulty with this question, one also has difficulty understanding what it might mean to become a senior. Throughout the first three Interpretations of the human condition, “truth” named a special event — one in which human beings turned toward, took the side of beings in their Being, overcame their own predilections, preconceptions, prejudices, and could thereby discover the

* With a difference: while Homer portrays the hero as testing himself (whether he can endure the primacy of the divine over all human interests), Plato and Aristotle elicit the heroic dimension within the processes of making and preserving (one who knows something must stand both within convention and for nature). The difference becomes evident in the contrast between Virgil's *Aeneid* (where the hero must make a new city for his people) and Homer's *Iliad* (where the heroes sacrifice themselves in a general destruction of cities).

“way things really are.” The age-old assumption was: beings can reveal themselves for what and how they are, can display their essential predicables despite the dazzling array of their more obvious but accidental features, only if and to the extent that we ourselves stand up and march out to face them, apparently to our own personal detriment, although ultimately to our own personal fulfillment in unison with the fulfillment of the revealed beings. Now in the Twilight of the Fourth Interpretation the reigning view of the human condition leaves little room for “service to the truth” in this sense.

First, language itself appears now as the adjunct of production rather than a dimension of action; and any suggestion that language “originally” belongs to contemplation appears fantastic. Read any standard account of our linguistic nature and you will find that speaking and listening are presumed to function primarily as means of communication, whether for transferring information (the primitive supposedly wishes to describe to others where he discovered a supply of honey) or for trading goods (one describes what one wants and offers something in return); and secondarily as a means of expression (just as primitives supposedly find emotional satisfaction in singing about their exploits, past or future, around the campfire after a hard day's work, so we too, in analogous circumstances, like to talk things off our chests). Whatever the details and variations of these theories of language from Hobbes to the present day, the deliberative, forensic, epideictic, eristic, and dialectical locations of language appear derivative: the basic location lies somehow in our mercantile nature, in the production and enjoyment of goods.

Similarly, freedom appears now as simply given or denied by one's factual condition, i.e. bestowed or refused by others. On the reigning view, one is not called upon to earn, prove, and preserve freedom in one's own speech. Thus education no longer consists in learning to take a stand in speech. Rather, one learns in school to work within standardized language to promote one's own mercantile interests — and the mercantile interests of the state. Freedom, it seems, is an issue only for those who suffer imprisonment or happen to live in some non-western country.

And, finally, truth itself appears reducible to accuracy of determination and validity of calculation, the two most obvious issues of industrial production. Yet public discourse, the speeches heard over the media (whether by journalists, advertisers, or politicians), hardly seem the appropriate opportunity for truth in this bifurcated sense. Language here
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appears as essentially deceitful: it is monological. Contrary to Aristotle, then, we tend to believe that the false and the worse are “more amenable to syllogizing and persuading.”

These historical shifts in our understanding of language culminate in cynicism, both literary and popular. Originally, a Cynic like Diogenes or Antisthenes was one who discounted the public realm as essentially crooked: the best life, that of contemplation, required that one withdraw into an independence, lead a life of utter simplicity and integrity. Much later, and now in the popular sense, a cynic is one who plunges into the public realm, agreeing with Diogenes and Antisthenes that this realm is essentially crooked, but preferring to twist public forces to one’s own private advantage—whether to one’s glory, wealth, or passing pleasure. In both the original and the popular senses of the word, cynicism amounts to a denial of the possibility of becoming a senior.

Our endorsement of liberal government calls attention to the inevitability of cynicism. For if the state must not tell us how to live our personal lives, how can we believe in the public realm per se? Of course, the state must provide the means of life, the channels of commerce and communication, production and distribution, and some form of internal and external protection. But only ends can command belief: we can believe in means only in the light of some purpose served by the means. And how can one speak to the nation, how can one listen to such speeches, if the end is avowedly private?

Such forms of social organization as theocracy, aristocracy, or monarchy could always stand, or pretend to stand, as models for their members: each individual could look to the social organization for a visible model of the invisible exigencies of his own soul (as Plato assumes in his Republic, 369A and 435E). But the liberal forms of social organization cannot rightly serve as such models. We in the West are still having to learn how to live our lives without any guidance from the State. England and America have progressed the farthest in this school of liberalism. Continental Europe has had considerable difficulty with the lessons—as the popularity of Hitler, Mussolini, and Franco attested. The East instinctively resists the movement. And in much of Africa and South America one finds only a thin veneer of liberalism covering a populace more or less oblivious of the imposition of our 18th-century ideals.

In the liberal West, one must learn to speak and to listen in public without any bridge between the obvious embodiment of public concern (the State) and the ultimate location of decision (the individual). One learns with relative ease the first lessons, becoming a sophomore and a junior with regard to the State. Only with difficulty does one learn what it means to become a senior despite the State, within its framework but without taking the framework as a model.

Within and despite the State, we still find ourselves in fleeting communities. It is in these latter where one can learn what it means to respond fully within public discourse. In the State itself, public discourse remains monologue: its prototype is the speech of the media, from which one might obtain abstract information but to which one cannot really respond except with skepticism, ultimately with cynicism. In a fleeting Community, on the other hand, public discourse may at any moment evolve into dialogue. To be sure, direct and live interchange within familiar circles of relatives, friends, colleagues, and neighbors tends to take the form of habit, of mutual accommodation: we tend to remain freshmen. Still, the discourse within a family (at first with one’s parents and siblings, later with one’s spouse and children), at work (among managers, foremen, employees, customers), at play (in sports, hobbies, and casual gatherings), in civic action (school, neighborhood, and community projects), and even in genuinely deliberative and litigative assemblies: in all these fleeting speech communities it is possible for us, often incumbent upon us, to stand up and speak out: to uphold the truth of the discourse, to do justice to the action. In short: to become seniors.

One of the crucial differences between a State and a Community lies in the differing meanings of leadership. In a State, leadership is nominal at best: a leader only makes announcements to the public, is a real leader only in his cabinet or his caucus; the factual “leader” of the people is the intervening bureaucracy. In a Community, on the other hand, leadership is resurgently necessary and fleetingly actual. As a parent, a foreman, a manager, or the like, one must both detect and employ underlying beliefs—just as a leader in the State must do. But in their respective communities, communal leaders remain in office only so long as they re-attune those beliefs, refocus the children, the workers, the neighbors onto essential issues: here, unlike in the State, one must create and preserve communal enterprise, instill a spirit of cooperation, truly lead and represent the Community. Such is the action at all times.
possible for human beings. And this action locates the full-bodied public discourse in which persuasion becomes a genuine issue, and in which we can fully wage and momentarily settle the recurrent battles between paralogism and syllogism.

Some such distinction between State and Community, monologue and dialogue, bureaucracy and leadership, will help us as logicians to understand the differing views as to what counts as a fallacy; the differences between moderns and ancients are quite remarkable in this regard. But it also and more importantly helps us as philosophers to contemplate what it means to overcome fallacies, what issues are at stake in speech, what it is that we human beings all too easily betray in our daily productions and actions.

§2. Snow-job fallacies: facts

A recitation of facts can be impressive. It indicates, we would like to think, a door onto the matter at hand. Truth, we would like to think, consists of an extensive set of accurate determinations. And one who can recite such a set seems to be drawing upon his own intimacy with the matter. For how else could he have such an array at his disposal?

Yet we eventually learn that factual accounts can also mislead us—can weave a veil concealing, or at least obscuring the matter, even if each fact is accurate in itself. For however strong each thread may be, taken together the threads may lead nowhere outside their own fabric. Without at all lying, a salesman can deceive us, snow us with facts about his commodity and thereby detract us from a direct consideration of it and of its relation to our own genuine possibilities of choice. Similarly, a big talker at a sports lodge might impress us, especially if we are novices, whereas a real sportsman will notice that the array of facts, accurate or not, stands in the stead of genuine intimacy with the matter under discussion. Indeed, at the artisanal (productive) level of life, talkativeness often compensates for incapacity.

We find the most pronounced examples of snow-jobs in public assemblies, in courtrooms, and (nowadays) in the speech of the media. In a Town or Faculty Council meeting, where it is permitted to bore the captive audience, one may recite statistics, data, and accounts of past events ad nauseam to convince others to vote as one wishes: the audience assumes that the speaker knows what he is talking about. In a court trial with judge and jury one must be more circumspect: the judge might question the relevance of the recited facts and the jury might sicken from the diet. And in campaign speeches and other advertisements one must exercise special care to please the audience when reciting otherwise dry facts.

In intellectual work, we readily acknowledge that there is a notable difference between reciting facts and understanding them, i.e. understanding the matter they supposedly represent. A teacher must read many a snow-job (innocent, snide, or cynical). A student might have to read or hear month-long recitations which fail, at least for him or her, to add up to anything. Professionals themselves meet at conferences, often only to display their learnedness, their having determined multiple threads within a fabric having no significance: it is, after all, possible to know all or much about works, authors, and schools of research, thought, or literature, without even trying to know the matter to which the originals were responding—perhaps even without believing that there is such a matter.

Yet we obviously need facts. A good carpenter and a good skier must know the facts about materials, e.g. about the properties of different woods and joints, about the different kinds of snow and wax. A good statesman and a good judge must know what laws in fact prevail and what people and nature herself have been doing. Similarly, a good teacher and a good scholar must know exactly what his predecessors have said and done. Such exactitudes are necessary. Yet they do not suffice: however firm our grip on them, we may still fail to respond to what is itself most exacting in joining or skiing, governing or judging, contemplating or understanding. And when such exactitudes are employed as though they were not only necessary but also sufficient, they are deceptive: we rightly learn to distrust them, i.e. the arguments based on them.

How can these friends turn out to be our enemies? What are these friends-cum-enemies? One can quickly detect a narrow meaning of “fact” —and just as quickly notice that the word more often, and quite legitimately, has a broader meaning as well.

Most narrowly: a fact is a formulated determination of how things already are in our repertoire, i.e. how they were at the time and place of our determination of them and now are as savings from the past. We obviously formulate and rely on such determinations in the course of
making things, leading organizations, and contemplating human endeavors. It is a fact that this beam is rotten, that such-and-such a wax allows the skis to slip well over such-and-such snow, that there are so many people in such-and-such area, and that they consume so much water and so much electricity each month. It may be a fact that the accused possessed a rifle and was seen at the restaurant on such-and-such a day and hour. It may be a fact that Socrates says such-and-such on a certain page of a certain book. And in the rise of modern science facts become of special interest: determinations expressly formulated in anticipation of discovering the mechanisms of nature, and formulated as data suitable for use in mathematical formulae.

On the narrow sense of the word, nothing counts as a fact that we cannot formulate as a thing of the past. It can never be a fact that a beam will hold or that the house will stand firm, that the legislation will have the desired effect or that the sun will rise tomorrow, that the man who shot the victim did so with criminal intent, that Plato meant to say X when he in fact wrote Y, that Napoleon wanted A while he was doing B, or that a work of art is beautiful.

Most broadly: a fact is any formulation of an affair on which we choose to rely. In this sense, I tell my wife that it is a fact that we have enough gas to get to Moncton, or a woman tells her boyfriend that it is a fact she will leave him as soon as she can get packed, or a businessman tells his superior that it is a fact that so-and-so cannot be trusted, or a science teacher tells the students that such-and-such a theory is now a proven fact. In all such conversations, the word “fact” refers to a settled policy, belief, or judgement: the reference of the determination is not directly something faced and done, it is rather a disposition with or in which one faces and does things — and asks others to adopt similarly.

Although thinkers of all Interpretations have pondered the peculiarities of our need for facts (that they are somehow necessary but not sufficient), we moderns are most familiar with the form the contemplation takes in the works of Galileo and Bacon, Descartes and Locke, Hume and Kant: the relation between concepts and percepts — between theory in mathematical form and data gathered from experimentation. In rough outline, the outcome of the debate, the modern verdict has become clear: facts in the narrow sense originally arise and continue to make sense only in and by our own commitment to facts in the broad sense, while tensions and discrepancies between these two forms of givenness inspire us both to revise the broader and seek the narrower — inspire in us a future. All this we shall consider carefully in Book Four.*

In the works of Plato and Aristotle, indeed throughout both the Second and the Third Interpretations, we find a simpler answer to the question why facts of either sort are insufficient. For any determination about how things have already turned out touches them only as we have been affected by them: such determinations fail utterly to catch beings in their proper power of acting and resisting, they name only accidents (συµβεβηκτά: what has fallen to things, and then to us). And any settled formulation taken as a base for facing and making things is only an opinion (δοξα: a more or less shared and workable expectation). In neither sense can facts themselves lead us either to the nature of any being in contemplation, to any well wrought decision in action, or to the exercise of any skill in production. Both accidents and opinions are remnants. The color of a person’s eyes or skin, the location of one building in relation to others, the temperature of the air outside — all such determinations make real sense only after we have gotten to know the person, the building, the environment as a whole. Similarly, what “all people, most people, or wise people believe” — that democracy is the best form of government, that health is the most important thing, that E = mc² — makes genuine sense only after we relate to what these people draw upon to form their opinions. Thus, while facts in either sense provide no foothold and even mislead, we can still save them — namely by returning them to their origins. To effect this return, we must no doubt familiarize ourselves with the affairs of the relevant craft (the πράγματα: the doings and undergoings). And in such familiarity we learn what in fact belongs to the situation (the διάρκεια: e.g., the strength of each army, the availability of funds, the disposition of allies —Rhetoric, 1396 a 15). But we must still learn to distinguish beings (τα οντα) as they seem to be

* “There are no facts, only interpretations,” Nietzsche writes in a fragment dating from the 1880’s (Will to Power, §481). The aphorism and its context fairly sum up the shock of modernity. At no time, however, has any serious philosopher disagreed entirely with the conclusion. The disagreements arise in regard to the reasons. Generally speaking, the modern reason is that facts are replaceable players within an ongoing game, while the ancient reason is that facts (predicates) rightly emanate from, but cannot by themselves take us back to, intimacy with beings.
from these “same” beings as they really are: for in such contexts we work to overcome the former for the sake of the latter, we help potatoes or human beings activate or fulfill themselves, we partly complete what nature is unable to finish and partly imitate her. As idle spectators wandering around a farm or a city we might gather many facts (accidents), but we are essentially deluded, both about where we are and about who we are (we are dreaming: Republic, 476C). Any genuine understanding of facts requires that we understand beings in themselves, i.e. participate in their actualization. To save facts we must first save ourselves, i.e. become seniors.

In past ages most people, and sometimes even wise ones, more easily believed in magic than in facts. Nowadays, in contrast, facts have a popular appeal which provides much more suitable soil for the cultivation of snow-job fallacies. Indeed, both popular government and popular education depend on a popular faith in facts.

Still, Aristotle comes close to addressing our own involvement in factual determinations when he discusses paralogisms “outside talk,” i.e. fallacies not so much dependent on our manner of speaking and listening as on our understanding of what we are talking about. Aristotle lists seven varieties.* Besides (1) the fallacy of accident (paralogism παρά τὸ συμβεβηκτόν) listed first and prototypical of all paralogism “outside talk,” there are:

(2) The fallacy of secundum quid: confusing what is simply the case and what requires qualifications of place, time, and relation (παρὰ τὸ ἀπλῶς καὶ μὴ ἀπλῶς). It is one thing to say that people should feel free to express themselves as they see fit — and another to interrupt a funeral or a lecture (time and place are relevant in the application of a generally accepted truth). It is one thing to believe that everyone has the right to buy and sell land — and another for a politician with inside knowledge of pending policies to buy land cheap and sell it dear (conflict of interest: an example of understanding action in relation to...). And to forgive people, or to give them a second chance, we often argue that there are “extenuating circumstances,” that we must be “reasonable, and make exceptions”; but of course this principle, too, easily leads to abuse. On a more fundamental level, we may like to believe something like “Prometheus gave fire, hope, and all crafts to humankind” — and forget that no individual is born with these (the story may be reciting a possibility for each of us, or even a burden).

(3) The fallacy of ignoratio elenchi: arguing in ignorance of the (nature of) elenchus (παρὰ τὴν τοῦ ἐλέγχου ἀγνοιαν). In Greek rhetoric, opposition to another requires that one undo his or her argument, i.e. make it seem unworthy, shameful, or self-contradictory; the “refutation” (elenchus) must take shape as an “inside” job (most famously illustrated by Socrates). Ignorance in argumentation may simply take the form of changing the subject, wandering off topic. Thus, if your opponent argues for protecting wolves, you might try to refute this argument by talking about all the damage wolves do to live stock and the like. Aldo Leopold counters this refutation by calling attention to the reality of wolves:

No one can understand an animal by learning only its parts, yet when we attempt to say that an animal is “useful,” “ugly,” or “cruel” we are failing to see it as part of the land. We do not make the error of calling a carburetor “greedy.” We see it as part of a functioning motor.

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(4) The fallacy of consequent: taking a consequence, a result, as a necessary sign (παρά το επομένον — Aristotle elsewhere calls this fallacy “non-necessary sign”). We often argue this way, especially in judicial proceedings, where we call such results or signs “circumstantial evidence”: we misunderstand such evidence if we take it as conclusive. Alcmeone identifies her husband Amphitryon, judging by Zeus’ impersonation of him. We judge a person to be elderly because he or she has gray hair. We assume it has rained recently because the lawn is damp. We feel sorry for people who have tears in their eyes.

(5) The fallacy of begging the question: assuming at the beginning what one must conclude (παρά το εν αρχη λαβανειν). We often do this, as when you tell me that Joe is no friend of mine because he lied to me, and I say he could not have lied, since he is a friend of mine: we exchange minor premise and conclusion. Also when we simply employ synonyms: Joe is unreliable because we cannot trust him. And even in strictly formal deduction we easily fall into assuming what we are trying to prove: I try to prove Barbara per impossibile, and proceed in the usual fashion, relying on Bocardo (which is proved with Barbara).

(6) The fallacy of non-cause: trying to understand something by focussing on a non-cause as though it were a cause (το μη αιτιον ως αιτιον τιθεναι). In his Rhetoric Aristotle illustrates the fallacy in temporal terms: politicians, he says, look to what happened before as the cause of what is happening now (our opponents raised taxes several years ago, and look at how the economy has slowed down! — later, this fallacy was called post hoc ergo propter hoc: after the matter, therefore owing to that matter). However, any effort to understand the inner workings of something may run afoul: we may try to “define” a good teacher as one who behaves in certain ways — these ways supposedly getting to the bottom of what it means to be a good teacher. In more contemplative debates, one might have to propose a cause or ground of something — or, much more easily, show that someone else’s proposal does not do justice to the matter. In our classical tradition, the cause of something must contribute to our understanding of its fulfillment: we even name something according to its good version, and understand lesser versions as failing to live up to their fulfillment (so that the true cause of many events is a failing of some sort). — In Book Three we shall consider the stringent requirements for discovering causes.

(7) The fallacy of many questions: posing as one question what is really several (το τα πλειω ερωτηματα εν ποιειν). Aristotle's examples refer to contexts where one asks and the other must answer yes or no: a colleague asks me whether the students in one of my classes are good or bad, and I can't answer yes or no, since some are good and some are bad; I ask you whether you support the President and will campaign for him, and you insist there are two questions. In a legislative context, policies come forward that try to solve a number of problems at once, and we might insist on “dividing the motion.” In contemplative discourse, I might want to argue that “facts serve to obfuscate issues,” and you might wish to divide the question: What shall we mean by “facts”? What sorts of things count as “issues”? How does such “obfuscation” take place (so we might we avoid it)? Situations do arise in complicated ways, so that it proves very difficult to keep topics separate. Yet fruitful discourse must lead to some separation, even if only to interrelate the issues. The New Yorker once referred to a dramatic instance of failure during a televised debate:

A young black woman expressed her contempt for whites who adopt black babies. ‘Anyone who really wants to do something for blacks,’ she said, ‘should give us jobs. If whites want to play God by taking black babies into their homes, let them do it after they open up their businesses and factories to blacks.’ The whites on the program were obviously reluctant to put themselves in the position of arguing with blacks on racial questions. Consequently, no one exposed the obvious fallacy in the young woman’s statement. She had assumed that all white parents who adopt black children are business people in a position to hire blacks.

It may well be that any given example of “missing the point” could fit into several boxes, since the exact fit depends on how we ourselves construe the invisible line of fallacious inference. In any case, the first of the seven, which Aristotle calls the “spurious enthymeme proceeding by means of accident” (Rhetoric, 1401 b1 5), throws the brightest logical light on snow jobs.

For example, one delivers an encomium on mice, concluding that these beings are wonderful because they destroyed the enemy’s weapons, stores, and crops. For exercise in public speaking, you could take most anything and concoct an argument of praise or blame: you only need
gather predications serving as middle terms, all of which are accurate but none of which touches the essence of the subject. The major premiss may even be plausible (“anything which destroys our enemies deserves our gratitude”). But there is something wrong with the conclusion: the predicate term does not really apply to mice; rather, it reflects our own interest in victory. But then the predications constituting the extended middle term also fail to talk about mice—in their essential nature, that is. The encomium concentrates attention on a smoke screen of accidents.

Aristotle’s strict definition of “paralogism based on accident” introduces a further subtlety (Sophistical Refutations, 166 b 27): the error consists in “deeming something to belong in like manner both to the matter and to the accident.” Mice are wonderful because they are creatures that destroyed our enemy’s goods: but the destruction here points essentially to our enemies, or to our glee at their misfortune, and only incidentally to the mice.

In ancient times a favorite game of sophists was to show that a given predicate both belongs and does not belong to a given subject. “Do you know that man over there with his face covered?—No. — But look now, he is your long-known friend.—Oh, yes.—So you both know and do not know your friend (or your friend is both known and unknown by you).” From this whacky argument we might extract a valid AAI-3:

That man with his face covered is your well-known friend.
That man with his face covered is a person unknown to you.
∴ Some people unknown to you are your well-known friends.

However, the predication of the minor premiss is strictly accidental; it says something about you, nothing about “the man over there.” Obviously, the covering over the face prevents the recognition of the man: the subject literally vanishes in the course of the predication. Yet both Plato and Aristotle take this form of argumentation seriously: the very whackiness of the conclusion stands as proof of our need to predicate beings, to uncover them in our predications, rather than trying to predicate other predications: the middle term must reveal beings in their being, and not simply parade incidental determinations.

In festive encomiums and eristic debates “deeming something to belong equally to the matter and to the accident” does no great harm, and it can be amusing. However, as Plato and Aristotle both complain, in courtrooms and parliaments such otherwise playful exercises lead to decisions based on the virtuoso skill of speakers and not on anything revealed in the speeches—they lead, that is, to political disaster. More yet: these exercises militate against the kind of self-knowledge required for dialectical discussions (philosophy). For such discussions require at the outset an acknowledgement of our tendency to lose both ourselves and reality in a phantasmagoria of accidents. Do we not all too easily predicate honor of what simply “attaches” to the honorable being rather than of the honorable being itself? We think “being invited to the boss’ home” or “being promoted to a full professor” or “being awarded a Nobel Prize” is honorable, whereas we must learn that it is the being’s performance leading to these “accidents” that is truly honorable. And do we not try to “understand” non-human beings very indirectly, namely by relating a few of their incidental predicates to our own affairs? Mice, we discover, eat holes in our belongings stored in the attic, chew off the bark of the saplings we planted last spring: mice, we say are ugly animals, useless animals, vicious animals. At best, we could say that the subject has changed: M-1 (destroying) belongs to S-1 (mice); but P-1 (ugly) only indirectly (apparently) belongs to M-1, it more truly belongs to another subject, S-2 (we are indeed discomfited by losing our goods): accordingly, the paralogism consists in concluding something about S-1 (mice) which really belongs to S-2 (us!).

Freshman may not mind. Sophomores may object. Juniors may capitalize on the cover. But seniors will wonder how one can penetrate the cover, predicate beings themselves rather than other predications—how one can “see together the same and the different” (167 a 38), how one can “propose one [predicate] about one [subject]” (169 a 8). Such penetration is indeed possible: there are people who understand the honorable performance underlying other predications, the mice underlying the predications evident to the frustrated farmer or his wife. There are even books designed for contemplating these matters: Homer’s Iliad or Plato’s Republic on the question of honor, any number of popular books on mice (e.g., on raising them in captivity: on their feeding, breeding, and nesting habits). But what turn of events, or turn of mind, allows us to concentrate on beings in this way?

Besides the obvious requirement that we desire to perform honorably or understand mice fully, i.e. that we be willing to activate ourselves and to see in any being “something of a wonder,” Plato and Aristotle repeatedly formulate an ontological distinction as it pertains directly to
our own vocational efforts as makers and doers. For instance, in Plato’s 
*Statesman* (284E) we read:

Clearly we should divide the art of measuring, severing it in two. One part comprises all the arts of counting: how long, how deep, how wide, how fast something is — always with a view to opposites. The other part assesses things with a view to the measure itself: to what is seemly, what is fruitful, what is obligatory — always with a view to a mean between extremes.

The examples are the art of weaving cloth (production) and the art of leading people (action). Any art requires one to learn two sub-arts: locating beings on various scales and relocating them according to where they should be on the scale. The relatively easy sub-arts are those of number: factual determination. The ultimate sub-art is that of the guiding measure itself (*τὸ μέτρον*): the standard. The first kind pertains more to the “materials” of production or action. The second kind pertains more to the “purposes,” that must “emerge” in the full exercise of the art. The agony of learning any art fully is that no matter how much one applies oneself to the first part one may still fail at the second. Whether weaver or statesman, cook or captain, carpenter or chairman of a philosophy department, one must not only be able to ask and answer how matters quantitatively stand: one must also ask and decide what constitutes a good fabric or a good society, a good meal or a good ship, a good house or a good program. We may initially try to answer these troublesome questions by simply referring to what we happen to desire: we try to reduce the good to what we happen to want for ourselves. But actual involvement teaches us that we must rather learn to desire what is good — what it is that we must help actualize: we must induce ourselves to want the good. Everything depends on what Plato calls the “emergence of the measure.” This emergence will then define what is too much or too little (too much glue in the joint, or too little; too much punishment, or too little). The apprentice learns to his sorrow that he cannot start with the extremes to find the mean; Aristotle sums up the same point in telling of the man who was asked whether he wanted his bread baked soft or hard: “What? replied the man, Is it not possible to bake it well?”

According to both Plato and Aristotle (indeed, our whole classical tradition), the central question divides into three: Is the “measure” at issue what is seemly (*τὸ πρέπον*): beautiful, noble), what is fruitful (*τὸ καιρὸν*: timely, advantageous), or what is obligatory (*τὸ δεόν*: just, fair)? These three correspond roughly to the three temporal dimensions of our dealings and to the three natural directions of our desire: we desire to make and do things for fulfilling the present (our house at the moment, our athletic performance at the moment), we desire to make and do things for fulfilling the future (insulating the house for the winter, exercising our bodies to get in shape), and we desire to make and do things for fulfilling the past (we had promised the customer we would insulate the house well). Whenever the question arises about the “emergence of the measure” we can, at the very least, ask which of the three is at issue: Should we decide upon the institutional policy with a view to nobility of human action, economic benefit, or the fulfillment of agreements already in force?

In making something (in production: forestry, carpentry) the question of the measure ultimately comes down to the nature of non-human beings (what it is that trees really need, how lumber really behaves — in themselves and also in given times, places, relations). To learn an art fully one must ultimately learn to relate to the first of the three measures. For the ultimate beauty of the beings one makes (trees or forests, cabinets or houses) also serves as the standard (truth) of what will prove advantageous (future yield) and even just (fulfillment of past commitments). This relation of the craftsman to the measure defines the knowledge essential to his art. It requires that one promote the beings one knows: the determination of how they happen to be (facts about beings) becomes and remains relevant only in light of the measure emerging to define how they (in their individuality) must be (in their universality); and this emergence (*γενεσις*) depends upon our own response to it. One can neither perform an art effectively, nor even take full note of the beings one encounters, until one has developed a “sixth sense” for their good — and has ceased trying to satisfy human

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*Rhetoric*, 1416 b 30; the image of the “right measure” falling somewhere between “two extremes” receives lengthy consideration in his *Nicomachean Ethics*, Book Two, Chapters 6-9. Interestingly, both Plato’s and Aristotle’s pretext for talking about the need for a non-quantitative (non-factual) relation to the measure is the question of how long a speech should be. See also Plato’s *Laws*, 722A.
In the learning and performing of a productive art errors abound, both of the basic type (failure to relate to the good) and of the incidental type (failure to determine givens correctly); apprentices, for instance, recurrently take non-necessary signs as though they were necessary. However, there are obvious “moments of truth” when errors of both types become glaringly evident, even to outsiders — namely, in the ineffectiveness of the performance.

In action, matters are both the same and different. As teachers or foremen, as judges or jurors, as chairmen of committees or members of some parliament, we will always work not just with facts about circumstances (human or otherwise) but also with assumptions about what is good for people: human being is our subject, or rather what is truly noble, truly beneficial, truly just in and for human action. But the performance of a practical art differs from the practice of a productive art. First, our medium is not directly any material or tool (neither earth, air, fire, nor water, neither hammer, knife, helm, nor reins). Rather, our medium is speech. Secondly, the immediate issue is no product directly given in experience (neither crops nor ships). Rather, it is persuasion, joint decision (ultimately the preservation of the city: see Republic, 465D). And, thirdly, the real effect of any decision is the propagation of the major premiss enthymematically instituted by the drawing of the conclusion: this it is that sets the standards invisibly governing the community.

True, the art of speech appears neatly divided into three kinds, corresponding to the three kinds of decisions and goods: epideictic, deliberative, and forensic (Rhetoric, I, 3). But one of the tasks of a master speaker and master listener is to discern which of the three is genuinely at issue, and to cultivate such discernment in others. This preliminary task already poses serious difficulties, since people as a rule are not yet seniors, at most juniors: “One openly praises justice and nobility but secretly wishes advantage for oneself” (1399 a 30). But even apart from this basic failing, one often confuses the three, handing down a judgement in a court of law according to the charm of the various personalities; or awarding a prize for an inferior musical performance because an earlier superior performance went unnoticed. And even if we all know which of the three goods is at issue, we must still decide the tricky question of what minor premiss, what predicates of the subject, truly serve to warrant the conclusion in the given case.

In short, there are seldom if ever “moments of truth” in action as there are in production: the truth lies behind, not before the eyes, and debates rage precisely about what counts as “behind.” It seems that only insiders can judge the truth of action, and these often pay the price exacted from Cassandra for refusing Apollo the customary intimacy. In any case, speech in the domain of action will always be rife with snow-jobs. And we can, as logicians, formalize them as efforts to conclude that \( P \) (beautiful, advantageous, or just) applies to \( S \) because \( M \) (a number of factual determinations) belongs to \( S \) — where \( M \) serves to distract us from a consideration of whether and how \( P \) belongs to \( M \). In contemplation, then, we recognize the necessity of raising this generally neglected, in snow-jobs intentionally obscured question in its various forms — the relevance of acknowledging the wonder implied in the question and accepting the responsibility for decision despite the lack of factual clarity regarding the major premiss, whatever the clarity of the minor. Herein lies the only “moment of truth” for action.

* See Plato’s Republic, both the concrete polemic with Thrasymachus (concluding that, for the master-artisan, “justice is the advantage of the weaker”: 338A to 346E) and the abstract claims about the “good” (concluding that the vision of the good is the ground of both our coming to know and the Being of the beings we can know: 508E to 509B). What modern readers find puzzling is the ontological status the Greeks ascribe to the “good”: such a status makes no sense from the standpoint of afoolish observation, which provides us only with experiential givens enwrapped in intellectual construction. Starting with Galileo, Bacon, Descartes and the like, observation replaces recollection. Indeed, at the end of the Third Interpretation “recollection” had no doubt degenerated into the recitation of Aristotelian and Ecclesiastical decrees, and philosophers had become “historians, or memory experts”—just as Galileo testifies (see his Dialogue Concerning the Two Chief World Systems, especially the first five or six pages of the Second Day). Still, the Fourth Interpretation generated profound reflections on the ethical status of the good: the meaning, agony, and drama of learning to will the good. I think of Kant’s Foundations of the Metaphysics of Morals and Kierkegaard’s Purity of Heart is to Will One Thing.
§2.1 The question of beauty

An art historian lectures on the Cathedral of Chartres, an English teacher on Melville's *Moby Dick*—in each case trying to convince me that the work is beautiful. Or I myself am already convinced that one of these works is beautiful and would like to get a friend or some other audience to share in my conclusion. In any of these cases, descriptive details (DDs) may fly through the air like snowflakes which the wind's blast, whirling the dark clouds, drives in their abundance along the prospering earth. *Iliad*, XII, 156.

But only if “All DDs are B” does “S is DDs” justify the desired conclusion. Will we, as listeners or speakers, raise the major principle into explicit consideration or not?

If not, we may fall prey to facts. As freshman we may for a while even accept the argument. As sophomores we may give up in disgust. And as juniors we will likely drop the conclusion altogether and make a profession out of the minor premiss.

If we do raise the question, we might aspire to fix a description of the cathedral or novel, i.e. formulate an M, which both matches the subject under consideration and, in our opinion, warrants the major judgement that *any* work fitting this description is beautiful. Pursuing this aspiration, we may either grant that beauty differs in kind from example to example, or insist upon discovering a common denominator for all different kinds—cathedrals, novels, and concertos, women, men, and horses, dancing, skiing, and singing. In either case, we try to formulate middle terms necessarily implying the predicate “beautiful.”

Can we rest content with the reduction of beauty either to personal taste or to general description? The first may serve the purpose of some classroom work—the purpose, namely, of more or less cynical pedantry, devoid of decision. It immediately fails, however, in contexts of action. A football player, a play during the game, an entire game: these may be beautiful or not. And while a fan in the grandstands may or may not judge these matters well, the coach and the players need to know the difference. Similarly in most any affair: there are beautiful performers (who eventually set the standards for others), beautiful performances (subsequently both remembered and anticipated), and beautiful situations (families, neighborhoods, towns, even departments, committees, or universities). For those engaged in such affairs, beauty remains a pressing issue, hardly reducible to solely personal taste. Yet the effort to settle the question by formulating a general description betrays the issue: henceforth, any upcoming S must simply obey the rules of M in order to warrant the predicate “beautiful.” And precisely the effort of any S simply to obey established rules, and our own effort to force S into such Procrustean Beds, generates the opposite: ugly performers, ugly performances, and ugly situations.

What is the pressing issue of beauty? Effectiveness? But things can be beautiful despite their marked failure to serve a purpose. Let us not confuse this good with future advantage (utility), just as we must not confuse it with past standards (justice). Rather, the issue is directly one of presence: what it is that genuinely concentrates us on present performers, present performances, present situations. These are beautiful (or noble) when they stand as complete for a moment, when they reveal themselves as containing their own standards and purposes at the moment. They then become memorable and desirable (past and futural), but only derivatively from the present. Such presence is indissociable from action and from the heart of action, decision. We might say that works of fine art, as distinct from artisanal works, athletic events, and communal enterprise, “specialize” in presence: not only create a presence but articulate the conditions of any genuine presence. Linguistic works like Sophocles’ *Ajax*, Shakespeare’s *King Lear*, and Conrad’s *Lord Jim* expressly portray the agony of decision within action. Yet the Cathedral of Chartres likewise articulates decision—in ways we now try to understand under the rubric of Church doctrine.

Now, if “beautiful” truly predicates only those subjects which embody a presence of decision and action, we can understand why no factual determination about an art work, a person, a game, an institution, a landscape, or whatever, can ever truly point up its beauty. For facts in either the narrow or the broad sense are remnants from better days: they call attention only to done-nesses, was-nesses. Worse, the exclusive concern for facts embodies a stance of non-involvement, non-action, non-decision: the aloof observation antithetical to presence, the peculiar desire to know at a distance, to establish a position leaving one dissociated from the subject appearing in the offing.

Returning to the lectures on architecture and literature: What might a professor, what might we do? How can one call attention to something
as beautiful? Since any effort at distancing (mere recollection, mere anticipation) contravenes the issue, we might answer the question by reversing the direction of such effort: in order to bring home the conclusion that a person or an action, an art work or a game, a landscape or an institution is beautiful, our speech must not so much refer to the beauty as embody it. For only then might we prevent its dissolution into past and future — and present its presence.

So... need we only dress up our speech with a rich tone of voice or a pretty handwriting, exotic words and suggestive metaphors, proper symmetries and regular rhythms? But such accouterment serves very well simply to lull others into consent, reduce them to freshmen, obscure the subject... snow the audience or the readership.

Granted that, as in everything else, “beauty” may only stand in for the pleasure of first impressions, we can still ask wherein the genuine beauty of speech might lie. And surely such beauty must bear some resemblance to the beauty of a person or an action, a cathedral or a novel: it must display complete presence. But how can speech display such presence? Especially on the more familiar Fourth Interpretation, the function of speech appears to lie in the double movement of referring to established determinations (always past, even if very recent) and expressing desires (directions for the future, even if almost immediate). Indeed, the marvel of speech seems to lie in its (our) power to orient ourselves and others to elsewherees and elsewherees. Far from embodying its own meaning, its own beauty, speech seems committed to pointing to it in the distance, to recalling and procrastinating. A person or an action can appear genuinely beautiful by embodying a harmony and spontaneity of movement, a complicated but simplified interrelation of decisions. But how might the speech of a professor or an essayist undo its apparently natural, certainly more usual dissipation into what has been and what will be?

Let us review the syllogistic structure of epideictic speech: S is beautiful, we want to argue, because S is M — where M says something definite about S. Both facts and accouterments contribute to the presentation of the minor premiss. But the crux evidently lies in the presentation of the major premiss. Can this presentation effect the embodiment?

Certainly not if we simply mention the major, or hear it mentioned. Rather, we would have to consider it, discuss it, question it — take it as a framework for bringing the subject to light. Does “beautiful” truly belong to S (cathedral, novel, action, person) once determined to be M? What determinations (formulated in M) belong to S while truly leading to P? The interrogative mood at least overcomes mere assertions (opinions and determinations which are at best shared and accurate but never sufficient), and converts them into proposals. And proposals, unlike mere assertions, must be completed: they remain meaningful only on the condition that speakers and listeners remain in hot pursuit of what is being proposed — the subject both as primary being (the cathedral, or whatever) and as secondary being (what makes cathedrals really be). Assertions refer or fail to refer: we count up the references in the minor premiss. Proposals invite us to participate in decisions: we interrogate them in the major premiss — this premiss being a framework for our attention to the subject rather than a possession to be defended or refuted (as in eristic). By engaging speakers and listeners together, speech becomes indeed an affair with presence, a shared action, a truly beautiful game: beautiful because reactivating the participants and actualizing the subject under discussion. Such speech embodies harmony and spontaneity, complication and decision, to the utmost."

It seems that a snow-job aims to secure a conclusion at all costs, to end the discourse, to have the last word, whereas a true speech takes us back to the beginning, thereby converts the conclusion into a question, and claims only to utter the first word. But, then, does speech aiming at the beautiful remain incomplete, inconclusive? Must a professor or an essayist sit on the fence? What is accomplished in speaking and listening if one only starts but does not end?

Perhaps a discourse truly ends by locating us in the middle of the subject matter itself. In any case, speech about presence only makes sense as having presence: like music, it can only be in the playing, in the speaking and listening — in the harmony and spontaneity, the complication and decision of ontological attunement. And just as we are

* Plato’s *Phaedrus* must surely count as one of the most beautiful works on how works can be truly beautiful. The third speech (Socrates’ second) proposes and illustrates the point in contrast to the first two (which are clearly snow jobs), and the dialogue continues to comment on it. With this dialogue in mind, one can also read Plato’s *Laws* (around 719-720) as arguing that beautiful laws are basically proposals rather than assertions — draw their subjects into reality rather than simply force them into conformity.
neither playing nor hearing music if we fall into merely waiting it out, so we are not really speaking or listening if we only wish to conclude the affair, get it over with.

A good discussion does not really end, it carries over, if only in silence. Epideictic oratory culminates in contemplative speech. Yet a good book culminates in much the same way. In his *Mont Saint Michel and Chartres*, for instance, Henry Adams seems to lecture us on the beauty of cathedrals. Yet the apparent lecture quickly takes the form of a puzzlement: How could the Middle Ages articulate such presence (beauty, power) while we today, for all our steam engines and attendant excitements, can bring our energies to focus only on the past or the future — and lack all sense of such presence? Similarly, in his *Moby Dick* Melville instructs us in the details of whaling, reciting fact after fact about whales; yet the book explicitly leaves us in the middle:

But now I leave my cetological system standing thus unfinished, even as the great Cathedral of Cologne was left, with cranes still standing upon the top of the uncompleted tower. For small erections may be finished by their first architects; grand ones, true ones ever leave the copestone to posterity.

For the truth of speech lies in the discovery of the subject (whales and whaling, say) and not in any list of predicates only, no matter how extensively ranging or intensively accurate. And since this discovery of the subject must also be a self-discovery (a discovery of oneself in whaling, say, and as a whaler), the copestone must eternally be reset: the book never sets it because it consists of an articulation in which setting it is precisely the issue. The truth of reading lies in the setting itself, just as the truth of music lies in the playing.*

To one unable or unwilling to engage in completing the first word, the proposed conclusion appears to attach “beautiful” directly to the subject: snow-jobs, concentrating one on the minor premiss, encourage one to look for such after-the-presence predication. Yet any such attachment fails to stick: the predicate will flee back to the speaker or listener and serve rather as a sign of a pleasant reaction or someone’s desire for approval. In the Twilight of the Fourth Interpretation this flight has been legitimized in the name of the “fact-value distinction.” One first assembles facts about beings and then asks what one can or should esteem: one assumes that the beings factually predicated stand in no need of revelation, and that one need not wonder how to contribute to or participate in any such revelation. By contrast, but with varying emphases, all three earlier Interpretations understood the basis of value to be precisely the revelation of the subject under consideration: if only I could understand whales, how wonderful and how beautiful! For such understanding requires that I first discover them, and such discovery requires that they, for once, be revealed and, in being revealed, bring me out of humdrum appearances and reveal me as well. The predicate of the conclusion does not so much attach to S as it signals the emergence of S. As Plato says of the good in his *Republic* (509), the predicate locates the source both of our own being as knowers and of those beings which we come to know: every effort to make it stick on something after-the-presence is bound to fail.

If, now, what is truly at issue in a speech concluding that an S is beautiful is the emergence of the subject itself, we can indict snow-jobs on the grounds that they commit a perverse crime, a felony, rather than a mere prank. For they distract from what is most exacting for us — precisely by concentrating attention on all kinds of exactitudes pertaining to human constructions.*

But how can speech help the emergence of its subject? This question has been with us from the beginning — from the moment we considered the structure of categorial discourse and of the syllogism built upon this structure. In regard now to the specific effort to persuade others to judge something to be beautiful, we are considering the possibility of speech helping the emergence of the subject not simply as an individual by itself but also as embodying the “emergence of the measure.” Indeed, the central claim of the Second Interpretation is that an individual being can only emerge when its measure emerges. Oddly,

* Hermann Hesse’s *Glass Bead Game* (1943) creates a dialogue on this question of completing beginnings in an age in which pure construction seems to set the standard for all intellectual work. See especially the epistolary exchange between Knecht and his teacher in Chapter Three: Knecht tries to articulate the beauty (meaning) of the Game and his teacher rebukes him: one should concentrate attention on the eighths and sixteenths of a musical piece, on the linguistic and metrical strategies of poetic works, and leave the rest to take care of itself. Yet the book as a whole helps us to take care of it.

* See Plato’s *Statesman*, 284D, and his *Republic*, 504E, for these opposing meanings of “exact.”
by concentrating us on the measure itself, the best speech contributes most to the revelation of individuals.

So what can we do if we wish our speech about beauty to remain true to its subject? How might we learn the second measuring art—and become seniors?

Interestingly, there is a crossroads at which the best of the Second and the best of the Fourth Interpretations meet: the beauty of something becomes manifest only inasmuch as we ourselves are entering into it truly. Our “biggest mistake,” the one paralogism underlying many more obvious ones, would be to speak and listen, read and write, as though some things were simply beautiful and others simply ugly—apart from our participation in their revelation. Indeed, to argue in this way would be to embody the first condition of ugliness: to place us outside.

Snow-jobs concluding that something is ugly illustrate even more powerfully our need to enter. Say I want to argue that the house you have built, the essay you have written, the game you... in the Faculty Senate to argue in favor of a university regulation requiring every student to complete at least one course in philosophy before graduating. In either case the conclusion likely takes the form of a proposed advantage: the federal law will benefit the public at large, the university regulation will benefit students and therefore the society in which they live and work. Why? Because the law will lead to a reduction in the incidence of lung and heart disease, the academic regulation will lead to greater reflexivity of the sort necessary for significant research in science and wise decisions in society. The speaker must, of course, illustrate the middle term in each argument: “reduced incidence” or “greater contribution” simply summarize facts—statistics showing how active and passive smokers suffer the diseases much more than do those living in a smoke-free environment; histories showing how students drilled in philosophical questions have made greater contributions to science and society than those untouched by such questions.

What can a listener do? Let us assume that the speaker has accurately paraded the facts. Moreover, let us assume that one does not place one’s private advantage above the public advantage (personal addiction to tobacco, personal dislike of philosophy). Must one vote in favor of the one or the other policy?

Schematically formulated, utility arguments are twofold: This Policy is Advantageous because it Fosters desirable future conditions (with the major premiss: All policies Fostering desirable futural conditions are Advantageous), and it does in fact Foster these because it Imitates desirable factual conditions (with the major premiss: All policies Imitating... are policies Fostering...). Or, reconstructed as a sorites with “policies” as the parameter:

1. All Fosterings of... are Advantageous
2. All Imitations of... are Fosterings
\[ \therefore \text{ All I are A} \]
3. This Policy is I
\[ \therefore \text{ This P is A} \]

Here, “imitation” has the classical meaning, as in Plato’s insistence that the constitution of the city “imitates” our life at its center (the soul).
Concretely presented, a utility argument will likely justify Premiss 1 with a probability such as “Health serves both to heighten the quality of life and to curtail social costs.” And it will justify Premiss 3 by a recitation of facts: the policy must appear as imitating facts about the interrelations among smokey environment, lung disease, and physical, mental, or social performance. Agreeing with all this, what grounds could we have for doubting the truth of the argument? Close examination of the sorites reveals three such grounds.

First, Premiss 2: it promises a relation between the past and the future, a relation mediated by present action on our part. Questions of utility are by definition questions of the future. But we form our present actions for the future by tracing out the past: what has in fact happened in smoke-free vs. smoke-filled breathing spaces, or what philosophically educated vs. unphilosophically educated people have contributed to society. Oddly, every argument about future advantage must dip deeply into the past for its evidence. In contemplating such arguments, we can therefore always wonder whether the future will indeed resemble the past. In working within purely natural phenomena we assume that “time is a moving image of eternity” (as Plato says: *Timaeus*, 37D), and we are seldom deceived: night will follow day, and day night; spring and its flowers will follow winter and its frosts; male and female rabbits will mate when placed together; human beings will slowly grow up and slowly wither away. However, humanly decided arrangements notoriously lack such uniformity: just as the words and flowers which succeeded yesterday fail today, so what worked in Athens fails in Rome, and the monetary and fiscal policies stimulating the economy today may very well kill it tomorrow. It seems that in human affairs the future, far from resembling the past, requires ever fresh attention, ever new arrangement—perhaps because it belongs to human nature to respond differently to any situation that has become old, i.e. because it is old. In any case, no number of facts about what has already happened in human affairs can entirely displace wonder about what will happen.

Second, the policy itself: although in substance simply naming an action of human beings, the proposed policy consists of an extensive verbal formulation which, once adopted, becomes a reference point backwards. In the future, we ourselves and our posterity will call upon it to decide matters of the moment. Especially in cases of infraction of the regulation, one must interpret the verbal formulation: What constitutes a “public space”? What counts as a “philosophy course”? Policies debated now with a view to utility (the future) will later be debated with a view to justice (the past). Thus the verbal formulation must reflect the truth of human action: a regulation *skeletalizes* action, sets the conditions for human decision. At the very least, then, one should attend carefully to the formulation and not get lost in the recitation of facts about the effects of smoking in public, or of studying philosophy.

And, third, the ultimate predicate term itself: “advantageous,” perhaps, but for what, for whom, when and where? We might grant that a smoke-free breathing space and a philosophical education are advantageous in the abstract. But are they advantageous in a given context? Policies, actions, decisions, are often isolatedly beneficial, but holistically pernicious. If the vast majority find life worthwhile only if permitted to smoke, or if a large sector of the economy (tobacco industry) depends upon rabid smoking, a law restricting tobacco consumption might result in lower medical costs, but it might also lead to economic collapse. If the vast majority of students attend university simply to obtain a technical training allowing them to slip into a routine job, or if society as a whole requires merely routine expertise in order to function smoothly, a regulation requiring consideration of philosophical questions would likely prove highly disruptive. Furthermore, one can always ask whether people will really obey the law, satisfy the requirements. For ineffective laws, ones which most or even many simply disobey, ignore, or evade, sow the seeds of injustice: not only infraction but also disrespect—as did the prohibition of alcohol in the U. S., Norway, and Finland during the 1920’s. In short, one must ask not only what specific advantage may lie in the offering, but whether and how the proposal integrates that specific advantage with other desirable things and other proclivities of the human beings affected.

The arguments of deliberative oratory inevitably land us in a cauldron of illusions. For they all place us at the edge of the great unknown: the future. Yet where knowledge leaves off hope necessarily begins, and pretension quickly follows. A parliamentarian or a professor speaks or listens to his peers debating a practical question, but confuses the advantage at issue with ones closer to his own concerns: being allergic to cigarette smoke, he would like his wife and friends to cease smoking in his presence; afraid that his position may be terminated for lack of enrollments, he hopes to force students to enrol in his courses.
Or, more crudely, he argues and votes in a way that he believes will stand him in good stead with his constituency or his peers — good stead for receiving future favors. The very indeterminacy of the future leaves an open field for the imagination to play on fears and desires.

Fully aware of the indeterminability of questions of utility, one may even fall into a melancholy skepticism. A seasoned parliamentarian knows all too well that no law will guarantee health, economic prosperity, or even obedience, let alone a good life. A mature university administrator knows all too well that no structural changes in the curriculum will guarantee a more thoughtful or even more effective education. Wise parents know all too well that good marks in school, disciplinary punishment, even a certificate in law or medicine will not necessarily lead to a good life for their children. In the Hindu classic The Bhagavad-gita, Arjuna knows all too well that victory in battle will bring as many if not more disadvantages as will defeat.

As logicians, even as sophomores, we can formally state the paralogisms which, once recognized, likely give rise to the melancholy. Most simply: concentration on facts yields at most conclusions about the past. Thus any argument concluding a future advantage from a consideration of facts alone treats two different questions as though they were only one: it commits not only the fallacy of Accident, but also the fallacy of Many Questions. Furthermore, it introduces considerations (e.g., statistics connecting tobacco consumption with the incidence of lung disease) which, despite their interest, do not bear on the subject (what is most desirable and how to obtain it): it commits, in addition, the fallacy of Non-Cause. Such an argument also takes what has followed (later achievement from earlier education) as though it necessarily followed: it commits the fallacy of Non-Necessary Sign. Most obviously, perhaps, it tends to drop the relevant qualifications (under what conditions would the prohibition of tobacco or the requirement of philosophy benefit people?): it commits the fallacy of Secundum Quid. Perhaps, then, it also assumes what it claims to demonstrate (that reflexivity in decision-making is good; that longevity, mere survival, is best): it may commit the fallacy of Begging the Question. And, from the standpoint of contemplation, at least, frustrated responses to such criticisms very easily betray an ignorance of what it means to argue: the fallacy of Ignoratio Elenchi.

Now, as juniors we spot the inconclusiveness of such arguments, admit to ourselves and our cohorts what advantage we would like to eke out of a given situation, and discern as best we can what arguments will most likely snow others: we then proceed to speak (and to vote) at two levels — basically for some envisioned private advantage, but all the while in the name of some public advantage. This pretense of nobility and justice concealing merely private utility constitutes the “material” of any given situation of action and speech: “What people want and what they say is not the same; rather, they compose the most elegant speeches while simply wanting appearances to pay off; e.g., they say that one must prefer a noble death to a pleasurable life, a just life in poverty to a shameful life in wealth, yet they want the opposite.”

So how might we become seniors in the bazaar of illusions into which every consideration of advantage casts us? Some have argued that in this domain there are no seniors, that public life (politics, action) offers nothing but illusion, operating as one here does on the principle of satisfying desire rather than on that of embodying truth. The Cynics make a point of this “nothing but illusion,” and Christians appear to Romans as likewise preferring personal salvation to public engagement. And no doubt juniors often fare better in this domain than do seniors.

But we can still ask what it would mean to become seniors in deciding upon policies of action for future advantage. And as philosophers we are naturally inclined to answer: it would mean to include in our consideration the reflexive question — to ask not only how we can achieve obvious advantages, but also what we mean by “advantage,” what really counts as “advantageous.” Morally formulated,

* Sophistical Refutations, 172 b 36. Perhaps Heidegger is right when he says (Being and Time, §22) that Aristotle's Rhetoric is the first thorough analysis of "inauthenticity," i.e. the human disposition to evade the whole of one's situation, to live rather in a desperate indeterminacy, forever unclear about what one wants. Aristotle calls such disposition simply "weakness," the Christians call it "sin," Kant "heteronomy," Kierkegaard "double-mindedness," and Sartre "bad faith." In any case, we can wonder whether people suffer most because what they want is unattainable or because they do not really know what they want but rather vacillate and procrastinate. If the latter, then the art of public speaking and writing would lie partly, perhaps largely, in redirecting volition, whether for better or for worse. Plato's Gorgias despair.s of this art as mere trickery and flattery (463A).
the reflexive question reads: What really deserves our desire? What should we desire by way of future benefit from present decisions?

Evidently, the reflexive question has its greatest impact on us when we are in positions of responsibility and leadership. As a parent, a store manager or a department chairman, I must incessantly direct attention, my own and that of others, to goals. As a private person, I may simply desire a goal, some obvious advantage like a clean house or a percentage increase in sales. But in my public capacity I desire something else as well: that others contribute to the attainment of the goal. I then desire at least two things: some particular advantage and some sort of cooperation. Moreover, the attainment of the first depends upon the attainment of the second. Any envisioned particular advantage projects a future which essentially eludes our grasp — which not only may prove unrealizable but might also prove disastrous in its realization (as Oscar Wilde poignantly remarked: “When the gods wish to punish us they answer our prayers”). But even minimal progress toward the realization of goals in a public space depends all the while on the creation and preservation of an esprit de corps, a sense of community. This latter it is that a genuine leader will promote first of all — whether as captain of a team, a chairman of a committee, a parent, or a town mayor.

But easier said than done. Aristotle defines seniors (“those in their prime”) as those “who live not for nobility alone, nor for advantage alone, but for both,” “who neither trust all nor distrust all, but decide according to the truth,” and “who are moderate with courage and courageous with moderation” — in contrast to the excesses of the very young and the very old (Rhetoric, ii, 14). Yet how can we live simultaneously for the present and for the future, and for the truth? Plato and Aristotle answer: by understanding the communities in which we live as themselves beings which we must, in our art, promote. If we ask what the Being of such beings as a family, a team, or a university is — what it is that we can and must promote in any given case — we find both Plato and Aristotle answering: φιλία, friendship. A group of people who work together to form a unit has, for each member, both presence and promise, both beauty and utility, and a truth of its own, namely the fulfillment of human action.*

* Plato’s Laws can be read as an effort to lay friendship at the basis of political action, as opposed to force. Similarly, Aristotle’s discussion of private property climaxes in the principle that the unity of a polis, its Being, lies in friendship (Politics, 1262 b 10). In The Prince (1513), Machiavelli acknowledges the principle of friendship as second to that of force, yet his “friendship” means not a relation of mutual affirmation among people but a feeling followers have for their leader: here the issue is no longer a Community but rather a State (a term Machiavelli introduced to describe the modern body politic: a condition to be faced rather than a being to be cultivated). By the time of J. S. Mill’s Utilitarianism (1861) all questions of friendship have evaporated in the effort to understand the State as only a collection of means to be utilized by individuals rather than a unity to be created and preserved. Hegel and Marx are the last great thinkers to attempt saving the principle of friendship — for the State.

There is, then, a duality of issue in every debate on the question of utility in action: the very specific issue to be decided and the general issue of communal spirit. In contemplation, we can see that the first kind is empirical: one argues on the basis of facts (the past) in order to foresee the fulfillment of some desire (the future). And that the second kind is ontological: it seems that one must recognize solidarity (unity) as the basis of deliberative oratory. For community provides the basis of action, is therefore the one Advantage underlying all other possible advantages, the one Measure giving sense to all empirical measures of the future.

And we can likely detect in the dual meaning of utility not only “something of a wonder” in our social being, but also the near inevitability of snow-jobs whenever the question arises. Facts we can get our hands and eyes on, while Community we can only acknowledge and embody. Paralogism consists not in directing others into the void or into the dark, away from the firm and the clear. It consists rather in calling attention to what is firm and clear while obfuscating the full question: in playing up what easily allows of determination and drowning out what naturally falls into the background anyway. Unless we do something to regenerate proper syllogism, we automatically fall into paralogism.

But what can we do? We cannot, in public, debate the ontological question: this topic makes sense only in contemplative discussion. And we certainly cannot legislate communal spirit: we legislate specific policies. Yet neither can we simply rely on the spirit of cooperation: unity (of a family, a team, or a university) proves to be extremely fragile, much more so than any exotic orchid or tropical fish.
Evidently, the most a speaker can do, or a listener can expect, is a presentation of S, P, and M that simultaneously represents communal spirit. But how can we represent solidarity? Most obviously, perhaps: we can present the exact formulation of the policy (S), as well as the facts in support of it (M), with a certain amount of reservation; we can admit that we must all attend to the interpretability of the formulation in future times and recall the shaky relation between the past and future. By involving others in these practical questions, we already do much to create a sense of community. More subtly, though: we can somehow make it clear that what counts most of all is not the law in itself but shared trust in it. A senior would prefer that a proposed policy, however good in itself, should be rejected if those deliberating the question cannot, for whatever reason, place their trust in it. In short, to represent the full question of utility, we must place a premium on persuasion—understanding it not merely as an empirical means for achieving specific advantages, but as the ultimate end embodying ontological Advantage, namely the unity of a community.

At our best, Aristotle says, we are courageous with moderation, and moderate with courage. The double meaning of “advantage” in deliberation may indeed require us to be at our best in this double way: courageous, because we must stand up in a public assembly (whether in an open meeting or in a locker room) to propose and defend a specific advantage, a policy the fruition of which depends on much more than any formulation (S) or factual determination (M) can guarantee (just as in battle one knows that no maneuver guarantees victory, or even survival); and moderate, because we must pull back in favor of the most important issue (here: friendship, φιλία), restrain ourselves from merely forcing the policy through, from inveigling others into endorsing it factionally. Of course, it is much easier to argue in only one of these ways.

If, now, a snow-job consists generally in parading factual determinations (accidents) to form a smoke screen, and if a paralogism in deliberation consists specifically in obscuring the ontological issue of utility (community), we can detect at least two principles we assume whenever we endorse this understanding: (1) that factual determinations tell us only about what has already happened and not about what will happen, and (2) that what people first believe to be advantageous (health, wealth, power, reputation, pleasure) will eventually have to give way to a belief in some ontological advantage (here: communal unity). On the Second Interpretation, these assumptions appear necessary. But are they?

The Fourth Interpretation boldly challenges both assumptions. In its Twilight, we can still hear it asking us to believe in expertise: to believe, on the one hand, in the existence of an exact science of nature allowing some people to foresee the future that will follow upon present circumstances and decisions; and to believe, on the other hand, in a science of man allowing some to assert with authority what people basically want and need. Both expertises assume that all beings behave the way machines do: if only we can understand their inner workings (as of clocks, one used to say) we can determine accurately their future behavior—and also, in the case of human beings, the conditions of happiness and all its opposites (unrest, lassitude, and so on). Once we believe that the undetermined and indeterminable (anciently called “chance” or, as late as Machiavelli, “fortune”) plays no essential part, we expect a good public speaker to list all the relevant facts and either square the future with pre-established needs or wants of the people concerned, or leave such squaring to a vote (since some listeners may prefer health, others immediate pleasure, still others wealth). The Fourth Interpretation thereby replaces communal action with collective production, the necessity for seniors with the necessity for experts. Since, on this Interpretation, the most we can ever ask from others, or provide for others, is a collection of determinations (empirical data and rational calculation), a snow-job can only amount to a kind of filibustering: a recitation of facts that fails to add up, fails to reflect one or both of the mechanisms at issue."

For a clear example of absolute faith in factual determinations and a candid denial of both classical assumptions, consider B. F. Skinner's work Beyond Freedom and Dignity (New York, 1971). Skinner proposes a “technology of human behavior” which teeters at the outer edge of the Fourth Interpretation. On the one hand, it follows in the footsteps of J. S. Mill inasmuch as it reduces the question of nobility as well as the question of justice to the question of utility. On the other hand, Skinner's proposal correctly locates and effectively attacks the Fourth Interpretation's faith in the “inner man” as the main spring of the entire mechanism (e.g., Mill's “inner sanction” of social utility, viz. the “subjective feeling in our own minds” or the “desire to be in unity with our fellow creatures”). Retaining the faith in mechanism while eliminating the reflexive question of human autonomy, Skinner proposes an understanding of human behavior (no longer of action) which corresponds
As the art of including others in communal action, of instilling friendship, cooperation, and unity, rhetoric has no place in the theory of mechanism. Instead of naming the concrete context of syllogism (and thus the study of logic), rhetoric now names its surrogate, an art belonging to the domain of demagoguery. How could it be otherwise, once faith in information has replaced faith in Community? The modern State must, by its very nature as a condition (means) rather than a being (end), promote expertise (modern science) and its dissemination (popular education), on the belief that individuals need merely be informed, not converted. In the Broad Daylight of the Fourth Interpretation, Locke, Rousseau, Jefferson and others all hoped that cooperation, solidarity, and unity would naturally follow from modern science and popular education: each would realize that the fulfillment of his or her own interest depended upon an exchange of favors among the constituents of the collectivity, so that a member of a minority will provisionally lend a hand to the majority, while those in the majority will still respect and include the interests of the minority.

To some, the defeat of John Quincy Adams by Andrew Jackson (in the 1828 United States presidential election) was the first sign that something was wrong. While the first five Presidents could disagree among themselves and still assume solidarity to be the chief advantage, naturally fulfilled in democratic government, the sixth was edged out of office by a man who spoke only to empirical advantages, who introduced the spoils system and also the primacy of economic expansion, the forerunner of colonialism and the commercial State. The accounts of John Quincy Adams given by his grandchildren, Henry and Brooks Adams, show the defeated man as the last of the Revolutionary Presidents, as one who believed in the creation of a nation rather than the organization of personal gain. This belief, however, rested on the remnants of the Third Interpretation:

John Quincy Adams had dreamed that, by his interpretation of the divine thought, as manifested in nature, he could covenant with God, and thus regenerate mankind. He knew that he had kept his part of the covenant, even too well. In return, when it came to the test, God had abandoned him and made Jackson triumph, and to Adams, Jackson was the materialization of the principle of evil.

[He] was forced, by bitter experience, to admit that science and education offer no solution to our difficulties, but possibly on the contrary aggravate them.

That is, the development and dissemination of expertise might very possibly encourage self-seeking at the expense of cooperation. Adams has not taken the pains to incite solidarity. Jackson promises a variety of empirical advantages. Adams does not even bother to reply, trusting rather in the automatic faith on the part of the voters in the ontological advantage. But these latter recognize the advantages they have been trained to recognize, and God does not intervene to help them recognize the ontological one, as Adams had hoped: “His own career did not offer proof of the worldly advantages of docile obedience.” The Enlightenment thinkers had hoped for an automatic faith on the part of those having the relevant facts at their disposal, and perhaps for divine intervention. In contrast, Aristotle’s claim that “by nature the true and the just are stronger,” that “by nature the true and the better lend themselves more to syllogizing well and to persuading,” challenges us to exercise our art with greater care. For “nature” here is no automatism, no mechanism, but rather “loves to hide” (Heracleitus).

§2.3 The question of justice

A student disputes a mark the teacher has assigned. A professor argues with a dean that he deserves a promotion in rank. Or I try to explain to my mistress, perhaps also to myself, why I owe it to my wife and children to remain with them rather than start a new life with her. In each case, one may recite facts: the student and the professor may point to the work he or she had done over the weeks or years, I may review the commitments I have formed through past words and deeds. Yet the teacher, the dean, or my mistress may rightly remain unconvinced — no matter how accurate and how extensive the facts.

* Henry Adams, The Education of Henry Adams (1918), Chapter One. The two quotations are taken from The Degradation of the Democratic Dogma (New York, 1920), the introductory chapter by Brooks Adams.
Similarly, though more formally, in courts of law: one lawyer will argue that Jones was responsible for the broken leg Smith incurred while descending Jones' front step, and another lawyer will argue the opposite; the defense will argue that Mrs. Stafford did not criminally kill her husband, admitting that she did in fact kill him, and the prosecution will argue that she did act criminally. The facts recited in such cases still leave room for a decision on the part of a judge or a jury.

Facts, it seems, structure at most a space for just conclusions or their opposites; they cannot ensure that justice will be done. And not simply because teachers, deans, mistresses, judges and juries may fail to heed the facts. On the contrary, the more one accounts for the facts in any given case, the more one notices the open space they leave.

In the West, at least, courtroom procedure forces into the open the complexity of the measure called justice. Following Plato's distinction, lawyers distinguish questions of fact from questions of principle, the de facto from the de jure question — the minor from the major premiss, the discussion of the measured (what in fact transpired) from the discussion of the measure to be employed (how the law applies). It may be that Jones fell and broke his leg in all the ways described, but by what standard of civil responsibility do these facts lead to a settlement? It may be that Mrs. Stafford inserted such-and-such instrument into such-and-such vital organ of her husband at the time and place described, but by what standard of guilt can these particular facts lead to a criminal conviction — rather than to an acquittal on the grounds, say, of self-defense? Cases which likely appear single in their original impact on our sensibilities must split in two as we strive to discuss and decide them justly.

Once we decide a given case of justice, the judgement will take syllogistic form: Our giving or withholding (a mark, a promotion, a living arrangement, a settlement, a punishment) is just, because the giving or withholding are actions based upon such-and-such facts and actions based on such and such facts are just:

\[
\begin{align*}
\text{All actions based upon these facts are just} \\
\text{This G/W is an action based upon these facts} \\
\therefore \text{This G/W is just.}
\end{align*}
\]

At least this is the syllogism we must ourselves stand by. It reflects, notice, not the performance of those who are judged, but rather the action of those who judge. The student addresses the teacher, as the teacher addresses the dean: “It is only just that you decide...” or “...that I receive from you...”; and I tell my mistress: “It is only just that I decide to stay with my family” and perhaps also “...that you so decide as well.”

In criminal cases, and to some extent also in civil cases, the question of justice may get blurred by the question whether or not the person judged had himself or herself performed justly: “Mrs. Stafford's act of killing her husband was not unjust, because her act was one necessary for self-defense, and no acts necessitated by self-defense are unjust.” In the courtroom, however, this second formulation obviously short-circuits the temporal progression of the judgement: only the decree of the court itself engenders the predicate “just” or its opposite. During the debate itself, the question is whether the act is “excusable” or “established,” whether the accused is “not guilty” or “guilty” (in some countries: either “innocent” or “not clearly guilty” or “guilty”).

One source of snow-jobs in questions of justice is the ease with which we forget that the question is whether our own action (our own judgement) in assigning rewards and punishments is just. For once we assume that the question is whether the predicate belongs to the original performance, we assume that some set of determinations about the performance can settle the question, and we tune our ears solely to hear such determinations. Judges and juries so attuned would clearly undermine our liberal judicial system, since they would not open themselves to the de jure question.

And in non-criminal cases our tendency to forget the proper subject of the predication likely begets a confusion of measures: since teachers cannot decide a student's mark by looking to the “justice” of the student's work, they might mix into their judgement some considerations of nobility or utility. Such confusion opens the door to totally undifferentiated influx of facts.

To preserve the question of justice intact, it seems we must acknowledge that we ourselves, or rather our actions as judges, bestow rather than report the predicate in question: the student's work becomes a first-class one by the teacher's action of assigning him or her a mark of “A”; the distinction of becoming a full professor is conferred by the dean's or a committee's action of promoting him or her; a criminal's guilt is decided the moment the judge or jury declares the verdict (at least in liberal polities). The bestowal, we generally assume, should account for,
even sum up the facts. But the quality itself will not rank among the facts.

So where is the quality of “justice,” if not among the facts? Let us look more closely at the act of judgement. I decide now to reward or punish a person (individual or juridical) in reference to something he, she, or it did or failed to do some time before. I look backwards to past deeds, as when deciding questions of nobility or utility. Yet in reaching a decision I look even further back into the past: the performance of the person judged squares or fails to square (as I determine it now) with standards established prior to the performance—standards of excellence in institutional affairs, of loyalty to promises and agreements in family affairs, of routine responsibility in civic affairs, and of respect for the life and property of others in criminal cases. These standards, available in written or oral form to those judged prior to their performance now being judged, engender the major premiss: “All actions based on such-and-such facts are just”—because “All such actions are restitutions of the available standard.” The full sorites:

1. All restitutions of established standards are just
2. All decisions based on such-and-such facts are restitutions of established standards
   \[ \therefore \] All decisions based upon these facts are just
3. This G/W is a decision based upon these facts
   \[ \therefore \] This G/W is just.

In a court, the question is whether each of the premisses holds. Premiss 1 once again displays the ultimate principle. Premiss 3 must become clear in the course of the deliberations. And Premiss 2 locates the possible source of snow jobs. In liberal polities, however, there is a broadly factual criterion for first dealing with Premiss 2: only those facts are allowed into consideration that bear upon the case, and only those bear upon the case that are referable both to the performance in question and to the standards antedating that performance.

Justice, it seems, has its source very much in the past — the past, namely, of the community, institution, or state in which you and I and our fellows have performed the actions under review.

Only in the past? In the midst of a debate we of course look to the future. For instance, I argue with myself about whether it will be just for me to leave my family. But here the crux of the question still lies in the past; e.g., the promises I made long ago, and the standards of loyalty which will continue to haunt me in the future. But we also debate the formulation of a new policy in the Faculty Council; e.g., whether it is just to require all students to pass a course in philosophy before graduating. Here the question of justice, apart from the possibly concurrent question of utility, asks us to foresee how the new course of action will apply retroactively: having already accepted some students into the university under the old regulations, how can we enforce the new policy, the new standard, at the upcoming registration? In the interest of justice, we likely exempt the current students and apply the new policy only to incoming ones; for only to these latter will the standard have been available at the beginning of the affair in which they will be judged.

In sum, we institute the quality (measure) “justice” by returning to some enacted policies of the past and to the performance judged (measured). And snow-jobs consist largely in short-cutting the return: we look back only to the deed judged, get lost in the determinations (facts), and allow some unattended premiss to do its work unnoticed — whereupon a logician (and, we hope, the judge) can easily recognize such specific fallacies as that of Accident, Secundum Quid, and the like. Moreover, as philosophers, and likely as judges (teachers, deans, husbands or wives or parents, occasionally even members of a jury), we naturally ask: How can one respond to the flood of facts initially demanding decision?

As freshmen, our only question is whether an action is (or was or will be) correct: whether it conforms to established norms (statutes, customs, temporary agreements). These norms are readily available: rules of behavior announced by a don, policies of procedure printed in the faculty handbook, or injunctions repeated time and again by parents, teachers, and priests, not to mention signed and sealed agreements. The major premiss is clear, and only the minor premiss remains an issue: one behaves and talks without questioning. Yet the clarity reigns only because one’s role of judge consists solely in trying to live by the rules. Ensnosed in the minor premisses, one remains a minor — a life of marvelous innocence, but also an easy prey to snow-jobs of all kinds, and to paralogism generally.

As sophomores, we accept the role of judge, and take on an entire syllogism: we insist on focussing not only on the action judged (measured), but also on the principle (measure) at work in the judgement.
A sophomore likely contends: “Look carefully at the rule and you will see that the deed in question did (or did not) conform to it.” By definition a sophomore assumes that attention given to the rule (measure) should suffice to establish a mechanical relation between the measure and the measured—so that decisions would run automatically (apart, that is, from some human weakness, such as lapse of attention, conflict of interest, or emotional disturbance caused by some excess in love or hate). In effect, one then treats the measure as though it too were simply a fact—as though all concerned could relate to it as one more fact in the computation. Considered as simply another factual formulation along with the facts of the case judged, the rule eventually either fails to fit some case we think it should fit, or even appears as a source of downright injustice. One then asks for exemption from the rules (pleading extenuating circumstances), or one simply makes exceptions (on the grounds of the uniqueness or the harmlessness of the deviation). As a sophomore, one is not quite prepared to admit that the principle at work in the major premiss is not a fact at all.

A junior takes careful note of the Facts of Life. One such Fact is that rules fail to fit cases precisely to the extent that they specify detail. But a more decisive Fact is that the judges ... smoke clears a sophomore will likely suspect that the rules have been violated. But a junior, like Thrasymachus in Plato’s Republic (343A), knows very well that sophomores are just runny-nose versions of himself, trying chiefly to get a conclusion while only incidentally appealing to the “proper” premisses. But the cynic will not often trumpet this knowledge, aware that his artistry depends in large measure on sophomores remaining sophomores and, for even easier pickings, on freshmen remaining what they are. If “justice is the advantage of the stronger,” then the preliminary task for me is to locate myself within the hierarchy of forces—overwhelming freshmen and sophomores with words and, among my own kind, jockeying for first place—all the while carefully distinguishing between those who will help and those who will hinder me.

And a senior? Here as elsewhere, and schematically stated, a senior is one who can turn directly to the major premisses. The others might seem to turn toward these principles, each in his own way, but not one of the three dwells on it. What sorts of appeals to facts are just?

In higher courts of law we find a close and formally structured analogue to the question. One can ask: Is the law otherwise serving in the generation of verdicts itself lawful? In the United States, the Butler Act read: “It shall be unlawful ... to teach any theory that denies the story of divine creation....” But is the Act itself just? Does one (in the U.S.) act justly in applying this major premiss? In pondering this question the litigants construct one or another sorites, the one arguing to hold up and the other to cast down the major. In arguing either way one appeals to a higher law, whether immediately available (as in the U.S. Bill of Rights and in the Canadian Charter of Rights and Freedoms), indirectly so (in precedent), or simply in the principle of consistency and clarity (given other statutes at the same level). Whatever else it may mean to become seniors, and whether or not we find ourselves in courts of law, we can see that it means at least this: the ability to put any directly applied principle on the docket for cross-examination. But not as a sophomore does, namely as a question solely whether others are acting properly: when questioning the law in this way, one is really asking, as Euthyphro does in Plato’s dialogue carrying his name, what one can get out of the situation for oneself.

But to what can we appeal when cross-examining a principle which has in fact been put in place and now appears questionable in application? We seem simply to appeal to some higher law (whether a Constitution or a Custom or simply Consistency). However, in pointing the finger at the words we find ourselves having to interpret what we see: What do those words in the Charter about “freedom” mean—not abstractly but in application? The need for interpretation shows that these “higher laws” are also conclusions derived from yet Higher Laws: they stem from basic decisions about human being. In the Fourth Interpretation the basic decisions structure our understanding of the relation between the individual and the State. At one extreme of this tradition, the State
appears as a merely convenient arrangement covering commerce, communication, and common defense (as George Ignatieff, one-time Canadian ambassador to the United Nations, once clearly argued*): on this view, lawyers and judges, teachers and parents, need only ask whether the arrangement serves the individuals. At the other extreme, the State is taken to be a Community (as in the Third Reich, but also in Russia and China during the first decades after their Revolutions): on this view, one must ask whether and how the individual serves the State, presumably as a minuscule co-creator.

While justice at the level of States remains rather remote from most of us (except abstractly, as when we contribute to Amnesty International), justice at the level of Communities touches nearly all of us in very concrete and often urgent ways. We are working together at a university, we are playing together in a game of basketball, we live together as a family: in all such cases there are rules, performances, and judgements—much as in a State. Yet here, unlike in a State, one purpose of the affair is to help each individual participate in it, to create something like a team: both the nobility of the individual and the utility of the existing context are at issue. And here, too, any senior will know that many of the others (e.g., the children or the rookies) are not seniors at all, but rather come under his or her own power (as parent or coach) to develop their power of becoming seniors. Justice in a State may nowadays lie chiefly in fairness with a pinch of sympathy. But justice in a Community lies in the cultivation of seniors: not in simply treating people as free, but in helping to free them as active contributors, to free them from being passive subjects of other people's desires and calculations, perhaps to free them from their own prior desires and calculations as well. A senior will know that his or her application of rules to cases is just only if the decision contributes to the emergence of the individual and of the team: a double emergence of nobility and utility.

Easier stated than fulfilled, of course. And we likely fall back on the consequences of justice: we strive for clarity of rules and evenness of application, we resist the onslaught of mere facts and we distinguish as best we can the question of justice from the other two questions. But we are always caught in at least one bind: two beings are at issue in every concrete decision, the individual and the community. Justice

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Paralogism: Truth and Perversion

Proposal's veracity, especially its promotion in modality, depends on our own observations and calculations: on the way each of us rises to meet and assess the situation to which the proposal refers us. And such independent (free and individual) judgement requires above all that we assume an aloof position from which to assess what we hear and read. This aloof position in language reflects the aloof position envisioned as necessary for our being truly in nature: distance from, no longer intimacy with beings, is here a necessary condition of truth.

On the modern view, then, the basic failure of human being is to fall out of aloofness. For, once having succumbed to the pressures of custom or nature, we can no longer “observe” in either of the two ways essential to preserving our own integrity of judgement: we can neither obtain a bird's-eye view of factual givens, nor reason carefully about possible formal interrelations among them. Once we are merely involved, any “decision” we take can only amount to a witch's brew of basic desires and fleeting wishes, incidental pressures of the moment, and sweet or sour memories of the past.

In our modern skepticism, then, we easily recognize those paralogisms which aim to compromise our aloof position. For instance, we easily recognize and likely scorn arguments based on sheer Force (“You had better believe this, otherwise you will suffer”: *ad baculum*). Likewise arguments based on mere Popularity (“It must be right because so many people think it is right”: *ad populum*). Likewise ones based on Pity (“The teacher deserves a promotion after all his misfortunes”: *ad misericordiam*). Those based on the other speaker’s or writer’s Character (“Look at him—there’s obviously something wrong in what he says”: *ad hominem*). And those based on deference to, or reverence for Authority (“It must be so, because Einstein said so”: *ad verecundiam*). Given our modern conception of free and proper judgement, such appeals appear not so much subtly misleading as grossly perverse.

Yet throughout both the Second and the Third Interpretations these appeals received attention not as fallacies but as topics: as places of communal discourse (“common,” i.e. shared places). In public speaking we deal with what “all people, most people or wise people believe”—so that Popularity and Authority may very well be at the point. Similarly, we are dealing with human fates and human reliability—so that Pity and Character may very well be at issue. And, although we try to persuade one another we must be prepared to respond to recalcitrance—so that Force somehow looms in matters of action. Of course, speakers and listeners, writers and readers may mishandle or misrepresent affairs when making such appeals: they (we) may certainly commit fallacies. But, on the classical view, not only the commission but also the recognition and any eventual rectification of such paralogism will take place within the shared context and therefore within the shared pressures. In other words, paralogism within the topics of Force, Popularity, Pity, Character, or Authority here remains in close association with syllogism within the same topics.

Are Ancients and Moderns then at loggerheads on this question? Not necessarily, or not entirely. For the differing conclusions stem in part from their different questions.

Plato and Aristotle generally envision the context of both syllogism and paralogism as initially one of dialogue and eventually one of action: the question, both philosophical and human, is whether the Measure will emerge (especially in contemplation) and how it will affect the measured (most obviously in action, but then also in production); the context of this double event is Community, and the formal question under debate lies in...
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the predication of a “quality”: Is it beautiful, this example of being-in-action? Will it prove advantageous, this policy before us? Is it just, this decision to reward or punish?

Galileo and Descartes generally envision the context of right or wrong thinking as initially one of monologue and eventually one of production: the individual learns by standing alone, by observing and calculating on his own, and he proves his learning to himself and to others by constructing a system which will both account for and lend support to observations; the context is not Community, but Method, and the questions under debate are whether fire has weight, whether the matter of the Moon differs essentially from the matter of the Earth, whether the blood moves only from the heart, and whether (in Machiavelli and Hobbes) leaders can properly form policies with a view to what human beings should be and might ultimately emerge to be. The Big Battles raged originally over the whethers, and then gradually developed into skirmishes over the quantities involved in the victorious views: the quantification of heat, the speed of sound, the atomic weights of the elements, the metabolism of the blood (and now the detection and prevention of cardiovascular diseases), the factors of human behavior (and now the techniques of reinforcement), and so on.

Both in the determination of facts (Aristotle’s “accidents”) as well as in the calculation of reasons (Aristotle’s “formal necessity”), appeals to Force, Popularity, and the like are clearly, even ridiculously irrelevant. We certainly find ourselves fooled by many determinations and calculations, especially when we only hear or read them; but we rectify our mistakes by turning away from the pressures of action and contemplation, and by performing the observations ourselves. On the ancient view, such turning away into monologue still leaves us with the task of returning to the questions of the Measure itself — either to a context structured by the pressures we had earlier abandoned, or to a context where these pressures simply serve as the food for contemplation. On the ancient account, speech arises primarily here, in action or in contemplation, and only incidentally when simply communicating among ourselves while observing and producing things; for speech manifests itself only in the tensions of shared syllogism, whereas observations either remain at the level of the minor premiss or unfold in monologue.*

* Even Rousseau, at the High Noon of the Fourth Interpretation, would not allow the reduction of speech to the communication of observation or volition — an office better served by silent gestures, he argues in his Essai sur l'origine des langues (published posthumously in 1781).

Reading Galileo and Machiavelli, Bacon and Descartes, one gets the sense that academics in those days were trying to settle disputes of fact by consulting authoritative books. Galileo tells the story about a learned man who attended an anatomy lesson in which the paths of the nerves were literally laid bare for inspection: the learned man expressed gratitude that he had studied Aristotle’s account, since otherwise he might have fallen into error by believing his eyes. Whether or not academics in those days were all so naïve, we today can safely say that only freshmen will henceforth ever advance an argument concluding such facts on the grounds of authority or popularity. Culturally speaking, we assume that disputes about facts rightly begin only with sophomores: with those who read or hear the observations of others in order to obtain some directives either for determining facts or for calculating relations — for doing so by themselves.

Yet both freshmen and sophomores find themselves at a loss in assemblies — in the exhortations of classrooms, parlaments, or courtrooms, not to mention in the rehearsals of these in books and intimate conversations. They therefore become easy prey for juniors — for the cynics who never succumb to arguments based on those appeals but who advance such arguments without scruple. The cynic knows the secret of the others, and plays on it.

Why do freshmen believe what they are told? Because they only want to be members of a group. Why do the sophomores insist that speech has public claim on them only in questions of fact and calculation? Because they want Measures to lie within the domain of their private lives. But why do freshmen whimper so easily? Why do sophomores protest so much? The cynic answers: because they are afraid. The first fear expulsion from the safety of received opinion, the second fear that some other agency will take over their lives. The cynic accepts these fears in others, as well as in himself, and follows Thrasymachus’ advice: he plays on them in others in order to arrange circumstances to suit himself; indeed, like a clever market analyst, he studies the subtle variations in these fears, be they those of his peers,
Paralogism: Truth and Perversion

masters, or subordinates. The cynic can be very effective at the level of production.

And the senior? Well, we shall need some examples. In preparation, however, we can recall the formal definition: a senior knows that in speaking or writing to conclude that S is beautiful his own speech must represent the S, must itself occasion the emergence of the Measure, must itself be beautiful. That in speaking or writing to conclude that S is advantageous his speech must account for the solidarity grounding the effectiveness of S, must itself occasion the emergence of the Measure, must itself be useful. And that in speaking or writing to justify S deserving reward or punishment his speech must recall the standards which rise from the past to bind communal action, must itself encourage and exhort and even persuade others to allow the Measure to emerge. And the speaker or writer, the leader who acknowledges these reflexive dimensions of speech will likely have to respond to and even appeal to Force, Popularity, Pity, Character, and Authority. For these represent pressures within the action at issue. But in responding and appealing to these pressures the senior will, by formal definition, aid the “emergence of the Measure”—where the Measure in this case measures our human nature.

Sounds very dramatic. And so any example of it will be. Or: so the Second Interpretation asks us to understand our condition. For us today, familiar as we are with the Fourth Interpretation, the drama becomes even more intense: while the original protagonists claim that Force, Popularity, and the rest can be quite legitimate, the new antagonists claim that they are highly perverse. Obviously, each side represents an extreme possibility which we ourselves can experience and contemplate. As late-comers we must trace the pressures, one at a time, through the entire spectrum from absolute persuasion to resounding relevance. And as moderns we best begin at our own end, approaching the other end testily, wondering what it might mean to face and incorporate these pressures in the emergence of Measures—rather than simply shirking, dismissing, or manipulating them.*

* The word Aristotle employs for the successful response to paralogism is λύσις (Rhetoric, 1402 b 23, and Sophistical Refutations, 179 a 27, et al.). The common meaning of this word is “release” (of a ship from port, of semen from the body, of an accused in court: the chorus in Sophocles’ Antigone, 598-9, says that “one generation cannot free the next, but some god will enter, and there is no λύσις”). But Aristotle uses the term for what is translated as the “untying” essential to tragic myth (Poetics, 1454 b 1 and 1455 b 25). In Plato’s Phaedo (67C) we may translate it as the “release” of the soul from the body, and in his Republic (532B) we may translate it as our “deliverance” from bondage to the shadows: the intended result of dialectic. Centuries later, Philo says that there is no λύσις of sin,” but rather a re-enactment of it, if we perform a sacrifice not in the spirit of justice but with a view to how our performance looks to others (Loeb edition, Vol. 6, p. 500). Perhaps, too, Heidegger’s Gelassenheit (translated as Discourse onThinking, wherein the word itself is translated as “releasement”) attempts to re-contemplate λύσις and its relation to the λόγος conditioning our nature as the “place of forms” and as being “in a way all beings” (On the Soul, 429 a 27 & 431 b 21).

§3.1 Force

Children may find themselves hoisted bodily from the floor and strapped into their seats, prisoners may find themselves chained at the wrists and dragged roughly to solitary confinement: and in advance of such enactment of force the child or the prisoner may hear the threat of it in the words of their wards. Ordinarily, however, threats remain vague, as when a teacher “forces” pupils to complete their homework assignments on “pain” of receiving disapproval. Here, arguments leave considerable latitude for response: most obviously, one may undo the pretended conclusion by scorning the major premiss; more subtly, one can ignore the stated middle term and supply one’s own (e.g., the pain of ignorance or cowardice rather than the pain of disapproval or poor marks). Evidently, most if not all ad baculum arguments depend upon a background of assumptions about the purpose of command and obedience in action—enthymematic principles, often inadvertently accepted by listeners as well as by speakers.

When force does effect conclusions, someone is playing on or succumbing to fears. The most effective appeals lie embedded in snow-jobs. To conclude that such-and-such reforms in education are necessary, one may recite one terrifying story after another about the millions of people in a given country who have led pathetic personal lives, or found themselves unemployed or unable to participate in the democratic process—all because they never learned to read when in school; but perhaps the proposed reform contributes to causing illiteracy rather than providing any cure for it. Or to conclude that adherence to a given religious sect is desirable, one may recite the terrors lying in wait for us all—at one
time, those terrors enunciated in Revelation, nowadays those of current research into the effects of the military industrial complex (e.g. the danger posed by the deterioration of the ozone layer) or into the spread of cancer.

Reviewing such fear-based arguments, sophomores will conclude that proper syllogism is one thing and appeals to force are another: that all arguments arousing fear are paralogisms. But both juniors and seniors disagree. For how can we ever persuade others to take education seriously unless we can draw up close for inspection the disasters already manifest or looming on the horizon? How can we ever get ourselves or others interested in medical research on a specific topic, unless dangers loom to threaten us all? In debating questions of utility our most common topic will be the probability of good and bad results. In debating questions of beauty, we inevitably consider the terror of dispersion, formlessness, lack of focus. In debating questions of justice, we must recall the terrors of anarchy and despotism. Not factual determinations or mathematical computations, but certainly communal action and persuasive speech take place against a threatening background: “fear makes us deliberate,” as Aristotle says, and “those who take thought of the polity must contrive fears so that others will, like guards in the night, stay on guard and not relax their vigilance for the polity: they must make the far come near.”

Evidently, the logical question is whether a given appeal to force, i.e. arousal of fear, fits in with the proposed conclusion; whether the subject matter of the appeal and of the conclusion remains the same throughout the argument. Sometimes the two are clearly the same, as when I propose that we surrender because our enemies have completely cut off our supplies and we assume that our main objective is the best form of survival rather than, say, a glorious death to establish honor for our families. Sometimes the two are clearly different, as when someone tells me that I had better join their sect because otherwise I will pay for my sins in Hell. And sometimes, perhaps most times, the two are different in principle yet confused in fact, as when terrorists propose that a government answer their demands because otherwise the 237 airline hostages will be shot one by one until these demands are met: here we are caught in the bind of trying to preserve the polity for innumerable people vs. trying to preserve the lives of 237 individuals.

Confusions of subject matter often arise simply from a confusion of questions: What is the issue — nobility, utility, or justice? For instance, we commonly undergo appeals to force taking the general form: “You should do X (i.e., it is good to do X) because Someone has the power to damage your interest if you do not.” If the promotion of advantage is the issue, and if you as a listener are prepared to admit that you are weaker than Someone, then submission seems appropriate: the argument is perfectly sound. If, on the other hand, nobility or justice is the issue, then some sort of defiance seems to be appropriate — assuming one has the requisite courage. Juniors, of course, enjoy a clearer field of vision, since they immediately translate all questions into the language of utility, and all debates into battles wherein they seek to promote their own advantage. Such clarity generally pays off so long as action can comfortably dissolve into production. But juniors make notoriously self-destructive leaders when they reach levels where one of the other questions becomes imperative: hitherto they may have relied on some remnant of the one Advantage, the communal base, allowing for production, but as the last remnant of this vanishes so too does productivity.

For an example of how a senior might undergo an appeal to force, we may turn to the classical example, Socrates. In Plato’s dialogue Meno Anytus has claimed that one learns to be an excellent human being in the same way one learns to be an excellent physician or an excellent wrestler, viz. by keeping company with people already accomplished in the matter (rather than by paying outside teachers). In reply, Socrates points out that a number of prominent Athenians, and also historical personages such as Pericles and Thucydides, though excellent human beings themselves, had sons who remained famously mediocre. Thinking that Socrates is charging these noble personages with neglect or incompetence, Anytus bursts out (94E):

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* Aristotle’s Rhetoric, 1383 a 7, and Politics, 1308 a 27. The imagery of “the far and the near” not only dominates Aristotle’s consideration of deliberation (action), where “the far” seems to mean “lurking but hidden, i.e. futural dangers.” It arises also in the context of contemplation: what will eventually make most sense to us, namely “universals,” lies farthest from us, whereas what is in fact nearest us, namely “particulars,” makes least sense (Posterior Analytics, 71 b 34)— where “the far” is precisely each individual being in its full Being.
Socrates, you seem to me too ready to speak evil of people. In other cities, too, but especially here [in Athens], it is easier to do evil than to do good to others. And I do believe you know that yourself.

Attention to these words, and to the entire context, yields a sorites:

1. All people easily harmed by the Athenians ARE well advised to hold their tongues in public.
2. All people in Athens who criticize others ARE easily harmed by the Athenians.
∴ 3. All people in Athens who criticize ARE well advised to, etc.
3. Socrates IS a person in Athens who criticizes.
∴ 4. Socrates IS well advised to hold his tongue in public.

Premiss 2 simply states the case: the Athenians “proved it” by executing Socrates. Yet Socrates evidently disagrees with the intended modality of the first premiss: Is it simply the case, or necessarily the case, that we should pull in our horns whenever threatened by superior forces? Here and elsewhere Socrates represents the possibility and even the necessity of standing up to such threats — following the tradition of Achilles who, when offered a long life and easy death, chose rather to do battle and gain everlasting glory (Iliad, ix, 411 ff., and xviii, 95 ff.); and of Aeschylus’ Prometheus who, promised reprieve if he would only bow to Zeus, chose rather to abide by his decision to side with human beings. Some people easily harmed by others are not well advised to pull in their horns.

But why not? Because, in a way common to both the First and the Second Interpretations, one finds one’s being in what one does and how one rises to circumstances — and not, or only indirectly, in what one undergoes. On this account of human being, neatly divided in two by the categories of acting and undergoing, there are two kinds of fear: most commonly, the fear of undergoing pain and destruction; more rarely, the fear of acting half-heartedly, double-mindedly, privatively: fear of merely reacting to and therefore being enslaved by what happens to one. The first is a fear of what someone or something else will inflict on one. The second is a fear of disgrace, the terror of falling back and inflicting on oneself an eternal postponement of life itself. To act fully is to live, to be oneself, to be noble. To react merely is already to die: this is ugly and disgraceful both in the eyes of others and in one’s own eyes. Thus, while Anytus raises the question of utility, and this at the level of private advantage and in a way suitable for slaves, Socrates raises the question of nobility, and this at the public level and in a way suitable for freemen. In general, Socrates shows that a senior cannot be bullied: for he or she fears more the loss of self than the loss of money or reputation or promotion — more even than the loss of life itself, considered as a merely futural and passive condition.

Seniors certainly do prove themselves in acting rather than merely reacting when adversely affected by force, and all three of our earlier Interpretations provide examples of such proof: Achilles and Odysseus, Antigone and Socrates, and then the Christian martyrs, i.e. witnesses unto death. Yet seniors often find themselves at the other end of the stick as well. What do you do as a coach having to mould a team? a director having to form a troupe or an orchestra? a parent having to raise children? a teacher having to guide students? a dean having to encourage professors in their intellectual work? In such instances, you may simply play the power game, for whatever private advantage you might imagine. But as a senior you want above all that those you govern will affirm the goal you place before them. You might on occasion have to drive them to pasture with a blow (Heraclitus), or to drive them away from some pasture altogether, and the prospect of this possibility naturally makes others fear you. However, unlike Machiavelli, who plausibly argues that in a State such fear becomes the primary factor of unity, you will, as a senior, want your constituents to grow out of the fear of what will happen to them personally into a fear of failing to act (of failing to

*Every Good Man is Free* is the title of a discourse by Philo. This thesis, common to Seneca, Epictetus, Marcus Aurelius and most thinkers at the Dusk of the Second Interpretation, draws its inspiration from the more subtle analyses of Plato’s *Gorgias* (suffering injustice is preferable to acting unjustly: 469B ff.) and *Phaedo* (the philosopher does not fear poverty or even disgrace merely in the eyes of others, but rather finds most terrifying the possibility of self-incurred bondage and ignorance, of failing to face beings as they are in themselves and merely glimpsing them through the bars of his own desires: 82E ff.). For a detailed account of fear (“pain or trouble coming up in imagining a painful or destructive evil”), what one fears, what kind of people fear what kind of things, its relation to hope and its opposite, confidence, one may study Aristotle’s *Rhetoric* (II, 5). In his *Ethics* (1675), Spinoza dwells on the same themes, distinguishing between passive and active emotions (III, 58 ff.) but rejecting both fear and hope from the active, reality-relating ones (IV, 47).
be themselves) and of failing to preserve the Community. As the last resort, Force always looms, and right from the beginning: in this sense a leader incessantly appeals to force. Yet one does justice to oneself, to others, and to the Community only if one places the premium on Persuasion in the creation of Affection (ϕιλια), as Plato reiterates throughout his Laws—on talking and acting with others to recall them to their own wits, to attest them both to their own possibility of acting rather than reacting, and to the necessity of doing so for the sake of the Community in which they have a part and in which their own participation is at issue.

It often seems impossible. Fortunately, however, we have not only philosophic disquisitions exhorting us to realize the possibility of seniorhood, but also poetic descriptions. For instance, Herman Melville's Billy Budd tells of Captain Vere having to hang the title figure. Billy is impressed from a merchant ship onto a warship in 1797, a time of conflict with France and mutinous unrest in the British navy. It is wartime: “the abrogation of everything but brute Force.” The officers are sometimes forced to “stand with drawn swords behind the men working the guns.” Billy witnesses a lashing and resolves never to do anything which would bring about such violence. His chief characteristic is an air of innocence, yet of a type that makes him effective in his tasks and creates fellowship among the crew. However, one officer, “naturally depraved,” manifestly hates a nature so contrary to his own and eventually perjures himself in order to incriminate the sailor. Billy, unable to make verbal answer to the false charge (“innocence was his blinder”), strikes and kills the officer on the spot. The Captain, knowing the sailor's basic innocence of character and suspecting also his innocence of the original charge, nonetheless recalls the naval ruling regarding striking, let alone killing an officer, recalls also the unrest aboard ship and the necessity that “the consequence of violating discipline be made to speak for itself,” orders the hanging. But not without first rebuffing his fellow officers who, aware of the perversity of the perjurer and the basic innocence of the sailor, recommend that he refer the case to the Admiral. And also not without a “closeted interview” with Billy Budd himself—whereupon the sailor’s last words, startling the whole crew into a sympathetic echo are: “God bless Captain Vere!” Somehow, despite the overwhelming pressures of Force, Captain Vere manages to engender Persuasion—at a great price to himself, who “was heard to murmur” the sailor's name on his own deathbed not long afterwards.

§3.2 Popularity

I bring a proposal to you with the comment that some friends common to us both have already endorsed it. A salesman tells you that lots of people have expressed satisfaction with the product he hopes you will buy. You exhort me to adopt or eschew usages of dress, vocabulary, hairstyle, or smoking—on the grounds that these usages are approved or reproved by others: “the other kids all....”, as children say; “everybody else....,” as grown-ups say. A very powerful tactical maneuver.

Even granting a certain accuracy of the facts advanced in any such case, one still enjoys a certain latitude of response. For one need only wonder whether one's own decision must replicate the general view. Inasmuch as one wonders, it becomes evident that ad populum arguments depend in any case on background assumptions about one's role in communities.

Popularity, too, arouses fear: fear of exile. Voices sounding out in every direction normally carry us along. And to doubt, to raise objections, to speak differently leaves one at least momentarily alone. Freshmen, dreading above all else the prospects of such solitude, readily fall for snow-jobs drawing upon “what everybody else is saying, doing, thinking.” In adolescence, of course, and especially in the modern age, one soon learns to doubt the general soundness of such arguments. After all, since Descartes and Galileo the whole educational and political project of our civilization has consisted in persuading people to think for themselves: to determine for oneself how the nerves in fact run through the body, to prove the Pythagorean Theorem oneself—in either case appealing to “what all people, most people, or wise people believe” only as suggesting possible directions of thought.

However, with the best of intentions, adolescents generally remain freshmen: abandoning the old, they flock into new groups, submit to the pressures of their own peers. The freshman's argument reads, in general form:

All things the group does are right.
Saying, believing, doing X is a thing the group does.
∴ Saying, believing, doing X is right.
Stated in this way, the major premiss expresses indirectly a fear. The half-buried principle supporting it reads: “All things the group does are things the not doing of which leaves me dreadfully isolated.” Moreover, a logician may notice that the subject term of the argument does not yet focus on any being, but rather on an action dwindling into a condition or even into a posture; anyone falling for this kind of argument has an eye solely to appearances, with no grounding in an actual agent, e.g. in oneself. And, finally, the predicate term expresses only approval: “right” has no bearing on any Measure: it merely means “what I wish to do.”

A sophomore focusses on the major premiss and finds such arguments so shameful that the explicit enunciation of it suffices, he hopes, to shame anyone into rejecting every appeal to popularity. But do all such appeals lead us into paralogism? If so, we shall reject all arguments based on the need to participate in families, teams, orchestras, communities. Perhaps it is no mere coincidence, no mere weakness on our part, that we cannot easily shake the addiction to peer pressure. One condition of action is a shared context: “what all people or most people believe” or, alternatively translated, “what appears to all or most,” circumscribes, in action, precisely what is.” In dressing, speaking, and behaving as others do, we participate in a minimal way within a community. And I may honestly admit the reason — but now a significantly different, a healthy one: I wish to take part in communal affairs (shared appearances). Obviously, we need not be ashamed of the major premis: we can easily broach it as a question, and decide one way or another, given the behavior and the community at issue. How are people generally behaving? Do I wish to take part? Such questions can become “common topics,” both in action and for contemplation.

Still, the junior recognizes the passivity of the healthy version of the argument: it calls for community as mere conformity, and cites at most a necessary condition for action. The group will always be pressured by the healthy version: a mutual recognition of what is.

* Nicomachean Ethics, 1173 a 1: “What appears to all, this we say is, and he who dismisses the persuasion of all people will not be very persuasive himself.” Aristotle’s context is that of contemplation, viz. the question of the good. Hannah Arendt comments on this principle (that “what appears to all” = “what is”) in The Human Condition (Chicago, 1958; p. 199), locating the question in the context of action and slightly mistranslating the Greek εἶναι, “to be,” as “Being.”

question is commonly divided roughly into three: What is noble? What is useful? What is just? But he also knows that “most people (οἱ πολλοὶ) tend to be slaves to greed and cowards in danger, ...ready to do wrong to others when able to do it” (Rhetoric, ii, 5): the cynic then pretends to endorse what people think they want (the popular solution), persuades any doubters by appealing to popularity, and turns the accumulated power to his own advantage — all the while speaking of public nobility, public justice, and public advantage.

What else might one do with reigning opinion? One might withdraw from it into one’s own work or into one’s private life. In such cases, one pays the price of the original Cynics: a mutual exclusion of individuality and community. Instead of floating, conforming, or manipulating, one withdraws, submits oneself both to Force and to Popularity at the level of appearances, and cultivates one’s own garden in abstraction from these pressures: all one can ask of Alexander is that he not block the sun; better to be slain for such insolence than to play the public game.

Is Cynicism the only alternative to cynicism? As a leader, whether a cub-scout master or a university department head, one can do, or try to do, something else: re-orient group opinion. While we may, in contemplation, despise any so-called leadership which simply forces its constituency into line, we also despair of any leader who simply accepts the initial opinions of those in the group and attempts merely to represent them in the making of policies, the distributing of rewards and punishments, and the establishing of standards. Rather, in questions of utility, justice, and nobility the leader’s task is that of persuasion: to bring the constituency to re-focus on the issues, namely on ontological Advantage (unity), on Justice (freedom), or on Beauty (performance). Such re-focussing may prove impossible in a modern State. But it is imperative in any Community.

The bind of liberalism actually illustrates the need for an alternative. While it is correct that, in liberal democracies, any policy that violates popular sentiment deserves rejection, how can one argue for the acceptance of a policy or a judgement? To obtain a conclusion to the effect that policy $P$ is advantageous or judgement $J$ is just, a liberal statesman must somehow argue that the majority upholds the action in question. However, the major premiss governing the eventual conclusion then reads: “All popular decisions are good”—and appears to decide out of hand, by a cowardly appeal, a question of utmost importance to all of
In objecting to Protagoras' dictum that "of all [practical] things the measure is man — of what is, that it is, and of what is not that it is not," Plato lets Socrates (in his *Theaetetus*) paraphrase the thought generously: "that man is wise who, if things appear and are bad to us, changes them so that they appear and are good to us" (166D) — so, that education consists of a wise teacher using accounts (λογοί, just as a doctor uses drugs) to change people's condition (167A). Plato in effect accepts the project of "changing the condition of the soul," but demands that we embark ... not initially our own at all: that we learn to face beings in themselves and not simply to manipulate practical affairs.

With the maturation of the State as the factual "role model" for questions of governance, the classical formula for genuine leadership has lost its public credibility. Why should, how can an individual devote himself or herself to reorienting the opinions of (the appearances to) others? Why should the individual do so — since such enterprise requires great personal sacrifice? How can the individual rightly do so — since such an effort interferes with the private lives of others?

Yet human beings have one concern suggesting that the lack of official credibility reflects not a genuine victory over, but rather a painful longing for seniorhood: the concern that what we do and how we are can serve as a standard for what others can do and how others can be. This concern appeals, in a sense, to popularity; not to a popular opinion supposedly fixed prior to one's appearance, but to the opinion of others after one's performance — and even then not to simple approval of the performance as meeting their standards (as a child may perform for the approval of its parents), but rather to the engendered approval of the performance as now setting the standards for others. The common name for the stakes of this peculiar popularity is, or rather has been: *honor*.

It all sounds so strange to us today. Achilles refuses to fight because Agamemnon has usurped a captive mistress, has violated Achilles' honor; and Pindar, Aeschylus, and Sophocles agree that only honor in public performance makes life worth living. Socrates, on the other hand, claims that excellence in individual performance, therefore honor not as "what appears beautiful to the many (οἱ πολλοὶ)" but as what appears beautiful in the eyes of masters and disciples, makes life worth living; and the Stoics, Cynics, and other Hellenistic thinkers agree, so much so that the basis of excellence, namely the intimate contact with beings themselves in one's art, tends to fall out of consideration. Jesus then stands for the glory not of any creature but of the Creator, and his followers eschew the Greek concern for honor as encouraging vanity — since now all efforts, save that of being a witness unto death (a martyr) appear as futile, empty, sinful; in the Christian era, excellence of performance (action) gives way to virtue in the soul (condition) dependent for its actualization on Divine Grace: in our essential impotence, we must realize that all honor, all glory, goes to God.

In a world structured by grand enterprises of science and commerce, we may find honor in any of the three previous senses impossible to live by: the most one can do is follow Kant when he gives a formal definition of what it *would* be like: so to act that you can want the maxim of your action to serve as a standard for others in their actions (with two consequences: so to act that humanity itself becomes the overall purpose of the action, and that self-legislation becomes the overall standard for each individual). However, as Kant himself warns: "It must
not be overlooked that one cannot show by any example, i.e. one cannot empirically decide whether or not there ever is such an imperative [at work]; rather, we must suspect that all imperatives which appear to be categorical [necessary in themselves] may really be hypothetical [expedient for satisfying some other desire].” Perhaps, as Clamence confesses in Camus' The Fall, one “specializes in noble causes”—helping widows and orphans, guiding blind people across the street, giving alms to the poor—simply in order to pass for generous, for honorable, in a kind of massive or even pathological appeal to popularity. Much intellectual work corroborates Kant's suspicion and Clamence's accusation to the extent that the “wise” of today generally expel honor from the ranks of legitimate concerns.¹

Yet communities can only arise if we can appeal to the opinion of others: in some fashion help create a shared affirmation both of goals and of the One Condition for attaining goals.

§3.3 Pity

Sad stories often go a long way. Wishing to be awarded a promotion, I can insinuate my misfortunes into the conversation—not just at the moment of application, but over the days and weeks or even years preceding the judgement. In general, those who find themselves subjected to the power of others—children and prisoners, students and employees—can influence the decisions of those in power by highlighting their own sorry plights. In the courtroom, too, lawyers contrive to focus the eyes and hearts of the jury on the brutal treatment suffered by the victim (the man mugged, the wife beaten), or on the sad condition of the accused (atrocious childhood, society-inflicted poverty, present contrition): a judge must often rule whether a given ad misericordiam argument contributes to or distracts from the question at hand. Perhaps a given story does cast some light on the issue: whether it does or not, obviously depends on what the issue is.

So long as we find and accept our position as one where circumstances simply toss and turn us, and especially where others are distributing rewards and punishments, praise or blame, we likely recognize that one major recourse lies in getting others to feel sorry for us. However, discovering that some of our peers likewise take such recourse, we soon object: the rules specify that such-and-such performance deserves such-and-such judgement, and each exception to the rules erodes the game itself. Pity, usually arising in response to sad stories well packaged with flattery and perhaps some services rendered, manifestly interferes with justice, makes a mockery of nobility and, as in industrial and banking enterprises, undercuts utility.

Are we then to declare that, whatever value pity may have in our private lives, we should ban it from our public judgements? Such division of Fact and Feeling receives much acclaim in modernity. Interestingly, however, we seldom if ever live up to the division, no matter what our view on it. And the junior knows one reason why: by the nature of human affairs, rules never quite fit cases. Any given play in a game requires a double interpretation: we must interpret what is happening here and now, and we must interpret the meaning of the rule—which rule to apply, if any, and then how to apply it. I tell my class that every student must submit the required essay by next Monday, yet one member of the class falls seriously ill in the interval, or suffers the loss of a parent: since the rule accounts only for routine developments, I likely make an exception. Yet in this instance I need not pity the student: I simply make equitable (or “fair”) allowance, and in being equitable I remain (I hope!) just. As Aristotle says in his Nicomachean Ethics (v, 10), “equity is just, but not as corresponding to law: rather, as correcting what is just by law; the reason is that law is always universal, while we cannot state [in advance] every individual case correctly in reference to universals.”

But what do we mean by pity? Aristotle answers: it is a form of fear—the fear inspired in me by the sight of the sufferings of others, a fear that such could happen to me as well. In Homer’s Iliad (x, 314 ff.)
we read of Dolon, the Trojan captured by Odysseus and Diomedes, pleading pitifully for his life; unmoved, Diomedes

... struck the middle of his neck
with a sweep of the sword, and slashed clean through both tendons, and Dolon's head, still speaking, dropped in the dust,

whereupon the two Argives, with the information supplied by Dolon, proceed to the Trojan camp and slaughter dozens of valiant and defenseless warriors in their sleep. A terrifying story — terrifying for us, while we also pity Dolon and the other Trojans: What worse fate than to be struck down helplessly like these men?

But construed as a form of fear, pity does not bode well for either side. For in simply pitying others I interpret them as essentially passive: I place the premium of importance on what I and others undergo rather than on what I and others can do. If not Dolon (who in fact betrays his fellows in a slavish attempt to save his own skin), at least the sleeping Trojans simply undergo the disaster — so much so that, unlike Dolon, they never suffered from foreseeing what they underwent. Perhaps this (the merely undergoing) is most dreadful to behold, even though hardly dreadful for those to whom it happens; in any case, on the First Interpretation, our only recourse in life lies in the activity of standing up to fate. To be sure, on the Second Interpretation we have another recourse, one not supplanting the first but rather supplementing it: here we can complete as well as follow what we undergo, and we therefore misunderstand both ourselves and others so long as we simply feel sorry for them. Aristotle can still say that an unhappy person rightly loses our esteem — because any “truly good and mindful person will bear with dignity whatever fortune brings” — so that people noble or beautiful, therefore happy, prove themselves precisely in the “multitude and magnitude of ill fortunes.” Pity is out of place.*

* See Aristotle's *Nicomachean Ethics*, I, 10 (especially 1100 b 22 ff.); then also his discussion of pity in his *Rhetoric*, II, 8. Our condition is obviously one in which we pity others, just as it is one in which we fear what might befall us. Aristotle's point is that in actualizing our condition we somehow overcome both pity and fear. Much of his *Poetics* (the passing mention of “catharsis” in Chapter 6 and the discussion of pity and fear in Chapters 13 and 14) interprets the human interest in tragic drama as founded on the passage from passivity to activity under the most terrifying of conditions. — In his astute

In the Third Interpretation something like pity, namely charity, does have a place. In parables like those of the suppliant prostitute, the good Samaritan, and the lost sheep (*Luke* 7, 10, and 15) we clearly read that the needs of a fellow creature should suffice for the decision to help him or her. The Christian principle reads, roughly: “All people I meet who really need my help are people who emphatically deserve my help.” Christian “pity” (really: charity) appears active and affirmative rather than unengaged and fearful. Yet care for others certainly goes beyond merely feeling sorry for them, and there seems to be some emphasis placed on the *meeting* of people: our neighbors, in contradi distinction to those in faraway places or in the yesteryear of books.

As philosophers we might still wonder what the parables of Jesus mean. I meet someone: How am I to understand his or her needs? I care for him or her: How am I to understand what he or she deserves? Another parable, that of the Last Judgement (*Matthew*, 25) seems to answer these questions: “I was hungry and you gave me food; I was thirsty and you gave me drink; I was lonely and you made me welcome; I was naked and you clothed me; I was ill and you tended me; I was imprisoned and you visited me.” But is this a recipe or a parable? Do people most need satiated bellies, human solace, warm clothing? Do I do justice to them by providing these things? Or do my fellow human beings, like myself, most need deliverance by grace and, in the interim, the giving and the receiving of love in a Community as remembrances of our shared plight and as preparations for salvation? In any case, the principle reads: what one needs defines what one deserves — so that the way I understand Ultimate Human Need will specify, and find expression in, what I do in response to my fellows: what I give. On the Third Interpretation I give myself, my presence, the admission of my own need for deliverance and grace and love. And I likely find myself torn in such giving — not least because those I care for tend to expect much less, calling it much more, namely that I relieve them of their own responsibility to overcome, with the help of Jesus, their fixations on food

analysis of the desire to incite pity, Nietzsche follows but also deepens the Ancient understanding of the event itself, and also of appropriate response to it: people down in their luck, and impotent to handle it, moan in order to bother those around them, since their only pleasure consists in the conceit that they do have this one remaining power of causing affliction in the world. See his *Human, All Too Human I*, (1878), §50.
and drink and clothing and the like. And torn also because, left to my own devices, I myself would rather travel on to Jericho.

A Christian of the Third Interpretation will not likely succumb to the sad stories of others. Just as Plato and Aristotle supplemented but did not supplant the Homeric exhortation to stand up to fate, so the New Testament adds something to our load: the giving of ourselves — our presence, our love, our admission of weakness. This giving, although providing the tender basis of a divinely based human Community (City of God) inclusive of all individual vocations, remains quite compatible with, may even lead seniors toward, decisions appearing cruel, heartless, pitiless to one stuck on his belly, his clothes, his promotion or his mark. The Old Christians had a rather demanding view of what others basically need and what I must do in response to their needs.

Some Christians of the Fourth Interpretation searched for and found a more tangible meaning of charity: good works. From Galileo, Bacon, and Descartes came the project of a New Science, a new meaning of knowledge based on empirical determination of givens and mathematical calculation of relations — but also based on and aiming for the human mastery of circumstances (a “completing” in which we “follow” only so long as is necessary to get the upper hand over what we encounter). This aim interprets what people most need: things of the stomach and, generally, of health (thus the new hope for medicine). Genuine learning (contemplation) and even political engagement (action, as interpreted by Hobbes and Locke) must now serve production. No doubt the Puritans also upheld the subtler doctrines of human need, but these New Christians conceived of actual responses to their fellow human beings in such tangible terms as we now understand under the name of “charity,” namely “relief work” as quantifiable in donations of money and time."

As we now move uneasily beyond the Fourth Interpretation into Consumerism, pity passes from a Fact of Life into a Principle of Action. Pity is now a feeling, an emotion (such as J. S. Mill called the “psychological fact” of sympathy) that leads us on occasion to satisfy other people’s needs in exactly the way others already understand them — instead of helping others to understand their needs differently. This new form of pity likely derives some honorific impetus from the earlier “charity of good works.” But we find a less doctrine-bound justification for it in the economics of the New State. As its sole function the State now provides the conditions of life, one of these conditions being that people generally, therefore also the poor, have the means necessary for consuming goods and thereby sustaining their part in the cycle of production and employment. Similarly, States even help other States with massive injections of credit at times of natural or other disasters — here for the purpose of sustaining the international cycle. Following our most visible role model, we too feel bad when confronted with sad stories about faraway individuals or peoples, and we send money, clothing, or whatever. Here, appeals to pity make no total claim on us, as Christian charity does; nor do we check whether the reports of the suffering are accurate, and much less do we ask what the people most need. Closer to home, however, we more likely try to uphold the division between Fact and Feeling, at least in a general way: at work and in public, objective Fact rules, while at home and in private subjective Feeling may sway us. Psychologically speaking, it seems that pity is our contemporary way of compensating at the private level for the atrociously competitive and terrifyingly frigid exigencies to which we must respond at the public level — in modern banking, large industry, political bureaucracy, mass education, and scientific research."

* For documentation on the relation between the Puritan concern for good works and the philosophical development of the New Science, see Richard Jones' *Ancient and Moderns*. Spinoza’s judgement on pity in his *Ethics* (e.g., IV, 50: “a man who lives according to the dictates of reason endeavors as much as possible to prevent himself from being touched by pity”) follows the older Interpretations but — as usually happens in such cases — leaves something out, here the drama of the Ultimate Bind: as for the Stoics, so for Spinoza, the emerging State obliterates any concrete sense of Community, and pity appears merely as the correlate of envy (III, 32).

* In this one instance, modern poetic literature often contributes to, rather than militates against the presuppositions of consumerism — namely, by encouraging the all-too-handly split between Fact and Feeling. In a theoretical vein, the poet Tolstoi endorses it in his *What is Art?* (1896; “communication” is either of knowledge or of feelings: Chapter 16). In a poetic way, Charles Dickens endorses it even more strongly, e.g. in his *Hard Times* (1854; the conflict between Thomas Gradgrind and Sissy Jupe). If we were to accept such accounts, we would believe that the only way to avoid snow-job fallacies would be to flee into pandering fallacies.
So what can a senior do in sight of human misery and in hearing of appeals to pity? Perhaps only this: to keep the faith so well stated by Faulkner in his Nobel Prize acceptance speech: “I believe that man will not merely endure: he will prevail.” As a leader of any sort, a senior stands in the Ultimate Bind of helping not only a community to take and retain shape, but also individuals to emerge as themselves in the community. In the shadow of consumerism we need no longer worry about how one happens to interpret this emergence—whether as simply facing what one undergoes, as learning to complete while following what one undergoes, as giving oneself completely over to what one Ultimately Undergoes, or as insisting (à la Kant) on the primacy of a non-empirical autonomy. A mature Fifth Interpretation will likely tolerate and even justify all the older Interpretations of what the individual most needs—perhaps even that of the Fourth Interpretation inasmuch as it speaks of what it means to do good works (as distinct from what it means to receive goods). However, the relation between Community and State will appear much more elusive.

§3.4 Character

I tell you that the man who is proposing this year’s university budget has failed to keep his private business ventures solvent. You explain in detail to me how the woman praising or blaming another mother has inflicted outrageous miseries on her own children. We hear that some judge has been heavily in debt, could have had easy access to cash through accepting bribes, and was obviously seeking his own advantage when a backbencher in his youth. In such ways one seeks to blacken the source of an argument—the person, his or her character—in order to obtain a rejection of its conclusion, namely the person’s own judgement. By appealing to the character of the source, we at least momentarily bypass the proposal itself, as well as its proposed justification. If, on the other hand, we remain more concerned with, say, the budget itself, or with the standards of motherhood, or the law at work in a given judgement, we may choose to reject the rejection, refuse to concentrate on the source, and seek a passage back to the subject itself. Still, the very frequency and power of ad hominem arguments raises a fundamental question for us in contemplation: What is the relation between a proposal that P belongs to S and the person who rises to affirm or deny the belonging? What does the character of a witness have to do with the truth of his or her testimony?

Not likely in determining whether a solution is acid, or in calculating the third angle of a triangle after having determined two: and perhaps not necessarily in reading an author of another age or locale: but certainly in any live discourse: we relate not only to subjects and predicates, but also to those who propose the subjects to us, or to those who consider the subjects we ourselves propose. Person-to-person-to-subject discourse is fraught with the question of how others and how we ourselves stand in the face of the subject. And fraught in various ways.*

Freshmen naturally find other people the most frightening factors in their lives. Indeed, freshmen reconstrue every utterance, even one carefully formulated as a proposal in the mode of possibility, as a command to be obeyed or skirted, depending on the perceived power relations in which the utterance occurs. As a result of such timidity, freshmen will immediately shrink back from a proposal made by X if Y tells them a convincing and colorful story about the evils of X. Of course, the success of Y’s slandering of X also depends a good deal on Y’s ability to suggest that the freshmen can best protect themselves from such evils by rejecting X’s proposal and siding with Y, who now stands firmly arm-in-arm with the freshmen.

Sophomores object: predicates applying unfavorably to the character of the person proposing that P belongs to S do not, logically, carry over unfavorably to the belonging relationship itself. The person is one thing, the subject (budget, or whatever) is another. In the face of the predication of the person’s character, one wants to cry out: “No bearing on the subject!” or “Misconception of refutation!” or “Confusing a multiplicity of questions!” Logically, the predicates “personally insolvent” or “terrible mother herself” or “having had the opportunity to

* Traditionally, ad hominem argumentation had nothing to do with shady character. Rather, it was recognized that, to argue against a proposal one must argue with those who endorse it. The crucial tactic was to show that the proposal (regarding nobility, utility, justice) conflicts with what they themselves uphold. This way of “appealing to the man” can be transposed into arguing for a proposal, but the traditional name of this positive strain of argumentation was “dialectic” or “Socratic induction” (the topic of Book Three).
accept a bribe” are accidents. Or the predicates focus only on non-necessary signs.

Interestingly, Aristotle and his tradition agree with juniors in disagreeing with sophomores on the question of character. Three of Aristotle’s “topics of probability” explicitly endorse what we would call *ad hominem* arguments: (1) apply to the other what he has applied to you, “for in general it is absurd to reproach another for doing what one oneself does or would do, or to urge another to do what one neither does nor would do”; (2) point out the difference between the openly avowed and the secretly wished, this being the “most powerful way of upsetting initial appearances,” and (3) take careful note of the purposes and opportunities of others, this being the main tactic in both accusation and defense. Until the Dawn of Contract Theory, and therewith the assumption that public discourse ought to consist largely in the transfer of information and calculation, the character of the speaker remained front and center, a feature essential to the discourse and somehow inseparable from its subject."

What is character? Aristotle answers: the initially fixed, only dramatically alterable way we choose. Each of us has, basically, one of three possible characters: whatever empirical item one happens to choose, one chooses it either for nobility of action, for eventual personal advantage, or for immediate gratification. In dealing wisely with others in communal circumstances, we obviously need to judge which way they choose. For only the noble person really keeps his word, upholds the words (laws) already structuring the Community — whereas the cynic keeps words of his own and follows words of others, past and present, only in his calculation of future advantage for himself; and the hedonist neither really has nor effectively follows any words at all. As we still say today, it is rare and marvelous that someone shows he has character. In this normative sense of the word, we likely agree with Aristotle: “character is the most persuasive thing of all.” For communal action depends on trust, and our trust in others depends on how we assess the basic choice governing the lives of those with whom we are working. Action spreads over time, and only nobility of character allows one to speak well for and over the entire spread. In a sense then, juniors (Cynics as well as cynics, and what Plato and Aristotle called “sophists”) keep and follow only half-words — have not yet fulfilled their nature as λόγος-conditioned; and the hedonist veers toward being a child or mere animal, i.e. toward abandoning choice altogether and therefore both action and speech.

Yet we must admit that judgement of character is risky: words and deeds are only signs of character, aesthetic indications of psychic dispositions, and therefore non-necessary. Although over time it may become necessary for me to issue a judgement of those with whom I work, the judgement itself does not rightly enjoy the modality of

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* Anciently, “speech acts” were always face-to-face: one did not hear about somebody and his λόγος, and then discuss him or it. Character remained that of either the speaker or the listener: each could immediately rebut any “slander.” In his *Sophistical Refutations*, the tensions and resolutions of paralogism are those arising in eristic: live exchange where one has agreed to ask and the other to answer questions (see Gilbert Ryle’s “Dialectic in the Academy,” *New Essays on Plato and Aristotle*, New York, 1965). Even forensic oratory was strictly first-person: an accused had to speak for himself, and witnesses similarly (see Plato’s *Apology* and Aeschylus’ *Eumenides*). And epideictic oratory required that one’s own speech embody the beauty either present or absent in the subject of the speech.
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necessity. Still, the limitation on our judgement need not terminate my concern for character. On the contrary, it can spur me on to consider the role of character for myself—precisely because I cannot make much headway in worrying straightforwardly about the character of others.

As philosophers we must of course ask what character has to do with reality itself, what human beings must somehow face. Is character not simply a personal concern, at most half the story, a half separable from the other half, namely the learning of what is (τα οντα)? At the Dawn of the Second Interpretation we hear a resounding “No!” The question of character here appears as a part, but an integral part, of the question of truth. The ability to stand in the face of beings, to stand up to threats against one’s person, to focus upon the ontological basis of beauty, of utility, and of justice—to act as well as to undergo, to affirm communal solidarity as well as the fruits of communal enterprise, to uphold the freedom of individuals in their relation to natural ultimates as well as the constraints of human formulations of those ultimates: this ability, character in the normative sense, is a necessary condition of truth itself. Only the good person can stand in the truth—and can as a result formulate and appreciate true sayings, relatively accurate determinations of circumstances at a given time and place. A person deficient in character will not have the ability to differentiate the essential and the accidental, will not act consistently when engaged in communal enterprises, will not retain, over time, the vision of the good that reasoning in action requires.*

In contemplation we can learn to appreciate that genuine speech addresses the stance of both speaker and listener. As Aristotle remarks,

* Plato explicitly locates the question of truth as a question of the condition of the soul: “the truly false is the ignoring arising in the soul of the one who is falsely oriented, while the false that arises in words merely mimes the undergoings arising in the soul” (Republic, 382B; cf. also 535E). The truly true would then be the heeding arising in the soul of the one who is truly oriented, and the true that arises in words would be a rehearsal of the doings as well as the undergoings arising in the soul. Such heeding and such doing take us out of ourselves and into things: the soul is then in truth, whereas falsehood is in the soul. Later, Plato's subtlety gets lost; e.g. the Stoics “claim that truth is only in serious people; whereas true [views] can be in foul people as well, since it is possible for foul people to say true things” (Sextus Empiricus, Outlines of Pyrrhonism, II, 83).

in the speech of action we must do everything to prove our own character and to test the character of others. Yet we can also observe that, and discern how such address can degenerate into paralogism. When applying to the other what he has applied to you, you might only be admitting that you are both scoundrels. When arguing that vile motives underlie another’s profession of nobility, you might simply be invoking facts of a personal nature having no bearing on the stance at issue in the proposed action, or even inventing a character to serve your own vile hopes. And when calling attention to the purposes, the stakes, embedded in the stances and words of your interlocutors, you might be presuming either that they should have no stakes in the proposal, or that their interests contravene the truth for which they stand. In short, all three “topics” may degenerate into slander.

In the speech of the State, especially in the monologues of the media, we have very little ability to test genuine and degenerate appeals to character: little if any occasion to steer the topic away from paralogism and back to truth. However, in the speech of Community, face-to-face as it is, character remains so much a live issue that the first person a slanderer discredits is generally himself.

At the end of his Topics (164 b 8-15), Aristotle sums up very well the chief daily consequence of a philosopher’s contemplation of the role of character in the human condition:

Not with all people is discussion [dialectic] feasible, nor is debate [gymnastic] with anyone who chances by. With some, arguments necessarily become foul. For against one who will try anything to avoid defeat it is just to employ any means of syllogizing—which is not good form. It is therefore well not to come together indiscriminately with chance people, since vile argument necessarily results. For those who merely debate cannot end a discussion without fighting.

We intellectuals easily appreciate these observations and recommendations. Yet we should not pass over the way they encourage us to withdraw: not absolutely, as the later Stoics and Cynics recommended, but at least when leading the best, namely the contemplative life. The Greek concern for character as a condition for truth seems to have in fact sown seeds which later flowered into a life of...
individuality precluding participation: character becomes the only real issue.

§3.5 Authority

I can tell you that an utterance has logical meaning for us only inasmuch as we can focus on a subject, ultimately a being, predictable in the ten ways described. Why is this so? Because Aristotle and his tradition said so. In singing the praises of ancient Greek architecture or poetry or philosophy or statesmanship, a professor might cite the testimony of venerable authors of more recent vintage—what Lessing or Goethe or Hölderlin or Nietzsche or Heidegger have said about these things. In law courts one calls in experts to testify on questions of handwriting, blood chemistry, psychic dispositions, environmental pollution. And in everyday matters I seek out knowledgeable people to answer my questions: what my lawn most needs, how to insulate my house or invest my savings, what kinds of gear to take on a climbing expedition, whom to trust as a guide in the mountains, when to attempt a climb, where to buy the best equipment. In all these ad verecundiam arguments one leans on the knowledge, feigned or genuine, of others, by-passes or at least postpones a direct contact with the beings at issue. Strange but true: we delegate part of our lives! In the face of such appeals to authority—of such veneration, delegation, circumnavigation—we might well wonder: In what ways do we, or can we wisely allow the testimony of others to stand in for our own?

In appealing to others either for answers or for proofs, we seem to place between ourselves and what we want to know a “third thing,” most obviously the words of our chosen authority. We thereby either create or acknowledge a distance, our ignorance. In either case, we lack intimacy. For a philosopher, at least, this intervening factor looms as the greatest terror: Will it chain us at a distance, or will it send us down a path toward intimacy?

In their crudest form, appeals to authority take the shape of predicating proposals rather than allowing proposals to predicate subjects. Schematically put:

\[
\text{Statement X is said by so-and-so,} \\
\therefore \quad \text{Statement X is true,}
\]

with the implied major that everything said by the chosen authority is true. But already a sophomore knows that a proposal will not be true because an authority, no matter how reliable, has uttered it; rather, the authority will have uttered it because it is true, i.e. because the predicate belongs to the subject. The freshman confuses these “becauses”: after all, he is only looking for relief, so that for him the predicate “true” means “what I should follow”—follow in order to move on past (rather than toward) what the authority is talking about.

What is the alternative? To ignore authorities? Many do, especially after being sated with, jaded by the directives of parents and teachers and books—and “experts” on this or that cranny of the universe. Or rather to take them as directives toward something? For we can learn from others, i.e. from wrestling with the challenges they set for us. Just as when someone who should know tells me that it is raining outside, I may dress accordingly and step outside to see for myself, so when a logician tells me that the EAO-4 is valid or a physicist tells me that $E = mc^2$, I can pause to consider what one or the other proposal means and proceed to “step outside.” But in this case I drop the authority and actualize myself in calculating whether, and under what conditions a Felapton necessarily generates its conclusion—or whether and how energy makes sense as the product of mass and the square of the speed of light: in either case I follow the original utterance back to its source.

You will object: no one has the time, few have the ability, to step outside on every occasion; trust in others, respect for their reports, is precisely one of the great benefits of communal life. Furthermore, even Aristotle advises us in some detail on how to appeal to authorities to make or break cases. He points out that one set, the long dead, often supply our only clues to what has happened in the past; and these witnesses have the advantage for us that present events cannot corrupt them; furthermore, they have often proven their credibility by having foretold events accurately. And then, as he says, there are the authorities close by, whether disinterested or directly involved (economists and politicians in questions of utility, experts or eye-witnesses in questions of justice). Once we realize how insecure our own opinions are, how little we have been “outside” in the matters under consideration, we rightly seek out the testimony of others, both with regard to the major premiss (the wisdom found in various literatures of the past) and with regard to the minor premiss (“what did or did not happen, what now is or is not”):
for an account of “appealing to witnesses,” see Rhetoric, I, 15). Certainly the best would be to know directly, but second best is to heed the words of those who do know, and the worst is simply to go one's way in the dark.

To be sure. But may we not betray both ourselves and others by simply leaning on authorities? Precisely Aristotle insisted on several distinctions paving the way out of total dependence on others. For one, most authorities direct us rightly only to the minor premiss: the quality at issue (justice, utility, nobility) remains outstanding as something only we, the judges, decide — although disinterested witnesses may speak very persuasively to the major premiss, and long-gone authorities can argue very persuasively on the principles of such judgments. And Aristotle also remarks that we must assess the testimony of witnesses in reference to their character: we must ask whether the cited author or authority deserves our veneration as a guide in our quandaries.

Elementary textbooks in logic often advise us to ask whether our veneration is relevant to the case at hand — whether, for instance, our respect for a baseball hero or movie star justifies our accepting his or her testimony on the quality of a whisky or the worthiness of a charity. But are we then not begging the question in a rather comical way? To know whether a reputed authority is really an authority, would we not have to know already the matter about which the authority testifies — and be authorities ourselves? You might say that the examples cited clearly illustrate a disparity of fields. But does our respect for an academic economist, versed in the past, justify our accepting his view on a proposed policy for dealing with unemployment at the moment? And perhaps a given baseball player does know a good deal about whisky, a movie star very much about the need to support charities. How can we know to whom to appeal? Often, in fact, we rely on or employ a freshman’s ad populum argument (everybody knows he is an excellent economist); or, in the more sophisticated discourse of parliamentary hearings, we rely on or employ a list of credentials or certificates referring to earlier judgments of the authority’s expertise (a snow-job of sorts: non-necessary signs of pending performance).

Returning to the crudest form of appeals to authority, we logicians can point up some further difficulties. First, the proposals of such arguments predicate not beings, but utterances. Furthermore, the predication in the minor premiss is clearly accidental: Statement X is (has been) uttered by so-and-so. And thirdly, in the conclusion the predication appears redundant: What is the logical difference between saying “The EAO-4 argument form is valid when \( M \neq 0 \)” and saying “That statement is true”? The content is the same. The two only differ in that the second seems to suggest a modality — in this case, perhaps “necessarily the case”, although in others a more fitting modality might be “simply the case.” When adding “... is true” (or prefacing “it is true that...”) to a statement, we in any case express a modal decision on our part, and perhaps emphasize the rejection of the opposite modalities associated with the contradictory appendages like “false” or “wrong.” In short, both minor premiss and conclusion come down to the same content:

So-and-so said: S-P
It is necessary/factual that: S-P.

From our standpoint as logicians, the references to the author and to the modality of the proposal only contextualize the proposal that \( P \) belongs to \( S \). They provide no focus on \( S \), nor any reasons for affirming \( P \) of it. If the speaker pretends, or the listener assumes otherwise, we have a snow-job on our hands.

Yet we might all agree that much of our speech-life leaves us in limbo. Indeed, Socratic wisdom consists in knowing ourselves distanced from the intimacy which we know to be imperative. To avoid utter impotence, namely a cowardly reliance on authority, we would then have to acknowledge that our ultimate task lies in becoming authorities ourselves. That is: in trying. That is: in keeping the proposals of others in the mode of possibility until such time as we ourselves can “step outside.” That is: in becoming not wise-guys for our own sakes, but witnesses of what is “outside.” The task would be to consider the testimony of others now, and our own testimony later, as lenses through which one can focus with one's own eyes and which make sense only upon such looking.

Although we now, in contemplation, focus on the speech of action, we can also see, perhaps more clearly, the role and task of authority in production — in learning an art. For here one must, as Plato has Socrates insist over and over again, attach oneself to a single master, one who knows the art (horse training, medicine, or whatever) and follow his or her words and deeds closely, eschewing the words and deeds of “just
anybody” (οι πολλοί). Why? Because one wishes to move out of the distance articulated by general opinion and into close contact with beings (horses, bodies, or whatever; in any case beings more obviously delineated than communities, the focus of action). We wish, in other words, to become masters (authorities) ourselves, and we obviously need help. Here, veneration of a master becomes emblematic of an eventual contact with and veneration of reality itself.

In the literature of the Third Interpretation, “becoming an authority” takes on added poignancy. For earlier Greek thinkers (whether Plato or Thucydides) “authority” as position and power, possibly even arrogance (ἐξουσία) is one thing, “authority” as knowledge and testimony pointing to something (μαρτύριον) quite other. In contrast, for the Christians the de facto meaning of “position of power” gives way to the de jure meaning of “bearing testimony”: genuine power, even in daily life, appears as grounded in, derived from divine power, and one can participate in this Ultimate Power if and only if one consents to represent it—to speak and act from and in the name of the Divine, and in order to spread its Word. Thus in listening one can hearken to discover whether the other speaks with authority, whether the speech stems from the Divine. Not the scribes, the pedants and lawyers quoting scripture to prove their points, enjoy the resonance of authority. Only one who speaks humbly, from a position grounded in the Divine (and not self-grounded, but grounded by grace) evidences such resonance: “...and he astonished them, for he spoke as one with authority” (Matthew, 7:29; Mark, 1:22), “for his speech was embedded in authority” (Luke, 4:32). And such a one bears witness at all costs: from God, for men, and unto death—so that commentators no longer render the Greek μαρτύριον as “witness” but rather transliterate it as “martyr.”

In becoming authorities (witnesses), Christians also become speakers. But the speech of martyrs has usually survived in the form of legends in which the personages already appear sanctified and more or less imaginary. In contrast, the historical figure of Thomas à Becket, murdered by the cohorts of Henry II in 1170, has occasioned detailed secular renditions by such poets as T. S. Eliot (Murder in the Cathedral, 1935) and Jean Anouilh (Becket ou l’Honneur de Dieu, 1959). History and poetry agree that the man was something of a playboy friend of Henry II (a friend of pleasure), and that he eventually became a clever advisor of the King as well (a friend of use). In hopes of securing his power over the Church in England, Henry II made his roguish friend Archbishop of Canterbury. Hitherto, Becket had never known authority except in the pagan sense of power, namely submission to and benefaction of the King. Now he finds himself at the other end, nominally in a position of power but, to his own surprise and to the vexation of Henry II, more fundamentally in a position of service—as suggested by Jesus (Luke, 22:25-7). As Archbishop he is an authority in the obvious sense that others—priests, bishops, laymen—entrust themselves to his care. But instead of playing the power game with the King, Becket becomes a representative of the Divine Source and so the preserver of the Church as the earthly reminder of that Source for others. He becomes, in effect, a Christian who speaks with authority, whose speech is embedded in the Source. His syllogism reads: “These actions of Henry II are not actions in which I shall acquiesce” because “These actions of Henry II are actions deterring the maintenance of the institutions in which I represent the possibility of salvation through grace.” Much of the story consists of a portrayal of the anguish of the two men. Becket especially tries to become for the King a “friend of contemplation”: failing, he still continues testifying to the Truth, himself a witness unto death.*

* For a parallel instance, see Robert Bolt’s A Man for All Seasons (1963)—the story of Thomas More, beheaded by Henry VIII in 1532 for refusing to sign a document acknowledging the King as head of the Church (where, however, there is no dramatic change from rogue to witness). Cynics, of course, attack the poetic version of these historical personages, claiming that they simply lost out in power plays. But then, as Aristotle says (Poetics, 9), poetry tells us...
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For a Christian, it seems, the only proof that he really bears witness lies in his continuing to do so despite his loss of power over the course of events: only being a martyr unto death proves that his λόγος is in authority. Sounds rather extreme. Yet any genuine appeal to authority (whether in production, in action, in contemplation, or in salvation) draws upon, refers us to, represents a reality, a power beyond the one who genuinely speaks with authority. A senior, at least, knows that the authority at issue is not his own.

§4. Linguistic fallacies: words

As Heracleitus remarked, “The water of the sea is both pure and defiled — for fish it is drinkable and life-preserving, for humans it is undrinkable and destructive.” Much the same we might say of talk: it serves as a path toward reality, also as a path away from reality. “The path up and the path down is one and the same.”

When you and I talk about something we both have contact with — horses, say, or computers — our talk stays within hearing distance of the subject, our words function like arrows in flight, and we follow them toward their targets: words signify the things we are talking about, and our talk remains rather transparent to the target as a whole.

On the other hand, when you talk to me about something that eludes contact with me the talk does not, for me, resonate with the subject, your words shoot off in directions I cannot follow with my eyes or ears, neither at the moment, nor in memory, nor in anticipation: I then grasp at the words in flight as they first issue from your mouth or lie printed on your page, and I try to keep track of them; eventually I may even succeed in retracing the flow of words heard or read, repeat them fluently, whereupon I forget the difference between up and down.

The root of all paralogism lies in the failure to make contact with reality while seeming to syllogize anyway. One can concentrate on accurate determinations at the cost of contact. One can pervert the danger naturally associated with such contact. Or, now, one can grasp at the words which might otherwise serve to focus attention on the subject of the talk. The root of linguistic fallacies lies in the hope that words will do the work for us — in the unwillingness or inability to do our own work within words. This root phenomenon has many exemplifications: students simply remember assorted key sentences they have heard from their teacher or read in books; I learn from my parents all the right things to say on social occasions; a teacher learns over the years the standard things to say in lecturing; braggarts overhear and repeat what others have said about love affairs or climbing expeditions.

Yet the game can be played both ways, both toward and away from contact. And since in public debate we will find ourselves talking with people who play it the other way, we will play it better our way if we practice ourselves in playing it their way as well. And this other way will not be that of those who innocently and blindly hope that words will somehow do the work for them. Rather it is that of the sophists, those who intentionally play to win, to gain not reality in contact but points in a contest. The context here is eristic: not what we today might call unconscious slippages in the meaning of words and phrases, but rather the highly intentional maneuvers of those playing a hard and fast game in which the purpose is to outdo others.

§4.1 The original six tricks

Our talk in fact wavers on the path, and Aristotle proposes six ways. First, and most prevalently, a word can be equivocal, can look or sound the same and yet have two or more distinct meanings (in Greek: homonymy). Second, the parts (clauses) can be so arranged that the sentence as a whole allows of two or more meanings (amphiboly). Thirdly, we can compress elements within sentences to generate a meaning different from the original (synthesis) or, fourthly, we can separate elements to alter the meaning (diareisis). Fifthly, we can change the meaning of words by slight variations in pronunciation or spelling (prosody). And sixthly, the grammatical shape (scheme) of the talk (lexis) can suggest wayward meanings by analogy (lexischeme). Paralogisms based on one or another of these six waverings of meaning will make sense primarily in exchanges between two speakers."

more about potentialities, history more about facts. — For the three kinds of friendship, parallel to the three kinds of choice, see Aristotle's Nicomachean Ethics, VIII, 2-3. And for an insightful account of authority during our Fourth Interpretation, see Chantal Delsof's L'Autorité (Paris: P.U.F., 1994).

* See Aristotle's Sophistical Refutations, Chapters 1-4 and 19-23. Plato's Euthydemus illustrates many of the paralogisms in dialogue form. Again, for a scholarly review, see Hamblin's Fallacies, especially Chapter 2. For
§ 4.1.1 Homonymity

You say to me, “Is not something yours — a dog, a house — only when you are in a position to dispose of it as you wish — use it, buy it, sell it?” I agree, and you go on: “And Canada — is this not your country?” I must agree, and you conclude: “Then you must be able to dispose of it as you wish!” A joke, but one reminding us that possessive pronouns can indicate possession in either direction: perhaps I can dispose of my dog as I wish (although pets often run the lives of their masters), whereas my town or my country more obviously disposes of me, and perhaps a talent or a love is “mine” in both ways.

If I ask you where Jim is and you tell me, “He's over there working on his bow,” I might reply, “So now he has taken up the violin.” But I would be making a joke if I had come to a party in which people were competing to see who could wrap Christmas gifts most beautifully; “the bow” can mean the instrument with which one plays a violin, or it can mean the result of tying ribbons together in a certain way.

Or, I ask you, “Are not elephants animals?” You agree and I ask you, “Is not a sick elephant a sick animal?” Surely you agree, and I conclude, “Then a small elephant must be a small animal!” Here the joke may be explained categorically: ordinary terms of quantity (small and large, short and tall, brief and long, even hot and cold, wet and dry) make sense only when conjoined with a consideration of relation (small compared with other elephants, long for an after-dinner speech, lots of moisture when compared with the Sahara Desert, little moisture when compared with Toronto in August).

Homonymity not only allows us to play tricks on one another. It also poses very real barriers in intellectual talk. For instance, we read Aristotle claiming that some people are slaves by nature; if we do not pay careful attention to the entire argument we will likely assume that Aristotle simply defends the economic and political arrangements by which some people do all the work and others rapaciously enjoy the fruits of that work; yet Aristotle’s “slave by nature” refers to a disposition toward passivity, and his “freeman” refers to the activity of assuming responsibility: “In a household ... the freemen are least able to act as they simply will, for all things or most things are already ordained for

§ 4.1.2 Amphiboly

Not only single words, but also entire sentences waver in their meaning. At a cocktail party I instruct you, “Serve the guests with drinks,” and you look around for the “guests with drinks” and supply them with yet more drinks. Again, a kind of joke: the phrase “with drinks” (or “with courtesy”) would seem to modify not adjectivally the direct object (“people”) but rather adverbially the action of serving. Yet you can look to the words rather than to what I mean by them and justify, by reference to rules of grammar, either interpretation.

“Nowadays,” you say, “students and teachers may freely discuss sex in the classroom.” I reply, “But I for one have never seen any there.” A grammarian might argue that the modifying phrase “in the classroom” should appear closer to the verb if we wish it to qualify where the discussion is to take place. In writing and reading legal documents we must, it is true, abandon elegance for the sake of self-defense.

“People who are allergic to eggs and children under 13 should not be given flu vaccine,” writes a well-meaning journalist in a newspaper article. “We're a little sensitive to both,” replies a wise guy. Does “children under 13” count as an addition to “people who are allergic to eggs” to form a two-part subject? Or does it go with “eggs” as a second thing to which people may be allergic? Conventional English grammar leaves the question undecided (although German would decide the matter by employing either the nominative or the dative case of “children,” and

Lighthearted illustrations in the context of comedy, see Henri Bergson’s Laughter, Chapter 2, Section 2 (‘The Comic Element in Words’).
French by either repeating or not the preposition à). As it stands in English, the clause “and children under 13” functions in either of the two grammatical fashions, and the whole can therefore waver in meaning.

In live exchanges, we may build on the words of others, adding units (modifiers) as they occur to us: “He has left Montreal, then?” Yes. “After saying good-bye to his wife?” Yes. “On the train?” Yes. “So his wife was on the train, too — since he said good-bye to her on the train!” In a well formulated monologue we would place the phrase “on the train” close to the verb “left” rather than to the noun “wife.”

Like homonymity, amphiboly poses alternative foci, thereby allowing us to play tricks on one another. And not just silly or dirty tricks, but even contemplative ones. For instance, we read Mallarmé saying of the practice of writing: “He who accomplishes it completely cuts himself off.” Grammatically, the adverb “completely” can modify either the verb “accomplishes” or the verb “cuts off,” and we may wonder which modification Mallarmé intended; perhaps, however, the intention is that we try them both, each separately or even together. Similarly, Dylan Thomas’s lines (unpunctuated):

Though wise men at their end know dark is right
Because their words had forked no lightning they
Do not go gentle into that good night

may mean two quite different things according to whether we read the “because” as reflecting why “men know dark is right” or why “men do not go gentle into that good night.”

§4.1.3 Synthesis and diairesis

Ancient Greek was written without spaces between what we now call words, and also without what we call punctuation: spacing and punctuation appeared as the responsibility of readers. Only later did scholars add these to help readers speak the lines with the appropriate pauses and rhythms. Today we take for granted that words and sentences fall naturally into well defined units, so that what we should take together (combine) and what we take as separated (divide) seem established from the start: confusions in these regards appear merely fortuitous. We may nevertheless devise analogies.

You say to me, “Save soap.” I agree, whereupon you add “...and waste paper.” If at this point I reply, “I shall do my best to waste as much paper as I can, all the while saving soap,” I play on the possibility of dividing the phrase “waste paper” into a verb and a noun, even though you likely meant it as one (in German one would in fact write it as one word).

I overlook some prisoners: Does this mean I keep them in my line of sight? or that I fail to notice their presence? One might imagine similar confusions arising in utterances containing compound verbs such as “overrule,” “upgrade,” and “run over”: “I’ll run you over” might mean “I’ll drive my car over your prostate body” or “I’ll take you in my car to your destination” — and one can imagine a conversation playing on these possibilities. Punctuation in writing, or intonation in speaking, help decide what should be kept together and what kept apart in such run-ons as “She has three year old babies,” “He liked the house (-) call on Wednesday,” “I read the newspaper (...) he left with the secretary,” and “He’s a stable boy.” Hyphenation would seem especially urgent in the announcement that “The ladies of the church have cast off clothing of every kind and they can be seen in the church basement Friday afternoon.”

In the careful (vs. impressionistic) reading of texts the placement of a comma can determine the meaning. For instance, a sentence in one English translation of Nietzsche’s Birth of Tragedy reads:

The image that now [in the Apollonian dream] shows him [the Dionysian singer] his identity with the heart of the world is a dream scene that embodies the primordial contradiction and primordial pain, together with the primordial pleasure, of mere appearance.

The placement of the final comma in this translation assures us that the three “primordials” (Urwiderspruch, Urschmerz, and Urlust) are all “of appearance.” Consulting the German, as well as Nietzsche’s argument as a whole, one would likely drop the final comma, keeping the phrase “primordial pleasure of appearance” together as a third factor of experience after the other two. The translator encourages readers to
divide the elements wrongly: to ponder how one might understand the contradiction and pain at issue as being “of appearance.””

Aristotle's example of synthesis is: “One can walk while sitting and write when not writing.” The reasonable meaning is that one has the ability to walk at another time while one happens to be sitting now; but the grammar also allows for the outlandish meaning that one can be both walking and sitting at the same time (or writing and not writing at the same moment). The Port-Royal Logic exemplifies synthesis also in reference to time differences: Jesus says, “The blind see, the lame walk upright, the deaf hear”—and the authors claim that “this cannot be true unless we take the things separately and not conjointly,” i.e. the subjects as named according to how they were and the predicates as naming how they are or will be.

Aristotle's example of diairesis (taking apart) is: “The number 5 is both odd and even. Why? Because it is made up of the number 2 and the number 3—an even number and an odd number.” The Port-Royal Logic exemplifies diairesis concretely: St. Paul says, “The slanderers, the fornicators, the misers will not enter the Kingdom of Heaven”; however, the authors go on, “this does not mean that none of those who have had these vices will be saved, but only that those who remain attached to them, who do not shake them by converting themselves to God, will have no part in the Kingdom of Heaven.”

In his *Rhetoric* Aristotle supplies us with some more jokey examples of synthesis: “You are in Toronto and you know Carl, therefore you know Carl in Toronto.” “You know all the letters of the alphabet, and you know that the word ‘xanthinose’ is made up of those letters, so you must know (understand) the word.” Not at all jokey is the example of Orestes, to which Aristotle often refers: “It is just that a woman who has killed her husband should be put to death, and it also just that a son should avenge his father, therefore it is just that Orestes should kill his mother (who had killed her husband, Orestes’ father)”; as Aristotle goes on to say, synthesizing the two principles of justice onto Orestes, the son of both persons, yields an injustice (or: one is arguing elliptically, leaving out the name of the agent—perhaps committing the fallacy of *secundum quid*). As an example of diairesis Aristotle proffers a very common argument, or at least slippage: “Taking X amount of medication at once is unhealthy, therefore taking half that amount must also be noxious.”

The examples of synthesis and diairesis proffered by Aristotle in his *Rhetoric* do not insist on a sharp distinction between paralogisms within and paralogisms outside talk: for the confusions regarding the justice of Orestes’ action and the healthiness of medical dosages depend as much upon misfocussing on beings as upon latching onto words: purely linguistic fallacies are only those in which there is no focus on beings at all, therefore not even a misfocus. When I argue that an orchestra will play beautifully because I have heard each of the musicians play beautifully, I not only let the word “beautiful” slip casually from beings taken separately to the new being synthesized by the conductor, I also fail to notice that a new being (an organism, an organization) is born out of parts (organs); similarly, one learns that a dish can be very tasty or a perfume very aromatic, while the ingredients are, taken separately, either indifferent or revolting.

§4.1.4 Prosody

Consider again the word “bow.” In English we do not have accents to distinguish the pronunciation of this word, even though we vary the pronunciation according to whether we are speaking of the instrument with which one plays the violin or the act of bending at the waist after playing it. In writing, we could play on these meanings: You scribble a note saying, “Jane is working on her bow this afternoon,” and I scribble back: “I did not know she was so stiff.”

Then, too, there are words (homophones) which are pronounced the same although they differ in meanings and are also spelled differently. A teacher wants to talk about the “whole” of something, and a student wonders why he should bother with a “hole.” You say you want to “complement” my efforts, and I reply that I can use all the “compliments” I get. I tell you that a certain “idol” is beautiful, and you tell me you are not interested in “idylls.”

* “Reading well, that is, reading true books in a true spirit” (Thoreau) requires such grammatical sensitivity, and the positive side of all these paralogisms inside talk is the development of such sensitivity. One of Aristotle’s favorite examples of a property (vs. both essence and accident) and of a quality (a category applicable to the substance “man”) is “grammatical.” What is at issue is what we nowadays call *literacy.*
Variations in pronunciation also occasion waverings in meaning. A child says he wants a pretty “pitcher” (picture) to hang on his wall, and his mother asks (sophistically) “with or without his mitt?” (or even “with milk or with water?”). Or consider the word “aunt”: as a child I once said to an American teacher of mine, one who prided herself on her Mayflower ancestry and on her retention of British mannerisms, something about visiting my aunt, and she remarked how strange it must be to spend time with such a tiny animal.

In its original sense, prosody has to do with pronunciation. In Homeric Greek the rhythms (shorts and longs) and pitches (ups and downs) were fixed once the words themselves were decided. The waverings of prosody then ceased the moment interlocutors agreed on these matters of prosody. However, modern languages (already the Greek of the New Testament) allow for another kind of wavering, that of laying stress on certain words when pronouncing or hearing them. I tell my son that he should not run after girls, and he says he agrees — and that he shall now walk after them (as though I had stressed the verb) or that he shall now run after boys (as though I had stressed the gender of the object of his desire). Or you tell me you are debating whether to fly out to Banff to go skiing at Christmas, and I reply, “Beats walking.” Stanley Cavell points out that the whole of Beckett’s Endgame might depend on how the actors stress the line, “You’re on earth, there’s no cure for that”: a stress on “cure” would suggest that there might be other alternatives, other nouns, to describe the task of our condition (acceptance, love, cultivation), whereas a stress on “that” would suggest that one might as well turn from one’s “being on earth” entirely and concern oneself for something else (a life of retreat, of despairsed resignation, of waiting for lucky breaks in heaven).

In speaking, we stress words or phrases in order better to focus others on what we mean, just as in writing we underline or italicize; similarly in listening we interpret and in reading we impose stresses in order better to get in tune with what others mean. Intelligent speaking and writing, listening and reading, requires that we act the lines.

But the mere mention of something—an utterance as a whole—can also constitute a kind of stress. My wife tells me that the trash container is full and I reply that, yes, there were lots of bottles left over from our party: my explanatory response might also be a kind of joke if the context of the interchange suggests she intended the mention of the fact as signal for me to dispose of the trash. Or I call my brother and ask how his wife is getting on, and whether she is still sleeping, my brother rightly interprets my question as suggesting the possibility of his wife frequently sleeping. One soon learns in life that the very choice of things to say already means something: in this sense, one learns that there is at least a pinch of irony in every saying, a “universal set” at issue in our stresses. And it is this “unsaid within the said” that gives us some free rein either to develop a contemplative sensitivity to the unsaid or to undermine (jokingly, ignorantly, or perversely) discussions by pursuing sayings in ways changing the universe of discourse.

§4.1.5 Lexischemy

You say you will do some work this afternoon “in the library” and I suggest that, on the contrary, you should do it “in a hurry”: I thereby play on the scheme of “in,” which often does suggest a place but may also imply a time or even a condition or an action (as when Plato speaks of courage as being “in the soul”).

In the Bible (Mark, 2:23 ff.) we hear the Pharisees arguing, in effect: “Is not field plowing work?—Yes. — And is not house painting likewise work?—Sure. — And is not paying our bills also an affair of work?—Yes, because money matters are basically labor matters. — And are you not forbidden to work on the Sabbath?—Indeed. — And so you see that this feeding and healing of yours on the Sabbath is contrary to the Law!” At this point Jesus might have objected: fallacy based on lexischemy! For the Pharisees were taking verbs like “plowing” and “painting” and “paying,” all of which intend an expenditure of effort to achieve some material gain, and were arguing by analogy that “feeding” and “healing” also mean “work”—failing to note that “feeding” and “healing,” at least in Jesus’ sense, do not mean acts to obtain material advantage in everyday routines, but mean precisely the overcoming of everyday routines for
which the Sabbath was intended (the Pharisees might just as well have argued that “praying” was unlawful on the Sabbath).

Or (and to illustrate Aristotle's own examples of lexischemy) I argue that mountain guides in France are men and in Italy they are women, while in Germany both men and women may ply the trade —because in French one says LE Guide, in Italian LA guida and in German both DER Bergführer and DIE Bergführerin. In English one cannot so often argue from the gender of names; however, objections to the designation “chairman,” to giving only feminine names to hurricanes, and to the use of “he” instead of “she” as the indeterminate pronoun, would probably suggest to Aristotle that we have a peculiar tendency to latch on to the conventional forms, the lexical schemata of the English language. The sophists were fooling, trying to catch their interlocutors unaware. We, on the other hand, speak the only western language in which gender plays so little a role that the slightest suggestion of one in a name seems to us to imply a sexual difference, or even a sexual preference, in the being named.*

In marking essays teachers often employ a number scheme to express a judgment of quality: a student might then conclude (jokingly?) that to improve his mark from 75% to 82.5% he should submit an essay 10% longer. Similarly, in the study of logic we “quantify” proposals as speaking of either “all,” “some” or “this one” — and students often assume that these designations refer somehow to numbers, whereas they refer to the depth of the insight the proposals demand of us: their point of reference is the category of being. Aristotle notes that lexical schemes also confuse action with quality or condition: “We shall both keep ourselves busy after lunch; my husband will mow the lawn and I will digest my food” — as though “digesting food” (Aristotle says: “flourishing”) were an activity. In Amsterdam I once saw a particularly playful example showing how the surface shapes of talk may remain the same for grammatically much different utterances — a sweatshirt on which was printed:

Time Flies Like the Wind but
Fruit Flies Like the Heat

whereupon a sophist would say you must conclude that fruit can fly, and that it does so in much the way heat does: he thereby commits the fallacy of unwarranted division (on “fruit flies”) but also plays on the scheme of “like the wind” and “like the heat” (also on the homonymity of “like”).

Lexischemistry becomes a major philosophical issue in the use of grammatical subjects drawn from categories other than that of being: “Ethics depends on religion,” “Speech is given to man to hide his thoughts,” “Deep feelings of affection are like respectable women.” In such cases the unwary assume that “ethics,” “speech” and “feelings” are somehow things in themselves which then receive predications. Perhaps because we in fact talk most of the time about human being we may rightly pass over this logical subject in silence. Yet the scheme of such talk also allows us to forget what we are talking about.

In his Rhetoric (1401 a 1-8) Aristotle mentions one more form of lexischemistry: in public assemblies speakers often enumerate their “points” (firstly, secondly, thirdly, ...) and triumphantly conclude “therefore ...” — following the surface appearance of premises necessitating a conclusion when in fact there has been no syllogism at all. Apart from the pomposity of such talk, we might here also detect the fallacy of Non-Cause, some form of snow job, or faulty diairesis — as in the following (adopted from Plato’s Hippias Major): “You and I are honest, therefore I am honest; you and I are good looking, therefore I am good looking; you and I are a marvelous couple, therefore I am a marvelous couple!”

§4.2 Three solutions

In the 2nd century A.D., Galen could still devote an entire treatise to the six sources of sophisms (six kinds of wavering meaning).* And he retains the identical philosophical spirit after those several intervening centuries. The only difference is that he systemizes the six. The purpose and therefore excellence of speech, he argues (contrary to Stendhal), lies

* In his “Defense of Raymond Sebond,” Montaigne remarks: “Nostre parler a ses foiblesses et ses defauts, comme tout le reste. La plus part des occasions des troubles du monde sont Grammairiennes.” Many of his examples illustrate lexischemy: a skeptic says “I doubt all things” and his adversary replies “You then affirm this at least.” Most of his examples touch upon the insistence on speaking about God in forms of speech suitable only for human affairs. Today, we can think of the efforts to reform English usage to reflect current political concerns: lacking faith, one retreats into lexical schemes.

in signifying something, in pointing something up, and failure may occur in two ways: either a word or a sequence may point in two different directions. Homonymity consists in one word actually pointing in more than one direction at once, and amphiboly consists of an entire sequence doing so. On the other hand, prosody reminds us that one (unfinished) word may potentially point in more than one direction — whereas once we add a mark (the “e” of “complement” or the “i” of “compliment”) or a sound (the quality of the “o” for “bow”) some of the wavering is eliminated. Similarly, synthesis and diairesis remind us that one (undeciphered) sequence of words may potentially point in more than one direction — whereas once we make it clear that “waste paper” is basically a single word rather than a verb and a noun, the wavering ceases. And, finally, lexischemy recalls our own ability to recoil totally into our talk and to forge directions fantastically — whether for words taken singly (the “in” of “in the library” implies a place, therefore it does the same when we speak of something “in the soul”) or for sequences (as in simulating a syllogism with the use of “therefore” after an enumeration of findings).

Galen’s systemizing of the six tricks depends on there being two locations of wavering (single words and entire sequences) and three ways of being and responding: at a given moment, we respond either to an actuality (a being presently functioning), to a potentiality (a being as it can or might function), or to a fantasy (our own memories left over from, and now cut off from beings).

But why bother, why did philosophers from Plato through Galen and beyond bother to dwell on such linguistic foolishness?

It may help to remember that the words “homonymity,” “amphiboly,” and the rest do not name fallacies. Rather, they name facts of our lexical (linguistic) condition, features of the tongue we happen to speak, features which allow two of us to heed the same words and sequences (the same in pronunciation or spelling) while each of us goes off in a different direction. Linguistic tomfoolery proves the duplicity, the unreliability, of words. The moral of the six-part story is clear in general outline: we must learn to do our work in words, all the while eschewing the false hope that words will ever do the work for us.

But what can the moral mean for us as talkers — as readers and writers, listeners and speakers?

Freshmen, of course, do not heed the moral at all. For them, words are words, and everybody knows, or should know, what they mean. Indeed, early in our lives we simply follow in the footsteps and wordsteps of those around us; but here both steps go together, and the difficulties of the one hardly differ from the difficulties of the other. Only later do other troubles set in, namely when we step outside. If at work I read a sign saying EMPTY GASOLINE DRUMS, then the drums must be empty, and I can freely smoke and cast matches, ashes, cigarette butts around as usual.* If I read in the Bible that one should care for one’s fellow human beings, “everybody knows” that this means we should provide others with the material and sentimental comforts of life. In general, one hears what one’s parents, playmates, and teachers say, or reads what authors say, and assumes that one need only listen to or look at the words (or look them up) in order to understand or to learn something; one appeals, in effect, to popularity or, at most, to authority. And it works — so long as one does not have to talk oneself, i.e. so long as one can simply parrot words heard, and can self-righteously complain that the sign should have read GASOLINE DRUMS EMPTIED OF FLUID GASOLINE BUT POSSIBLY FILLED WITH GASOLINE VAPORS.

Sophomores heed one half of the moral: words do not always do the work for us, they often prove unreliable, and even when formulated very carefully they waver in meaning. This half encourages us to look for more than “mere words” — it urges us to think and to feel for ourselves, and ever anew. It even leads us to eschew carefully and expansively worked out formulations, our own as well as others’, on the grounds that they necessarily pretend to an adequacy impossible by the very nature of words. I become sloppy in my speaking habits. I find reading and writing very cost-inefficient — requiring more energy than the results ever warrant. And when I listen to others I become impatient with what they say — and ask them simply to tell me what they mean. In general, the conclusion of this half of the moral leads to silence.

Juniors pay special attention to the other half: we must learn to do our work in words. However, for a junior the work consists of organizing others to promote his own designs. A cynic says to others what he figures they want or need to hear, either to pacify them or to

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* B. L. Whorf, both a fire-insurance salesman and an accomplished linguist, comments on these and other examples (of the harmonies and disharmonies between footsteps and wordsteps) in the essays collected under the title *Language, Thought, and Reality*; he discusses the “empty” of EMPTY GASOLINE DRUMS on p. 135 of that volume.
scare them — whichever makes them willing to contribute to the fulfillment of his own designs; similarly he takes what others say as signs of their own interests and dispositions, bluffs and threats. As Hobbes and Machiavelli so clearly see at the birth of the Fourth Interpretation, the evolution of modern social and political structures requires more, not less attention to the power of words, or rather to the ways in which the power-plays within and between States take place in words. But words here no longer function as illuminators, as responses illuminating beings. They are what Hobbes calls “counters” in negotiation, quite rightly contrasting their use here from their use by freshmen. “For words are wise men’s counters, they do but reckon with them: but they are the money of fools, that value them by the authority of an Aristotle, a Cicero, or a Thomas, or any doctor whatsoever, if but a man.”

And seniors? How might we understand our position in words as one of responding to and illuminating some reality — as distinct from the designs we happen to have? The literature of our traditions answers this question in two drastically different ways, both of which have recently again been challenged by a third, itself rather uncertain in its direction.

§4.2.1 Aristotle’s way

On the first page of his little work on communication (conventionally translated with the slightly misleading title On Interpretation), Aristotle says of both nouns (names of things) and verbs (qualifications of things):

They are, in sound, symbols of the affections in the soul, whereas written ones are symbols of those in sound. And just as the written ones are not the same for all people, so neither are the sounds the same. But what these are originally the signs of, viz. the affections of the soul, are the same for all. And the things of which these affections are similitudes are also the same.

One must read these statements carefully. Words are “symbols”; that is, they go together with, make sense as arising with or from... What? With (or from) “affections in the soul”; that is, with (or from) what happens to us in our responsive nature. “Affections” (παθηµατα) are, in precise categorical terms, our undergoings. But what do we undergo? We read: “things,” whatever we in fact face; but this word (πραγµατα) also has a decidedly categorial meaning: “things in the making, things in the doing” — “affairs,” we might better translate. Anyway, these two, “affections” (undergoings) and “affairs” (things in the making) are, Aristotle says, the same for all. The same, that is, for those who “have” them — even though the sounds and the writings differ from person to person, from people to people. More: the first, our undergoings, are similitudes (Διαµετικα) of the second, the affairs in force: in our responses (our souls) we attune ourselves, more or less, to the things we undergo. And our talk, our nouns and verbs, manifest this attunement of ourselves (our undergoings) with what is happening (affairs). Our words are then in some way with and not just about things — when, of course, they are really ours.

Abstractly put: on the Second Interpretation, the meaning of a word is the thing it symbolizes (is the symbolizing of the thing by the one uttering the word and affected by the thing). Or: words mean things (as meaning things). Somewhat more concretely: words are our embodied (in-sounded) responses to affairs. But then we can see the “weakness” of words, namely our own weakness: it is possible to settle for only half the symbol, that half which echoes in our ears (or, halving it again, that quarter which lies static before our eyes); a word can all too easily become what the late Scholastics call a flatus vocis, mere gas — and our talking can degenerate into mere gassing. As seniors, our task in talking with others is to counteract this tendency (rather than encourage it, as the sophists often did). We must, as it were, mend the broken symbols. Careful attention to the “six ways talk wavers in meaning” shows us how easily they do break, leaving us with gaseous halves (or constipated print). As Aristotle says in his Topics (I, 18), sensitivity to such wavering also braces us for our never-ending task:

It is useful to examine the variations in the meaning of sayings — both for the sake of being clear oneself (for one better knows what one is proposing when one has shown how the saying of it varies) and for the sake of the emergence of the thing itself (so that one does not base one’s syllogizing on its name).

Here, you might say, we have Aristotle’s solution in a nutshell: when discussing something, take the time out to record the various possible meanings of the key words, i.e. the various possible responses-to-things. But not just for one’s own clarity, since the question is precisely whether
the clarity is genuine, i.e. grounded. Rather, for the “emergence of the thing itself”: the πράγμα, the thing or the affair, must emerge to ground one's clarity, mend the broken symbol, resitute the word as a word rather than a flatus vocis. Only then does our sylogizing focus on the thing itself rather than on our own (gaseous) half, the name which is only, i.e. no longer truly a name.

But how? How can we be lovers both of talk (naming things, qualifying things) and of what we are talking about (the emerging of the matter itself)? Or: How can we, in our love of speech, overcome the tendency to let speech run away with us? How can we love talking as an illumination of affairs? Aristotle answers this question many times, but let us consider one he proffers in passing (On the Heavens, 294 b 8):

We all share the habit of directing inquiry not toward the matter itself, but toward contrary sayings. One even inquires by oneself only until one no longer has anything to say against oneself. Thus it behooves one who is to inquire well that he stand up boldly to what properly stands forth within the field, this stand requiring that he contemplate all the differences.

Two forms of opposition, then. The one, to which we as intellectuals find ourselves committed in any case, is the opposition between sayings: sparks fly, speech catches fire, when one saying opposes another and when we rise to the stand-off. The other, the source to which we must return to inquire well, is the opposition engendered by the matter itself (τὸ πράγμα): our horses or houses, our friends or spouses, our university or team. Now, this second form requires that I stand up to what stands forth in the field itself (in the γένος). And I do so not passively, but actively: by heeding the differences in what here stands forth. The formulation of this stand is the familiar one for the way we focus on beings: we achieve this focus (and simultaneously and not only among the various sayings: the latter forever waver in meaning until we stand up to what stands forth.

In his Cratylus, a dialogue devoted entirely to the question whether names stem from “nature” or simply reflect arbitrarily human “convention,” Plato has Socrates conclude that “we must learn and research beings not by starting with names, but by starting with beings themselves” (439B); “surely no man of sense will assign his soul to the care of names, trusting them and their positors, confident that he has come to know something thereby” (440C).

Of course, philosophers want to talk precisely about this “emergence of the matter.” We cannot help wondering what it means to start one's “learning and researching” from “beings themselves.” Following the Second Interpretation, we must ponder the “matter itself” as double: both an affair of individual beings (that horse, this girl, my community) and an affair of its Measure (what it strives to be: being in the secondary sense, form). We must follow Socrates contemplating, in his talk, “packasses and the work of smithies, those who work leather and those who tan hides” (Symposium, 221C): the art we as human beings develop to deal with beings so that their nature emerges. And we must consider the proposal that talk itself is grounded in this doubleness of beings. In Book Three we shall turn directly to these questions.

§4.2.2 The modern way

But let us hasten to contrast the first solution to the second—to the one proposed by William of Ockham and others already at the close of the Third Interpretation, worked out arduously by Descartes and Leibniz, more or less assumed by Hobbes and Locke, and nowadays perhaps appearing the most familiar to us. This second solution could be read as a parody of Aristotle's comments on nouns and verbs:

They are, in sound, signs of conceptions in the soul, whereas written ones are signs of those in sound. And just as written ones are not the same for all people, so neither are the sounds. But what they are originally signs of, namely conceptions in the soul, are basically the same for all. But the things of which the conceptions are similitudes vary from instance to instance.

This parody replaces Aristotle's “affections” with their categorially opposite, “conceptions”—taken broadly to include ideas, images, even desires and feelings (but all these considered as having a legitimate life of their own somehow “inside” us). It replaces “symbols” throughout with “signs” (indicators). And it also replaces the original basis of “sameness” (the emergence of what we face as the immediate source of our undergoings) with the modern basis (what we ourselves “have,”
whether confusedly, as in the case of our feelings, or clearly, as in the case of well structured thoughts). What we actually face — this the second solution consigns to the realm of the changeable and unreliable: no “sameness” here, and therefore no anchors for our words, no solid grounds of meaning."

Crudely stated, the modern theory reads: the meaning of a word is the concept for which it is a sign; words mean concepts (in the loosest possible sense: whatever thoughts or ideas I have, structured or not); they are purely conventional, incidental and fortuitous indicators of inner thoughts. The phrase “in the soul” comes to suggest an independent "place," namely one's own individuality withdrawn from circumstances. In any case, this theory arises in strange opposition to the Greek understanding of human being as fulfilled only in the learning of some art — in learning to “complete what nature is unable to finish” — so that the soul is only a “place” for other beings (On the Soul, 429 a 27 and 431 b 21). The modern theory fits the condition of intellectuals who have withdrawn both from action and from production into a contemplation that no longer requires them to take the commitments of the other two as the subject matter of their own talk.

The most obvious result of letting words mean concepts is that we understand our talk as a process of trying to find out what we ourselves mean — as a process of transferring thoughts and feelings. We no longer

* The “rewrite” of Aristotle’s original version summarizes Hobbes’ account (Leviathan, “Of Speech”). For an intermittent version, see Thomas Aquinas’ discussion of “The Names of God” in his Summa Theologica (1265-1272), I, 13. Aquinas still accepts something of Aristotle’s account, yet he also changes something: “voiced words are signs of intellecions, and intellecions are similitudes of things.” When A. C. Pegis inserts “ideas” instead of “intellections,” he takes the reader all the way into the lap of Descartes, Hobbes, and Locke — familiar to us but foreign to Aquinas. As Etienne Gilson puts it, “what Thomas Aquinas calls the interior word does not signify its object, but manifests it by the act of the intellect, while the voiced word is a sign and signifies directly the object that the intellect knows” (Linguistique et Philosophie, Paris, 1969, pp. 142-3). One discovers in reading Aquinas carefully that the “objects” manifested in the one case and signified in the other are neither the affairs of human concern (πραγματα) nor the things encountered (τα διντα): they are God’s forms of creation, of which individuals are mere signs.

Three Solutions: The Modern Way

A third result is that the drama of excellent talk changes theaters. The original interest in clarity becomes a desire for certainty. For Aristotle and his tradition, any clarity I may enjoy results from the emergence of the matter itself. The fullest drama then lies in the soul’s undergoings adequating themselves to the goings-on affecting it: the theater is life itself, the full encounter with beings, where the soul only flourishes as the place of other things. Both poetic works and philosophic works bring that drama home — purify the παθηματα and show us that and how this purification can happen, Aristotle says (Poetics, 1449 b 28 and 1451 a 37). But once clarity becomes a self-contained issue, once we aim for certainty within our own conceptual frameworks (each of us alone and only incidentally together) poetic narratives appear as fantasies and distractions, and even philosophic discussion appears rather inept, at best preparatory for more serious endeavors. Henceforth, mathematical discourse sets the standard of excellence in talk — a form of discourse considered by both Plato and Aristotle to be only preparatory.

The second solution — that we clarify, that we adequate our words not to things but to our own thoughts — has its own history of development, albeit brief in comparison to the history of the first solution (which embraced both the Second and the Third Interpretations). At its heyday, during the 18th and 19th centuries, it appeared to explain the whole phenomenon of meaningful talk — not only that of intellectuals (in which the solution easily becomes self-confirming), but also the talk of bakers and hunters, mothers and daughters, lovers and saints, poets and philosophers. It seemed to provide a cure for the muddles arising in conversation (we must pause to define your terms), a reason explaining why parrots do not really talk (they do not vary the words to express thoughts), and the basis for a new community among human beings (although the things we face are never the same, our affections, intellecions, or inner concerns are the same: the basis for a secular humanism).
§4.2.3 A third way

A paradox: we demand conceptual clarity for the sake of “objectivity,” yet to fulfill the demand we must retreat into “subjectivity.” The paradox certainly bears fruit in the domain of modern science. Precisely where it also explodes the supposition that talk “naturally” amounts to giving voice to the “insides” of speakers. Powerful talk is a decidedly “outside” phenomenon, and human beings are most “naturally” and at best “outside” in their talk, i.e. within shared circumstances.

In any case, a third solution now challenges the one supposition common to both the preceding ones. Does our ordinary, our basic talk with one another require a “sameness” at the basis of the talk — as a reference we attend to (whether as beings in their being or as concepts in our souls)? Much of our talk takes place as a part of an activity, part of some procedure of getting something done: our work in talk is ordinarily the same as, i.e. indistinguishable (for those engaged in it) from the work we happen to be doing. If you and I are not doing the same work, we may stumble a bit when we first start talking to one another, but the only solution is to keep talking until we find ourselves doing the same work: “How did you find the book?” I ask a student, who replies, “My roommate put it on my desk”; “No,” I say, “I was asking whether you enjoyed reading it.” Patience, acceptance of delay, persistence in working and talking: these are the keys to meaningful talk. Only sometimes, only after we are firmly tilling our garden together, might we fathom what we are talking about, what we are facing, might we pause to define our terms.

What can we now, in contemplation, say about those linguistic conditions founded on work? Our work/talk takes place in (perhaps as) a context: at a place and at a time. Not just anyone is speaking and listening: you and I are. And not in a void: there is a map on the table or an engine in the workshop. And not from scratch: we have already said things. And not in ignorance: we have had certain experiences. And not indifferently: we have our projects. All these may present “samenesses,” but they allow for many divergences as well. Each conversation is an occasion in itself, and our responsibility is to engage in it as it arises, keep it running and therefore meaningful, but in a strictly ad hoc manner: the refusal to respond ad hoc leaves us with unattuned words, flatus vocis."

Aristotle argues that there is something absurd about dividing up our talk into “names” and “thoughts”: the division destroys the focal point of any discussion (Sophistical Refutations, 10). Similarly, one of the foremost representatives of the third solution, Ludwig Wittgenstein, argues against dividing the “sign” from the “meaning” — it leaves us with something incurably dead on the one side and something impossibly private (mysterious thought) on the other; opposing this division, he writes: “if we had to name anything which is the life of the sign, we should have to say that it was its use” (Blue Book, 1934, p. 4). Words make sense as we use them in context, just as the keys of a piano and the tools in a shop. On occasion we may have to call a halt to the use of a word (a peddle, a wrench), namely to fix it or to discard it; but we will never understand it by examining it in idle abstraction. And, even less, although perhaps not so obviously, will we make sense of a flow of discourse by stopping it and asking of it whether it can “stand for the truth,” or “correctly represent the way things are”: for we then put both our language and ourselves out of work, falsifying our entire linguistic condition. To be sure, the specialized talk of natural scientists and formal logicians does require some momentary suspension of activity (for double-checking and preparation, besides fixing and discarding). But these suspensions take place within larger contexts which continue to function.

Still, the second solution keeps protruding. It asks us to apply its own conceptual analysis to all domains, to give this analysis priority as revealing the best form of talk. And, according to Wittgenstein, herein lies the cause of so much frustration in contemporary intellectual work: where we should be responding contextually we inadvertently keep abstracting conceptually. He understands his own contribution to lie in helping us intellectuals to overcome this source of frustration: “the confusions which occupy us arise when language is like an engine idling,

* For a linguist’s account of the role of context in meaningful talk, see John Lyons’ Introduction to Theoretical Linguistics (Cambridge, 1968), especially pp. 412-427; or the entirety of his two-volume work Semantics (Cambridge, 1977). Lyons concentrates on the factual workings of talk rather than on “solutions,” but his account speaks out of full awareness of both traditional and contemporary philosophical accounts.
For a convincing illustration of the claim that lexischemy poses the greatest danger not only for intellectuals but for citizens generally, see Stanley Cavell’s “Ending the Waiting Game” in Must We Mean What We Say? (Cambridge, 1969 & 1976). One may profitably contrast Cavell’s account, which shows not when it is doing its work” and “my aim is to teach you to pass from a piece of disguised nonsense to something obviously so” (Philosophical Investigations [1953], §§132 & 464).

Bearing this third solution in mind, we can reassess the six ways in which our talk wavers. Homonymity and amphiboly work on two (or more) contexts, both of which are immediately feasible, and we resolve the wavering by working in one or the other. Prosody, on the one hand, and synthesis and diairesis, on the other, require us to take our stand within the words, singularly or in sequence, as acts, while we put them to use in a context. And lexischemy works on pure fantasy: here we are really abandoning all contexts and retreating into words.

This last (lexischemy), Wittgenstein and his followers claim, poses the greatest danger for us in the 20th century — for us as intellectuals primarily, but then also perhaps for us as citizens of a world run largely by bureaucratic speech. One instance of lexischemy in fact dominates our culture and leads to all kinds of distortions and frustrations. It runs like this: writing occurs, and I can see that the hand and fingers are doing the writing; speaking occurs, and I can see that the lips and tongue are the agents; listening occurs, and I can see (by plugging my ears) that the ears are the agents; thinking occurs and I can only suppose that there is something else, call it the mind that is thinking; feeling occurs, and I can only assume that there is something else, call it the heart, that is feeling. But the last two “follow” only from the “scheme of the talk” in which I formulated the first three. I have lost myself in the words, been misled by the analogy — by the “grammar,” Wittgenstein says. And we must wage battle against this “bewitchment of our intelligence by the mediation of language” (Investigations, §109) because it can actually lead to serious distortions: for instance, while I can pause to treat my hand, my lips, or my ears to promote an activity, ... of activity (although one can imagine a context in which such “treatment” — like taking time off to read or write a novel — does make sense, but in a different “grammar” or “scheme of talk”).

As Plato would say, we have to cease trying to “learn and research” the mind and the heart starting with the names — whereupon we may discover that these words do not name beings at all, except metaphorically. As Aristotle would say, we must beware of supposing some individual being (τὸ δὲ τί) to underlie what strikes our ears as one thing (Εὐ) when we talk about it (Sophistical Refutations, 169 a 35). As Wittgenstein says: Beware! — “a substantive makes us look for a thing that corresponds to it” (Blue Book, first page).

Yet the third solution sets severe limitations on our own philosophic work: once we abandon everyday work (tending a garden, coaching a team, researching atomic reactions) we might conclude that our only legitimate task is to contextualize wayward formulations, to un-bewitch ourselves and others from idling language, or to poke fun at those formulations in which people insist on traveling on the backs of words rather than speaking and listening with their feet moving on the ground. Indeed, as Wittgenstein himself insists, the only “depth” possible for us in our work of contemplation is that of a “grammatical joke” (Investigations, §111) — a clash generating the sense of a missing but needed context, one wherein we can straightforwardly participate if only we would do the work ourselves and stop trying to get the words to do the work for us. Philosophers must confine their works to comedy.

And here is the mark of the third solution’s adolescence. The call to contextualization makes a lot of sense for getting our talk back to work in responding to an “actual exigency” (§108); it clears the air, returns us to the concrete space of our lives. It also sets the stage for conceptual investigation: for the specialized work of natural science and formal logic. And yet there are two questions, two actual exigencies evident enough as we grow up, both of which appear somehow “ineffable” to Wittgenstein and his mainline followers.

One such exigency is the admittedly strange-sounding need for what Plato and Aristotle repeatedly call the “emergence of the matter itself.” The first and the third solutions agree on the general imperative that we must somehow keep our linguistic responses on the grindstone of the affairs in which we are speaking and listening — or, in intellectual

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discussion, invent or recall some such affair. Yet the earlier philosophers note that the affair, the matter at hand, may or may not emerge fully even when we do attend to it: beings may present only their incidental sides, somehow hide their measure — and therefore elude us in their individuality as well. The evidence demonstrating the need for a fuller presentation, for further emergence of the matter, is the remarkable difference between the apprentice and the master of an art such as that of horse training and (especially in the works of Plato) the often agonizing difference between the enslaved and the free passion (ἔρως) for working up and experiencing beings as “good and beautiful.”

Another exigency is the need for re-entry itself. Is there no significance at all in sustained reflections, in our earlier traditions, about this return? Plato’s Socrates describes our condition as one of being prisoners at the bottom of a cave, or as one of wandering aimlessly in the foggy dales (Republic, VII; Phaedo, 109D) — in either case as one of needing to get out “into the light” and up “onto the earth.” In the Old Testament, Jonah tells us of his struggles to return to Nineveh to carry out his task. And today, at the end of the Fourth Interpretation, works such as Franz Kafka’s The Trial and Samuel Beckett’s Endgame structure very exactly and exactlying the need for re-entry, but here a need somehow frustrated, perhaps by one’s own ineptitude or perhaps by the total lack of guidelines such as our earlier traditions so abundantly proposed or tyrannically imposed.

Once we acknowledge these two exigencies, we likely wonder whether formal contextualization is enough. On the one hand, it seems to leave us with dull speech: there is nothing really at issue except the return; after our return there is nothing special to say. On the other hand, formal contextualization appears absurd: Why bother returning to a world of dull routine? Herein lies our most obvious agony today: we have not only emptied our world of gods, we have emptied it of beings

* See Plato’s Statesman (283-285: “preserving the measure” while exercising an art we “work up everything as good and beautiful”); Symposium (206E: “love is of engendering and birthing in beauty”); Phaedrus (243C: the difference between free and enslaved love). Heidegger’s works concentrate on the question of the “emergence of the measure”—in an effort to liberate it from the traditional evidence of knowing, completing, and manipulating; see his Being and Time (1927) and the collection of essays in Poetry, Language, Thought (1971).

as well: what we actually face no longer appears as a sign of its possible emergence and of our own responsibility for its emergence; the ghost of the second solution still weighs on us: beings are mere occasions for our determination and consumption of them.

All three solutions argue that our words embody meaning only as they flow from engagement on our part. The question is: What calls us into engagement? Answering this question, we decide both what counts as engagement and how contemplative speech contributes to it. The first solution proposes that the only real engagement is that of exercising an art, and that contemplative dialogue with others recalls, re-enforces, articulates and answers the basic call of any art, namely the “preservation of a measure.” The second solution proposes that the only real engagement is conceptual re-arrangement, and that mathematical monologue in one’s own study can result in a conceptual order reflecting God’s Plan of the Universe (“theory”) and answering the basic call of mankind to adjust the machinery of the universe to human needs as we all know them (“practice”). Both these solutions developed in response to calls. Nowadays, when these calls no longer appear self-evident, Wittgenstein speaks for many when he proposes: Don’t get involved in this kind of speech at all! It simply postpones re-entry, obfuscates what we can face, entraps us in lexischemy. We have enough trouble responding to the “calls of nature,” whether by eating or excreting, mating or fighting, sleeping or earning a place in the sun — or, if you wish, by attending to horses à la Aristotle or constructing conceptual schemes à la Leibniz. Don’t start talking about the emergence of what a horse really is, or about the order of the universe to which your conceptual order corresponds.

Can the third solution mature to the point where the question of a call to contemplation can make full sense once again? If so, the task may well be to respond to it without pinpointing it as our earlier traditions tried to do — and therefore without limiting it to any preferred form of engagement. So long as it does not mature, we are intellectually orphaned. For the adolescent version of the third solution does not really allow words themselves to embody meaning in a contemplative manner: we either get on with our affairs, forgetting words as we forget the keys on the piano we are playing, or we get lost in them and destroy their function. Without some depth other than that of a “grammatical joke,” logic becomes the rather slavish affair of promoting the moribund ideals
of technology. Comedy, yes. But, as both Plato and Aristotle insist, the laughable ultimately makes sense only in the realization of our destiny. Tragedy, then, too.

Book Three

The Workings of Neologism
Truth and Discovery

The Workings of Neologism

In the course of a life, one learns many things. One learns facts: telephone numbers, the location of towns, the population of a district, the dates of events in both personal and cultural history, the shapes of various chairs and trees, the names of various people and of their activities, of artifacts and of natural beings, the relation between the consumption of various foods and their effects on bodily health, and a multitude of signs and probabilities commonly accepted in one's social milieu. One also learns activities: walking, talking, and personal hygiene already as a child, sailing or golfing, posturing or calculating, preparing meals or treating diseases, breeding horses, building cabinets, or raising children. And one learns the comedies and tragedies, the “facts” of life as a whole: the difference between trust in others and gullibility, the roles of love and sex, the finitude or mortality of human affairs, the nature and limits of formal deduction, the exigencies of beauty, utility, and justice, the power and the seduction of language.

How do we learn these things? In at least two different ways: naturally and artificially. Natural learning happens directly in our involvements without any special intent on our part; we recollect it as having characterized our lives already prior to the imposition (on our part) of any method designed to assure it. In contrast, artificial learning results from carefully constructed involvements, from our designs; we anticipate it as evolving from the imposition of a method; modern scientific investigation typifies this kind of learning.

It is crucial to remember that the original bulk of our western intellectual tradition consists of a contemplation of natural learning. The
topmost, the most recent and obvious layer of the tradition asks us to contemplate the possibilities of artificial learning.

In dwelling now on the question of how we naturally learn, we must distinguish between how we initially and how we ultimately learn things.

Initially, we learn facts from the words of others. How do I learn my friend’s telephone number? He, the telephone book or a common friend tells me. Similarly, parents, playmates, teachers, newspapers and the like tell me about places and times, names of natural phenomena and of famous people. Later, a master artisan (woodsman, mechanic, physician) will tell me how one phenomenon (color of leaves, sound of engines, outbreak on the skin) stands as a sign of other phenomena. Long before we determine facts for ourselves, we hear about them.

Initially, we learn activities by imitating the gestures of others. How does a child learn to walk and talk? Others are doing these things, and they include the child in their world. Later, we may decide to work with wood or scale cliffs, to coif hair or dance a jig, and thereupon go to watch another performing the craft. Already in the watching, our muscles mime the motions, and we soon try our own hand or foot at them, sensing whether we are moving as our mentors do, feeling clumsy at first and noticing the disparity of the result, gradually improving the imitation. Whatever others tell us about the activity, whatever we read in books about it, makes sense only indirectly: as whetting our appetites for future imitations, or as recalling the intricacies of past imitations. One learns an activity only by doing it, and others help us directly by their gestures, only indirectly by their words.

Initially, we learn about “life as a whole” from the concentric communities in which we grow up: family, school, church, town, nation. Our tradition incorporates us into a direction of... of them. But then it is too late: the interpretation has already caught us up in the movement, we have already learned it.

Yet we also learn, eventually, that the way we initially learn facts, activities, and principles is inadequate. Have I really learned, come to know, the telephone number of my friend simply upon hearing it or reading it? Not really. Rather, I dial the number and, upon reaching my friend, fully learn the number: instantiate the fact, confirm it—and more likely remember it and pass it on to others with confidence. Have I really learned how to sail a boat or validate an argument by simply following the gestural patterns of those who already know these activities? It seems that here, too, I must “dial in” my own gestures, not simply taking them over from the master but attuning myself directly to the realities governing the gestures I initially only mimed. And have I really learned the life-doctrines traditionally associated with the Bible, with Freud, with Darwin by simply becoming familiar with the literature, with the intricately interrelated descriptions and prescriptions recitable in the form “it is written that...”? It would seem, once again, that full learning of these interpretations would require that we “dial” them in—“apply” them, if you wish, but first understand them. Understanding interpretations of the basics, the priorities of life, requires above all that we... interpret them.

The initial ways of learning are immature: so far, facts and activities and principles are second-hand.

Information we merely receive is hearsay, and we eventually learn to contrast such hearsay with the facts we have determined ourselves—determined in an actual encounter. As a telephone electrician, I wired the telephone connections, or as a user I dialed the number: in either case I have taken on an entire situation in which the fact becomes a fact and can be passed on to another who initially takes it on hearsay.

Similarly, activities which we perform solely with an eye or ear to other people's performances are wooden, and we soon learn to contrast such mechanical imitation with graceful performance—graceful because following what currently presents itself rather than other people's responses to previous presentations. In sailing a boat I must soon move out toward, and move along with, the wind and the water, adjusting the boom and the rudder in part to them and in part to the desired course: at the most crucial moments there will be no time for referring the question to the masterful gestures of one's predecessors. However, precisely my direct response to presentations at the moment may serve as a standard of grace for the initiates who happen to stand by.
And a doctrine that we use only to interpret, merely to subsume events dwindles into prejudice, and we generally learn to contrast such blindness, or opinionatedness, to the thoughtfulness and openness which any genuinely functioning doctrine fosters. As a genuine Christian, or a genuine Freudian, I learn that the literature embodying the doctrine sets the questions, not the answers, in which I shall enter events. At this later stage of learning, I recurrently interpret the doctrine itself rather than simply interpret events with the doctrine. While second-hand facts and second-hand gestures naturally collapse at some point and then naturally invite first-hand revision, second-hand doctrines have a long life, even an after-life. A Sunday-school teacher who imposes, with hatred and vengeance, the doctrine of neighborly love becomes a subject for comedy; a leader like Oedipus who insists on the virtue of finding and punishing polluters becomes a subject for tragedy. It seems that doctrines most easily supplant the priorities they name, and that one must constantly overcome them in order to actualize them.

Bothersomely evident inadequacy signals the need for growth. On the early modern interpretation, artificial learning requires us to reject previous determinations, simple imitation, and conventional overviews to allow for new growth (one reads Galileo, Francis Bacon, and Descartes for encouragement in this direction). However, the learning that most naturally develops evolves from second-hand facts, activities, and interpretations: natural learning recycles rather than rejects initial learning, so that the results of such learning are much different from those of artificial learning. As philosophers we are now looking not for new growth but for the natural growth which recurrently transforms the old into the new — which renews the old.

How does natural growth proceed? The first step is that of the sophomore: the realization that facts are relative to the activity generating them; in the abstract, facts are phantoms, left-overs from yesterday or yesteryear — even one’s own become outdated (my friend gets another telephone number). This first step liberates one from the tyranny of others. But what then? One may tarry a while on the sidelines before realizing the next step: that precisely this “relativism” requires one to plunge into an activity — to learn to train dogs or raise children, to coach a basketball team or test the validity of those 256 argument forms — precisely so that the ever-changing facts can rise and fall with a reference. A junior, you might say, is one who “gets down to business,” one who works the facts the way a salesman works a neighborhood; for such a one, all “principles” come down to the desire for success in the enterprise at hand.

Perhaps all three serve as conditions for growth: the soil (the freshman’s initial reception of facts, gestures, and overviews from others), the seeds (the sophomore’s realization that these ingredients are relative to the times and places, the actions and passions of people), and the water (the junior’s application of this relativism in his desire for success). Is there a fourth condition? Or at least a fourth stage of learning?

The birth of the West, the parturition most evident in the works of Plato and Aristotle, consists in the stipulation that what we must ultimately learn, and what serves as the “sun” conditioning all growth, is beings in their Being. A senior (the “philosopher” as first rigorously defined by Plato) is one who has learned that only by focussing on the being itself — on the boat, on the dog, on the child, on the team — do all the facts hold: all the passing determinations of relation, quantity and quality, time and place, doing and undergoing, being disposed and being prepared. A senior has learned that graceful activities flow only from direct contact with beings and not with an eye to standardized gestures of others. And only a senior has learned that theories intend not to erect a picture replacing our attention to reality, but rather to occasion a focus directly on reality. At this fourth stage of learning one learns what human beings must ultimately learn, the center that human beings must uphold if it is to hold, if the multiplicity is not to shatter into chaos, if conviction is not to become bigotry.¹

Sounds abstract. And the formulation of it certainly is. However, the decisive proposal that what we ultimately learn is beings in their Being intends to bring into relief what most concretely and most naturally happens — and what marvelously undergirds the more obvious learning of facts, activities, and doctrines. One can read Herman Melville’s Moby

¹ The word “philosopher,” devotee of wisdom, has shifted its meaning. In the Second Interpretation, one devotes oneself to the σοφια evident “at the games” — in the perceptiveness, the fluency, the overall grasp enjoyed by others (the best). This devotion may lead to an articulation and elicitation of the center already at work in σοφια and implicitly upheld by the “players.” In the Fourth Interpretation, one devotes oneself to developing a σοφια of one’s own: to upholding a center one has elicited solely in reference to one’s own competence.
Dick (1851) as working out the same proposal in a story—an account of the activities associated with the pursuit of whales at sea. The narrator, evidently a Platonist, puts the point bluntly in Chapter 94:

For now, by many prolonged, repeated experiences, I have perceived that in all cases man must eventually lower, or at least shift, his concept of attainable felicity: not placing it anywhere in the intellect or fancy; but in the wife, the heart, the bed, the table, the saddle, the fireside, the country; now that I have perceived this, I am ready to squeeze case eternally.

Yet this more concrete formulation poses its own questions to an attentive reader. What does “in the heart” mean in contrast to “in the intellect or fancy”? One significant contrast, in keeping with the others cited, would require that we shift from a concern for our own inner achievements into a devotion to what we encounter. And what does “readiness to squeeze case eternally” mean? Certainly, the readiness to tend to the beings at hand, in contrast to scurrying on past them. But perhaps also (on the double meaning of “case”) the readiness to extract from “all cases” the human drama of shifting or lowering—the vocation of bringing this basic drama into relief.

Indeed, the learning of the concrete is essentially dramatic. Most formally stated: predicates (facts) pave the way toward subjects (beings), and yet they get in the way as well; they are necessary but not sufficient, while putting on airs of sufficiency. More concretely, we make contact with beings only in an activity, one that has its own claims upon us, its own goals, its own standards of success (capturing and killing whales, often a very special whale as well); we both make the contact and lose the contact, since the pursuit of goals contravenes the shifting or lowering. And on the more theoretical plane: even if we do tend to beings we are caught in the cross fire of beings in the primary and beings in the secondary sense; for I must handle my boat as a boat, as one of a kind, and my understanding of boats can unstring the attention to the one boat I must understand. Of central interest to philosophers has been the tense drama of “ideas” themselves: an εἰδος intends to provide a full view of the being it names, but it can also obstruct the view.

In ancient Oriental literature we also find the proposal that “man must eventually lower, or at least shift” his conception of consummate learning to the level of beings. In such texts as the Chinese Tao Te

Ching and the Hindu Bhagavad-gita, and in the recorded stories of Japanese Zen, we incessantly hear that illusion consists in believing that the determinations even we ourselves make can do justice to the beings we encounter, or that the heart of our activity can lie in our understanding of the fruits of the activity, or that the principles we explore linguistically can make any sense at all apart from the principals coming and going in our daily encounters. About this last, the articulation of doctrine, the Oriental recommendation appears much more drastic than anything well planted in the West: Don’t talk about it, do it!

It is here that West and East part company in ancient times. In Oriental thought, the horrifying prospect seems to be the possibility that talk, and especially talk concerned with ultimates, will pre-empt the return to beings themselves (a possibility evident in any effort to say what beings, e.g. whales, are—so that one of the first enjoinders is: stop trying to distinguish between what is and what is not!). But the most horrifying prospect for the Greeks, already prior to Plato and Aristotle, the prospect engendering Occidental philosophy itself, is the possibility of living in the accounts, the λόγοι of yesterday or yesteryear, whether one’s own or another’s. An old λόγος, a formula or a speech simply taken over, appears as a λόγος in name only. For it no longer constitutes a response to the present, but rather one from the past. The Greeks bequeathed to us a passion for current life: a passion, a desire for making something out of life here and now (the fire which, our Oriental brethren say, we should quench), and more exactly a passion for re-achieving this presence in λόγος itself. The Greek, genuinely western maxim reads: Talk again to keep the talk as a doing. Do not fall back on old talk, but above all do not abandon talking! For we ultimately learn beings in their Being by developing a linguistic presence.

If syllogism names that assembly of talk which reveals what gives rise to what (deduction), and if paralogism names that assembly of talk that diverts us from following the talk into reality (perversion), then we can let neologism name that assembly of talk which discovers presence (embodies it: incarnation). Such talk shows what it talks about and thereby provides the basis of syllogism as well as the impetus for overcoming paralogism. Aristotle called such “showing talk” αποδειξις, conventionally translated as “demonstration” (which for us means merely a thorough deduction). In addressing ourselves to beings in their Being, we not only present ourselves to beings, we not only find beings revealed
to us in their fullness, we also renew λόγος itself, overthrow (for the while) the ever-threatening but counter-natural tendency to fall back on antiquated talk. There may be no denying that we are, as Aristotle formulated it, “the only living beings conditioned by λόγος.” But the category of condition sets us a task: we are those living beings for whom the actualization of our condition remains constantly an issue. We fully are only in being neo-logical. Initially, our condition is one of paleologism—of λόγος long pre-set. Ultimately, then, learning consists not only in discovering the presence of beings but also in responding freshly to it—in keeping with the freshness essential to genuine presence."

§1. Socratic Induction

As logicians, we notice the formal necessity of universal premisses: no syllogism is possible (valid) without at least one such premiss (or two singulars, as in “exposition”). If deductive inference is essential to the actualization of our condition, then universals likewise are. Similarly, “practical” enthymes, i.e. communal discussions of questions of nobility, utility, and justice, draw upon universal premisses undergirding the community itself and lurking in each decision. Freshmen and others may believe they are tending only to individual givens in the course of their daily inferring and deciding, but a trained eye has no difficulty spotting the universal principles at work in such tending.

How can we, how do we learn the much needed universals? A natural question for one who has recognized the functional inevitability, the formal necessity, of such “major premisses.” Yet we would denature the question if we were to proceed, as key thinkers in the Fourth Interpretation have done (from John Locke down to the academic psychology of today), by asking how we obtain universals “from scratch.” In naturally asking the question, we acknowledge that it is too late to play the game from zero. For by then we are already lodged in universals, and the natural question reads: How can I reside in them properly and fully—rather than obliviously relying on, leeching off them?

How fully and properly? Aristotle explicitly offers one very general answer: by reworking (refining, re-attuning) an already functioning universal so that it becomes transparent to the being it represents—to being in the secondary sense, to what our tradition subsequently called its species. Already as children we learn to identify (recognize) horses and to talk about them in a general way. The same with circles, mothers, and fathers (Aristotle’s examples in the first chapter of his Physics). And actions (playing, walking), passions (being burned, being petted), conditions (being tired, being happy), and the rest. We “understand” these things, i.e. refer to them in our talk, recognize the reference to them in the talk of others. Yet in another sense we do not understand them at all: we do not know what they mean, we cannot articulate the detail of horses, of playing, of being petted. And as a result we easily miss the actual thing when it appears, simply pass it by, fail to grasp its full significance, or mis-assess it entirely. The singular eludes us; the given horse in its horse-ness, the playfulness of a given activity, and so on.

Our initial condition is Oedipal, as Aristotle (not Freud) interprets the tragedy: resident in opaque universals and so blind to, eventually stumped by, the actual case in its detail. Full and proper residence requires that we undo the opacity.

But how might one rework one’s initial ideas so that they become transparent to beings in their Being? Despite their love of talk, despite their faith in the power of λόγος, the Greeks of the First and Second Interpretation resisted the suggestion that one person can successfully tell another anything which would settle questions bearing on what must ultimately be learned. Even the self-styled Wise Men of the Periclean Age (the Sophists) claimed that they taught their pupils how to relativize discourse, whether in a destructive or a constructive manner (the possibilities Plato illustrates and criticizes in his Euthydemus and Protagoras, respectively). If, as Plato says in his Republic (518D), the art of education consists not in instilling sight into the soul of others but

* Martin Heidegger has shown that and how Greek philosophy set western intellectual work on the road of a “metaphysics of presence” (see, e.g., his “What is Metaphysics?” and Nietzsche). The ancient passion for presence (ουσία, Anwesen: estate) gradually led to an insistence on the presence of λόγος: on a presence of human response at the expense of that to which one responds. It has led to formalism—and to nihilism (as an historical predicament, not an avowed creed). In highlighting the western commitment to “metaphysics of presence” Heidegger has set us the task of renewing the respect (as distinct from the passion) for presence—by acknowledging but also articulating both the possibility and the necessity of absence—the withdrawal of beings in their Being.
rather in redirecting, even converting the soul already having sight, then much less than either of these two (instilling or redirecting vision) can it consist in passing information on to others. Children, of course, banter about horses, pass rumor on to one another; but they exchange empty universals for empty universals, and the result is still zero even if such exercise helps them become fluent in their talk.

According to Aristotle (Metaphysics, 1078 b 28), Socrates was the first to offer the decisive alternative: inductive speech (ἐπακτικός λόγος). Induction names a kind of discussion wherein one speaker calls to the attention of another speaker two or more examples of something and requires the other to recollect the essential sameness (the abiding) as distinct from the incidental differences (the variable); if successful, the discussion culminates in a carefully constructed encirclement (definition) of the universal—it focusses the initial idea of, say, sophistry, justice, courage, expertise, or statesmanship onto the being ultimately at issue. Such discussion appeared to Plato and Aristotle as showing the truth of the otherwise merely competitive debates already popular at the time. It also signaled the possibility of meeting cynical Sophists on their own ground—as in Socrates' debate with Thrasymachus early in the Republic: Thrasymachus must admit (contrary to his personal conviction that all the rigmaroles of justice come down to power plays in which the sole question is who can get the advantage over the others) that power depends on developing an art (witness the shepherd's power, the physician's power,... any craftsman's power, including a political leader's), and that the development and performance of an art require one to promote the good of the beings tended to (witness the examples again, starting with the simpler and ending with the one at issue).

Induction in the strict sense takes place between two speakers, most likely between teacher and student, master and disciple, parent and child, but possibly also between two equals, providing that one ask and the other answer questions (a common exercise in the halcyon days before “media” supplanted the live voice). In written form, however, it amounts to a style of both writing and reading, and we can speak of induction in this broader sense as well. As a writer, one brings in examples and points the way, in an interrogative mood, to their significance—all the while wary of allowing the formulation of the significance to pass into just another piece of supposed information. One can write inductively in dramatic form (presenting, as Plato does, personages wrestling with the

questions) or in procedural form (enumerating, as Aristotle does, the questions and the examples in a kind of draft for a script). As a reader, then, one has a choice. One can settle down comfortably on the sidelines to keep score (wondering who will win out, Socrates or Thrasymachus). Or one can move into the dialogue oneself—act out the development, in which case one must, as an actor must, interpret and interpolate, even orchestrate and choreograph: one must here be the one who learns, who sharpens a focus. In either the strict or the broad sense, inductive λόγος intends to help only those who help themselves.

Yet induction evidently makes no sense except as an outgrowth of temporally prior theaters of learning. For the recollection presently induced, whether in live exchange or in reading printed words, is none other than a recollection of what one has already learned. Inductive λόγος provides an occasion for solidifying at the level of contemplation what has naturally taken place in one's “previous lives” of production or action. Consider the sketch of a script Aristotle offers in his Topics (105 a 12) to exemplify the workings of induction as “moving along a path from cases taken severally toward taking them holistically”: the unchoreographed script reads: “if the pilot who is knowledgeable is the most effective [among pilots], and similarly with the charioteer who is knowledgeable, then holistically the one who is knowledgeable is the best in each [i.e., in every domain].” When acted out, this mini-script not only drives home the enthymematic principle fundamental to the Second Interpretation (and again, although much differently, in the Fourth) that “bestness” lies in effectiveness within a vocational domain. It depends on the second speaker's (or the reader-actor's) ability to recollect the activities of piloting ships and racing chariots (today, perhaps: of driving tractor-trailers). If one has not learned these things already (at least as an amateur) nothing is brought into focus and one does not really move along the path. We may hear a simple-sounding “yes” to the question “Is not the knowledgeable pilot the most effective in the long run (and so to be trained in company with a master rather than elected by the will of the sailors)” Yet the simple answer to this complex question must represent not docility in the face of a new suggestion, but rather acknowledgement of a thread running through many encounters over the years. Obviously, in a live induction, the first speaker must find examples suitable to the condition of the second speaker; and in a literary induction, the writer must choose one kind of audience and assess its condition to the best of
his recollection — and hope that his readers will focus on parallel examples of their own.*

Production? As Alcibiades complains in Plato’s Symposium (221C), Socrates had the strange habit of asking his interlocutors to contemplate “pack-asses and the work of smithies, those who work leather and those who work hides.” Why contemplate these “lowly” things? Already in the Apology Socrates reports that artisans, makers of tangible things, display genuine learning — unlike many poets and politicians, makers of worlds in words. In learning how to handle pack-asses, in shoeing them or in extracting and tanning their hides, one enters into the tension between empty and full universality. On the one hand, one can only handle again and again a pack-ass or its hide if one has come to understand pack-asses generally, their hides generally — if one has learned its εἶδος (“form”). On the other hand, each given instance puts to the test any generality one already has (on hearsay, from watching others, even from remembering what one learned yesterday). Artisanal production naturally and intensely throws us, has already thrown us, into a relation with beings themselves — the first condition of learning and therefore also of genuine speaking and genuine listening (“real words”: Cratylus, 339). Examples of such production then serve well as grist for our contemplative mill, i.e. for inductive speech.

Action? Most ancient literature addresses primarily the ruling classes, people who by birth or disposition find themselves having to lead others in communal effort — to take charge of foraging expeditions or hunting parties, to command a ship or run a household-farm, to hold appointed or elected or usurped positions in a city. A Socratic induction generally assumes that one has played a part in this second theater of learning — that one can recollect the essential tension between human beings as they present themselves with all their variations and incidentals, and as they need to be, in their essential fulfillment. Furthermore, such induction assumes that one can reclaim the strictly human, all-too-human exacerbation of this tension: the conflict between the fulfillment of the individual and the fulfillment of the community. For throughout Greek literature we find the supposition, first challenged by Machiavelli at the Dawn of the Fourth Interpretation, that the social context in which one lives is also a being, something requiring development in its own right (rather than just a condition, a “state” belonging to individuals collectively and deserving cultivation only for the sake of individuals). Action, i.e. ongoing decision-making that creates (or destroys) human enterprise as communal (well knit, unified, functional — as a football team must be) throws one, has already thrown one, into human reality itself, and into both dimensions (both individual and communal fulfillment). Examples of such action constantly serve as “subject matter” for recollection in Socratic induction, especially as Plato transformed it into a literary style.*

Induction itself comprises a third theater of learning — the one Aristotle credits to Socrates. The stage-setting is the structured discourse of questioning and answering, a game already familiar to the Greeks both in the marketplaces and in the law courts. The drama itself, however, is the recollection of what one has already learned: the drama of re-learning, of giving an account (λόγος) of one’s “previous life” — of the encounter with beings in production and action — so that the Being of beings becomes an issue (at the very least, the tension between singularity and universality, and the strange, even wonderful dependence of each upon the other). And, in giving an account of what we otherwise only experience, we not only realize what we encounter, we also actualize how we encounter beings, the distinctly human way of being (as λόγος-conditioned). Induction is then a way we come to “know ourselves” — following the injunction inscribed at the Oracle of Delphi, but in a new

* Plato’s Phaedrus poses the question of the speaker’s responsibility in apparent monologue and concludes, in part: “so long as one does not account for the natures [intrinsic powers of growth] of those who hear... one will not be an artisan of λόγος according to that very potentiality of human being” (273D). Jean-Paul Sartre’s What is Literature? (1948) answers the question in a similar way for the writer of novels, essays, and the like: “It is by choosing his reader that the author decides upon his subject.”
way (the older meaning of the injunction likely had more to do with accepting one's limitations than with actualizing one's power).

Evidently, induction too is an art—just as making pots and coaching teams. The art is first of all that of a teacher: one who directs attention by asking questions. But it is also that of the pupil: one who, in considering and answering questions focuses his attention. Onto what? Most obviously, onto the examples cited, on the paradigms the master draws from the theaters of production or action. How attend to these examples? By searching out the Being of the beings encountered. But what does that mean? Aristotle answers in the name of both Socrates and Plato: one searches within the examples for what remains the same throughout them all, separating this out from what varies from one example to the next. One searches, then, for what is durable and essential, as distinct from what is transient and accidental: “one (the master) induces the universal (for the pupil) by induction of particular examples with a view to their similarities” (Topics, 108 b 11). The similar (τὸ μοιοῖον) is what makes each example “like” the others. Indeed, the philosophic disposition consists precisely in the ability and the passion to “contemplate the similar even among very diverse examples” (Rhetoric, 1412 a 11; cf. also 1394 a 7). The art of induction is then the art of analogy: the ability to handle the helm with a view to wind, etc., is “like” the ability to handle the reins with a view to the race track, etc. — both require knowledge of beings. In the course of an induction one arranges two parallel λόγοι to focus on the core of each event, a shared core. The λόγος of the analogy then reveals each holistically. The key to the revelation is the detection of the durable and essential as distinct from the transient and accidental. And the test of the induction, of the analogy, is whether the resulting λόγος allows one henceforth to fly from old to new examples, to events as they transpire in the future course of one's life, landing in each case on the core, i.e. articulating anew (and effectively) each without getting lost in the incidentals.

But what counts as “similar” or “like”? Simply those determinations which all examples share — their “lowest common denominator”?. But then examples of most anything share many incidentals. A human being and a plucked chicken are both featherless and two-footed: Does calling them “featherless bipeds” help us understand either one? Most any human being you bother to anatomize has something called an appendix extending from the caecum of the large intestine: Does recognition of this similarity help us understand human beings holistically? Whatever else “similarity” means, it must touch directly upon, must put us directly in touch with, the essential — the Being of the beings in question. It will be durable because it is essential, not the other way around — just as the incidental features are transient because they are accidental, not the other way around. If induction is to work, it will plunge the pupil back into the subject, whereupon the subject can re-emerge holistically: it will not provide a safe and gradual approach to the subject by way of simply accumulated and somehow synthesized predicates (as Francis Bacon and J. S. Mill will wish to understand induction in the Fourth Interpretation).

If we carefully consider the two examples of induction already cited (Plato's choreographed script “Socrates vs. Thrasymachus” and Aristotle's sketch “effectiveness in piloting ships and racing chariots”) we notice that the being at issue is also human being. They ask us to reconsider what it means to be human, i.e. what we must realize (and, even more, actualize) to reveal our being holistically: that we must complete nature by keeping in step with her. Presumably, one naturally learns this destiny by performing an art, whether of production or of action. One then re-learns, solidifies this learning when performing the art of contemplation. Only then can one proceed to the next step in contemplation: to the realization that one has been heeding, already in effective shepherding and the rest, what beings (sheep, or whatever) are holistically, how they all need to be and therefore can (with our help) be, in their Being. Socratic inductions typically work primarily on eliciting this pre-condition of learning. The rest, namely the recognition of universals within one's own art, follows naturally. The recollection of horse-ness is possible only for, and relatively easy for, one who works with, or has worked with, horses — providing only that he has fully developed his own power. It is no wonder that Antisthenes could see only horses but no horse-ness; or, rather, we can wonder whether anyone can see horses simply by looking at them.

In any case, Socratic induction provides a third theater of learning — by all accounts a new one at the time, a form of life we call contemplative, and one originally drawing upon the two established theaters of production and action. In our late position as heirs to this form of life we might forget that, originally, it directly served the two other theaters as well. Plato especially, but Aristotle also, understood this new form of life as a style of education intending not a withdrawal from
practical affairs (although the masters of the art of induction may themselves appear to work on the sidelines) but rather a kind of “sensitivity training” for upcoming leaders of society: having already been engaged in the rigors of various arts, the young of aristocratic families could learn from a sojourn in the contemplative life that the essence of leadership (action) lies in the cultivation of human destiny (i.e., the cultivation of others in their destiny) to develop an art devoted to nature (which loves to hide) and in this way to contribute to the making (completing) of a community. The \( \theta \varepsilon \omega \rho \alpha \tau \alpha \) cultivated by induction remained bound to the \( \pi \rho \alpha \zeta \zeta \zeta \) essential to everyday affairs: its purpose was to highlight the conditions for completing what the everyday is unable to finish, and to do so by keeping track of our own powers. Only in the Fourth Interpretation did the contemplative life, did “theory” evolve in abstraction from the evidence of production and action and then turn around to serve the exigencies of industry and capitalism.

Still, learning does not begin in time with Socratic induction, even if the beginning becomes clear only during such induction (a crucial distinction in Aristotle’s *Metaphysics*, IX, 8). Indeed, Socratic induction explicitly assumes that the pupils (the eyes of the learner) have long ago, in previous lives, naturally begun; the inductive \( \lambda \delta \gamma \omicron \zeta \) only brings out the ultimate direction of the beginning (the Being of beings) and helps one to complete the journey. As philosophers, however, and entre nous, we naturally desire to locate and expand upon the temporal beginning as well—the learning original to the theaters of production and action. We ourselves must learn to tell stories in which learning gradually takes place. Only then can we detect the crucial stages, and formulate these stages into a logic of discovery, a logic which Plato and Aristotle supposed to undergird the logic of syllogism and to supply the positive counterpart to paralogism.

§2. Storytelling

How can one tell a story? (Or listen to one?) What, exactly, is there to tell? (Or to hear?) On this question the West was conceived and born. First there were the Homeric stories showing warriors learning to die. Then there were the stories of the Periclean poets, Pindar and Aeschylus and Sophocles and even the comic Aristophanes, showing something of both the beginning and the end of the gestation. And finally there were the Platonic and Aristotelian stories showing human beings learning beings. These greatly differing stories all show what one must learn, all show “particulars” and also represent the universal, the basic human condition: for in every apparent story there lurks a basic story as well. And since Plato explicitly offers a basic story to replace those of his predecessors, let us look at one of the latter first.

Aeschylus’ *Prometheus Bound* retells the story of the Titan who abandoned his old (earth-based) kin to join the new (sky-based) gods under Zeus to ensure the victory of the latter in the battle between the Old and the New. But Prometheus also gave mankind the three divine prerequisites (fire, arts, and hope) for resisting Zeus’ decision to eradicate the human race as well. In a fury at Prometheus’ betrayal now of the New, Zeus has him chained to a cliff in the wastelands, far from both gods and men. In his chains, Prometheus explains at length, to the sympathizing Chorus of the Waters, the one gift he stole from the New to give to our forefathers—the arts (\( \tau \varepsilon \chi \omicron \omicron \omicron \omicron \) : crafts, abilities):

Mortals at first had eyes but saw to no avail, they had ears but did not hear.

Like the shapes of dreams, they dragged through their whole life and handled all things in bewilderment and confusion.

To undo this sorrowful condition, Prometheus bestowed upon mortals the art of building houses in the sun (to get out of their dark dwellings under the ground), the art of detecting the coming seasons (to allow them to plan ahead, as in agriculture) and of reading the stars (to allow them to navigate by night) and of numbering (for measurement, the basis of all arts), the art of writing (to preserve a past for memory), and the various arts of handling oxen and horses and ships (for easing labor, facilitating transportation, and engaging in sports). And the arts of medicine, too:

...if ever one fell sick, he had no defense, neither healing food nor drink nor ointment, but for lack of drugs wasted away before I showed mortals the blending of mild simples wherewith to drive out all manner of disease.

Finally, Prometheus claims to have taught our forefathers the arts of interpretation (of dreams, of animal cries, of bird flights, of entrails, of shooting stars) and to have shown them the blessings of the ores under the earth. “One brief word tells the whole story: all arts mortals have come from Prometheus,” the god of Forethought.
There is more to the story: Prometheus' other gifts to mankind, his defiance of Zeus, his encounter with Io (a mortal woman also plagued because of Zeus). But the story tells only of the source of the gifts in time: at some cost to him, Prometheus gave all these arts to our forefathers. How fortunate we are!

...Or are we? If we can now see and hear to some purpose, if we can now open our eyes and unplug our ears, we will immediately note that neither we ourselves, as individuals, nor the young we bear and raise, have the arts, have any art, actually: rather, we have to learn, we must develop one or more. What we in fact already have are only the results of Prometheus' gifts: the houses somebody else built, the medicine some doctor has given us, the copper, iron, silver, and gold others have mined and forged for us. Only a freshman can listen to the tale complacently. It does not congratulate us on our achievements, it informs us of our tasks. The price our imagined forefathers had to pay for not learning the arts, not receiving the gifts, may have been a life withdrawn into dark caves, limited to foraging, and plagued by disease—a life of blindness and deafness with no calculation of the future or recollection of the past. For us, however, such wilderness and confusion takes place amidst the results of arts: in “houses built of brick to face the sun,” in the marketplace where we purchase goods brought in from the farms and the ports, and in bed or at the druggist's—and in the din of pre-arranged numbers and words. Until, that is, we ourselves learn an originating art or two.

Aeschylus' story is a story for stories: it tells about the preconditions for the learning of an art on the part of an individual. One precondition is the recognition that the art to be learned is not possible as a self-wrought and self-serving possession: divine in origin, an art makes sense only in service to the divine. Prometheus himself illustrates one significance of this precondition. As the Chorus points out to him, for all his own powers he cannot free himself. And the Chorus seems to represent us in our undeveloped condition when it advises Prometheus to cease serving mortals and to apply his arts to serve himself.

A second precondition lies in Prometheus' answer to the Chorus: “art is far weaker than necessity” (514). To learn an art, one must eventually recognize that it does not allow (indeed, it henceforth prohibits) one to do simply what one wants. Necessity, not choice, rules: “the power of necessity is incontrovertible,” Prometheus had already said (104). It may seem (to freshmen, at least) that an art allows us to pressure things, but Prometheus tells us that it rather puts us in a position to be pressured. The “inside” story Prometheus tells the Chorus appears to illustrate the significance of the options: the old earth-gods applied only brute force in their battle against the new gods, whereas their mother, the Earth Goddess herself, had foretold victory for those who exercised art with “device” and “guile.” Response with mere strength is indeed tempting when one has to respond to strength, but then any individual human being is weaker by far than that to which he must respond. Yet in responding with art to strength one must still acknowledge that what one responds to overreaches one's own strength; for otherwise one falls back into the freshman's option. And this acknowledgement locates the bitter necessity pre-conditioning any relative victory possible for a mortal (the victories of Prometheus, himself an immortal, are also only relative: his predicament illustrate ours).

The subplots of Aeschylus' story draw upon and underscore the basic pre-conditions of learning. Io, the one mortal appearing, must endure her own very peculiar allotment, or µοῖρα: she had been touched by the divine (Zeus) and now she will be plagued for ages (by Hera); such is our condition, we might hear allegorically. And Zeus himself represents the tendency of the new gods (of Greek modernity in the Periclean age) to forget the second pre-condition and to try to apply mere force (to complete nature without following her, as Aristotle will soon put the point in secular language); the moral of the story might be that such forgetfulness will lead to its own destruction (as Prometheus predicts for immortal Zeus, so a fortiori for us mortals). And, of course, the political dimension of the story is undeniable: λόγοι, Prometheus says (380 ff.), are the doctors of our condition—“providing they soothe the heart rather than suppress agitation by force” (a theme receiving extended treatment throughout Aeschylus' Oresteia, a trilogy highlighting the role of persuasion vs. force in a free community).

But those two pre-conditions—divine intervention, commitment to device—do we learn them first in time? Hardly. Only after having learned the rudiments of an art, whether of splitting firewood or of conducting kindergarten classes, might the pre-conditions become learnable. In the meantime Aeschylus' story, like many others, appears to tell merely about farfetched personages suffering their trials and tribulations. Plato and Aristotle, on the other hand, aim for elucidating
the temporally prior conditions in conjunction with some such ultimate but ulteriorly learned pre-conditions. They ask: How does a youngster learn an art in the first place — so that the pre-conditions can become learnable as well? Their stories aim to tell the whole story. In Aeschylus we find the Second Interpretation in nascent form only, a version to which Homer can still claim the paternity. In Plato and Aristotle we find the mature version in full swing, and in explicit contrast to Homer: What exactly must an individual go through to arrive at a full and genuine, an artful encounter with beings?

The question is addressed not to those who must still learn the pre-conditions, but to those who must address such potential learners: to parents, teachers, creative writers — to whomsoever must help others learn. The context of Socratic induction remains, then. And from our modern standpoint the answers Plato and Aristotle offer appear, logically speaking, to beg the question or, socially speaking, to embody the aristocratic mood. Even to hear what these authors have to say we must strain to accept the logical anomaly and social position. For we moderns would like to consider answers about beginnings as ones intended for ourselves and at the beginning, while our intellectual forebears proffer answers about beginnings primarily for others and from the end. Perhaps the chief agony of the modern approach is the lack of direction for learning. In contrast, then, the chief difficulty of Platonic-Aristotelian storytelling is to retain a sensitivity to the initial stages despite our foreknowledge of the subsequent stages.

So the question for parents, teachers, creative writers, and the like, is: How are we to tell the first stages of learning? Aeschylus has Prometheus saying that mortals at first had eyes but saw to no avail, that they dragged through their whole life handling all things in bewilderment, and that they lived “dug down, like swarming ants, in sunless reaches of caves” (452-3). Plato has Socrates sketch out a very similar image in the Republic (Book VII):

... compare thusly the effects of guided learning [παιδεία] and its opposite on our nature: picture human beings dwelling underground as in a cavern which has a path, as broad as the cavern is wide, toward the light a long way up. They have been there from childhood [ἐκ παιδόν], with necks and legs fettered, so that they remain in the same place and can only see ahead of themselves, since their bonds prevent them from turning their heads.

The tale then continues, both with a description of what life is like “under the earth” and with an analysis of what happens should anyone find release from the bonds, climb out into the sunlight, and then descend to talk with the others. The first stage is again the human condition of darkness, and the issue is once again a condition of light. Yet the Platonic story stands in remarkable contrast and (one suspects) intentional competition with the older ones — both in style and in content.

"A strange image,” Glaucon says upon hearing the beginning of the story, “and strange prisoners.” “They are similar to us,” Socrates answers (διόμοιοι ἡμῖν). Plato’s version of dark vs. light immediately contains a request that we square the picture with our own condition: right at the outset, we are asked to compare. The story itself, we are told, is only an icon — an image, a likeness, a figure: in the tale, we as readers or listeners should note manifest elements (dark, chains, path) in order to draw out the unmanifest (the question of light, of freedom, of change of place). We may start with searching out the unmanifest in reference to the manifest, but eventually we must, as Socrates had already stated earlier, reverse the direction of the comparison: at the end we shall understand the elements of the image by understanding what is at issue in the recitation of them. But the moment we accept the task of comparison we cease searching out or concentrating on how others behave and fare, act and suffer — how Prometheus or Io, Socrates or Glaucon, our friends or our enemies, are. Rather, at that moment we search out the sameness in the story that pinpoints our own condition. We may, of course, remain unable or unwilling to make the comparison, and therefore unable to engage in the search. But the story has manifestly done what it can, viz. indicated what must be done.

* Socrates had already spoken of our commitment to images when discussing the “Divided Line” (511A). See also Plato’s Phaedo (76E): we routinely make sense out of what we immediately encounter by drawing upon principles (or principals). The stories Plato elsewhere tells expressly illustrate the new style of storytelling; e.g., in the Phaedrus (246A) Socrates proposes to talk about the meaning of immortality by likening our condition to a pair of winged horses and a charioteer. In Greek, the word for “likeness” (εἰκός) lies visibly (rings audibly) in the verbs to “liken” and to “compare” — and even more so in the noun translated as “likelihood” or “probability.”
"Truth is entirely and absolutely a matter of style," as Oscar Wilde repeats Plato's point after over two millennia. Fully grown people (seniors) may hear or read Homeric stories allegorically (in υπονοια) because they have the “intellectual intuition” (νοης) to detect in them the tensions of the actual human condition: they can make up for the deficiency of style, the missing cornerstone. Others, however, and especially children, take such stories at face value and remain marked for life by them. Our task as seniors (parents, teachers, leaders of communities) is to oversee “all stories which are first heard to make sure they are told in the best way so that those hearing them will be directed toward excellence” (Republic, 378D-E). “Life imitates art,” Oscar Wilde also says, again repeating Plato. And the difference between good and bad imitation depends on whether or not the stories inciting the imitation expressly ask us to question our own condition: if they do, there is a chance we shall come out into the light; if they do not, only seniors will be saved—i.e., those who already know. And this difference hinges first and foremost on the way a story is told.*

From the difference in style between the Old and the New storytelling flows one difference in “content” that has determined all western thinking. Both styles raise the question of learning from a concern for human efficacy: What happens in learning that distinguishes effective from ineffective response to circumstances? An appreciation of the Platonic answer requires a careful consideration of the “pre-school” stage:

The people below have praise and honors for each other, and prizes for the one who sees most clearly the shadows that pass before them and can best remember which one usually comes earlier and which later, and which come together, and therefore can most ably foretell the ones to come. [516D, but I have altered the grammatical mood.]

Who can doubt that this image aptly catches our initial stance in life, with its specious criterion of efficacy (based on pattern-recognition in the flux of incidentals)? But true competence, that of any genuine artisan, consists not in the ability to keep track of the various ways beings happen to appear (the history of their predicates). It consists in the ability to face beings themselves, the ground of those otherwise incidental appearances (the subjects of all those predicables). And this efficacy, the more reliable of the two, follows naturally from asking oneself how and where one is—rather than from remaining at the level where the bestowal of prizes by others is the standard of success (“what appears beautiful to οἱ πολλοὶ”: 602B).

The Older, the Homeric stories also raise the question of human efficacy. But the Homeric answer differs remarkably from the Platonic one. In the Older stories what accounts for human efficacy is not the recognition of the difference between beings and their shadows, but the acknowledgement of the need for divine intervention. The Older difference bears on human responsibility, but not on a secular development of the soul and the learning of an art: rather, it bears on receptivity (secularly stated: luck) as against pretended self-sufficiency (commonly called arrogance). For instance, in the Iliad (v, 846 ff.) we hear that Athena “pushes aside” a spear cast at Diomedes, her favorite, and that she “leaned on” the spear Diomedes himself casts back so that it would “gash the fair flesh” of his adversary. And in Pindar’s 7th Olympian Ode we hear that while “respect for Forethought implants in men excellence, and delight also,” the people of Rhodes were “stalked by the mist of Forgetfulness” when they journeyed afar to establish sacrifices for Zeus and Athena: they had forgotten to bring the necessary fire; still, despite their own failings, Zeus “rained much gold on them” and Athena “granted them arts of every kind, so that they would surpass all others in the work of their hands.” That efficacy is a gift receives poignant illustration in the story of Ajax, son of Oileus, whom Poseidon first saved from drowning after a shipwreck but then struck with a boulder when he (Ajax) vaunted his power to save himself, all by himself (Odyssey, iv, 499 ff.; cf. the story of Thamyris, the singer who lost his voice, sight, and memory after boasting that he could surpass the Muses themselves: Iliad, ii, 595 ff.).

After Plato, we find in western storytelling the incessant insistence that the precondition of all learning is that we stop waiting: stop waiting for another to do the job for us, stop waiting for inspiration, outside help,
But how to stop waiting? How can I learn for myself, get out of the dark into the light? How can I help you learn under these arduous conditions of having to “go it alone”? We start in the dark, and know nothing else: we have no reason for turning around. In the story as we have it so far, only force accounts for one’s ascent out of the cave and for one’s return into the cave: “force” is the very same word describing the plight suffered by Prometheus and the illusion of both the other Titans and of Zeus. We are familiar with the force exercised by others upon us — by parents, teachers, priests, policemen, and enemies. But outside force leaves us with shadows. Is there no inside force, one of our own, and one already at work to help us out into the light?

There is such a force, namely the attraction, the magnetism certain beings inevitably exercise on us. This force is that of ἐρως, the passionate love that drives us toward, or rather names the drive toward a person, a horse, a mountain ... anything beyond ourselves. Plato’s Symposium and Phaedrus especially, but also his Ion, expressly tell the story of ἐρως — of human learning as consisting in following this peculiar force through. In any case we are pulled. Superimposing the stories of these other Platonic dialogues upon the image of the cave in the Republic, we can suspect that the “turning around” from mere shadows and the “journeying up” into the light which one cannot posit as a goal (not yet having enjoyed it) — that these crucial events are caused by what lies outside (beings in the light) and are felt by us down below (in dark passions): in a sense, we are human only because of our prior commitment to beholding beings in the light (Phaedrus, 249B-E). Thus there is a “practical lesson” we can draw from this composite story, a lesson touching both on our own learning and on our helping others to

learn: Don’t shy away from passionate love. If you love horses, as Pheidippides does in Aristophanes’ comedy The Clouds, don’t dismiss this passion as a mere aside in life, but go to them, groom them, ride them, raise them... If you love another person, don’t be ashamed, but....

Yet there is a danger, a risk we all run: a danger and a risk any helpful story will progressively illuminate. For love not only names the draw toward beings. It also names the power we easily pervert, twist back toward the shadows. As Platonic seniors, we already know how this perversion works: one identifies the goal, the source of the attraction, with the givenness, the alreadyness of a being — what we simply see flitting before us as a shadow on the wall, the leftover of the being attracting us outside the cave, out into the light. We pervert the draw of ἐρως the moment we assume that it bids us simply to gawk at that horse, to have it around as the one it already is — rather than to help the horse be a horse, help it live up to its horse-ness. We pervert the draw another has for us the moment we assume that it bids us simply to possess the person as he or she already manifestly is — rather than to participate with that person in the dynamism of coming into our own being, living up to the meaning of being human (namely, by becoming alert and active in responding to and accounting for our shared circumstances, e.g. our horses). In categorical terms, genuine love names the pull toward a primary being, but a pull in which that being also participates — a pull toward actualization as given direction by secondary being. Love anchored in an individual gets bogged down in accidental features, while love for an individual (horse or person) flourishes when anchoring both the lover and the beloved in the universal (horse-ness, human-ness). One of the things we must learn and relearn is precisely this hairline difference in the draw of love. However, we love already before learning the difference: that is a boon, no matter how dangerous. It provides (on the Second, and even on the Third Interpretation) the motive power for getting out of the dark. The central task of the New storyteller is to tell the story of learning as a love story in which the dangers also become clear."

* Plato explicitly objects to the Older stories as encouraging whining, whereas we exercise λόγος, do justice to ourselves and to our circumstances, by rising to any misfortune, taking stock of the givens, envisioning what can and should be done, and getting down to work: one should “replace lamentation by therapy” (Republic, 604D). Of course, there are different modalities of waiting: Diomedes is hardly passive, like the characters in Samuel Beckett’s Waiting for Godot; nor does Simone Weil recommend a whining passivity in her Waiting for God. For a subtle commentary on (and a story about) merely passive waiting, see Franz Kafka’s The Trial, Chapter Nine.

* For the Platonic plot summary on love, see Symposium, 210A ff.: Diotima informs Socrates that “he who proceeds rightly in the matter will begin in youth to encounter beautiful bodies and, if guided rightly by a mentor, to love one such body only, in which he will engender beautiful discussions (λόγοι); but then he will notice that the beauty of this or that body is sibling to the
§3. Plot developments

A youngster learns horses: learns about them, learns to groom them, to feed them, to ride them, to breed them. An adolescent learns basketball: learns to handle the ball, to play the game, to participate in and to engender in others the teamwork essential to the game, and finally perhaps to coach others in the playing of the game. How are we, as philosophers, to understand this learning in detail? How can we tell the story in each case to reveal the significance of this learning?

The overall story is not enough. An effective understanding of Socratic induction, and of the natural learning presupposed by such induction, requires us to pinpoint a number of crucial twists in the natural learning process — elements of plot structure, as it were.

§3.1 From what is prior to us toward what is prior by nature

All teaching and all learning arise out of what makes sense already. This is evident if we contemplate the instances. For both the strictly disciplinary forms of knowledge [e.g., arithmetic and geometry] and each of the arts evolve in this way.

Such are the opening lines of Aristotle's Posterior Analytics. A more conventional translation of the first sentence reads: “All teaching and all learning arise out of previous knowledge.” But “previous” means more than merely “at an earlier time”: it means “already functional, already informing the present.” And “knowledge” here translates γνώσις, a word denoting the generic event of “making sense”: such “making sense” preconditions the possibility of επιστήμη, fully matured knowledge. The starting point is “what makes sense already”: let us contemplate some instances — ones that we ourselves might find. (Aristotle's examples bear on the arts of special disciplinary interest to Greeks: arithmetic, geometry, rhetoric — the arts preparatory to contemplation and action.)

The one youngster has seen cowboy movies, has read stories about fox hunting in England, has followed the rodeos, has ridden ponies at the fair. The other has watched a sibling play basketball, has shot baskets when the court became free, has followed collegiate games in the gymnasium. Whatever one or the other will learn in a strict way about horses or about games has already made sense in a loose way. The very first stage, the initial contact with horses or games, has no real precedent: a youngster happens, by chance, to see a movie or go to a fair, to have a sibling playing basketball, or whatever. If someone shows up at a riding school or in a gym class without any familiarity with horses or games, the instructor must somehow juxtapose the newcomer with the being in question: a horse or two, a game or two. This juxtaposition marks a pre-school stage. Still, some such stage is absolutely necessary: what one must learn is beings, and without some contact with the beings to be learned, without such familiarity with them, there is nothing to focus on, nothing to learn.

Following Aristotle's precept, one who listens to lectures or reads books about Canadian history would have to settle back into, focus onto whatever involvements one has already had with Canada: dealings with institutions, stories in the newspapers or on the air. Only in such reversion to what already makes sense does one have something to learn — here, one's political circumstances. Only then can the words heard or read mean something, focus on something, work on the familiarity to draw one deeper into the matter itself. Evidently, the teacher's and writer's task in such cases is to draw attention to such familiarities, encourage listeners and readers to revert to what already makes sense to them. Without this initial recollection on the part of the learner, the teacher's or writer's words have no hold. Without recollection, would-be learners can at most familiarize themselves with the flow of words —
memorizing these (usually someone else's summary of them). And this surrogate focus destroys the possibility of learning anything at all.

But wherein lies the development? Starting with what makes sense already — recollecting this familiarity — what next? We must start with what is first for us — what is “prior to us.” But the point is to learn the thing itself — horses as they are, games as they are, what is first for them. In retrospect, we realize that what first made sense to us was, for all its drawing-power at the time, shadow-like in comparison with the nature of horses, the nature of games.

For these are not the same: what is prior by nature and what is prior to us, nor what most makes sense and what makes sense to us. Ibid. 71 b 35.

The goal of the development is illumination of the beings themselves — as they require us to make sense of them. The illumination of nature, if you wish — but “nature” understood as the power inherent in what we are dealing with (in horses, in units of human effort) rather than as an “area” next door to our own domesticity. Indeed, the sense we make of our own domesticity must evolve into a sense of the things themselves.

Learning always arises in this way: from what by nature makes less sense to what by nature makes more sense. Just as we work in practical endeavors [in leading people] from what is good for each in order to see to it that things wholly good are also good for each, so too we work from what makes sense to the self in order to see to it that what by nature makes sense also makes sense to the self. Metaphysics, 1027 b 5

Herein lies the challenge for both teacher and learner: the end, the illumination of the beings themselves, must emerge from the initial familiarity. We learn something so that the beings (horses, or whatever) make sense to us: ever more fully. Here, as elsewhere, we can detect the birth of the liberal West: the task of the leader (here, of the teacher) is to include others in the event, not to impose the event on them. In pedagogical terms: the riding master will not immediately try to direct the youngster's attention to the inner nature of horses; rather, he will highlight precisely those features of horses that already mean something to the youngster: astounding size or speed, cuteness of the ears, dangerousness of the hooves, glory in riding on parade, or whatever — accidents or shadows. It may well be that the youngster will never pass beyond the fascination with these “externals.” But without some such contact the prerequisite fascination or love never takes hold to allow an “inside force” to propel the learner; or the fascination gets destroyed by the instructor's insistence on bypassing it for the sake of a deeper, but to the learner unfamiliar love. Forcing the learner prematurely beyond the shadows attracting him, the instructor turns the “deeper” things into ... mere shadows! Or, rather, the learner hears only rumors about those things.

But what does the “prior by nature” mean? Horses themselves, games themselves. But what does that phrase “themselves” mean? Full revelation of the individual horse, the individual game. Such revelation allows, and takes place only within full encounter with the being. But what makes the encounter full rather than partial?

And now we hear the answer that can so easily turn into mere rumor: in ποιησις, encounter with beings as they fully are requires us to encounter the given being in its being-ness (ousia) — the horse in its horse-ness. In πραξις, leaders must encounter their community (πολιτεια, “polity”). — It is our task, now in contemplation, to think this verbal answer through.

Antisthenes spoke for most of us when he remarked that he could see a horse but not horse-ness. But did he speak for horse-trainers, horse-breeders, and horse-doctors? These people do not simply gawk at horses, they handle them, help them, treat them. Artisans deal with individuals — their peculiarities, possibilities, and shortcomings — by relating these not only to what we humans want them to be (what makes sense to us) but also to what the individuals themselves can be and must be. Indeed, these artisans of horses adjust their own desires to the destiny of horses — unlike children, who seek to adjust horses to their own desires. Artisans can then move with effectiveness and confidence from horses with which they are directly familiar to other horses, elsewhere and when — unlike apprentices, who can only handle the initial set of horses and find themselves stumped in having to handle, help, or treat a horse they have never seen before. Master and apprentice alike assume that there are such things as the anatomy of a horse, the nutritional needs of a horse, and the possible developments of a horse — all of which constitute the horse-ness of horses. And they assume that they need not cut each new horse open to discover its anatomy, or test each horse afresh to determine whether it is carnivorous, or develop brand-new
procedures for breaking in each new horse. The artisan becomes a master by “seeing” precisely the horse-ness of horses—what Antisthenes on the sidelines will never see. But the artisan learns to see the horse-ness not in the abstract, but while learning and exercising an art. In contemplation we may generalize: progressing toward what by nature makes sense requires us to learn the art relevant to helping the being actualize its nature.

For only in learning and exercising an art do we turn toward beings in their Being. Otherwise we simply suffer them, guess at them, avoid or consume them—remain with what they mean to us. The progression “from ... toward ...” names a turn-about on our part: we cease directing beings toward ourselves, we now embark in their direction, we put our selves in service to their selves. This reversal of direction locates the fundamental tension, the stress of learning. For one initially resists the pressure of beings upon one, all the more today when the official ideology of the waning Fourth Interpretation informs us that we must twist every being into serving us.

Why be so “selfless”? Why learn, why worry about horses—about what is good for them? To one convinced that life consists in discovering or constructing arrangements which are good for oneself, which make one feel good, which are pleasant—to such a one the suggestion that learning consists of moving from “looking out for oneself” to “looking out for other beings” appears counter-natural, downright perverse: a suggestion most likely made by a clever bastard who is trying to fool others into serving him.

Plato and Aristotle recurrently reply to this question by asking others: What is this self that we so eagerly wish to promote at the expense of the beings around us? What is genuinely pleasant to us? The answers to these two questions go hand in hand to answer the first. In the mild language suitable for modern sensibilities, we can hear our Greek forebears saying:

* Our own art, the art of contemplation, might stand as yet another example. The nature of human beings is our topic—human beings as they are by nature caught up in the dramas of syllogism, paralogism, and now neologism. How can we learn what human beings are destined to learn—unless we participate in the process of learning itself?

§3.2 From encounter to memory and beyond

“The most startling thing about human beings is that they must account for their circumstances. This accounting requires the development of an art. The exercise of an art of dealing with beings requires that one adjust oneself to the claims of these beings—not just any claims, but those claims stemming from their need to actualize their own nature. The greatest pleasure, indeed the only genuine and lasting pleasure consists in successful performance of this art. Formally put, the soul is (in a way) the beings it deals with (On the Soul, 431 b 21): either passively (in the sway of the way beings happen to affect one) or actively (in the sway of beings in their οὐσία). And one eventually learns that pleasure lies in the actualization of the self, that the pleasures we experience passively are placebos helping us forget our own lack of self.”

The basic development of learning consists, then, in a movement from the reduction of beings (their predicates) to one's own self (and interests) toward the induction of one's self (and the transformation of one's interests) into the beings themselves. The drama of this development repeats itself interminably throughout life. Once we have learned to respond to ice with skates, we try our feet at responding to snow with skis—trying at first to ski in a manner appropriate to skating (our previous knowledge). Similarly, parents learn to respond to children and subsequently try to respond to adolescents as though they were still children. Or one learns sheep and tries to handle cattle similarly; learns bicycles and passes on to motorcycles, or from motorcycles to automobiles. Even within one's familiar domain one may approach the new reductively, trying to ski on an unfamiliar type of snow as though it were the familiar type, trying to treat other children as though they were the same as one's own. In such cases we learn (if we learn anything) that our efforts to reduce the new to the old falsifies the new. Thus the genuine development consists not in a quantitative growth but in a qualitative transformation: we learn, time and time again, to respond to the claims beings make upon us—*this* is what is prior by nature.
These lines from the final chapter of Aristotle’s *Posterior Analytics* answer in large part the question how we learn the origin (αρχη) of syllogism (reasoning of any sort) and demonstration (reasoning that reveals to us the circumstance at issue). This origin lurks in the major premiss of each of the four “perfect” forms. And this premiss presupposes (for one who talks knowingly, freshly, first-handedly) a contact with the universality of the beings at issue. The sequence “encounter → memory → experience” maps an itinerary of the soul. Once travelled through, it culminates in the kind of universality basic to, prerequisite for art and knowledge, the two conditions of the soul which complete the learning. Such completion enables us to learn individuals afresh each day—it originates, does not terminate the affair.

From encounter, Aristotle says. What does “encounter” mean? The Greek word is αισθησις, from which we derive our “aesthetic,” “an-aesthetic,” and “kin-aesthetic.” More common translations of the Greek are: perception, sensation, or sense-perception. ... while easily leaving us at the level of rumor, may instigate exposure more effectively than does a seat in the grandstand.

Arises memory, Aristotle says. What does “memory” mean? The Greek word is μνημη, from which we derive our “mnemonic device” and which likely relates to our “mental,” “mind,” “mention,” “monitor,” and “ad-monish.” We are most familiar with “memory” when someone asks us a question about what we had earlier seen, heard, smelled, tasted, or touched—frequently also about what we heard another person say. On such familiar occasions, we cease being where and when we are (waiting on a street corner for the light to change, sitting in a classroom) and we strain to see again, to hear again, or whatever, and to put into words the results of our efforts. Memory in this sense is abstract (since we cease responding to present bodily exposures) and intentional (since we search for the appropriate old sensation). In contrast, memory can also be concrete and still intentional: I am trying to mount this horse and I search for the way I saw the instructor do it; I am trying to scale a familiar cliff and I search for the footholds I remember having seen there. And memory can be spontaneous while remaining abstract: I am dribbling the ball down the court and suddenly my troubles at home or my appointment at the dentist comes back to me, even crowd in upon me, perhaps interfering with my responses to the game at hand (the involuntary memory which Marcel Proust understood as the αρχη of pulling a life together into a whole, i.e. a work of art).

The itinerary Aristotle describes evidently requires us to distinguish the memory at issue in learning the universal from all the more familiar occasions. For only memory as both concrete and spontaneous leads to the next stop on the itinerary, namely “experience.” What might “memory” mean, what is the occasion of such memory, so that it makes sense to say and to consider that “out of manifold memories of the same thing emerges experience”?

It is one thing to find oneself for the first time seated on a horse, reins in hand, seeing the ground so far below, smelling the sweat, and hearing its snorts and its restless hooves. It is a second thing to be sitting there after many times: now a wavering familiarity informs each moment. And it is a third thing to find oneself going through the routines of mounting, settling in, handling the reins, kicking the beast in the ribs, and managing to move with the horse without a premature termination of the encounter and an undesired commencement of another. This third thing names the experience of horses.

The Greek word for “experience” is εµπειρια (in-surge), from which we obtain our English “pirate” and “empirical.” It names the ability to sustain an encounter with some kind of circumstances—and must, in the reading of ancient and even early modern literature, be distinguished from the initial encounter itself (from αισθησις). In distinguishing the next
Plato complains that the public speaking so central to Greek life at his time was practiced by those having only επειρια in the business of impressing others, whereas the whole context was begging for a τεχνη of public speaking, an art of bringing issues into the light and into focus (Gorgias, 463, & Phaedrus, 270).

Plato, especially, insists that all art and knowledge evolves from a modification of memory: “our learning is nothing but recollection,” he says (Phaedo, 72E, and Meno, 81C) — nothing but ἀναμνησις, “reminding”. But “recollection” here is not quite the same as “memory,” nor does it have the same things in view. After dealing with horses for a while, we may become “blocked” or “stuck”: we cannot simply go on in our routines (we suffer an ἀπορια). At this stage the whole affair bothers us, haunts us, comes back upon us. Memory has become total and involuntary — as in the shame we may feel in facing others whom we respect, and whom we suspect of knowing that we have been handling horses poorly. The question is then: How can I ride horses well, how can I excel, develop excellence in my encounter with horses? The answer: intensify the memory, work on this natural “come back” so that you yourself can make a come-back. This intensification of memory, directed to the being itself (unlike Proust’s), is called ἀναμνησις. It is the task of Socratic induction to instigate the intensification — an instigation only possible for those who have previously taken beings (horses, or whatever) to heart, who have natural and spontaneous memories, and who are “blocked” and therefore hauntible by the whole affair.

On Aristotle’s itinerary, manifold memories naturally evolve into experience — into routine that already provides the basis for the possible culmination in actual art and knowledge. How can experience provide this all-important basis?

Once we are experienced in something (horses or games, lawns or paperwork), our encounters have a unity: we handle things in a single way. The single way now transcends the variable beings encountered: these appear as kinds, and we respond to them in keeping with the kind we think we perceive. Aristotle paraphrases this feature of experience by saying that in experience the universal has come to “rest in the soul.” In his Physics (247 b 11) we read the same imagery describing the knowledge that somehow issues from experience: “We consider ourselves to know, to fully mind something, when our intellectual response to it [to each being of a kind] has come to rest, when it [our response] has taken a stand with regard to it.”

But how are we to understand “coming to rest” and “taking a stand”? We likely recognize immediately the difference between first, even memory-informed encounters and sequential, routine-infused experience: the difference is well described as one between restlessness and tranquility, bothersome instability and reassuring stability. The restlessness and instability seem to mark our own insecurity, a nervousness, a lack of confidence. Then, already at the stage of experience, tranquility and stability set in: we now do the same thing, repeat our responses, we manage to remain comfortably on the horse and get it to do what we want it to do. However, Aristotle’s illustration of the
difference (between restful standing in the face of beings and the prior restless instability) shifts the emphasis away from our plight to the appearance of beings themselves. The conditions of the soul we call art and knowledge are, he says, neither handed over to us at birth or later (as miraculous talents), nor do they emerge from other conditions wherein things fully make sense to us (as we might believe we could move smoothly from knowledge of sheep to knowledge of cattle). Rather, art and knowledge take a direct route:

They emerge from encounter (αἰσθησις) itself, in a manner like the following: in a battle, when a rout has occurred, if one takes a stand so does another, until there is a formation [an ἀρχή, a new start]. The soul subsists in a manner enabling it to undergo things in this way. Restating what has already been said: whenever one individual being takes a stand, the universal first arises in the soul—for the simple reason that although we encounter only particulars, the encounter itself derives from the universal (e.g., we encounter Callias as a human being, and not just as Callias). Then a stand occurs in these until things not just assembled from parts, i.e. universals come to a stand (e.g., some given animal takes a stand, then “animal” does: this latter just as the former). Thus it is clear that we necessarily get to know primary matters by induction: for αἰσθησις implants the universal accordingly. Posterior Analytics, II, 19

However we eventually choose to understand the loaded imagery of this passage “from rout to stand,” we must ponder two proposals: that the much-needed universal emerges for us only as we plant our eyes and ears, hands and feet, firmly in and on the beings to be learned (even if some Socrates will help us intensify this implantation) and that the drama of restlessness vs. tranquility, of instability vs. stability, touches upon, becomes evident in the beings we encounter as much as in ourselves.

Of course, a rout can distress us, unsettle us, and perhaps Aristotle suggests here that such unsettling (like Plato’s ἀπορία) is a precondition of learning (another, but unnamed stop on the itinerary). But it may also uplift us, and mark a beginning in an upbeat manner. Imagine a grandfatherly marine biologist walking with a young boy along a deserted and rocky seashore in the early morning hours. Crabs, snails, sea urchins and other creatures teem in the tide pools, exhibited as though for inspection and capturing the attention of the curious boy. Hopping spritely from foothold to foothold among the slippery rocks, and calling out excitedly each time he makes a find, the boy provides a living picture of response to a riot of colors and shapes—likely sounds and smells, too, although peripherally at first, recognized only on later occasions. Trailing cautiously behind, and offering quiet commentaries in reply to the boy’s questions and exhortations (naming the creatures discovered by the boy and noting the peculiarities of how they move, eat, relate to each other and to the dining room at home) the old man provides the perfect foil for the riot—for the rout in which the boy cheerfully participates. The boy may be struck by the color of the spines on a sea urchin, the old man explains that the color may vary in the performance of their function; the boy may think they are for attack, the man explains they simply serve for protection. Or an exposed sea anemone strikes the boy as some sort of plant, the man explains that it is an animal which, properly submerged in its element, displays stinging tentacles to paralyze and consume its prey—and that the thing actually moves about. Later, when the boy looks at the panorama at high tide, himself submerged with a diving mask, many of the things he saw before will literally run away from him, while others pass quickly into and out of view. But all the while the old man finds that all things stand still: he sees “with the eye of the soul” what is still there throughout all the variations, all the riot.

With the two poles in mind we can ask what developments in the middle could illustrate the maturation of the boy: How does he progress to experience, and from there to the translucent universal? Well, the boy must return to those tide pools, most likely alone, or with a friend who will not outstrip his own responses. He can now respond mindfully (“rememberingly”). But even this manifold of encounter might terminate the affair. He must learn to get on with the creatures in the tide pool: at first he plays with them, plots to catch the crabs, dams or drains the pools. In the extreme, he might turn some of the pools into “farms” for mussels that he will harvest to sell for bait. He may then reconstruct the circumstances with the intent to facilitate his routine, experiment with ways of harvesting the creatures, learn by trial and error and hearsay when to harvest them, and so on. Once established in a routine for getting on with creatures in the tide pool, the boy has experience of these things: he experiences, moves in upon and along with them, rather than
simply encountering, being affected by them. And such experience already places him above any given mussel or tide pool: he perceives each as one of a kind. The boy then responds holistically, he moves within the initially universal. This “holism” he can (if he finds himself so inclined) deepen — to discern mussels as they are in themselves. Without this initial holism, rooted in and springing from actual encounters over time, there is no basis for such deepening.

How can we illustrate the “depth” possible for the boy? Most often we simply extend our routines so that we can handle a greater variety of circumstances. As comical as it might seem to outsiders who assume that the only legitimate purpose of intercourse with nature is to devise a way of selling off parts of it, the boy might in fact take an interest in the various organs of the mussels, in the conditions of their growth, in the purpose of each part of the mussel and the purpose served by the mussel in the tide pool as a whole, and finally perhaps in all the varieties of mussels the world over (in which case he might write books about them). The boy himself, then, focusses increasingly on the being as what the Latins called a species, and becomes all the more alert to specimens. One mark of this development, as Aristotle says about Thales, is that the boy can, if he chooses, prove the value of such alertness by arranging for a bumper crop of mussels and even cornering the market, “thereby proving that it is easy for philosophers to become rich if they choose, although this is not what they care about” (Politics, I, 4).

§3.3 From opinion to knowledge

Knowledge and what is known differ from opinion and what is opined inasmuch as knowledge bears on the universal and derives from necessity. Whatever is necessary does not allow of being otherwise. But some things, while truly being, are so in a manner that allows them to be otherwise: clearly, knowledge is not about these things. ... Rather, opinion is about what is true or false, i.e. about what allows of being otherwise. In short, opinion is the assumption of a proposal that is unmediated and not necessary. Posterior Analytics, I, 33.

You see, don’t you, that opining correctly without being able to condition it with an account (λόγος) is neither to know (for how could knowledge be an affair without an account?) nor is it unlearnedness (for how could unlearnedness be at all in touch with a being?). So correct opinion is surely somewhere between complete mindfulness and total unlearnedness. Plato’s Symposium, 202A.

Knowledge differs from opinion. We start with opinion. From correct opinion we may progress toward knowledge. Both opinion and knowledge name ways we find ourselves involved with beings. Obviously, Plato and Aristotle claim that one way is better, more fulfilling, than the other way. But wherein lies the difference? How do we make up for the difference, progress from one to the other?

I start out believing that horses are brown-colored, well-behaved, corral-confined, ride-able things. I believe that horses need fodder of the exact sort I see the stable boy giving them, that they need exactly those shoes I see the blacksmith forging for them. All these opinions may be correct for those horses I happen to be experiencing and during the temporal span of my experience. Indeed, these opinions had better be correct: if I have not related correctly to these particulars, if I have not noted the way these horses happen to be at the place and time of my experience, I shall not progress any farther; rather, I shall remain trapped within the fantasies I have formed while hearing fairy tales or watching television. However, I will soon learn that horses — other ones or even the same ones at other times — “allow of being otherwise”: may take on other colors, other manners, other environments, and require other treatments and develop other abilities.

Formally put, opinion names a relation to accidents and (since these vary from place to place and from time to time) relates us both correctly and incorrectly to the beings at issue. Experience thrives on opinion in the ancient sense of these two words: I become more experienced in horses as I extend the range I can recognize and handle. And the range naturally expands if I stay with horses: such is one meaning of time, namely the unceasing variation of the beings one experiences. Humanly put, variations easily frustrate me: they jolt me into changing my own habits of recognition of and response to beings; I must strain to keep up with the flux.

The Greek work for “opinion” is δόξα, from which we obtain our “ortho-doxy” (right opinion), “doxo-graphy” (written history of opinions), and “para-dox” (deviation from opinion). It refers to received opinion. Whereas we today speak of private opinions as though an individual
formed his or her own, the Greek sense suggests that opinion first of all informs us: I accept a “current view” of horses, one provided by those already working in the corral or the stables (or the television studios). Horses appear to me through the eyes of others — at best, through the eyes of those who relate directly to horses, since their eyes might then direct me to the horses themselves (without, of course, assuring that I will follow up). For all I know, however, the horses may appear to me through the eyes of merchants trying to sell me horses, breakfast cereals, or cigarettes: these people have no real eyes for the horses at all but only for their pocketbooks, and mine. In any case, δόξα refers to appearance via others.

Opinion is a “condition of the soul,” that kind of readiness to respond to beings (of the sort we experience or hear about) which relates us at best partially to the beings. Not relating us fully to them, it is essentially unreliable. I opine a bundle of predicates that perhaps contains but also conceals the subject in its fullness. If I stay with horses, the predicates I insist on using to relate to them will someday likely cut me off from the beings themselves: for horses themselves differ from the predicates I ascribe to them, and they may well appear as radically, strangely, and frustratingly other than what I was conditioned to face. The otherness may be occasioned simply by another predicate (a new behavior, a different shoe), but it may also be occasioned by a horse itself. Perhaps every violation of our opinions initiates the possibility that the being itself may break through — break through our “defenses”, we might say today (opinion being a kind of fortress of routine, of apparent safety).

Opinion proves especially unreliable as a basis for helping others. If I only have opinions about horses, even correct ones, what I say about them lacks resonance, draws another not toward me but away. I may try to adorn my speech with all the trappings of self-assurance, above all the tone of attunement with horses. Apart from play-acting, the λόγος of opinion is old, left over from better days. Speaking and listening, writing and reading within the confines of opinion are paleological, essentially and therefore someday distressingly so, no matter how new and glistening the bottles may be into which the speech is poured.

And knowledge? As a condition of the soul distinct from opinion, knowledge names our relation to the universal: I may come to know what pertains to the horse as a whole, what makes the horse whole. And my knowing of this wholeness draws upon, stems from, necessity. So we read. But we must once again ask: What does it mean to say that “the known” as distinct from “the opined” is the universal? And this question arises along with another: What does it mean to say that knowledge, unlike opinion, is “from necessity”?

Already at the level of learning called “experience” I float over horses: I relate to one and then to another horse in a routine as though I were responding to horses as a whole. Basing myself on the routine, I talk with others about horses, this one or that one, very generally (generically, we can today say). However, tied as I am to predicates (accidents, even properties and genera), I skim by the subject, what underlies: I miss both the individual horses and the species horse. Horses appear only in their particularities (in parts, partially; I opine them by focussing only on what pertains to each, therefore only to their parts: such is the meaning of the Greek words for particulars, τα καθ‘ εκαστα).

Whatever its meaning, knowledge does not require us to move farther away from beings. I am already far from horses. Knowledge will differ from opinion partly by establishing an intimacy with beings: in coming to know horses I will discover that my initial generality of opinion (experience) becomes translucent to the specificity of horses. I can then still float above any horse here and now, and float on to other horses at other heres and nows, but I can also focus on each one, achieving an intimacy with each individual because I can relate my routines to its wholeness rather than simply relate its parts to my routine. Knowledge names a condition of the soul, a condition of the soul's intimacy with beings.

Very formally stated: knowledge names a complete relation to, a relationship with beings: as species and as individuals. It names a relation to the subject, to what underlies predicates (what we formulate about beings: the usual categories), and to what takes us beyond the obvious ways we encounter beings (sights, sounds, smells, tastes, touches).

Humanly stated: knowledge names our ability to work with the necessities belonging to beings themselves. Such working, our own position, manifests itself in a τεχνη, an art: our ability to “complete
what nature is unable to finish, while also keeping in step with her.” Knowledge is part of our ability to participate, take and be a part in the genesis, the becoming of beings. Humanly understood, such participation requires that we acknowledge our own partial, incomplete (and therefore erotic) nature: that the soul is only as other beings come to be through human activity. Ontologically understood, such participation requires us to relate doubly to beings, and to know them as double: as being both individual and species. In both senses of “of,” knowledge is “of necessity”: we relate to the being knowingly by embedding ourselves in the necessity each individual has in living up to its form, and also by accounting for these necessities. Art and knowledge emerge in tandem as experience gives way to full participation: the one names our ability to complete, the other names our ability to envision the goal of the completion. Neither is possible without the other.

As formulated by Plato and Aristotle, the marks of art and knowledge are humanly clear, and are the source of wonder, amazement, admiration: effectiveness in the care of beings—or of horses or cornfields, ships or temples, communities or households. Whereas in ἐργαζόμενος and δοξή we flit by beings, in τέχνη and ἐπιστήμη we come in close to, participate in, aid and abet the dynamism of each being in our care, we actualize what it can be—not just incidentally and for ourselves (as we might train a horse to dance a jig) but also specifically and for itself (healthy, strong, whole, active).

Effectiveness? One who simply gets on with horses (an “empiric”) can be effective in some sense, and for a while: namely, so long as nature is doing a fair job of completing herself anyway. Doctors often claim that midwives are only empirics in this sense: when nature falters the midwife falters too, whereas the doctor can enter fully into the fray. The opinions had by empirics relate them to what can be otherwise, so that when the “otherwise” occurs opinions prove unreliable, i.e. ineffective. In contrast, knowledge is always true, Aristotle says. By definition. For we are contemplating this possibility—the possible “condition of the soul” as evident in the master craftsman as opposed to the apprentice. The “being true” predicates our relation (as knowers) to the universal. If I learn horses fully, I relate to the species horse. Truth is basically this relation. It accounts for the greater effectiveness of the master. But knowledge, the relation of the knower in truth, i.e. to the species, does not guarantee that I shall know this horse, here and now, fully and effectively: it may have features, incidental to the nature of the species, that baffle me at the moment (if there were only time enough, I might...); it may have a disease doomed it to perish. But the failings possible for one who knows horses differ sharply from the failings evident in one who only opines horses: one who knows relates to the universal and somehow fails only in regard to the individual, whereas the one who opines fails already at the start, relating to horses within the merely generic universality of routine and participating only in the fray of conflicting accidents rather than in the fray of the individual striving to actualize its own nature. Failing to make sense out of particulars differs radically from failing to make sense of universals (Nicomachean Ethics, III, 1).

The “effectiveness” of knowledge lies most basically in the assurance of participation in the genesis of a being, a genesis based on (grounded by, directed to) the specific nature of the being at issue; the relative assurance of results provides only a sign of knowledge. Genuine knowledge is then not simply had or possessed: only sophists identify knowing with having knowledge (Posterior Analytics, 74 b 25; Nicomachean Ethics, VII, 3, §7). Categorically stated, knowledge is not fully in evidence as a condition of the soul. We only really know something as we are engaged with the being in question—with a being in its doubleness as both individual and universal. As Aristotle says and discusses at length, we philosophers stumble over this peculiarity of the knowledge we contemplate in action and production: we note that knowers in these two domains relate to the universal; yet:

this account is in a way true and in a way not true. For knowledge, like knowing, is twofold: the potentiality and the actuality [the ability (the δυναμική) and the engagement (the ἐνέργεια)]. The potentiality is materially given as universal, i.e. non-definite, and it is of the universal and non-definite. But the actuality is definite and of a definite this-here, a unique something. Metaphysics, 1087 a 15.

Such inside effectiveness evidences itself fully only to the knower himself. The mark of this effectiveness, the assurance of participation in genesis, is the ability to create and preserve a presence within flux (so that beings reappear as moving images of eternity: Plato’s Timaeus, 37D f., and Symposium, 207D f.). Genuine effectiveness lies in the ability to be present to what one encounters, to present oneself to the
horse, so that what one encounters can become fully present, whereupon one's *activity* remains focussed at every moment and therefore graceful in movement. Furthermore, one's *talk* sounds out of direct encounter: it does not echo old accounts. It is relevantly exact: the exactitude follows not from memory nor from mathematical forms of expression, but from intimacy with what is most exacting (necessitating) about the horse here and now. Genuine exactitude stems from our own in-actitude into what ex-acts us, the exactitudes of the matter itself (see Plato's *Statesman*, 284D, on “showing what is itself exacting”).

The ultimate mark of knowledge is the ability to sustain a presence of encounter. On this account, knowledge undoes the dispersion of mere encounters by intensifying them, not by abandoning them. Enabled to enter into the fray of this horse here in relation to its species, I make my circumstances (generate them) all the more “aesthetic”:

> It is apparent that if any mode of ἀισθησις is lacking, some knowledge will be lacking as well. For we would be unable to obtain it. We learn either by induction or by demonstration: demonstration proceeds from matters understood universally and induction from matters understood partially. But we are unable to contemplate universals except through induction, ... and we cannot proceed with induction without conditioning by ἀισθησις. *Posterior Analytics*, I, 18.

Whenever we are deprived of our eyes or ears, our tongues or noses, or some organ of touch — whether from birth, by accident, permanently or temporarily — we are cut off: we either never attain to relevant knowledge at all, or (having earlier attained it) we are no longer able to actualize the knowledge that conditions the soul, and we live out our dotage with unactualizable potential knowledge (as happens to mountain climbers who lose the use of one or more of their limbs). For, deprived of ἀισθησις, we are deprived of immediate encounter — the necessary, even though not sufficient condition of presence.

Whatever else knowledge means in the literature of the Second Interpretation, it names the most exacting and amazing and fulfilling relation we can have with our circumstances, encompassing our direct encounters and intensifying these into full presence within flux. Obviously, the paradigm instances of such knowledge are found in those who work with human beings (in action) or with non-human but naturally double beings (in production): leaders of communities who must know what human beings and human teams “most need to be,” and craftsmen who work on farms knowing animals and plants, who work in towns knowing garments or cabinets, who work on ships knowing sails and water and wind and stars.

The full contemplation of such knowledge becomes rather complicated: Must an architect know not only “houses” but also human nature, as well as the nature of wood and stone, cold and heat, wet and dry? Must the doctor not know parts of the body as well as the whole — organs as well as flesh? And if the captain knows ships, must he not also know human nature, wood, wind, water, stars? Is knowledge not a strangely multidimensional affair?

The logician’s question (How do we learn the basis of syllogism and the cure for paralogism?) limits the inquiry to consideration of the paradigm instances: to human engagement with middle-range, obviously double (“composite”) beings. For these are the beings we can learn directly, and as ones of kinds: ones having a unity of their own, and a unity we can help. It seems that we might also learn amorphous beings (soil, wind, fire, water) or beings seeming to have form but indistinguishable in their parts (blood, skin, sap, fiber); and it seems that we might learn beings we can in no way help live up to their form (planets, stars, gods). But perhaps we learn both micro-beings and macro-beings only indirectly, namely while learning the middle-range beings.

Similarly, it seems that we learn “things” that we call purely formal: mathematics in the narrow sense, namely geometry and arithmetic. Yet here we learn not beings at all, but activities that we ourselves perform and that bear on the category of quantity (in conjunction with continuity). Here we exercise our own being, without taking stock of it: an excellent preparation for any subsequent activity, precisely because it abstracts us from all encounters, prevents us from drawing upon incidentals to justify any decision we make. Mathematical activity frees us from the limitations of mere “experience.” But mathematics also cuts us off from nature, therefore from the provider of those universals which undergird concrete syllogism (*Metaphysics*, 995 a 15, 1073 b 7, and Book XIII, 1-3). Although we might today construe syllogism itself as a strictly formal affair much like geometry and arithmetic, Aristotle assumes that its reliance on universals for demonstration (the dialectic of all-some-one, and the conundrums of negation) indicates a grounding in nature — in
nature as evident to one who has learned to participate in the genesis of middle-range beings.

It has proven to be the task of modernity to investigate the micro and the macro, and to extend our understanding of geometry and arithmetic toward precisely these strange beings: from the elements (on the Periodic Table and beyond) to the heavens (our solar system and beyond)—as well as to the events, human or otherwise, hundreds to billions of years ago. The investigation of these matters contrasts structurally with the learning we naturally engage in. And the results of such investigation contrast so remarkably with the results of natural learning that the whole deserves another name in English: science (originally simply the Latin scientia, translating the Greek ἑπιστήμη). Science as we now understand it does not evolve naturally for us (we must devise it), it does not focus on middle-range beings (it analyzes these and recomposes the results into a cosmic system), and it does not find its evidence in action and production (it seeks its own evidence, available strictly to those undertaking the investigation). And its basis is precisely that mode of human response which Aristotle admired but criticized as a non-relation to beings: mathematics. Modern science, too, can be an amazing and exciting and fulfilling activity. But it does not serve as a paradigm for the natural learning of natural beings. Rather, it serves as a paradigm only for itself—and for the mathematical logic that will occupy our attention in Book Four.

§4. Questioning

We naturally find ourselves underway—at the very least, from encounter through memory into experience. Being underway, we are also caught in the middle between making sense of things for ourselves and making sense of them in themselves, between the approximations of received opinion and the exactitudes of art and knowledge.

But progress on our way depends on whether we can be sufficiently bothered by beings so that we can ask questions about them—or (as in Socratic induction) take as our own the questions directed to us by others. Complacency leaves us at the level of experience: we find a way of merely getting on with things, a routine in which we can skim by beings.

Questioning stems from an uneasiness, a blockage. If only for a moment, we stand outside, in some sense beyond our routine. Even an idle question, raised out of mere curiosity or courtesy or desire for attention, suggests a transcendence, an acknowledgement of the peculiarly human destiny to make more out of present circumstances than already simply meets the eyes, the ears, nose, tongue or hand.

To be sure, many questions beg for information (What's her number? Do I have appendicitis or not?), and answers simply send one on one's routine way. Similarly, academics get used to examination questions: a teacher asks questions of others in order to judge their competence or performance. And then there are the questions tourists may direct to their guides, or mothers to their grown-up children, and the answers bring pictures of other people's lives into view.

Much different are questions arising within one's own work, stemming as they do from wonder at beings themselves. The youngster asks, Why is the horse refusing to take the bit? Why does the horse limp? As a teacher I ask myself, Why are my students unable to compose narratives of learning? A farmer asks whether or why his crops are failing. An excited child asks an elderly companion why mussels are sometimes poisonous. An already credentialed musician or mathematician asks what really makes the difference between beautiful and unbeautiful music, sound and unsound demonstration. These questions are being-based: they formulate linguistically the pressures imposed by what we face; they are natural rather than contrived. And only those answers which arise in response to being-based questions can serve the development or preservation of τέχνη and ἑπιστήμη.

Genuine questions stem from pressure of circumstance: I pose a question genuinely only as I am called into question by the beings I otherwise simply handle. But precisely because it is my routine that I find called into question, I can choose to respond to it as though it simply called for an adjustment within the routine. For instance, the youngster finds that the saddle tilts to the side of the trotting horse, this circumstance appears dangerous, he checks the strap, cinches it up, and returns to the routine: he may keep all learning at the level of what makes sense to him—at the level of relieving the pressure. No doubt such deflection is often most appropriate to the circumstances. But the habit of deflection prevents the development of any art or knowledge—prevents the rooting of human response in what is prior by nature. For instance, the youngster may henceforth cinch up the strap much too tightly, assuring the position of the saddle but damaging the horse.
Of course, we usually think of questions as arising in discussions with one another. Children ask their parents where babies come from, a girl asks her mother why the cookies burnt, students ask how they might improve their writing, I ask a musicologist how exactly one might distinguish folk from long-hair music. And each expects an answer from the person queried. Yet the tertium quid looms to the extent that in our willingness to ask a second person (who, one presumes, knows) we already acknowledge that something is escaping. The pressure of beings, after all, is the pressure of their escape: the drawing pressure of their rout.

Among all the forms of questioning, Socratic induction stands apart: here, one who in some sense knows questions one who does not know. Strange but true! The obvious assumption is that the learner, the one who does not know, will come to know only by building up his own steam when recollecting prior engagements. The unobvious assumption is that the learner will learn only to the extent that he can be bothered by the nature of horses, the nature of music, or whatever: that the questions of a Socrates simply articulate what questions the learner. A parent plays Socrates not when answering the questions posed by the child, but by questioning the answers presumed by the child: the child assumes that the cookies are burnt, and the parent challenges the child (Are they really burnt?)? Similarly, a teacher plays Socrates when calling into question each effort on the part of the student to settle an understanding of something.

From Socratic questioning proper we must distinguish a number of look-alikes. One look-alike Socrates himself exemplified: since many people who do not know what they are doing or saying nonetheless claim they have the answers, quasi-Socratic questioning can simply aim to expose the fraudulence, to bring the paralogism out into the open for the bystanders to judge. This look-alike has many variations in the academic world, where one scholar may aspire to undo the claims of another — either for the sake of the tertium quid or for the sheer pleasure of verbal victory in public. The same range of look-alikes also appears in law courts and parliamentary assemblies, where decisions issue from speeches for and against. And of course teachers often aim to preserve the sayings encased in books, and therefore question the efforts of students to reformulate these sayings (so that the tertium quid lies somehow in language rather than in experience itself, and memory gets relocated accordingly).

Pure Socratic questioning, clearly a rare event, works best with children and with adolescents who admire the questioner for other reasons; grown-ups usually feel themselves constrained to claim they already know what needs to be known (since they generally occupy a position defined by their claims to have learned something). What in fact happens, then, is that spectators profit from imaginary dialogues in which people appear as struggling under fire. For instance, students listen to a professor reciting the pros and cons of a question. Or one reads a book, e.g., Plato's Republic or Faulkner's Go Down, Moses. On the sidelines, one has the choice whether to let oneself be bothered. One must then learn to question and to be questioned in solitude.

§4.1 The four questions

The things we seek [or ask] are equal in number to those we [eventually] know. And we seek four things:
1. the “that” [Is it so that S is P?]
2. the “by what” [By what is S in fact P?]
3. if it is [Is S?]
4. what it is [What is S?]

For when we ask which — this or that? — among a number of possibilities (e.g., Is the sun eclipsed or not?), we seek the “that.” A sign [of our asking this sort of question] is that when we have discovered that the sun is eclipsed we stop [enquiring], while if we already know that it is eclipsed we don’t ask which [of the two possibilities holds]. On the other hand, when knowing the “that” we ask “by what” (e.g., knowing that the sun is eclipsed or that the earth is moving, we ask by what the sun is eclipsed or by what the earth is moving). — While these two are obvious ways of enquiring, we ask questions in another way — e.g. if there is or is not [such a thing as] a centaur or a god (I speak here of whether such a being simply is or not, not of whether it is white or not). And once we know that a being is, we ask what it is — e.g., What is a god? What is a human being? Posterior Analytics, II, 1.

The first two forms make most sense to us. So long as the youngster simply assumes that the horse in the stable is fast or slow, healthy or
sick, or that horses (those he is familiar with) are to be shod or ridden in a certain way, he will learn nothing more. Only in wondering and asking, or in being asked by another and being prepared to be puzzled himself, will the youngster possibly learn the art.

Is the horse really fast? really sick? Are horses generally to be shod thusly? to be ridden thusly? This first question requires one to look more closely: like the other three, it makes sense only as a pull toward, back to, the being itself. Yes, the horse is fast (S is P) because the horse repeatedly galloped from one end of the corral to another in only five seconds (S is M). Yes, the horse is sick (S is P) because it has a fever (and not just because it is sweating). No, horses are not necessarily ridden Western style (some S are not P) because some horses are ridden English style (some S are M). Questions of the first type beg for an affirmation or denial of a proposed conclusion — by first turning the initial conclusion into a question, then initiating attention to how beings in fact present themselves, and finally inciting the formulation of a reason, a middle term that grounds the affirmation or denial of the conclusion attributing P to S. The reason for the horse being fast or sick (or whatever) becomes a reason for thinking the horse this way.

Now, and only now, can the youngster significantly wonder why the horse is the way it is — e.g. what makes the horse (S) sick (P). Once this second question bothers him, he must re-look, re-consider the horse. But what is there to see? At present, there is only a sick horse. The youngster can never see with his eyes, hear with his ears, feel with his fingers why P bears on S. What “brought it about” that S is P may be the feed, the exercise, the breeding, the exposure to cold winds or to virus-infected water. But the reason here (the M) does not appear fully to sense-perception (αισθησις). Rather, the reason becomes evident only in experience (εµπειρία): the boy re-collects, re-members multiple encounters now unified into a routine. Asking or being asked why S is P, the boy re-coups, re-covers his routine, re-installs himself within the sequence surrounding any immediate encounter, and searches for the “before” which belongs to the “after” (the M that belongs to S to render P presently belonging to S). This search sends the boy beyond the immediate. But in re-grouping the entire sequence around the encounter, the effort to respond to the question thickens the encounter: the horse now has a history.

While the first two questions would seem to be essential even to the quantitative expansion of one's ability to deal with a realm of beings, the next two forms (Is S? and What is S?) require a qualitative development. When would someone ask whether a horse is or whether horses are (and not simply how to predicate them)? Aristotle's two examples appear to touch upon beings out of the middle range: on centaurs (presumably not being) and on gods (presumably being). Moreover, and oddly, he insists that we can only ask what a being is (a centaur, a god) once we know that it is: only after we have found a being can we wonder what we have found. On this account (and also on the account in his Metaphysics, vii, 12) we run the risk of enquiring into nothing at all if we leap over the preliminary stage: the very words “centaur” and “god” and “human being” defy real definition until we settle the question of whether they are (of being in the preliminary sense: ἐϊναι “existence”). This prerequisite to the fourth question may seem strange to academics accustomed to talking in abstraction from actual dealings with beings. But it makes full sense when located in the processes of natural learning: How could one ever ask significantly what horses really are unless one had been involved with them? How can a man wonder what women really are, a woman wonder what men really are, parents wonder what children are, or teachers wonder what students are — unless the beings themselves had instigated the wonder?

How, then, does the third form of questioning work? In the case of a centaur, one has read stories containing descriptions of shapes and actions, and one has seen the friezes belonging on the Parthenon. The question might then arise: Has ever anyone seen in the woods or on the hilltops something in flesh and blood corresponding to the features familiar through the stories and the friezes? But then we are looking for something that fits a set of predicates, and our search does not correspond to the third kind. The question taking the form “Is S?” leaves predicates to one side.

Part of the difficulty lies in the homonymity of the Greek word for “is,” ἔστιν. It carries not only the sense of a copula (marking the occasion of a predication). It may also carry the normative sense of “makes its presence known” or “presents itself forcefully” or “lays claim upon our attention.” Thus, for our purposes, we can read the question somewhat more amply as: “Is S really there, really present or presentable?”
If the youngster has only read about horses, or heard about them from others, he must still ask whether horses are, i.e. whether he can present himself to their presence. People hear about battlefields without ever presenting themselves to the presence of one. People hear about climbing mountains in great detail, only to discover that the first mountain they approach literally escapes them. In Plato's image, one is still chained at the bottom of the shaft, seeing only shadows, hearing only echoes. By acknowledging the question, one sets the stage for a possible discovery of the being itself: that it is.

Even after long familiarity (experience) with horses the youngster may find himself asking: Are these horses — for me? Do they really command my attention? Do they really make sense? This ... “Is S?” signals a fundamental option in learning. Muffled, it appears merely rhetorical, and down we climb — or slip.

Once the youngster has learned the art of tending horses he must on occasion ask the third question in yet another way: Is this horse — or is it not? Is it curable, generable, really alive — or not? A dead (even a dying) horse is a horse in name only. But for one who tends the horse the question will occasionally sound out loudly whether the given horse has come to such a pass. Here the question of being (generation) arises in tandem with the question of non-being (destruction).

And finally the question “Is S?” may press in upon the youngster in the form “Is there such a thing as horse-ness?” One who has naturally learned horses helps bring given horses into line with what they by nature are, i.e. need to be. The knower measures a given horse (assesses and, where possible, corrects its manner of being), but can do so only if the measure itself, the standard, emerges. The knower’s participation in the genesis (growth) of the given horse depends upon the genesis (emergence) of the measure. The dependence is radical, then: one will always wonder whether there is a single measure. As a teacher fully aware of the diversity of students, and also of the diversity of philosophical accounts of human nature, I often wonder; I am never in a position to be absolutely sure, as a god might be. Even writing books about horses or students leaves one wondering. Indeed, good books articulate questions above all else — the questions naturally plaguing us. Even out in the sunlight, fully presenting ourselves to beings presenting themselves to us, we must acknowledge that the “knowledge and truth” we happen to enjoy is grounded, and the most we can hope for is that our vision of beings be fully conditioned by this ground, held by it: our task is not to know (command) the ground (the measure), but to know and to aid beings on its strength, and to draw our strength from it.

After affirming that S is, the youngster can ask what S is. On this account, a prior involvement with the forceful presence of a being provides the “reference,” the “something to look at” necessary for wondering significantly what it is we are dealing with. The question “What is S?” does not ask what somebody means by the word (e.g., “centaur” or “god” or “human being”): it asks what the being means. Every individual being, everything counting as a being in the primary sense, means (is) its own destiny, its secondary being: this we naturally learn in learning an art of production. That each human being also means (is) his or her own destiny, we also learn (and easily forget) in the art of action (leadership). In contemplation, however, one learns something further: that the destiny, the “what” of a being can never be rightly settled out of hand; it remains a question, one guiding the in-hand, hands-on effort “to complete what nature is unable to finish, while also keeping in step with her.” Indeed, in contemplation we contemplate precisely this question: “What is a being?” — a question which, Aristotle says, locates what human beings “of old, now, and always search out and find baffling” (Metaphysics, VII, 1). Learning to engage ourselves and others in the question “What is a being?” we learn to perform the craft of contemplation itself.

* Plato’s Republic, 508E f. In the story of the cave, while pursuing the image of the sun as the source of light, Socrates recurrently vacillates on the question whether we can see the sun itself. He does insist that the good conditions both the knower and the known, both knowledge and truth, and in that sense the sun cannot be directly envisaged.
§4.2 The inductive syllogism

The four forms of natural questioning will take on weight when we turn to the four corresponding forms of natural discovery. But how might “asking the right questions” structure the development of art and knowledge? Aristotle answers: In each of the four questions we search for a “middle term,” a focus on the subject that mediates the attribution of the predicate (in the third question: the presence). And every minor premiss suggests, structurally, a major premiss.

The horse appears to be sick (S is P). But I suspend this appearance the moment I ask whether it is really sick. I then determine more exactly that the horse is feverish (S is M). Yet this determination makes full sense, as a confirmation, only if All M are P. In deciding that the horse is indeed sick because it is feverish I may have introduced the wrong mediator (as I would likely do if I simply focussed on the color of the horse’s mane). Still, as soon as I suspend the immediate appearance, and re-focus on the detail, and propose a reason (an M), I am well on my way toward presenting myself fully to the being in its fullness.

What made the horse sick? The horse became sick because it had been drinking foul water (M). All horses drinking such foul water are bound to get sick. Again, the mediating focus may be wrong, but I am gathering up detail in a sequence, presenting myself to the whole horse.

The logical structure of the third question reads: S is (fully present) ... but is S (really fully present)? ... yes, because S is M ... then it would seem that All M are (forcefully present). Our question in contemplation is: What can count as an M here? We read in Plato’s Sophist (247E) that a being is its power (δύναμις), namely the power to effect the nature of something else, or to undergo such an effect, no matter how small and even if only once. In short, a horse is inasmuch as it is manifesting such power. But this manifestation of power is possible only inasmuch as I am participating in it (riding the horse, using the being, helping it function): παν τουτο οντως ειναι, “every such thing really is.”

The logical structure of the fourth question requires a slightly altered schema. This-here is a horse (has fully presented itself as such). What is this-here? It is D (a definiens). If D does account for the essence of the this-here, its destiny, then the enthymmetrically evident major premiss (All D are horses) is convertible — and we have a definition answering the question what horses are. Apart from the status of such an answer, we can see why Aristotle insists that we can only really ask the question after knowing that the being is. For we have to predicate “horse” (or centaur or god or human being) of a this-here (τοδι τι), and look to this-here-fully-present (τη ουσια) for an answer to the question. Lacking the affirmation of the this-here, we can only answer the question paleologically (namely, by drawing upon old stories). And lacking the full presence, we will look only to accidents (and formulate an erroneous middle term). Only the emergence of the measure in a present being allows for the emergence of a λόγος corresponding to what the being holistically is — allows us to answer the question neologically.

We moderns would still like to know what guarantees the movement from initial conclusion through the minor premiss into the major premiss. But there is no guarantee, neither for one who learns beings in production nor for one who learns human being and human community in action. In contemplation we ponder the possibility of the move, the destiny only rarely fulfilled.

Yet Aristotle does hazard, once, a formal statement of the upward move, what he calls the “inductive syllogism” (Prior Analytics, II, 23):

1. Settle upon an initial conclusion — (All) S are P.
2. Introduce an initial minor premiss — (All) S are M.
3. Intensify this minor premiss — only S are M.
4. Conclude the major premiss (from 1 & 3) — All M are P.

On a purely structural plane, this upside-down deduction works, provided only that (1) we affirm the initial conclusion (initial appearance) universally and (2) we find ourselves able to intensify (convert) the initial minor; a Barbara does the rest.

But as logicians we balk: How can one rightly convert the initial minor, an A-form proposal? Aristotle answers: “one must fathom (νοειν) the minor premiss as drawing together all that pertains to each being separately.” That is, the minor premiss must formulate such a radical insight into the subject that the middle term accounts for the entire core of the subject. Aristotle’s hasty example reflects the contemplative interest in nature that we now call biology: we notice that many beings (horses, mules, humans) typically live long lives; we then anatomize specimens of these beings to discover that they all have no bile (Aristotle must have anatomized only fetuses of human beings); if now we come to believe, with time and experience, that only those species we have examined are bileless, we naturally conclude a principle applicable to the genus: “All bileless beings are long-lived” (bile is a kind of poison, so
that lack of bile accounts for the possibility of a longer life: see *Parts of Animals*, iv, 2).

Still, the inductive syllogism is puzzling. Aristotle’s example suggests that we need only look to a specimen or two of each species, note a property common to all specimens (presumably all their species as well), and determine that the property is found in no other species (that bilelessness does not belong to short-lived beings in the same genus: fishes, snakes). But Aristotle is the first to insist that the “all” so essential to the development of insight (to fathoming a being) can never take the form of “all we have happened to see” (*Posterior Analytics*, 71 b 4). For how can we know from anatomizing a finite number of specimens of various species that M belongs to all of one kind and to none of the rest?

Aristotle exemplifies the inductive syllogism in reference to learning a species-genus relationship in the contemplation of nature: here, quantitative considerations do appear most compelling, even if tentative. However, the example of a youngster learning the species from dealing productively with individual horses illustrates the crucial juncture of the syllogism (the conversion of the minor premiss) much more convincingly. In asking each of the four kinds of question while dedicating himself to the horses, the youngster concentrates on primary beings: his subject is “this horse” or “these few horses.” And in looking again, more carefully and more fathomingly, the youngster rightly excludes other examples for the duration of his concentration: not only are these horses sick or well fed or fully present or really (οντως) horses, but only these are so. In old age, one may agree that such restricted affirmation, such exclusion, is madness (μανια): mere infatuation, or blinding love. For in some instances, especially those in which a distinct predicate is at issue (e.g., sick), the inductive syllogism leads to a false major premiss (not all beings drinking such water get sick).

But consider the third kind of question: Is my horse fully present (καλον: noble or beautiful)? Yes, because it is engaging me in its own dynamic doing and undergoing. And only my horse does this! The all-important major premiss, deduced in the Barbara-form inductive syllogism, reads: All beings engaging me dynamically in their doings and undergoings are (fully present). This major premiss may liberate me from the obsession with only these horses. But in contemplation we might well wonder whether we can ever arrive at a liberating major premiss without first dwelling erotically on one individual being. How else can one fathom the universal, except by fathoming individuals? How can we learn “human being” unless we begin by loving one as though he or she were the only real one? How can we learn community (family, team, city) except by presenting ourselves to the one that engages us, as though it were the only real one? Otherwise, we slide from one outside view to another, from one set of accidents to the next. We may need to break out of our obsessions with individuals, but we need even more to break out of our oblivion of them.

“By nature,” Aristotle says in conclusion, “syllogizing by way of the middle term is prior and makes most sense; but, to us, syllogizing by way of inducing the middle term is most evident.” Obsession with one being certainly breaks the indifference of opinion and routine. And if S is singular rather than general, the required intensification of the initial minor premiss signals not the exclusion typical of narrow-mindedness and complacency, but one essential to concentration and engagement. While “only women are capable of such and such” likely amounts to mere prejudice, “only my wife is powerful (engages me powerfully in her doings and undergoings)” has promise: if I proceed on this affirmation I may learn what it means for a being fully to present itself (All M are P), whereupon I may learn how to syllogize in a way that is “by nature prior and makes most sense,” viz. when judging and encouraging other human beings.

§ 5. Answering

We all share the same habit of conducting enquiry not toward the matter itself [προς το πραγματικον], but toward someone who says the contrary. And one even inquires by oneself only so long as one finds nothing to say against oneself. So it weighs on whomsoever is to enquire well that he stand up boldly to what properly stands forth in the field [το γενος]: this mode of enquiry comes from contemplating differences. *On the Heavens*, 294 b 8.

A verbal answer arises from a question posed by people (others or oneself), draws upon familiar or yet-to-be familiar formulations (received opinion), and intends to affect people (to facilitate a routine, to allay a fear, to create an impression, etc.). In contrast, a real answer arises from a question posed by circumstances proper, draws its formulation from the revelation of those circumstances, and takes one in closer to those
circumstances. A real answer is always neological (even if it takes us back to what has all along awaited us), while a verbal answer is basically paleological.

On the Aristotelian account, there are four kinds of real answers (mediators of S and P)—one kind for each of the four kinds of question. (1) The youngster learns to look closely at the present condition of the horse. Only in this sheer condition, his exposure to it, will he possibly find the answer to the question, “Is S really P?” This first kind of real answer is hyletic: the revelation of the material condition, the material cause in and of the horse, the detection of which assures the affirmation of P belonging to S. (2) The youngster must learn to recall the generating condition of the horse. Only in “seeing” previous conditions of the horse in the present condition will he find the answer to the question, “What makes S be P?” This second kind of real answer is kinetic: the revelation of the moving condition, the efficient cause in and of the horse, the detection of which already gives “invisible” depth to the affirmation of P belonging to S. (3) The youngster must learn to foresee the destined condition of the horse (in several, often strikingly different ways). Only while participating in the purpose (or purposes) of the horse will he find the answer to the question, “Is S really—in itself?” This third kind of real answer is telic: the revelation of the purposing condition, the final cause of the horse, the detection of which governs all subsequent involvement with horses (among all the imaginable uses and predications, only some henceforth appear appropriate to the subject itself). And (4) the youngster must learn to envision the horse in its completeness, its horse-ness. Only in striving to pull together all that belongs to horses, distinguishing between possible and necessary ways they appear and elicit response (the various material and efficient causes), between accidental and essential features of horses (in reference to the final cause); distinguishing also among the different kinds of horses (from Shetland pony to Belgian work horse, from fillies to stallions) and horses from near-horses (from mules and zebras), might the boy (or girl: likely now a man or a women) be able to detect and formulate real answers to the question, “What is S?” This fourth kind of real answer is eidetic: the revelation of the formative condition(s), the formal cause.

In productive relation to middle-range beings, the four kinds of real answers describe a reasonable agenda for the development of the intimacy essential to art and knowledge. Does not the would-be potter have to learn clay in all its variability (the material causes), the multiple maneuvers required to change the shape and consistency of the clay (the efficient causes), the wide-ranging purposes of pots (the final causes), and the differentiation of essential vs. incidental possibilities of pots (the formal cause)? And do not judges of pots (e.g., knowledgeable buyers) have to focus on each of the four causes?

The fourfold agenda of learning has dominated western thinking for more than two thousand years. Its emphasis on the paradigm of artisanal production lies at the basis of our passion for technology. Yet one might wonder whether the agenda fits equally well the learning of action and contemplation. As applied to the learning of leadership, it seems to require that we mould human beings the way potters mould clay or trainers mould horses. Is this the way we learn human beings and the formation of communities? And as applied to the learning of nature in general the agenda has been under attack since Francis Bacon—who rejected all but the fourth of the causes, and rechristened even this one as “law.” Do modern investigators learn about nature in consonance with the “matter-form” distinction? — But before one can fully appreciate alternative proposals one must fathom the original paradigm which such proposals aspire to modify, alter, or reject.

§5.1 Hyletic vision: the material cause

In one way, that out of which something comes to be, what is already there, is called a cause: e.g., the bronze of this statue, the silver of this bowl—as well as the kinds of these things [bronze generally, silver generally]. Physics, II, 3.

Letters are causes of syllables, wood is the cause of many utensils, fire and the like are causes of bodies, parts of wholes and premisses of conclusions: these are causes in the sense that they are “that out of which.” Ibid.

What is called cause is multiple ... one is the material, or what underlies [the subject, that to which accidents and properties belong]. Metaphysics, I, 3.

Many of the first philosophers considered only what is seen in material as the origins of all things. Ibid.
We consider ourselves to know something when we see its causes, and these are four in kind: ... one is that which is necessary for something to be at all. ... All causes are shown by means of a middle term. For nothing is necessarily so if only one proposal is grasped: at least two must be, and they must have one middle term [so that the material of a syllogism comprises two premisses sharing one middle term]. *Posterior Analytics*, II, 11.

If we are ever going to pass out of dealing merely routinely with pots or horses, human beings or human organizations, we will have to learn to wonder at, and to look at, what is simply there. I may be pleased or displeased with the meals I routinely consume in the school cafeteria, but I will only come to know meals if I start by asking whether the meal really is as I first determine it to be. I must look closer: the vegetables are fresh and moderately seasoned with parsley (or soggy and oversalted), while the meat is lean (or dripping with lard). In such wondering and looking, I attend to what is simply there, the materials out of which the meal comes to be, and I obtain a middle term that enthymematically confirms or overthrows the initial response. If I pursue this looking throughout the days I may settle into a holistic view of each new meal: No meals composed of wilted vegetables and fatty meat are pleasant (none such are good, I can now say, since immediate pleasure is no longer the criterion). A necessary condition for meals “to be any good” lies in the matière première.

What counts as material varies from project to project. The cook likely takes parsley to be material for the meals he prepares, but the farmer who grows the parsley understands water and soil and sun to be the materials out of which he cultivates the parsley. Similarly, a carpenter takes wood as his material, but the forester understands the trees as forms having other things as materials. The manipulator of human beings takes people as material, while the doctor treating individuals understands their muscle tissue as material. In contemplation, then, we can identify the matter at issue in any art as the potentiality of the being defined by its actuality: what might become a good meal, a healthy plant, an effective organization, a robust constitution. Material is the subject or substrate that we face immediately (“aesthetically”) and work up into something definite (*Metaphysics*, VIII, 1). But then we must determine the material of any given being by relating it to the form of the being: we understand matter in the category of relation (*Physics*, 194 b 8). From these considerations Aristotle draws the interesting conclusion that “those who contemplate nature must contemplate the soul as well, inasmuch as the soul is not without material,” i.e. inasmuch as we ourselves must take responsibility for what counts as matter (*Metaphysics*, 1026 a 5).

In Greek, the word we translate as “material” is ὑλή. Most narrowly, it means “wood”—be it what we build fires with, what we build houses with, or what we log to obtain these materials. The word then takes on the metaphorical meanings described by Aristotle, and covers anything we encounter as potentially something more. Thus, even the elements of speech (letters or syllables) are material which we work up with the pen or lips into something more; and proposals are materials for arguments: in the study of the trivium (grammar, logic, rhetoric) linguistic forms are the materials, and without careful attention to them one will never actualize human nature in communal enterprises (action).

The thinkers of the First Interpretation highlighted the hyletic conditions of beings, the four that appeared basic to the learning of any hands-on art: earth, air, fire, and water. Whether in farming or navigating, blacksmithing or potting, one must relate to the soil (including rocks and ores), wind (for breathing or for sailing), heat (from the sun, a forge, or a stove), and water (for drinking, for sailing, for irrigation, for hygiene). No one familiar with Greek myth can rest satisfied with the idle judgment that our ancestors were offering a merely primitive version of what chemists now consider to be the hundred or so basic elements. Rather, they were promoting a slightly secularized version of earth-and-heaven worship: something close to what we may call aesthetic materialism. Their voice addresses farmers and navigators, potters and blacksmiths: you may delude yourselves into thinking that reality consists in the affairs of the marketplace, but you owe everything to the soil, the wind, the sun, and the waters; to these you must basically attune yourselves, namely to the elemental presence of your circumstances: in this attunement you will discover the basic tensions and harmonies underlying those of the marketplace.

The same aesthetic materialism is still evident today in much early Greek architecture, chiefly in the locations of their theaters, temples, and towns. A person seated in the Theater of Dionysus on the southern slope of the Acropolis in Athens looks over the town to the mountain crests
and the blue sky; from the top of the hill of the north-facing Pnyx, where magistrates brought policies forward to the Athenian citizens, one looks south to the port of Peireaues and the vast sea beyond. The theaters at Epidaurus and Argos, and the whole Sanctuary at Delphi are constructed to command panoramic views of the four elements. Considerations of defense and disease might have played a part in the locations of such towns as Lato, near Aghios Nicholaus on the Island of Crete: a high-perched, now deserted town perhaps two hours by foot from the clearly visible harbor and sea below. But sensitivity to such troubles hardly accounts for the sensitivity to the immediate visual attunement with the earth and the sea, the sun and the wind.*

Aristotle and his successors accept the contemplative account of the four basic materials, but with an epoch-making proviso: in production and in action one must attune oneself to the relevant materials, namely to those which are in movement toward the telically envisioned form of the beings in one’s charge. Plato already argued (most famously in his Phaedo) that we cannot understand anything by simply accounting for the materials at hand. Production and action require of us that we learn to take responsibility for initiating the movements of crops and ships, of families and communities—that we learn the art of moving material into the form proper to these various beings. Still, the Platonic-Aristotelian tradition makes full sense only as a modification of the earlier Greek worship of aesthetic presence—the earlier aesthetic materialism resounding both in their literature and in their architecture.

Of course, both “material” and “materialism” are homonyms. And historically so, with the result that we likely find ourselves recurrently thwarted in trying to understand how Aristotle could raise the then-current aesthetic materialism into an ontological account of our commitment to learn beings in their hyper-aesthetic fullness.

Already during the Third Interpretation the philosophical understanding of the material basis of our condition appeared suspect. In the Old Testament the earth appears cursed inasmuch as our original forebears defiled their circumstances. Paul and Augustine interpret this curse as resulting from our ever-current propensity to love the creature more than the Creator—to cling to the givenness of some beings without regard for (even as a way of avoiding) the divine source of both ourselves and those beings. Yet to the pure all things are pure: our task is to learn to love all earthly things, but only as evidencing their Source—something we can do only if we gracefully accept our own createdness, our own creatureliness.

During the Fourth Interpretation a materialism evolved that is more familiar to us today: the givens of our circumstance appear as challenges to intellectual mastery. Bacon and Descartes call upon us to cease pondering the mysteries of divine origination and to begin experimenting with the presentations themselves. Nature becomes a resource, a source of “raw materials,” the basic varieties of which appear on the Periodic Table of the Elements posted in many classrooms. The development reaches a concrete climax in Karl Marx’s writings, where action (decision of communal policy) appears as based on industrial (no longer artisanal) production and as geared to the satisfaction of human needs (rather than to standards of nobility, much less to exigencies of holy salvation). Intellectuals today generally endorse the “dialectical materialism” envisaged by Marx, namely the heuristic principle by which we understand the evolution of social arrangements as based on the material conditions. Of course, few intellectuals endorse the additional claim that current industrial conditions will naturally lead to the overthrow of capitalism and the inclusion of all people in a universal community. In fairness to Marx, however, we must remember that his “materialism” was a theory of the basis of action, the goal of which was a new understanding of communal spirit.

Thales, Aristotle, Augustine, and Marx addressed those who were in hot pursuit of the materiality of their condition. Each interpreted the prevalence of ardent attention to beings in their givenness. In our age of consumerism, however, each of these Interpretations easily sounds as though it were asking us to take a stand in the conflict between spending our days at shopping malls and settling down to read books, or between satisfying our own private needs and donating energy and money to charities helping others to do the same—between “material” and “spiritual” concerns. Listening only through today’s ears, we not only distort the literatures of our tradition, we also lose sight of the starting point of all learning.

As Plato has Socrates remark in his Apology, artisans at least know something. Today we may think of a competent electrician, auto
mechanic, dog trainer, or forester. But how about ourselves in the Academy? Students of chemistry are made or unmade according to whether they “take to” the laboratory equipment: they learn the basis of chemistry by tending to materials, not in listening to lectures; the basis appears only where things present themselves to the eyes, nostrils and hands, not where the whithers and whences rumble in the ears. Before one can significantly look back to where things come from, or forward to where they are headed, one must learn to heed what is right there, in some sense even touch it. How else can one attend to the matter itself (enquire προς το πράγμα, and resist succumbing to rumor?)

Prior to the emergence of the welfare state, individuals automatically found themselves thrust up against their often harshly given circumstances. Learning appeared self-evidently as an affair of attending to the soil of the crops, the fire of the forge, the wind in the sails, the water for the horses — and as an affair of handling tools in exact response to the exacting demands of givens. Our own condition is nearly the opposite. As one eminent art historian has remarked (in defense of the visual arts in the university curriculum), we have become “so cut off from fundamental physical reality that we need now to learn how to get back to it” (and thereby to accept the “fundamental integrity of natural process — that is, in the broadest sense, the earth itself”):

So everyone in a university should have a drawing course, should be forced to focus, should be forced to see how the rock is made. Only by drawing it does one make physical contact with its reality. Only by drawing it does one look at it hard enough to begin to know it."

§5.2 Kinetic vision: the efficient cause

Then, too, the primary origin of change, or of rest, is called a cause. For example, the advisor is a cause, the father is a cause of his child. In general, the maker is the cause of what is made, the changer of what is changed. Physics, ii, 3.

The sperm, the doctor, the advisor, and in general the maker: every one is an origin of change, of halt, or of movement. Ibid.

I say, for example, that neither wood nor bronze is a cause by itself. Wood does not make itself into a bed, nor does bronze make itself into a statue: something else is the cause of change. To search after this cause is to search out another origin: what we may call the origin of movement. Metaphysics, 1, 3.

One cause is what gets something moving. ... Why did the Persians wage war against the Athenians? What was the cause of the war against the Athenians? The cause was that the Athenians, along with the Eretrrians, raided Sardis. For that first got things moving. Post. Analytics, ii, 11.

But what made the meal that way? Obviously, I can look to the cook. Or to the farmer who produced the vegetables and the beef. Or to the truckers who packed and transported the ingredients. Or to the soil in which the crops grew and on whose grasses the cattle grazed. Or, nowadays, to the fertilizers and hormones in some measure making the vegetables and meat what they are. The variety of possible answers illustrates the significance of the question: we relate an hyletically envisioned presence to factors which somehow were, but in a sense still are in the presence. Such factors have disappeared in their at-one-moment-present power and yet remain in the actuality of the present — as accounting for the movement into presence. If I will ever learn meals (how to prepare them and how to judge them) I shall have to sharpen my vision of “how meals come about.”

Yet a mere timetable of factors records only a sequence of shadows on the wall. Really to pass out of routine, I must locate more exactly the “primary origin of movement”: not simply the cook or the farmer, but what the cook or the farmer do. The primary origin of movement is not another being: here, it is art — the art of cooking, the art of farming, or whatever. What makes the meal the way it is, is the exact way of preparing the vegetables and the beef (perhaps also of procuring the materials).
Truly having learned humanly made beings, one recognizes the art that went into their production. At the race track, an automobile mechanic not only notes that a given car is performing well, he recognizes something of the *why* — and he detects more of this middle term the more he knows the make, and even more if he knows the individual car. Thus when Socrates and others proclaim the beauty of dancers in Xenophon's *Symposium*, they admit they are taking pleasure in the art of the dancing teacher who taught the girls (2.16 & 9.3).

But how can one learn to see *nature-made* beings kinetically? I learn that the scab forming on a wound is brought about by the blood coagulating upon exposure to the air, or that the fetus develops from both the ovum and the sperm. I learn that the rabies virus causes the vicious behavior and then death of a certain dog, or that acidic soil and high winds cause larches in the Canadian Maritimes to grow in stunted and twisted ways. In such cases I look not to art but to nature as an efficient cause: I come to see not human abilities but natural abilities at work.

In general, the two positive kinds of efficient causation are human power over beings and inherent power within beings: art and nature. In assessing events we have to decide first of all which of these two kinds is at work. Did the forest fire come about by the hand of man or by the hand of nature — by someone lighting a match or by clouds sending down lightening? Is the horse recovering from its disease by the veterinarian's application of medicine, or by its own powers? Did the wind blow the clothes off the line or did pranksters rip them off? Did the woman die of a blow on the head or of a heart attack?

Middle-range beings *are* in their movement. And we fail fully to encounter them unless we participate with them in their movement. Such seeing and participating requires the detection of either art or nature as the origin of the movement of the being — and therefore still in the being. It may be that we afterwards construct a sequence of discrete events (doings and undergoings of other beings, or conditions of one being at a variety of times and places). But kinetic vision itself must precede such construction if either one is to have much sense.*

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* Plato and Aristotle *assume* that efficient causes appear *in* the being. What is at issue is the movement of the being itself, not the interrelations among disparate beings. Xenophon has Socrates remark about dancers that their beauty comes out only in the dancing itself, when no part of the body is idle, when neck and hands and legs are working together (*Symposium*, 2.15 f.).

**Art or nature?** Human intent (possibly some negligence) or natural event (possibly an act of God)? At the beginning of Shakespeare’s *Hamlet* the ghost appears and announces that the origin of death (halt, rest) has been art, not nature:

Now Hamlet, hear:
’Tis given out that, sleeping in my orchard,
A serpent stung me; so the whole ear of Denmark
Is, by a forged process, of my death
Rankly abused; but know, thou noble youth,
The serpent that did sting thy father's life
Now wears his crown.

Hamlet’s agony henceforth stems not only from the loss of his father and his mother’s strange shift of loyalty, but also from the modified perception of the origin, the efficient cause, of present arrangements: his father will not finally rest, and neither will Hamlet, until the movement built into the present affairs of state becomes fully evident for all to witness. And it is his responsibility to bring the origin into evidence. The ghost begins to answer the question, “What made P belong to S?” and Hamlet must finish it. If only a legless serpent *had* done it!

And often we must learn the opposite as well. In Chapter 23 of *Huckleberry Finn*, Jim tells of the time he ordered his four-year-old ‘Lizabeth to shut the door: the girl just stood there smiling, and did not budge even after Jim shouted at her a second time; he then “fetch’ her a slap side de head dat sont her a-sprawlin’.” Shortly thereafter Jim learns that the girl was “plumb deaf en dumb”: scarlet fever has been the origin of her “just standing there, kind of smiling” — not the intention of challenging authority. Nature, not art. “De Lord God Amighty forgive po’ Jim, kaze he never gwyn to forgive hixself as long’s he live.”

Although jurors may have to distinguish sharply between art and nature in determining guilt and responsibility, the distinction is essentially blurred in the original space of learning. For in learning an art we learn to help nature: on that oft-quoted principle, both learning and knowing place us in the middle between how a being happens to be and how it needs to be. In the actual performance of an art, *both* art and nature are causes. Even when sitting in knowledgeable judgement on the sidelines of production and action, we might discover *both* causes lingering on in the movement, giving the being encountered its breadth and depth.
We today likely have trouble understanding efficient causation in the Aristotelian mode; or we should. Bacon already complained that, as commonly understood, both the material and the efficient causes are “slight and superficial, and contribute little, if anything, to true and active science.” We moderns would like to understand one being as causing another, and we search out laws, i.e., general descriptions of such sequences, to aid us in performing operations with them. Following Bacon, we understand the contemplation of causes as an effort to extract rules of operation for ourselves. On this scheme, neither art nor nature can count as “primary origins of movement,” at least not as Aristotle understood these words.

To learn to work with dogs or horses, wheat or spruces, one must still learn an art that meshes with nature. But we moderns ask: With what can art mesh? As Bacon already insisted, “in nature nothing really is besides individual bodies, performing pure individual acts, according to fixed law.” To mesh with givens so understood, we can only formulate the rules of operation to operate more effectively. There is nothing really to mesh with—at least nothing in beings. We investigate, discover, and explain the fixed laws—as befits, say, agribusiness. And without a nature with which to mesh, art itself has no hold. Artisans then appear as undeveloped (because banausic) businessmen.

Let us consider carefully the argument Aristotle proffers in his Parts of Animals, Book One, Chapter 5. Beings constituted by nature are, he says, of two kinds: the ungenerated, imperishable, and eternal ones (e.g., the moon and the stars) and the generated and perishable ones (i.e., all given plants and animals). The former we philosophers naturally find honorable and divine, and love most of all. However, we should also (so Aristotle argues) take a keen interest in contemplating the other kind, the middle-range beings. Why? Because we encounter them directly, we grow up with them (literally: we and they nourish each other), they are near to us and akin to our own nature, their multitude will always give us something to ponder, and we can find “untold pleasure” in doing so—since, being philosophers, we can make sense out of such beings by envisioning their causes. This last consideration Aristotle expands to clinch the conclusion urging us to contemplate the animals and plants with which we live “co-nurturingly”: we obviously take great pleasure in contemplating likenesses (paintings, sculptures, etc.), and we take such pleasure because we are co-contemplating their origin, namely “ demiurgic art” (the human art of bringing such likenesses into being); but it would then be “ fallacious and absurd” if we were not even more fond of contemplating beings constituted by nature herself—because (1) these are the originals of the likenesses and (2) in these originals we can learn to co-contemplate “demiurgic nature.”

In the Aristotelian tradition, art and nature appear in exact parallel: both are causes that must be learned (and such learning is most pleasurable—see also Poetics, 6). Nature here is not the mere collection of “individual bodies” describable by laws of behavior—an “it.” Rather, she works exactly like an artisan works: nature, too, is demiurgic, and all artful people must learn to work with her—the ultimate origin of movement, and of the beauty we learn to appreciate in middle-range beings.

In the Third Interpretation nature ceded her pride of place to God, the Source of all things. And during the Fourth Interpretation God withdrew from the world and left Man with the responsibility of initiating movement within the mechanisms the Source left behind. While in artisanal production we might continue understanding movement in a Greek manner, in modern contemplation we are stymied—and left with two spin-off look-alikes of kinetic vision that we must carefully distinguish from the original: lyrical vision and historical vision.

Lyrical vision we may define as the way we look at present things (a landscape, a person, a madeleine cookie—as in Proust’s work) whereby we find associated with the thing a whole sequence of our prior involvements. The landscape or the cookie occasions our vision of previous movement. The origin here is not that of the landscape or the cookie, but some other landscape or cookie. And even here the origin is neither art nor nature, but simply impressions (affections): we were affected by some circumstances, but somehow missed out, and now these affections return to thicken the presence of the present thing. In this one respect, lyrical vision very much resembles kinetic vision.

Resuming the topic of pleasure in the perception of beings, Henri Bergson describes the workings of lyrical vision, especially as we advance in age:

We too often speak of our feelings of pleasure and pain as though full grown at birth, as though each one of them had not a story of its own. Above all, we too often ignore the child, so to speak, that still lurks in most of our joyful
emotions. How many of our present pleasures, were we to examine them closely, would come down to remembrances of past pleasures? What would there be left of many of our emotions, were we to take them back to what we exactly feel, cutting out all that is simply remembered? Who knows, perhaps after a certain age we become impervious to joy that is really fresh and new, and the sweetest satisfactions of maturity might well be only the feelings of childhood once again revived—a balmy breeze wafted to us in ever fainter breaths by an ever more distant past? *Laughter, II, §1.*

Some of the best literature of the nineteenth and early twentieth centuries articulates the fullness of presence such remembrance allows.

Historical vision we may define as the way we investigate present things (institutions, rituals and customs) by searching out temporally prior troubles (undergoings) and decisions (actions) to explain present arrangements (postures) and dispositions (conditions). Articulating such precedents, we hope to reveal the inner workings of the present—a substance justifying their presence and perhaps suggesting a direction for the future. Now that action no longer obviously draws its substance from the formation of communities, historical vision offers hope.

In the Second Part of his *Democracy in America* (1840), Alexis de Tocqueville investigates historically the disposition of Americans. For example, why do Americans who meet each other by accident in foreign places speak with one another in a “natural, fresh, and open” manner and likely become friends at once, whereas Englishmen in like circumstances usually “stand so cautiously apart” and “converse with a constrained and absent air upon very unimportant subjects”? Because, we read in Chapter Two of the Third Book, although “the Americans are connected with England by their origin, their religion, their language, and partially by their manners,” they differ in one essential respect, namely in the way their nation came into being in the first instance. While Englishmen (in the mid-nineteenth century, at least) must still struggle anxiously to obtain or maintain social rank in a crumbling aristocracy:

In America, where the privileges of birth and rank never existed, and where riches confer no peculiar rights on their possessors, men unacquainted with each other are very ready to frequent the same places [professors, junkies, and janitors gather at the same bar], and find neither peril nor advantage in the free exchange of their thoughts.

Similarly, investigators look to the past to understand the present hatred of the Irish for the English or the present stability of Switzerland, to understand the present conflicts in labor-management relations and federal-provincial politics in Canada or the present harmonies at these levels in Japan.

At their best, both lyrical and historical vision share with kinetic vision the broadening and the deepening of the present: each time I walk through a landscape I see a whole life in it (*à la* Van Gogh or Proust), each time I deal with an American I apprehend political ideals of democracy at work (*à la* Whitman or Tocqueville). But neither lyrical nor historical vision has roots in the beings we encounter, and in this regard both differ radically from the detection of efficient causes, as Aristotle and his tradition understand it. We may even believe that the accounts of both Proust and Tocqueville bring only their *authors* to the fore—people who effectively project their own concerns onto cookies or Americans. The real “origin of movement” then appears at most alongside the thing occasioning our notice. Both look-alikes then degenerate into ways of *dismissing* the present: into mere lyricism and mere historicism.

Full-blooded kinetic vision enhances our commitment to, our love for present circumstances. For such vision places us at the origin and in the being (engages us in “demiurgic art” as artisans, or in “demiurgic nature” as naturalists). Lyrical vision can at most enhance our love, as Proust claims. And the historical vision of recent philosophers like Martin Heidegger can enhance our sense of predicament and commitment. But neither one places us at the “origin of movement” in beings themselves, at least not in the Aristotelian sense. At most, they articulate a kind of openness to beings in their Being—something quite different from efficient causation.

§5.3 Telic vision: the final cause

Then the end [τελος] is called a cause: that for the sake of which something is. For example, health may be the cause of someone taking walks. Why is he walking around? For health, we say—and consider ourselves to
have stated the cause. In addition, other things in between that contribute to the end are causes, e.g. for health there are excision or purgation, drugs or instruments: all these things are for the sake of an end, and may be divided into deeds and instruments. *Physics*, II, 3.

[There are two ways of considering what first moves things]: for example, Why did they go to war? Because there had been a raid. Or, For the sake of what? For gaining dominion. *Ibid.*, II, 7.

The cause of origination opposite to the origin of movement is that for the sake of which something is, i.e. what is good. For this is the end \( \tau\varepsilon\lambda\omicron\sigma \) of all becoming or moving. *Metaphysics*, I, 3.

That for the sake of which something is, is the [its] good. It arises within [belongs to] human acts and other things which are moving, and first gets things moving. For it is the end \( \tau\varepsilon\lambda\omicron\sigma \). *Ibid.*, XI, 1.

Then, too, the cause of something is its purpose. For example: Why is he walking around? For health. Why is the house? For the sake of dominion. Here, there is no difference between “Why must he walk” and “For what purpose does he walk?” *Posterior Analytics*, II, 11.

What is the meal supposed to *do*? What is the *point* of meals? The end, the good, the purpose of a meal determines my understanding, my learning of meals. For according to how I envision the purpose, I *choose* the materials at the market and I *work* them up with my art.

Before learning either to heed the material exactly, or to work it up carefully (let alone appreciate or assess the abilities or products of others), we in fact treat meals *solely* as means to an end — the end being extraneous to the means. Their purpose appears to be the satisfaction of hunger, i.e. relief from pain in the belly or from general physical unease. Such satisfaction or relief we call “pleasure.” In the penumbra, we glimpse most everything only inasmuch as it appears to render or impede a service of this sort: each shadow on the wall we interpret already as an adumbration of something to come, something that will relieve or aggravate our condition and that we might either pursue or avoid to some extent.

However, once engaged in the art, choosing materials and preparing the meals, we slowly learn to wonder about purposes doubly: we begin asking not only what the purpose of meals are *for us*, but also what the purpose of meals *in themselves* is — and learn that the second version of the question conditions the significance of any answer to the first version. Discovering, for instance, that the purpose of meals in themselves is really nourishment (rather than pleasure), I understand both them and myself much differently. A given meal will be *good* or not according to whether and to what extent it actually nourishes the body, makes us healthier (and not according to whether it happens to relieve pain). And I now prepare meals not only with an eye to the palate but also with an eye to the nutriments (balance and variation of proteins, hydrocarbons, vitamins, minerals, roughage, liquid, solid).

**Purpose**: what is pro-posed, put forward, as what the thing is *for* (*pour-quoi*, *per-chè*); on whose behalf, for whose sake the thing is (*war-um*, *wes-halb*). But what is the *status* of purpose? How do we learn the purpose of something? In what sense does nourishment belong to the meal itself and not to what the meal serves, namely the body (its health)? The question of purpose locates very exactly the crux of Greek ontology: how beings can emerge (or fail to emerge) into their full Being.

In the case of the final cause we once again focus on something supersensible in the beings we learn. The color of the carrots and the meat, their size, weight, shape, location may lie open to the gaze, and the master cook can point to them with his finger while talking about them to his apprentice. But the purpose — the inner power of foodstuffs the cook must learn to preserve and enhance during the preparation — defies such finger-pointing: it remains invisible, inaudible, unsellable, untasteable, and untouchable. Purpose names the supersensible *par excellence*, something the apprentice only hears about. Yet the master cook (today: the nutritionist) must somehow have learned it directly, and not merely from hearsay: the master can see the invisible within the visible. And of course many never become masters: they aim not at the good (health) but at gratification (approval) — a fact we find blatant in today’s fast-food industry and discussed at length in Plato’s *Gorgias*.

One solution is to suppose that purposes simply reflect human desires as these have structured both our perception and our construction of the environment: feeling thirsty, I desire water and eventually see water as serving the purpose of quenching thirst; I then dig a well, install
a bucket with a rope, and perceive the well, the bucket and the rope as serving purposes. On this account (that of the Fourth Interpretation) all purposes are strictly human—either mine or, in contractual arrangements, other people’s. Purpose names the objective correlate of subjective desire.

But can we really reduce purposes to human desires? The main line of defense against such reduction lies in the evidence of artisanal work: whereas children and fools may go after food and drink while basing their pursuit on their own desires and projections, one who knows meals—who prepares them with art—looks to the purposes of food and drink as embedded in the food and drink themselves. Children and fools know nothing, make no distinctions, take anything—and therefore depend on the art of others if they are not to ruin or even poison themselves. Children can reduce purposes to their own desires only because somebody else’s art is looking after them—looking to what is prior by nature as conditioning what appears prior to the children. The master cook knows, and the children might someday learn, that we must “redirect appetites rather than yield to them” (Gorgias, 517B).

Of course, once we do learn the innermost purpose of meals we can arrange our priorities. For instance, a cook may realize that meals also have the purpose of bringing a family together—or, as we read in Homer, Plato, and both the New and the Old Testaments, a whole community. Meals then become rituals in which individuals acknowledge the purposes they themselves must learn to serve. Or I may learn that the purpose of nutrition must bow to the purposes of impoverished students: as a cook, I must try to serve both the health of their bodies and the size of their pocketbooks. With an eye to the one purpose “prior by nature,” we can learn to appreciate the many purposes at work in concrete cases.

But how can we learn beings as having purposes in themselves? In contemplation we must view beings as they arise in production. For only here, in our prior engagement with beings, can we possibly follow Plato and Aristotle when they answer: a being is its function (εργον), reveals itself as it is only when drawing us into its function (only when we use it), and thereby proves itself to be its ability (δύναμις) to do or to undergo. Let us consider how various kinds of beings may display themselves in their purposiveness.*

* For discussions of the primacy of use, see Plato’s Republic, 601D, Cratylus, 389B, and Euthydemus, 289B. For the statement that a being is its power of doing or undergoing, see again Plato’s Sophist, 247E, and Aristotle’s

Consider tools. Within artisanal work we already recognize, criticize, procure, dispose, and improve tools with a constant eye to their purposes: that a hammer is emerges first of all in its function, as we are using it; and what it is becomes clear in what it can do or undergo, whereupon we become clear about how it is: too heavy or too light, the head too soft or the handle too long, or everything just right—for the work at hand. But while the beginner might just look to the work (to the extrinsic purpose), the master will also look to the hammer: because the master looks for purposes in the hammer he becomes effective in the work; because the beginner skims by the hammer, looking only to the result, he loses the work.

Even more obviously, the parts of any product are their services. The doors in a house, the hinges holding the doors, the screws holding the hinges, the doorjams holding the screws: each is what it can do and undergo, silently, as we go about our business, not noticing them unless we are making them or having trouble with them. The moment we do attend to them we assess each according to its purpose. The overall purpose is clearly that of the house, and to this whole we must look to foresee how powerful (dynamic) each part must be. Looking to the whole, we also look to the interest humans will have in it, including the desire to resist the onslaught of the weather (outside doors) or the passage of sound (inside doors). But in returning to the parts we work on them according to their purposes, not (at this point) according to our desires. Or, if we do look to our desires we simply kick the door, curse the hinge, bang in the screw—both losing the door and frustrating our desire.

Whole products like houses and automobiles easily assume the guise of tools once they are fully made. Yet when making them, and later when maintaining them, we discover them as they are—as organizations of parts. In making or maintaining such beings, we enter into their own directionality, their own purposiveness, their own “life”: we adjust our responses to their good—even if shortly thereafter we laugh and say that

Meteorology, IV, 12. To appreciate these thoughts, we today must overcome, first, the supposition that contemplation is an art uncommitted to the evidence of production and, second, the supposition that consumption provides the meaning of “use.” Our consumer society denies the “union of making and knowing how to use” essential to the argument (Euthydemus): today, we “use” everything, from automobiles and airplanes down to mayonnaise and hamburger without knowing anything about them, let alone making them.
we only adjusted ourselves to them so that they would better serve our own purposes. But then they vanish from sight and begin to deteriorate, whereupon we either come back to them again or pay someone to look out for them so that we can “use” them without knowing them.

And natural beings? Does a goat, for instance, have a purpose in itself? Again, we may believe that a goat serves our purposes—supplying us with milk and meat. But only children, fools, and other unfree people will proceed directly to the meat or the milk. Even a junior like Thrasymachus in Plato’s Republic will have to admit that one must first of all develop an art of tending to the good of the goats. Otherwise one will slaughter the goats immediately, and lose them—or take all the milk and destroy their progeny. The artisan of goats will have to learn the functioning, the inner directionality of the goats themselves: how to promote their own purposes, e.g. to allow the milk to serve the kids first of all, and to take only the surplus (as a recompense, so that the goatherd can continue helping the goats).

When Aristotle remarks (Parts of Animals, 1, 1) that “purpose and beauty” are more evident to us “in the works of nature [goats] than in the works of art [houses],” he must be contemplating these two kinds as they emerge in the concurrence of making and knowing how to use (in fully developed goatherding and carpentry). Certainly we today, taking the consumer’s standpoint, would claim that the “purpose and beauty” of a house is obvious (since we recognize immediately how it serves our purposes), whereas the “purpose and beauty” of a plant or an animal in itself appears inscrutable. But then today we are not really learning plants and animals. And those who do learn them appear strange to us—as when Farley Mowat claims (in Never Cry Wolf and Virunga) that one can only come to know wolves and gorillas by living with them, by adjusting oneself to their functions, dynamisms, and purposes. Aristotle could still say (ibid., 1, 5) that people in fact live with plants and animals, and learn them accordingly.

Action complicates the question whether and how the beings we face have purposes in themselves. For to be human is not simply to give an account (λόγος) of what we are doing: we must also plot our lives in general and choose details within the plot. We often try to concentrate on the detail, of course. But our concentration easily gets out of focus because we are not clear about the general plot—the kind of life we are trying to fashion for ourselves. As Aristotle says (Nicomachean Ethics, 1104 b 30), we cut out our lives in one of three basic ways: we aim for immediate pleasure, for long-range utility, or for nobility (τὸ καλὸν). At the meeting of these three roads we recurrently stand—so that three sets of detail loom ahead for us. The first two are purpose-ridden in an obvious way: in choosing a detail I am trying to satisfy either immediate or long-range desires. So long as I ponder only these two sets (and the conflict between them), action appears merely self-serving, and the beings I meet appear only as they serve or violate my interests. But when I embark on the third path (that of the senior) action appears as rendering service. And only then does action tie in with the standards of production: only then do I work for the good of human being—just as an effective shepherd must work for the good of the flock, or an effective navigator must work for the good of the ship. And to work for the good of human being is to promote the purpose inherent in human beings. To be sure, these are our purposes, but ones which we must discover—ones to which we must adjust ourselves, i.e. adjust our desires. The genuine leader (parent, teacher, governor) therefore not only knows the purpose of being human (namely, to be good at something, to promote the good of something), but also knows that human beings struggle at the place where the three roads meet: like any true artisan, he will help human beings along their proper way, i.e. help human nature fulfill its own purpose.

Actions of others we even name according to the purposes we detect in or ascribe to the agent: intentions. Is this woman ambling along the highway “taking a walk”? Yes (she intends to lose weight by exercising) or No (she is looking for the scarf that blew out of the car window). Many legal institutions recognize the significance of intentions when interpreting the recorded facts: Was that man hitting the woman “doing violence” to her? Yes (he intended to rob her) or No (he was defending himself from her attack, he was training her to defend herself, or the two were putting on a show to distract the bystanders).*

* Recent theorists like John Rawls (following J. S. Mill) have defined “rationality” as the ability of the agent to assess each choice with a view to long-range utility and to take into account contractual relations with others. A judge may acquit a person accused of the most atrocious acts—if the defense can show that the accused was capable only of acting hedonistically. Noble intentions then appear as odd instances of utilitarian intentions.
On the Second Interpretation, contemplation has a special commitment to telic vision. And not only to dialectical refutation of common views repudiating such vision. Aristotle recurrently asks us to contemplate the conditions of such vision (e.g., in his *Metaphysics*, especially XI, 1, and XII, 7). To become truly wise, he argues, we must learn beings in their Being. But to know beings fully we must come to know their origins: their causes (the material and the efficient, but now most crucially the final). To know the origins of beings (of meals, of horses) is a double event: to know them in *movement* (as coming to be and passing away, as changing, as temporal) and to know in them their *unmoving source* (what guides the movement, our assessment of them at any given moment, and our own contribution to them). To develop, in contemplation, this second half of knowledge we must be careful not to call upon the criteria of the first half. For the unmoving source of given horses does not itself have material, does not itself change, does not itself serve a purpose. Herein then lies the difficulty: while we must, in production, learn to see beings such as horses and meals hyletically, kinetically, and telically, we cannot rightly contemplate the *source* of movement in the same venerated ways. We must then wonder: What is the criterion of knowing the source, the Being of the beings we so obviously know in production according to the other criteria? After much discussion, Aristotle eventually answers: we must learn that the source of movement, not itself in movement, engages in movement both the beings we artfully know and our very selves. Unmoving itself, the Being of beings moves all else. Herein lies the criterion. We must learn that both human desire and human insight are *drawn* toward the Being of beings. We must learn that, while our passing appetites (wishes, inclinations, drives, attractions) are moved by what *appears* beautiful, desire becomes *steady* only when we allow ourselves to be moved by, drawn by, what is beautiful. Usually, we try to *alleviate* the draw. Wisdom consists in learning to move truly with the draw. Aristotle sums up the crucial point (of conversion, Plato says) in a grammatical distinction—in Greek, as a difference of “cases” expressible in English as a difference in prepositions (“of” and “for”). Although the fullness of beings, their Being, cannot be understood as itself serving a purpose (for we then have to look for yet another source), a distinction reveals that purpose is in what does not move: the purpose *for* and the purpose *of*. What does not itself move is the purpose of [other things and of ourselves], but not a purpose for [them]. What does not itself move engages [other beings] in movement by being loved. Other things [a given horse or a given ship, a given meal or a given knife] engender movement by being themselves in movement.

Here as elsewhere, love (Ἐρως) supplies the clue to the alignment at issue in human responses generally, and in contemplation especially. Only when we *rise* in love do we accept our position as moved by beings themselves (by their Being) rather than by the way they merely happen to affect us (by their shadows). Only in such acceptance do we open ourselves to the criterion by which we can “complete what nature is unable to finish, doing so while following her.” And only then do we cease trying simply to satisfy our own passing appetites (to alleviate the draw): as we do when only *falling* in love, running after shadows."

Following Aristotle’s distinction between unmoving and moving movers, we may distinguish full from partial telic vision. Full telic vision consists in attending to the unmoving within anything moving, and in acknowledging that the unmoving Being of moved and moving beings engenders the movement of these latter (including the performance of our own art). Thus the “origin of movement,” although initially understood as located in the art of an artisan or in the nature of a non-human being, appears ultimately as the unmoving Being of the beings, their “final cause.” Partial telic vision then consists in detecting within a given moving being an intent, a purpose beyond: focussing artfully on the meal, the hammer, the battle, we understand it as having the shape it has because it serves the “next” thing. In performing an art, we naturally tie one thing together with next. Only when examining the grounds of the tie might we be struck by the apparent incongruity of the “within” and the “beyond,” and wonder which metaphor most accurately expresses the location of the purpose.

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Plato’s *Phaedrus*, especially Socrates’ second speech on Ἐρως, expressly distinguishes between falling and rising in love. In Plato’s *Symposium* Socrates claims that he really knows only “erotics” (177E & 198D). Aristotle speaks much more pudically about Ἐρως than does Plato—as in *Parts of Animals*, 1, 5, where Ἐρως appears as a simile, and in *Prior Analytics*, 11, 22 (end), where Ἐρως is related to φιλία in an example of syllogizing.
On Aristotle’s account, telic vision will remain partial so long as efficient and final causes appear distinct. And of course they generally do so appear: the meal appears to be on the table both because the cook prepared it and because (so that) it can serve to nourish the body; the horse appears shod both because the blacksmith forged and attached the horseshoes and because (so that) the horse can fare better on rough terrain. Partial telic vision most obviously belongs to ordinary human dealings with beings. Full telic vision evidently requires a modification of human being, a modified installation into the draw.

When full telic vision fails, we easily confuse it with teleology, a look-alike we have inherited from the Third Interpretation. Teleology claims to account for the totality of beings: it rains so that the grass will grow, the grass grows so that the cows can thrive, the cows thrive so that human beings can survive, human beings survive so that they can realize Heaven on Earth. On this account, we come to understand each and every thing as part of a great chain of purposes. Teleology tells the basic story from the outside — even if it has allegorical, tropological, or anagogical meanings. Taken literally, it describes beings as a whole without accounting for the modification of human being essential to human learning. Such teleological vision does not spring from the τεχνη and the ερως Plato and Aristotle discern at the heart of all genuine production and action. As a doctrine, it springs from contemplation only.

In the respective understanding of chance and spontaneity we can detect some essential ways in which telic vision differs from teleology. Precisely in learning and performing an art we must cope with things appearing to have no purpose, often opposing inherent purposes. My horse steps into a gopher hole and breaks a leg, the handle of the frying pan suddenly snaps off, a tree falls across the path, the prime minister suffers a heart attack, the appendix of my friend bursts, policies enacted to increase productivity impair it instead, a plague devastates the city of Oran. How are we to understand these things? On the teleological view, we can only stand helplessly to one side and ascribe a greater purpose to them: nature intends to destroy the old to make room for the new, God intends to punish human beings to correct their ways, the World of Universals overrides the World of Particulars.

But for Plato and Aristotle the performance of any art not only takes us into a direct relationship with nature, it reveals the abiding possibility of failure. Indeed, in learning an art, and the nature on which art works, one first learns what really counts as a failure, both in art and in nature. In doctoring or horseback riding, skiing or piano playing, one starts learning when one begins to recognize exactly how one has failed: one is now in a position to proceed self-correctingly. Similarly, only in learning what a being really needs to be (its own innermost possibility) can one start distinguishing between apparent and genuine failings in the being. Just as a young girl may believe that her first menstruation, or a young boy his first erection, is a bodily failure of some sort, so later we might believe that a plant shedding its leaves, a cat shedding its fur, the body vomiting, or a muscle stiffening is itself a failure that should be counteracted, whereas in each case it might be a necessary byproduct we must let run its course. In learning the art of caring for the beings at issue we learn the real failures: infections, neoplasms, hernias — all instances where nature is not able to finish its own work, and where art might step in to help.

Unlike teleology, telic vision plunges us both into art and nature, and into the privations of these two: chance and spontaneity. Then, in moving erotically with the draw, with its necessity, we learn the material necessity of byproducts. Purpose is absent in all three of these: failed human effort, failed natural effort, and byproducts. In Aristotle’s discussion of bile in various animals we find an explicit rejection of teleology: “although nature may also make use of leftovers [fallen trees for mice, excrement for fertilizer], that is no reason for searching for purposes in everything.”

*Parts of Animals, iv, 2. For Aristotle’s discussions of chance as a privation of art and of spontaneity as a privation of nature, cf. Metaphysics, 1065 a 26 (they are accidental causes) and 1070 a 6 (they are privations). For a fuller discussion, see Physics, ii, 6. Aristotle does not always hold to the distinction between the two; sometimes chance seems to cover anything embodying neither purpose nor material necessity, and sometimes spontaneity appears as the generic term covering chance as well. Moreover, the word spontaneity (αυτοµατον, “automatic”) simply means “self-pushing”—and covers not only failures of nature to do or to resist (constipation, AIDS) but also any growth that appears to start from scratch rather than from kindred seed (as Aristotle believed that some insects are born not from parents of the same kind—the most natural and purposeful way—but from dung: History of Animals, v, 1). Much later “spontaneity” came to name the human ability to respond from scratch, i.e. creatively—the opposite of being an automaton!"
One reason for attending carefully to the distinction between telic vision and teleology is that all modern objections to “final causes” highlight the inadequacy not of Aristotle’s account but of the outside and overall story that grew out of that account. For teleology not only rationalizes chance and spontaneity, it also short-cuts both inquiry and responsibility. Teleology by itself erects a wall of “theory” protecting its adherents from concrete exigencies, especially those of action but eventually even those of production.

Consider, for instance, Machiavelli’s famous call to action: we are to understand virtù (Greek ἀρετή, English “excellence”) as an affair of facing fortuna (Greek τυχη, English “chance”). Machiavelli insists that genuine leadership consists in rectifying what actually happens, according to the exigencies of the moment—rather than simply bearing in mind ideal communities and assuming that actual happenings will naturally flow toward fulfillment. But conventional education, he says, has instilled into citizens and rulers the pseudo-virtues of contempt for actual conditions, of hope for paradise, of strength to suffer rather than to rectify injustices, of aloof contemplation of events rather than self-insertion into affairs. Above all, it has instilled the false belief that human achievements depend on fortuna (the inscrutable workings of nature or God) rather than on virtù—on chance rather than on art. In effect, Machiavelli notices (like Galileo, Francis Bacon, and other moderns) that teleology amounts to a story told to us at the bottom of the cave: even though the shadows coming and going on the wall appear “chancy,” there is a higher order outside—without human responsibility for connecting the two realms. In short, the interpreters of Christianity had promoted lethargy (ozio) rather than a drive for excellence (see Machiavelli’s Discourses, II, 1 & 2, and The Prince, 25).

When the moderns rejected teleology they also lost telic vision as a theme of contemplation. But Machiavelli still pleads for a return to at least part of the ancient view of human responsibility, and he often reads as though he were simply translating Aristotle. For Aristotle, too, claims that “both art and excellence always develop in face of what is more difficult” (Nicomachean Ethics, 1105 a 9). And:

Every art is about emergence (γενεσις). And to perform an art is to contemplate how things may emerge that may possibly be or not be—the origin of which is in the maker and not in what is made. For art is not about what is or emerges by necessity, or by nature, since these things have their origins in themselves.... And in a way art and chance are about the same thing, as Agathon says: “art is kin to chance, and chance to art.” Ibid., VI, 4.

The part on which Machiavelli agrees with Aristotle is the home of telic vision, the home that teleology ignores.

The Platonic and Aristotelian contemplation of purpose highlights the directedness inherent in any significant human encounter with beings. But the home of this directedness is precisely the “chaos” of the moment: the indeterminateness of immediate circumstances, the failures (accidents, privations) of human art and of encountered nature.

But as late-comers we must ask: Can we rightly consider purposes as belonging to beings separated off from our own artful involvement with them? We certainly determine, in speech, the purposes of a hammer or a house, of gall-bladders or horses. But can we understand our determinations as referring to something both in the being and separate from the encounter? Once we separate our determination of a purpose from the directedness inherent in the encounter, we seem to forsake the condition for its reference: immediately upon such separation, the purpose appears to express my own projection of the next determination. And the connection between one determination and the other (even between gall-bladder and horse) appears external, gratuitous, accidental. The intelligible basis of purpose henceforth appears to lie not in the being encountered but in ourselves. This shift of basis locates the modern propensity toward subjectivity.

Objectivity springs from the desire to understand beings as they are, but in separation from our own artful involvement with them. Subjectivity is its twin. Neither has any patience with final causes. Both stem from a contortion of the complexity of full encounter—a complexity appearing in the guise of τεχνη.

§5.4 Eidetic vision: the formal cause

Then the form (ειδος), or paradigm (παραδηγματικος), is also called a cause. This is the account (λογος) of what a being is meant to be, and its genera (e.g., the ratio two-to-one [states what an octave is meant to be] and then number
generally), as well as the parts that arise in the account. *Physics*, ii, 3.

In some cases the cause is what underlies (τὸ συνοικετυμένον), e.g., the parts of something, and in some cases the cause is what a being is meant to be (τὸ τι ην εἶναι): the whole, the synthesis, the form. *Ibid.*

The “Why?” of things not moving leads ultimately to “what the being is”—just as in mathematics, where one must ultimately come back to the definition of straight, commensurable, and other such things. *Ibid.*, ii, 7.

What we call cause is fourfold, but we say that one is the Being (οὐσία) [of the being], i.e. what it is meant to be. For the “Why?” leads to the ultimate account (λόγος), and that through which a being first of all is, is a cause and origin. *Metaphysics*, i, 3.

While the “what” does not emerge as a result of syllogism (or of demonstration), it does become clear in the course of syllogizing (or demonstrating); while the “what” of anything having a cause other than itself can never make sense apart from demonstration, there is not a demonstration of it. *Posterior Analytics*, ii, 8 (end).

What more may we learn about meals? What more than their materials, the ways to prepare them, and their purposes? Or about horses: What more than all the organic parts of the horse’s anatomy, the ways to care for and train them, their internal needs, environmental integration, and their external services? Surely one who can heed all these causes when dealing with such beings has learned the essentials—has learned to move with the draw of their Being.

Yet Plato and Aristotle detect one more exigency: once the nutritionist or the horse trainer can “produce and know how to put to work” meals or horses, i.e. participate in their teleically grounded movement, they can learn to account for the movement itself, i.e. the entirety of the draw. The account is human: it is something each of us must do. Yet the account reflects, belongs to, the Being of the being. As human, the account arises in thought and in speech: in a neological fluency. As belonging to the Being of the being (as ontological), the account reflects, brings out into the open, the glow drawing each being unto itself: glow (ἐνδοξ) and account (λόγος) are the same—they are the *cause* of each being of a kind. Indeed, now each being (each meal, each horse) appears as an “instance” of the glow, as a “that” simultaneously illustrating and indebted to (caused or originated by) a “what” as a “what it is meant to be” (τὸ τι ην εἶναι, abbreviated in a Latin coinage *essentia*, English “essence”). Only now does one stand firmly in the sunlight. Only now does one know what “universality” means (τὸ καθόλου, “bearing on the whole”), as distinct from an overall, over-bearing, pre-empting opinion.

And only now can one do full justice to instances, to individuals, to this-horse-here or this-meal-here. In learning the formal cause of horses, meals, or whatever, I most obviously become capable of recognizing and judging, making and using a wide *variety* of each species: I move freely and effectively in the variety of materials, of maneuvers, of purposes in evidence at various times and places. But, for all the variety, I learn to focus upon, to draw out and to draw near the glow appropriate to any given instance. Or, rather: I learn to gather the various particulars together while *following* the draw. The ability to follow up the unifying draw grounds the possibility of gathering the multiplicity together in given instances. I can draw out materials, draw up plans, draw near to beings in the stable or the kitchen, only because I am effectively drawn toward the beings—by their Being.

The formal cause is most evident in production and most urgent in action. Anyone who fully knows how to make and to use a kind of being—pots or houses, trees or horses, diamonds or wines—ultimately works within the purview of their glow, understands the glow as the origin of both the product and the art. And anyone who genuinely leads human beings in the formation of communal enterprise—parents or teachers, committee chairmen or university presidents—will stand as a “role model” for others. These others will be attracted (or repelled) by the leader’s glow (or privation of it). The glow, not the person, legislates, well or badly, long before the particular rules or laws get laid down—even if everyone pretends that the legislation awaits the decree or the vote.*

* Indeed, the Greek, Roman, and Renaissance acknowledgement of the prior legislation of the glow of human possibility coming *through* the leader allowed our ancestors to take seriously the roles of character and authority, even popularity, force, and pity, in public speaking and (enthymematic) argument.
In contemplation, both the evidence and the urgency of formal causation become issues. Even when engaged in production or action we “have no time” for the glow itself: we forgo the glow, in this special sense we “forget” it, let it do its work while we tend to this pot or horse, this act of formulating or enforcing regulations in this communal enterprise in progress. More: as Plato and later thinkers like Augustine and Kierkegaard and Heidegger emphasize, the draw toward beings leads us into temptation: we assume that beings themselves, as they already are (in αἰσθησις) do the drawing, have the drawing power. At our better moments, namely in eidetic vision, we realize that although we are drawn toward beings we are drawn by their Being. In our finitude we naturally forget this distinction: we attach ourselves to this or that being, identify it with the source of our own movement (we commit the fallacy of misdirected ερως: Republic, 476). Socrates, Plato, and Aristotle gave birth to the contemplative life by showing systematically how one can undo the forgetfulness of both sorts: how one can recall, call attention to, dwell upon human responses as glow-based. In Socratic induction we help one another recollect what has otherwise fallen into oblivion.

Imbued with the remnants of subsequent Interpretations, we today must strain to appreciate the vivacity of the original debates on the question of glows (forms, ideas). Against hefty opposition both from the defenders of the Homeric tradition recalling religious heroism and from the then-new Sages offering secular training in practical affairs (a curriculum tailored to juniors), Plato and Aristotle draw upon the then-current pride in the arts of production for evidence that human beings are what they are only in fulfilling a commitment to the perfection of beings lurking “within” these beings—or “beyond” them, but in a “beyond” to which the beings intrinsically belong, and into which human art helps them develop. They also draw upon the then-current ineptitude of secular leaders (their inability to form a stable city) for evidence that leadership itself requires an aptitude for contemplation: action, too, they argue, has its own commitment to glows. Against the defenders of the withering First Interpretation, Plato and Aristotle highlight human responsibility (rather than divine fate)—and appear akin to the Sophists. Against these latter, however, Plato and Aristotle argue that we fulfill our responsibility to beings by following up their ownmost draw—and appear akin to the Homerids. Eidetic vision issued from this vivacious warfare on two fronts.
In the works of Plato and Aristotle, the basic question hounding the consideration of eidetic vision reads: In what sense might we judge the ἕνδος of a being to be “separable” from the being (χωριστὸν: separable)? All along, we ourselves have assumed such “separability” — already in accepting the evidence that an artisan understands a this-here by understanding what it needs to be. And then the technical discussion of categories, of negation, of generality (both universality and particularity), of propositional and syllogistic form, and of the tricky distinction between para- and syl-logism: all require an “ontological difference” between given (finite) beings and their direction-giving (infinite) origins. Yet when we turn from the contemplation of finite dealings (e.g., with horses, or with the routines of schoolroom logic) and ask how we can, as philosophers, account for the difference, we run into a strange impasse: the glows seem to duplicate what we need to understand — even triplicate it (since we then need to understand the relation between the two). For instance, I must not only understand my wife, but also (it seems) wifehood — or even motherhood, womanhood, humanhood, animalhood (for where does the generic universality stop?) or facelessness, armlessness, liverlessness (for where does the material universality stop?). And supposing that I study such “-hoods” and “-nesses” (in the name of gynecology, obstetrics, psychology, anthropology, or physiology): How can I relate what I learn in my study to the woman who meets me at home?

Obviously, something is wrong. Like children listening to a fairy tale, we imagine different things of the same order (a being and its Being). We are construing both as finite. We are then left with, and plagued by, figments of our imagination. Contemplative discussion runs idly as soon as we drop production and action from view and start telling a tale all our own. For then an ἕνδος can only be an ἔνδολον, an image left over from previous encounters rather than the guiding star of encounters. At this point, the discussion had best be terminated, the thesis of glows refuted — as both Plato and Aristotle insist.*

* For definitive arguments against supposing “glows” as separate from beings, see Plato’s Parmenides, 130-135, and Aristotle’s Metaphysics 11a 3, 5, 7, 8 & 14, 10. When Aristotle says, “As for glows, that they are not is clear” (1059 b 4) we must carefully consider what the “are” here means: only beings really are... in their Being. Cf. also Posterior Analytics, 83 a 34.

How can we, now as philosophers, right the wrong? Only by assuming a position within the tension, a responsibility between the finite and the infinite. As in production and action, so in contemplation: only here does the ontological difference continue to make sense. Otherwise, and especially in leisure-cum-idleness, we foolishly aspire to a position outside the tension—a god-like position. Within and between, the focus of contemplation is then the human condition itself, our (and everybody’s) participation in the daily mix of the finite and the infinite, where each being appears as a mix of “this-here” and its “glow” (evident as potentiality). This participative position we can only rehearse in reading and writing and talking: an exercise in “thought thinking itself” (Metaphysics, 1072 b 20). Eidetic vision, so very akin to telic vision (which places us in the draw of the Being of beings) takes place only in stories of the human condition depicting not directly the Being of beings but the learning of the human position within the difference. Such stories appear as comedies inasmuch as the scenario allows for the humanly inevitable but deluded aspiration to learn something (whales, say) from a safe position on the sidelines (whereas “the only mode in which you can derive even a tolerable idea of his living contour, is by going overwhelming yourself.”). But ultimately they appear as tragedies: for in the end one must allow oneself to be supported by what one aspires to understand (“...but by so doing, you run no small risk of being eternally stove and sunk by him”—Moby Dick, 55).

Meanwhile, eidetic vision itself spawns two look-alikes which, already in the literature of the Second Interpretation, threaten to overthrow the original.

One look-alike of the original ἕνδος is clearly visible in the Latin translation of the word as species. Like the Greek, the Latin meant “glow, gleam, shine” — and could also mean “merely show” (as in “specious” and “idol”). But in modern languages today “species” means “fixed form” (of animals, vegetables, or minerals) of which individuals are indifferent specimens. During the Third Interpretation theologians gradually endorsed this look-alike, integrating it into the One Story of divine creation: God, the Source of all created beings (individual creatures) becomes also the Source of all species (types) of beings. He thereby establishes a transcendent and eternal Order of which each creature is a passing sign—an Order which, being the product of Divine Will, also accounts for the interrelations among different kinds of beings.
Neologism: Truth and Discovery

(rain for grass, grass for cows... ). On this Interpretation, knowledge (scientia) means the adequation of the soul to this Higher Order, independently of the Lower Order. In Plato’s image, the soul now dwells outside, while the body does penance inside the Cave. Any commitment we have to the instances of a species is only emblematic proof of our commitment not so much to their origin as to the Origin of species.

Perhaps because we generally read Greek literature through the eyes of the Third Interpretation, we find it difficult to distinguish eidetic from cosmological vision. However, eidetic vision remains a vision of the beings we deal with—it accounts for the unity (rather than only the duality) and the fullness (rather than the usual indifference) of the beings we encounter, one at a time. Eidetic vision grounds, on the “inside,” the possibility of transcendence: the possibility of knowing about horses in a way that does not confine us to the one horse or collection of horses we happen to be treating, training, or riding. But the transcendence evolving within eidetic vision, unlike the scientia that develops within the One Story of the theologians, remains a mere potentiality awaiting actualization: we know something actually only when transcendence bears upon a “definite this-here, a unique something.” The glow of eidetic vision is essentially a glow of an individual being, we can only know it in actual encounters (Metaphysics, 1087 a 15).

Or: the Third Interpretation aspires to a direct verbal account of the infinite, whereas the Second Interpretation leaves us with accounts of the finite which only draw upon the infinite. Originally, theologians understood that the Verbal Account at issue would have to be Divine—so that all human stories and sayings would have to reflect the One Story, or else lose their veracity—neologism being God’s, in all eternity. But something remarkable happened as the Third Interpretation ran its course and God finally withdrew from direct intervention in daily genesis. For then intellectuals assumed the task of detecting within passing instances the fixed forms (in their interrelations: laws) left behind by God. Thus modern science was born. One now bypasses the evidence (phenomena) of production and action and establishes an independent realm: calculation and experimentation. Without the intervening Verbal Account of Transcendent Order, Greek science could never have evolved into what we now understand as science: natural learning could never have engendered artificial learning.

Closely related to “species” is another look-alike: “number,” both cardinal and ordinal. Consider the act of counting. The youngster learning horses soon learns to determine how many horses are in the stable, the number of stalls, harnesses, saddles, bails of hay. In determining that there are two or three or four of something, the “of something” must mean “of the same.” The same what? Horses, saddles, and the like are beings. What is the same about the different horses? Ultimately only their “glow,” Plato says; their potentiality for an actuality, Aristotle says. But even “accidents” will do. Anyway, in counting beings the youngster has already related to sameness, and is currently mapping his learning out, i.e. fixing formally the arena of beings he is dealing with and recurrently understanding. Of course, children learn to count in the abstract, i.e. rattle off some finite portion of the natural number sequence; in so doing they construct sounds according to phonetic rules of the tongue. Actual counting entails identification: the youngster adds or subtracts (recognitions of) identities: “harnesses” (perhaps only old ones, or broken ones, or ones at a certain place) or “bails” (of hay for feed in planning for the winter, or of hay or straw in calculating storage space). Two apples and three oranges add up only by way of a generic identification: five pieces of fruit. Three horses and two stirrups per horse add up to six stirrups only by way of part-whole identifications.

And consider the act of ordering. The youngster learning horses will learn the “steps” to follow in brushing, feeding, mounting: first this, second that... And the “rank” of each horse: of three, this one is fastest, that one slowest, this one the youngest, that one the oldest. The ordering of beings—according to their nature, their properties, or their accidents—belongs to any art. Such ordering requires not only that we count identities, but also that we detect priorities in a pairing operation: not only do we count eight people (different yet the same) lining up for a taxi, but note that one is “first”, another “second”, and so on—where “first” is paired with “first taxi.” Similarly, one might order the bails of hay according to age or size, pairing each in a schedule of feeding or pricing. In our own domain of contemplation, ordinal numbering takes on special significance: we discuss the priority of eidetic glow over immediate givenness, as well as the priority (in λόγος) of actuality over potentiality, of the unmoved and stable over the moved and unstable.
Philosophy is nothing if not a concern for distinguishing “what comes first” from “what comes second.”

Numbering in all its forms is essential to coming to terms with beings. And in numbering we transcend beings in their immediacy: we relate to “samenesses” and pair each immediacy with a “beyond.” This transcendence comes into strong evidence in the exactitudes of calculation. We prove the Pythagorean theorem in a way allowing us to vouch absolutely for its result, for we have enacted the proof “all by ourselves,” i.e. “in thought” and not in reference to what we undergo (always variable, always out of our own control). And every right triangle we draw on paper, and every right angle in the house or cabinet we are building, henceforth appears as only an approximate, an imperfect instance of what we have contemplated in thought. In general, mathematics bears all the earmarks of eidetic vision — as a discipline of performance (not, of course, as a mere routine).

Or nearly all. Mathematical performance establishes distance at the expense of intimacy: it is a “dry run” which, as challenging as it is, easily waylays and bewitches us. Yet the “dry run” so much resembles eidetic vision that we can highlight the workings of the latter by thinking through the significance of the former. Perhaps that is what Plato was doing in his famous lecture “On the Good,” in which he baffle his audience with a meticulous discussion of the One and the Two.*

An appropriate translation of the Greek μαθησιν is “to learn”; and the broadest translation of τα μαθησικα, “mathematics,” would be “things (exactly) learned.” What does it mean to learn and what does one ultimately learn? We pose this question significantly only when we know that learning can be specious, or incomplete. In much apparent learning we merely absorb current opinion or habituate ourselves to some routine. A rhapsode (Plato’s Ion or today’s scholar) might learn Homer, i.e. to recite the texts, without getting into the realities at issue in such works as a whole. Or a Sophist might establish conviction in the soul of others without relating them to their individual and communal destinies. In such cases one stops short of complete learning, does not attain to an art. But in yet another way learning can remain incomplete, a way native to art itself: one backs off and “systematizes.” For instance, after learning music, e.g. how to play the harp, one can analyze the harmonies, as the Pythagoreans did. Although the best musicians might take some interest in, and derive some benefit from these systems, one can also make a profession out of systemization — rather than a vocation out of music. Similarly with measuring land or building houses — or, nowadays, managing a business: surveyors, architects and businessmen have their own tasks, being-based, but their arts can also be systematized, laid out in a rigorously defined format of “things to be learned.” In short, mathematized. Unlike polymaths, rhapsodes, and sophists, such “mathematicians” have indeed learned an art in which rigor counts above all, i.e. the ability to vouch for each and every determination in reference to clearly, previously adopted principles — rather than in reference to the affairs of the moment. Rather than ad hoc.

But in one of its two dimensions, real learning, learning that runs its course toward the real and its reality, is precisely ad hoc. Genuine inquiry, Aristotle says, is προς το πραγμα — a λόγος directed toward the “outside” matter rather than the “inside” difficulties of the λόγος itself. Whether in building a cabinet or coaching a team, in climbing a cliff or playing a piano, in reading Greek literature or writing essays, the crux of the learning lies not in recognizing and enforcing generalities, but in detecting and acknowledging the singular, the this-here, the πραγμα itself. This strange detection-acknowledgement marks both the master of a craft and the making of a moment as awe-inspiring (δεινον) and wonder-begetting (θαυμαστον). Occidental thinking may have sprung from the insistence that human being (human art) requires the systematic development of transcendent determinations of beings (subsuming individuals under describable species, counting and ranking them). But until the rebirth of the West in mathematical science, second-dimension knowledge, knowledge-at-a-distance, appeared only as potential, as merely necessary material awaiting the making of a moment,

* For the literature on this lecture, see Konrad Gaiser’s article “Plato’s enigmatic lecture ‘On the Good’” (Phronesis, xxv, 1 [1980]). Aristotle employs the same language in his Metaphysics, e.g. at 1081 a 14: “For number is from the One and the Dyad without limit.” In his Problems (XXX, 6) Aristotle asks why human beings are so much given to following orders given by others. He proposes three possible answers: (1) because only human beings can number (count, rank, deal uniformly with variable instantiations), (2) because only human beings believe in gods (acknowledge their debt to the infinite), or (3) because human beings are most imitative (take upon themselves the task of learning). Perhaps the three go together; in any case, Aristotle ascribes the first to Plato.
an actualization in which we truly know. The learning of the formal cause of something, of what the being is meant to be, its glow or “idea,” its Being or “substance,” takes place only at such moments and never as a self-wrought accomplishment — only in the course of reasoning and bringing into focus (“syllogizing” and “demonstrating”), and never as a result.

In comparison with eidetic vision, the dry runs of mathematical vision appear for what they are. Aristotle says: working purely with numbers and figures (arithmetic and geometry) deprives us of being in their Being (Metaphysics, 1073 b 8, & XIII, 1-3). Plato says: purely mathematical work proceeds from, never towards the Being of beings (Republic, 510 B ff.). Yet the affinity remains. Indeed mathematical vision so much approximates eidetic vision that Plato argues that in the education of those destined to leadership roles mathematical studies are necessary: not only do arithmetic and geometry plant one squarely above, liberate one from, the flux of encounters; such work protects one from the smart-alecky of adolescence and the attendant cynicism of middle age (Republic, 522 C). Only later in life (at 50 years of age?) can one embark upon pure contemplation: one then learns that how mathematical exactitude cannot rightly be taken as the standard of insight either in the study of human destiny or in the study of non-human nature (Nicomachean Ethics, 1094 b 13-26, & Metaphysics, 995 a 15-19).

Meanwhile, the difficulties arising in contemplation should not distract from the proposal that production, and possibly even action, already develop in the purview of glows. Plato and Aristotle argue so intensely for the then-new vocation of philosophy, and for its relevance in leadership training (action), that their successors have always been tempted to construe eidetic vision as an issue born in and confined to intellectual work (e.g., biology and mathematics, and finally philosophy). Most originally, however, we philosophers raise the question of eidetic vision with a view not only to our own vocation but with a view to any human vocation.

§6. Defining

A definition is an account (λόγος) of an essence (τὸ τι ἐίναι “the what it was to be”). Topics, VII, 5.

It is not enough that a definition formulate the way things are; it must reveal the cause. On the Soul, ii, 2.

Each being is defined by its purpose (σκέλος). Ethics, 1115 b 23.

A definition is of a universal, i.e. a glow. Metaphysics, 1036 a 28.

A definition is an account [emerging] from differences or, strictly speaking, from the last one of them. Ibid. vii, 12.

From a definition, i.e. the account, [emerges] an origin. Physics, 200 a 35.

There is no definition of that of which there is a demonstration. For definition is of what a being is, i.e. its οὐσία, and all demonstrations presuppose and take for granted what they talk about. Posterior Analytics, 90 b 30.

Since definition is said to be an account of what a thing is, it is obvious that one type will account for what a name or phrase means, e.g. what “triangular” means. ... Another type of delimitation is an account that reveals why (by what) a thing is. Whereas the first type means but does not show, this second type is like a demonstration of what the thing is. Ibid., ii, 10.

In learning beings, we naturally learn words. For instance, a younger attends to a horse giving birth and simultaneously hears his elders conversing about the “mare,” the “filly,” one thing and then another doing and undergoing this or that in such-and-such ways: and eventually the younger participates actively in such conversation. Or, the younger hears the coach talk about the proper way to “dribble,” to “pass,” to “shoot” while actually performing these operations: eyes and then hands on the ball, the younger will not only dribble, pass, and shoot, but also converse freely in the medium of such words as “dribble,” “pass,” and “shoot.” Most naturally, words take on meaning as we learn to respond to beings — as we learn the beings. The meaning of the words is our manner of response.

But we also ask what words mean. For instance, in fireside conversations about horses or games, one person utters a word strange to
another: the latter might then interrupt to enquire about its meaning—in order to keep up with the conversation (a “filly”?—just a young female horse). Or I may have heard and even spoken a word for years (say, “welsh”), and suddenly wonder what, more precisely, it means (not simply to “cheat,” but more exactly to “avoid paying debts incurred in gambling operations”). In either of these two cases we might just as well consult a dictionary: we are asking what people have generally meant in using the word.

When establishing or conducting communal enterprises we must often decide on the meaning of words. For instance, on the meaning of “doping” (when judging eligibility or achievement in athletic competitions). Or the meaning of “honors-level work” or “full professor” (when judging academic performance). Or the meaning of “courage” (when praising or blaming the deeds of people facing danger). Of “beauty” (at horse shows or art exhibitions). Of “murder” (as distinct from the killings called “manslaughter,” “accident,” or “self-defense”). Once our community has decided on the meaning of such measure-related words, we must look to documents rather than to dictionaries for their meaning.

When answering a question about the meaning of a word or phrase we may offer a definition. And here we can ask a question essential to our own domain of logical inquiry: What does it mean to define?

According to Aristotle, Socrates was the first to engage in “inductive λόγος and defining universally.” The second achievement culminates the first. Socrates engaged himself and others in a kind of leading speech, a “conversational learning experience,” which aimed for “defining universally” what (being and its predicates) the speaker knows. An inductive conversation refocuses, recollects, questions previous and technically competent encounters with beings: the finishing touch is a special kind of λόγος, one that defines... not what words mean, but what we can mean once we know what some being means.

The Greek for “definition” is ὄρισμα, from which stems our “horizon.” Aristotle sometimes says ὥριον: “limit.” Most tangibly, an ὥριον is a boundary stone. The usual meaning of the Greek verbal form is: to fix the limits of, to separate out one unit as against another (as in boundary disputes). Plato and Aristotle obviously stretch this everyday meaning into the domain of purely linguistic responsibility: in the new vocation of θεωρία, Socrates and his followers strive to mark out very carefully in words what exactly we can best mean by a word. At first, these definitions were always “ethical”: they focussed on the norms of human conduct and potentiality. In Plato's Republic we read toward definitions of courage and moderation, wisdom and justice. In his Statesman and Sophist we read toward definitions of genuine leadership and phoney oratory. Then, especially with Aristotle's works, definitions mark out domains of non-human nature: astronomy, biology, and the like (as the young boy fascinated by life in the tide pools might someday become a marine biologist and write a book bringing each form of tide pool life into focus—defining each universally).

Whatever else a Socratic definition may require of us, it marks the culmination of a conversation—and of our learning. It is neological. In this one respect it differs remarkably from ordinary definitions, i.e. those which serve primarily to allow us to get on with the conversation. In order better to keep the two distinct, let us consider the workings of the ordinary.

We normally define a word by miming the original learning experience. If the boy asks me what “filly” means I take him out to the stable to show him one and I talk to him about it. In the first instance, I point over to the filly (“That's a filly,” I say). In the second instance I reduce the word to other words familiar to the boy (“It is a child of the mare; it is young, and it is a girl,” I say). And I likely go on talking and pointing things out around the filly. Or, asked what “garish” means, I fetch a suitable shirt and say, “Like this: loudly colored”—and perhaps go on to talk about different color combinations, pointing to other garments.

Defining ostensively and reductively works well enough to keep the conversation going. Yet when we break down these two defining processes we might well wonder how they work.

Consider ostension. In pointing towards something, how can I help another to concentrate on the “right thing”? The farther I stand from the filly, the more kinds of things come within the scope of my gesture (mare, hay, ...), while the closer I stand the more detail (the mane, the croup, ...). Moreover: Do I mean a genus (animal, mammal) or an accident (domestic, small)? And my interlocutor might even understand “filly” as a proper name I have given to my pet: Filly? Or the reverse: I point to my dog and say “Fido,” and a foreigner assumes that “fido” must be English for chien, cane, Hund. And we all experience difficulty
when trying to “point to” sounds (in the woods or in a concerto), tastes, smells and even textures (I say, “This is silk” or “This is smooth”: which will my interlocutor pick when he touches it?).

And consider reduction. What do we achieve by replacing a familiar word with an unfamiliar one? Phonetic or graphic brevity may be desirable. Perhaps “garish” means neither more nor less than “loudly colored,” so that we can prefer the shorter version as more economical. And acronyms like NASA or foreign compounds like “octogenarian” we may define reductively with no semantic loss — so that we can return to the shorter versions on grounds of taste or convenience. Such definitions amount to synonyms — alternative names for the same thing. If the new word is to name something new, the reduction must remain incomplete — as in an analogy: A filly is like a baby-girl horse. Here one must discover the similarity and discount the difference. Perhaps an intellectual would prefer to define “filly” as “a recently born female offspring of a horse”; but such highly abstract definitions serve mainly to display intellectual virtuosity. Precisely those who deliver popular lectures on technical subjects must master the art of defining the relevant words analogously: those who fully know the subject will notice how misleading such definitions can be; yet those who know little of the subject can follow the lecture on a parallel course, borrowing some comfort from their familiarity with other subjects. Still, just as synonyms are uninformative, analogies are inaccurate.

When ostension and reduction work well — to keep a conversation going while also revealing something new — they work together, each undoing the shortcoming of the other. Talking about fillies while pointing one out, I help the youngster get the subject straight, and pointing out an example while enumerating familiar predicates I help the youngster open out to the newness at stake.

The benign complementarity of ostensive and reductive definitions highlights the temporality of the conversations in which they occur: we are moving toward beings while learning new words (we are creating a future); and the eventual meaning essentially grows out of a prior ability to relate to some beings in some words (we lean on a past). Ordinarily, then, we simply keep moving forwards.

But now Socrates asks us, especially those of us about to engage in action, to stop — even to move backwards. In the new vocation of contemplation, he asks us to recollect our encounters with beings, re-integrate “what makes sense already.” Why? Partly so that we may pass from obscurity to clarity of issue — just as when we talk or write about something familiar in its multiplicity we suddenly see its unity. The formulation of such insight is a definition. Not one that helps us merely to get on with the conversation. Rather, a Socratic definition is like a pilot’s license: it enables one to steer forward — into safe havens. And, like a pilot’s license, it is inherently non-transferable. Socratic questioning may help another, some author’s book may help a reader, to refocus on the being at issue, to formulate the lucidity. And a speaker or a writer might even do the refocussing and formulating as though others were questioning him. But any formulation, again like a pilot’s license, can expire.

Following Socrates and his pseudonyms at work (in Plato’s dialogues), we hear emerging definitions of courage, temperance, wisdom, justice, community, love, death, leadership, sophistry. The definitions of interest are all normative, positively or privatively. Like the IOC’s definition of “doping,” Socratic definitions lay down (at the end, however) what the words can mean (for license holders: what they shall mean — as beacons). And although what each of these definitions defines appears to fall under categories of condition or posture, action or affection, each actually defines human being itself: they reveal norms for the fulfillment of our own nature. Thus we have the ability to follow Socrates at work, since we in fact already make some sense of the subject at issue.

Formally put: a Socratic definition issues from, incites us toward, a vision of an essence, namely our own possible perfection, as lurking “in” or “over” each of us in our communal condition (as leaders or as followers). If we aspire to formulate a definition, or understand one that another proffers, we must not only “look” to the essence, we must move within its draw, toward it as the completion of our own being. As Plato’s dialogues often illustrate, one must design a philosophical definition in such a manner that it makes no real sense apart from such looking-in-movement. For if a Socratic definition could make real sense apart from...
our own participation, it would pre-empt the reality at issue, and therefore ourselves: everything could make perfect sense beforehand, as though we were gods.

Following Aristotle at work, we hear definitions not only of human being at the level of action (in his Ethics, Politics, Poetics, and Rhetoric), but also of non-human beings as they present themselves at any level and now in the new vocation of contemplation (in his Physics, Metaphysics, On the Soul, and so on). How are we to understand time? place? change? becoming? ceasing? sensation (both in humans and in dogs)? Indeed, some of Aristotle's works ask us to contemplate what we would nowadays call “purely natural” (as against social, human or supernatural) phenomena (e.g., Meteorology, II, 3: Why is the sea salty?). All such discussions and eventual definitions are categorial: they often appear to draw into focus what time, place, change (in quality, quantity, posture, condition), sensation (passion, action) mean. Yet Aristotle also insists that definitions are fully possible only in regard to the first category: in each case, we must define some “being in the secondary sense,” i.e. what a being is destined to be. And he illustrates this point in each of the discussions leading to a definition. For example, time means what it means because individual beings are essentially underway toward (or falling away from) their fulfillment: other meanings derive from this central, being-based one. In general, we truly (Aristotelianly) understand the nine predicables of being only in the process of truly understanding beings in their Being. And conversely, if we wish to understand non-human beings (e.g., the change in carrots or the saltiness of the sea) we may employ distinctions drawn from the nine predicables (often straightening out confusions by making categorial distinctions—as in On Becoming and Ceasing, 319 a 9-15).

In Aristotle’s categorial analyses we find emerging something like a theory of meaning in general, one intended (no doubt) to undergird the Socratic conversation aiming toward “defining universally.” Stated in somewhat modern terms, the theory might read: to understand the power and function of any element in the spoken or written language (from nouns to particles, tenses to moods), we must understand them as serving the human effort to focus on beings. Thus, on this theory of meaning, (1) concrete nouns obviously rank first, while abstract nouns (justice, speed, color) play grammatical tricks on us. Already with (2) verbs like “running” we shall have to decide on the being which undertakes this action: are we talking about noses, waters, clocks, horses, or athletes? To understand the meaning of a verb I just first recollect the being at issue—and allow the verb to vary in meaning according to the being we focus on. Most adjectives (e.g., “carnivorous” or “moribund”, “deceitful,” “fallow,” or “phlegmatic”) were long classified with verbs—much like present and past participles used adjectivally—since they indicate what beings are (or could be) doing or undergoing. Then (3) qualities like “green” we can understand only in reference to something like a stoplight or an apple, where the accidental nature of color in general may evolve into a property of an individual. On the other hand, a quality like “beautiful” may remain accidental to the being (this horse makes me feel good), or it may evolve as a sign of the essence of a being (the horse is beautiful because it shows what a horse can be). Obviously, (4) prepositions like “in” take on power according to what we are trying to understand: a person being in court, in trouble, in a coffin; a god appearing in time or God being in creatures. Perhaps (5) moods and tenses “mean” something about us in regard to what we are talking about: “the wood will burn if we light a match to it” indicates our understanding of factual possibility (of the wood, but also of our selves), whereas “the house would have burned down if we had not quenched the match” indicates our understanding of a counterfactual possibility. “I insist that you be on time” indicates a recognition that what is necessary at one end of the discourse remains still only a possibility at the other end. Tenses, conditionals, and subjunctives appear in some languages as powerful ways of articulating the human drama of intimacy and distance: in German, for instance, one employs the subjunctive mood to embody a modal difference when reporting what others have said. Finally, (6) the differences in articles, cases, pronominal adverbs, and the like subtly articulate the focus (e.g., only in the nominative case does a concrete noun announce the focus).

Definition becomes relevant when we recognize that we learn to speak and listen, even to read and write, long before we learn the full meaning of words. Children begin, and adults often continue, by learning

* For an overview of the gradual evolution of alternative “theories of meaning” (culminating in the Second and the beginning of the Third Solution, as sketched out at the end of Book Two), see Bibliographies A and B in my Linguistic Responsibility (1988), especially from p. 383 (Aquinas) to p. 387 (Saussure).
to move within conventions. A real definition (Socratic, Platonic, or Aristotelian) then represents an effort to cut through the conventional use of words to reground them in nature.

§6.1 Essences: genus and differentia

On the surface, the procedure of an Aristotelian definition requires simply that we state the genus of what we hope to define, and then work out the way or ways it differs from the other beings in the genus. Thus:

A chair? A kind of furniture for human dwellings, but furniture intended for sitting (unlike items intended for eating or reclining or hanging up clothes) — intended for one person, and movable (unlike benches and built-in arrangements).

A horse? An animal (unlike earth, air, fire and water; also unlike stationary living things such as carrots and trees, and unlike all man-made things such as chairs), a mammal (unlike chickens and lizards), usable for riding (unlike dogs), and allowing us to do so in style (unlike donkeys and oxen).

A piety? An action of human beings in response to indebtedness, but unlike other forms of justice, grounded in the superhuman (cf. Plato's Euthyphro).

In formulating a definition, we start with a genus, proceed by making distinctions within the genus, and end with an essence. But how can we really “find” a genus? How can we properly “make” distinctions? Only if we answer these two questions fruitfully can a third become more than rhetorical: How can a definition culminate in an “essence”?

The genus of “gold” appears to be “metal” or “element.” That of “chair” appears to be “furniture.” That of “piety,” “justice.” In the classical definition of “human being,” the genus is “animal.” A genus appears, then, to be simply a “larger kind,” one in which there are other instances: along with gold, there are silver and lead; along with piety, there are shrewdness, liberality, moderation, and courage. Along with humans there are horses and dogs, crocodiles and oysters. Nowadays, we are accustomed to classify each thing more thoroughly into its kingdom, phylum, class, order, family, genus and only then species (not to mention sub-species, etc.). So what is important about, indeed what is the meaning of “genus”?*

In the case of “human being” (and what do we more urgently need to define?), the classically proposed genus is revelatory: we (along with horses, dogs, ...) are “generically” animal. Unlike that of plants, our life is one of perception, αἰσθησις: we take note of the cold of the wind and the hardness of the road, the color of the sky and the sound of the wind, the smell of the fields and the taste of the apples; and according to what we note, we engage in movement, κίνησις: both intransitively (we proceed through the countryside) and transitively (we push the boulder aside, put on a jacket, throw the rotten apple away). In comparison with animals, plants simply root themselves in their elements and are moved by them: carrots in the earth and by the sun, algae in and by the water. Animality is essential to us: take this our genus away from us and we are no longer human beings at all (on the classical view), for we then either regress into plants (as when brain-dead bodies keep vegetating with the help of high-tech life-support systems) or revert to Adam (dust to dust).

In the case of “human being,” the genus suggests the necessary basis of re-emergent being. This basis the individual of the eventual species (you or I!) can indeed lose, but to his or her destruction. However, this necessary basis does not suffice: the individual “has λόγος, i.e. must actualize and re-actualize this “difference” — and come into his or her glory (εἶδος) when doing so. When sleeping, flying into a rage, or merely stuffing oneself with food and drink, one falls back onto one's generic being, retaining (mysteriously) only the potentiality of the glory. In the Aristotelian sense, the genus is the “home base” of the dynamism and the energy so much at issue for the individual of the species.*

In the case of “chair” (an artifact), the genus “household furniture” also serves to name the basis for the chair to emerge as a chair. What would a chair be without “furniture in general”— tables and beds, pots and pans, forks and spoons, plates and cups? A chair becomes a chair when distinguished from, because related to, these other beings in the genus. And, just like human beings, it often does not emerge — whereupon it sinks back into indifference, merely more clutter in the room, or turns into a makeshift stepladder, or even vanishes into

* In the archaic, Homeric sense, τὸ γένος meant “race” in the sense of the “clan” or “tribe” or “family” to which one belonged and which formed the basis one represented on the battlefield of life — and in the name of which one aspired to distinguish oneself.
firewood. The genus locates the potentiality of the chair, the condition for its actualization (ἐνεργεία).

Similarly with the qualities, conditions, postures or actions of special interest to human beings. For instance, Aristotle argues that qualities which we attribute to human beings inasmuch as they visibly excel (courage, justice, liberality, moderation, etc.) are generally conditions of our invisible Being (our souls), ready-nesses for choosing what is appropriate for each moment in the exercise of our various arts, readinesses also to avoid the shortcuts and retreats (based on mere pleasure or mere advantage) always tempting us away from noble choices. The genus of any excellence in human affairs (action) is then a special condition (a disposition to choose), out of which the individual can emerge as courageous, moderate, or whatever.

On this account, a genus belongs above all else to the individuals of the species. Each human being who really counts as a human being at all (whether sleeping, raging, fornicating, wrestling, leading, or contemplating) is animal. Each chair which really counts as a chair (whether gathering dust in the attic, serving as a stepladder, or seating someone at dinner) is household furniture.

If every member of a species includes the genus “as a starter,” and on this basis possibly emerges as it really and fully is, the quantitative version of genus, species and specimens fails utterly to represent the relation concretely at issue: the individual actualizing its species becomes greater than the genus. For the individual, the genus is a part of the species. The diagrams of Euler and Venn, and all subsequent set theory, reverse the original part-whole relation.

But Aristotle, at least, says: a “definition” is the λόγος emerging “from differences,” and the “completing difference will be the Being of the matter” (the οὐσία of the πράγμα, i.e. the “definition of it” (Metaphysics, vii, 12). Once we get a fix on the genus, we must still distinguish the being in question from the other kinds of beings sharing the genus. How are human beings more than animal? How are chairs more than household furniture? How are pious actions on our part more than just actions on our part? How is our condition as courageous more than simply a “condition disposing us to choose the mean between extremes”?

How does one make distinctions? Oddly, Plato and Aristotle claim that we make them significantly by learning how the being itself differs, stands out, comes into its own against the background of its genus. That is: when true, the human act of distinguishing follows upon the being’s act of differing. Children have to learn first that, and later how human beings themselves differ from teddy bears, roses, and cats: they learn the first by living with teddy bears, roses, cats, along with siblings, parents and eventually strangers. People, children soon learn, respond and require response much differently than do the others — despite their generic similarity with, say, the household cat or the neighborhood dog. And an apprentice cabinetmaker must learn first that and then how chairs differ from footstools and tables, beds and bookshelves — since in their function (ἔργον) the former require materials and maneuvers of construction, and even more a design adapted to human posture and human use, that differ remarkably from the materials, maneuvers, and designs demanded by the others.

The logical question of “differentia” then takes us back to the question of natural learning. Only a cabinetmaker or a householder really learns how chairs themselves differ from other beings in their genus. Only a horsetrainer or horserider really learns how horses themselves differ from other animals. That is, only one who uses, puts them into their function, can really learn these beings in their Being — not even the mere carpenter, the mere farmhand knows how they can really distinguish themselves (Republic, Book 10).

However, we ourselves contemplate the human condition of λόγος. Following Plato and Aristotle, we contemplate the workings of artisanship and leadership. And while we cannot truly usurp the prerogatives of cabinetmakers and horsetrainers, we can notice the “conditions of λόγος” built into these arts. Our own subject is not chairs or horses, but rather human being. As philosophers, we can only testify to the truth of definitions which issue from the ways human beings distinguish themselves. Any effort on our part to define “chairs” or “horses” we expend only for illustration — not for cabinetmakers or horsetrainers.

But how can one proceed to formulate a definition — given all the arduous pre-conditions of natural learning? As ever, Plato especially, and Aristotle by and by, suggest some mental “gymnastics” to help initiates. These exercises are now summed up under the general heading “method of division” — a procedure by which we cut each determination in two while searching for the ways a given being itself differs from others — in its being.
To instruct us in the new art of contemplation, Plato’s *Sophist* first traces down one kind of artisan: the angler, the one who spears fish (218E-221C). Instead of saying just that, Plato’s work takes us through an entire process of bifurcation: an angler stands out against idlers as one who has learned an art. He then stands out from many other artisans inasmuch as he acquires rather than produces things. He stands out from many other acquirers (e.g., negotiators) inasmuch as he employs coercion. He stands out as a hunter (rather than a boxer) in his coercion. He hunts living things (rather than precious metals); water (not land) animals; swimming ones (rather than water fowl); with an instrument for delivering blows (rather than with nets or other traps); and one having a barb for retrieving (rather, say, than depending on a dog).

The astounding feature of this kind of account is that it spreads over the entire human condition, the home base in which we all live (all who engage in an art). And the distinctions increasingly draw into focus not only the subject, the angler, but the materials one may put to use, the places one may be, the maneuvers one may employ, the purposes one may have. Finally, the account debouches both on the being — the entire functioning of the being — and the name: the two are brought together, at least momentarily.

If nothing else, the gymnastics of division in the *Sophist* and elsewhere train us in the new art of contemplation. Definition by genus and differences may then be appropriate only for philosophical considerations. It would seem then that if we asked makers or leaders to define what they deal with or how they deal with it, we would be asking them to rise out of their own vocations to contemplate the human condition. At least the “method of division” would require them to do so.

Let us then consider yet another example, the application of the method for which the division of the angler intends to prepare us: the chase after the “sophist.” Following now the inductive syllogism suggested by Aristotle, we take striking examples of sophistry (Protagoras, Gorgias, Thrasymachus: today we must settle for examples of those who claim to impart knowledge in return for pay): these (S) are sophists (P). We then search out an M by looking to S. We first place our sophists back into their genus — to witness their parturition in unison with our bifurcations. Plato then asks us to see that these sophists:

1. proceed with τεχνη;
2. produce (make, re-make) rather than acquire things;
3. aim for human rather than divine (or natural) affairs;
4. deliver ειδωλα (images) rather than “originals” of things; an architect draws houses while a carpenter builds “originals”; divine art creates both ειδωλα (e.g., day-dreams, night-dreams, and reflections in water) and “originals” (e.g., ourselves, animals, plants, fire, and water);
5. provide “fantastic” rather than “iconic” ειδωλα (these of an architect, presumably also of a philosopher, may be icons, likenesses: they can prepare us for “originals”);
6. embody their products in their voice and gesture (rather than employing instruments, as flute players and drama directors do — and as teachers show movies);
7. fabricate their fantasies of human affairs with a view to conventional opinion (approval) — rather than fabricating likenesses with a view to generating wholesome attention to beings in their Being; and, finally;
8. create speech (λογος) that intends to deceive (to persuade people to branch off into fantasies, leaving beings behind) rather than more simply to impress (to get people to applaud the speech itself, as a mere orator may wish).

If, now, we are satisfied that we have “caught” all sophists (not simply the three or four strikingly present), and only sophists (have let all look-alikes, e.g. philosophers, slip through the net), we may convert our minor (all people doing these things are either Protagoras, Gorgias, etc.), whereupon we can “deduce” the major, itself convertible: Sophist = M.

But what do we learn from such definition? The meaning of words? Plato and Aristotle insist that learning in this sense is highly and dangerously derivative from learning beings. Do we then learn what it means to be an angler? To act as an angler? The art of angling? Or to be a sophist, to act as a sophist? The art of sophistry?

We often read, especially in the followers of Aristotle, that we should find at the end of the division, at the center of the genus, the idea, the species the glow — as Theseus found the Minotaur down in the labyrinth. But in Plato’s works, at least, we recurrently read quite a different image: all along, while sifting through the genus, we are dividing, bisecting, bifurcating by glows (κακες είδη) — and we end not with yet another idea or glow, but with the matter itself, the πράγμα in its Being, its οὐσία (Sophist, 267D, cf. also 218C). On this image
(presumably iconic rather than fantastic) a definition by division will carry us to the end only if glows are already guiding us through our home base—only if we are already struck by the differing ways beings appear. Then the resulting formulations (of angling, of sophistry) merely set the stage for dramas of discovery which others can act out on their own. A classical definition sets us up, as it were: provides a game plan, or a script, but does not itself discover anything for us, does not pre-empt our own original encounter with the being at issue.

But formulations may in fact pretend to do the work for us—or we may presume that they intend to do so. Indeed, Plato's stated reason for defining the sophist is to reveal exactly how this pretense, factual or presumed, is possible. One formal reason that the Minotaur of this one definition cannot be an idea, a glow, is that sophists are fundamentally deprived: sophistry is a privation of human being, one that catalyzes in others a privation of Being in general. Sophists look very much like philosophers—but “by definition” they are mere look-alikes, failed philosophers: they branch off into non-being at a crucial juncture, one at which we ourselves recurrently arrive and depart. The “division of the sophist” scripts this fundamental possibility, among others.

Aristotle's sequence of four questions and four answers may help us to concretize the gymnastics of division. While an appropriate definition will account for the material and the motion (of angling, of sophistry, presumably also of chairs and of horses), it must take us through the dramas of purpose in order to culminate in a formulation of what a being is. And the purpose of a being (our full telic vision) requires something a verbal formulation forever fails to do: we must ourselves participate in it, follow its draw. Indeed, on the classical view, λόγος first arises for us because we are drawn toward beings in their Being (their glows): genuine λόγος is already in tune with beings. But we in fact start out with the λόγοι of yesterday and yesteryear: our condition is essentially (generically) paleological. The “techniques of λόγος” Plato and Aristotle proffer intend to help us re-enter fully into the draw so that our λόγοι can renew themselves: we actualize our condition neologically.

* For a helpful scholarly account further exemplifying the “method of division” and duly criticizing its apparent adequacy, see Kenneth Dorter's article "Justice and Method in the Statesman" collected in Justice, Law and Method in Plato and Aristotle (Edmonton, Alberta: Academic Publishing, 1987). The author convincingly argues that the method of division is a dianoetic (downward moving) exercise which can never itself be adequate to the noetic (upward moving) formation of normative definitions. Thus, while the method may work well in defining the sophist, angler and box-maker, it breaks down when applied to the philosopher, the statesman or human being: this latter appears comically as "a bird with the feathers missing" or "a pig with two legs missing."

§6.2 Concepts: intension and extension

Rather than defining what some being means, we may prefer to hear or to proffer a definition of what a word means. This preference belongs especially, even essentially to an age in which we wish to consider talk, and seek agreement in talk, prior to (and optionally leading toward) actual encounters.

But a word all by itself means what we in fact mean by it—whether the “we” comprises an already existing (possibly very large) group, or one that is “in the making” (and possibly very small). The “we” of the first kind expresses a natural community, the “we” of the second kind a contrived one—as when a chemist or a spy tells a colleague that he intends by word w some special meaning m (or as lecturers and lawyers stipulate what a term or phrase shall mean in the context of the lecture or the document).

Of course, what we naturally mean by a word such as “chair” or “jerk,” “fast (car)” or “beautiful (style of writing)” is often very loose. When tightening up our meaning we generally aim for two results: a list of features and a number of applications. These two stand in reciprocal relationship: the list of features (of “chairs” or “fast [cars]”) should help us identify cases of proper application, and in the course of applying the list we should find our list of features somehow workable and thereby confirmed.

Defining a word in this modern way, we are pulling together (and eventually interrelating) a number of properties: we are first of all defining a concept. When defining a concept we have likely (but not necessarily—on the modern view) encountered examples to guide us in the selection of the properties. But henceforth we no longer look to examples for advice on how to formulate (or even on how to understand) the concept. In the end, we identify examples according to the rules now available in the concept. In effect, we end by stipulating what our words shall mean—in which case there is no longer any genuine possibility of appeal (to “disagree” would amount to opting out of the community—}
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stipulating one's own meaning). We are well advised, then, to exercise great care in conceiving our meanings — both for the sake of subsequent identification of cases and for the sake of preserving a community of speakers and listeners, readers and writers.

The modern effort to define words rather than beings belongs to the larger modern effort to promote communication — both in everyday affairs and in modern universal education. Here a definition aims not for an articulation of our plunge into the labyrinth, let alone the discovery of any Minotaur. Rather, it aims to articulate some set of rules for a game: the game itself may be anything from buying bread to delivering an introductory lecture on psychology, from preparing a legal document to “formalizing mathematics, including logic, so that it becomes a game with symbols played according to fixed rules” (Hermann Weyl's depiction of David Hilbert's project). A definition may aim to help us play the game (buy bread, prove theorems), but it lies outside the game itself, i.e. we formulate it prior to the playing. Thus the sooner in a discussion we request or proffer a definition, the better we shall proceed together through the discourse — whenever, that is, one of us has not already taken the vows of the discipline.

Just as Aristotle's real definition, so too nominal definition leads to paradoxical results — and thereby raises questions inviting contemplation.

Roughly, the difficulty henceforth is this: on the modern scheme we have two kinds of meaning, and the relation between the two becomes complicated when we scrutinize the way words variously work in our discourse. Ultimately, the modern scheme expresses the re-interpretation of our λογος-bound condition worked out by such thinkers as Galileo and Francis Bacon, Descartes and John Locke. And then, when taken for granted at the outset, it no longer simply expresses but rather re-enforces the Fourth Interpretation, making it appear “perfectly natural.”

From the Scholastics and then the Jansenists, and on down to more contemporary logicians like Frege, the meaning we somehow gather together into a unity (take together: con-ceive) has received various epithets in various languages (compréhension in the Port-Royal Logic, Sinn or “sense” in Frege); and likewise the meaning we enact with regard to the given and often multiple instances legitimately named by the word in question (étendue or “extension,” Bedeutung or “reference”). Let us settle on the name intension for the former, distinguishing it from its homophone (we are not talking about the intention or “purpose” one may have when talking); and on the name extension for the latter, distinguishing it from its homonym (we are not talking about the “extension” of a deadline or the “extension” of a motel). In our logical nomenclature, the “intension” of a word is what we tend to in the way of conceptual unity, and its “extension” is what we tend to by way of examples of such conceptual unity.

Some words have intension but no extension. The name of a species we take to be extinct has no present or (we presume) future extension, at least not in αἰσθήσεις: we conceive of such things in reference to memories, paintings, photographs, and verbal descriptions. Similarly, we can conceive of something and give it a name having (presumably) no extension: “unicorn,” “Martian,” “speed faster than light,” “cold colder than minus 3000 degrees Celsius,” “finite and complete numeric expression for \(\sqrt{2}\) (or for \(\pi\)),” “complete determination of time and place of nuclear particles” (in Heisenberg’s quantum mechanics), or (according to Gödel) “complete and consistent axiomatic system governing all mathematical work.” The first two of these names and phrases in fact have no extension; the remaining ones in principle have none.

Many words appear to have the same extension: “larch” and “tamarack” and “hackmatack” may all mean the same trees (in some regions), and perhaps the “value” of commodities is the same as their “worth” — if all we mean is the figures written on their price tags. Perhaps the range of applications of the “grass” outside “houses” is the same as that of the “lawn” outside “homes.” Historically a very eclectic tongue, English has many examples of duplication like “sufficient” and “enough,” “drunk” and “intebriated.” If we insist, as we should, that such synonyms do in fact differ in meaning, we might (in some cases, at least) be including in the intension of the word something of our own attitude toward the items covered by the word. In talking about the “value” of a car we may be thinking of the effect of its purchase on our pocketbooks; whereas in speaking of its “worth” we may wish to emphasize its inherent quality of craftsmanship or its present condition.

In speaking about the “grass” outside my “house” I may be emphasizing a botanical fact surrounding something I own, whereas in speaking of the “lawn” outside my “home” I may be emphasizing my own responsibility for both.

If we agree that the extension of two different words is the same, and similarly their strict intension, we may then agree that the two differ
in connotation, namely in “associated human response”: strong examples of this kind of difference are racial/racist words (“Blacks,” “Negroes,” “Niggers,” and “colored people” have in some regions both the same intension and the same extension while arousing human responses of very different kinds). And Russell’s “irregular verbs” illustrate the extent to which we include our own attitude along with intensional and extensional meanings (“I am firm, you are stubborn, he is pig-headed”). Since live speech aims, among other things, to gather people together in shared responses to shared circumstances, effective speakers and writers compose their music with some awareness of the range of responses, the tonalities, built into the use of words at a given time and place (and effective listeners and readers will make allowances for deviations from their own expectations). But connotations in this sense are not part of the intension of a word, and much less of its extension; connotation is not an affair of meaning itself, but one of contingent (historical) association with meaning.

And of course there are words that sound or look the same, but have partly or wholly different extensions, and therefore different intensions: “race” as a sub-species vs. “race” as a sport event, and “score” as an achievement in a game vs. “score” as a transcription for a musical performance, have no extensional or intensional relationship. Since a word is ultimately a human response, and neither simply a noise in the ears or a light wave in the eyes, we can rightly say that in such cases there are two totally different words. However, “man” as distinct from “horse” and “carrot” is related to “man” as distinct from “woman” or “child”; and a “determined” as distinct from “unknown” fact is related to a “determined” as distinct from an “oscillating” will (so that “I am determined” might lead to misunderstandings). And by now it should be clear that “definition” as an action of one who aspires to clarity (or a condition of one who has formulated the clarity) differs in meaning according to whether one is defining a name or a being; yet the two overlap one another: both share in the same effort to overcome the blurry connotational meaning typical of much idle talk, and both are related to “definition” as what a photograph may or may not have.

While many of the foregoing examples of ways extension and intension interweave may appear non-contentious, we may wonder how two quite distinct intensions can have the same extension—as they seem to have on some occasions. Biologists, psychologists, and philosophers may conceive “human being” much differently, and yet point to the same examples: Do these conceptions somehow “complement” one another? Or is one right and others wrong? And consider “light” as the wave theory conceptualizes it and as the corpuscular theory conceptualizes it: referring to the same phenomena, does one have to deny the other? In his Physics and Beyond Werner Heisenberg discusses at length how he, Bohr, and Einstein have puzzled over what the “same” of “same phenomena” can mean when observing and calculating phenomena of sub-atomic or macro-cosmic proportions. But we likely have an example closer to home: might not Venn and Euler diagrams be made to represent the same examples of valid inference—while conceiving of validity in differing ways? We may then wonder what the status of an intension is: Is it possible to conceive of a meaning correctly if two different (even incompatible) concepts adequately identify an apparently single phenomena or set of phenomena?

In any case, we moderns have abandoned subjects and have thrown our lot in with predicates. In the nineteenth century John Venn wrote about his way of construing propositions with empty and non-empty circles: “the distinction between subject and predicate is robbed of its significance on such a scheme as this” (Symbolic Logic p. 7 of the 1971 reprint of the 1894 edition). Even the notion of a predicate wanes in its significance: it no longer intends to bring out anything, it simply expresses a concept we ourselves have formed. And the correlate of a concept can only be an undifferentiated instance—very suitably expressed in a diagram by a little x, the “significance” of which is that it allows us to consider the possibility of the set having a member (the concept having a reference, i.e. not being empty). We then ask afterwards (in or after the game) whether and how the concept has “existential import,” whether or not it can (we can) lay claim to an application. This crucial development likely has its roots in the Scholastic concern for the terms of an argument (of propositions)—S, P, and M taken one at a time (see Book One, §7). Still, a more contemporary logician like Wittgenstein must repeat the modern point over and over again, as though we can never quite get it straight: “‘Essential’ is never [really] the property of an object, but rather [only] the mark of a concept” (Remarks on the Foundations of Mathematics, I, §73). Nowadays, we would likely have to revert to the Deism of early
modernity to claim that we could “access” anything essential about what we encounter, i.e. subjects.

Under the conditions of conventional naming, what can we say in advance about the prerequisites of a proper nominal definition? At least this: (1) it must take place in an effort to form an agreement within a group, (2) it will draw upon features (properties, concepts, “predicates”) already familiar to the members of the group (or possibly familiar to them — with further definition or a bit of work in the field), and (3) it will establish criteria by which members of the group can decide whether or not a given instance fits the concept being defined. These basic prerequisites themselves provide criteria for deciding whether a proposed definition is good or not: one will certainly not be good if it is idiosyncratic, irreducible to familiarity, or non-decidable in application. The “wisdom” of a definition can never be one’s own, nor can it be dependent upon any plunge into the labyrinth, nor can it be puzzling (although it may appear in any of these ways to a novice who drops in on a discussion). Here, definitions are preludes to, not climaxes of, the game.

One interesting result of modern definition is that we can define a concept having in fact only one illustration. For instance, we can pull together enough properties of the planet on which we live (size, distance from the sun) and then “discover” that only one object fits the concept we have formed (at one time, astronomers thought there must be two earths). A Philosophy Department in Canada once advertised a position, citing a “job description” so tightly formulated (“... and must have many publications on the question of time in Husserl’s writings”) that some of us knew in advance that only one candidate could fill the position.

A related result of the modern division of meaning is that proper names like “The Eiffel Tower” or “Fido” resist intensional meaning and seem to have only extensional meaning. On the one hand, each refers to a single instance (if “Fido” also names your dog, we have an case of homonymity: the words look and sound the same, but they function differently as human responses). On the other hand, each does invite a number of predications (standing in Paris, designed by A. G. Eiffel, built in 1889, 300 meters high, etc.). However, we may gather together as many of the predications as we wish, we will still only formulate a meaning which in principle (1) may cover other instances (others may be built exactly so) and (2) may cover no instances at all (we may have made a mistake in formulating one or more of the properties). Thus, if “The Eiffel Tower” is to have the meaning it in fact has, we must assume the priority of extension over intension.

We see in the effort to define by predicates (to classify multiply) a dramatic blockage when we come to the question of how we can properly approach, in λογος, individuals — what Aristotle called primary beings.

That is, real individuals having proper names. Interestingly, fictional names (“Prospero” or “Little Red Riding Hood”), and also names which have become legendary (“Homer” or “Héloise”), in fact require us to pull together a number of properties. And while it may make a big difference if I mis-predicate L.W. (whereupon I rightly come back to the extension of the name), it would make no real difference if we discovered that “the author of the Iliad and the Odyssey” covered two people or someone the neighbors at the time and place called “Sappho.” Homer is for us “by definition” the author (and we may arrange our copies of those works under “H” in our libraries). Similarly, it may possibly make no real difference if scholars revised all the facts about the “historical” figure we now call Jesus in the New Testament: both “Prospero” in Shakespeare’s play and “Jesus” in the Gospels are now very literally figures of speech, what grammarians call “personifications” of sets of possibilities (concepts) having no pre-established extension. Proper names in great literature have powerful intensional meaning forever awaiting instantiation... by us!

These last examples highlight the easily forgotten basis of the effort to formalize meanings on the intensional-extensional scheme. The purpose of defining words rather than beings is to promote the development of knowledge-at-a-distance. We pass beyond this purpose if we make any claim to account for beings themselves, as individuals.

§7. Observational induction

Although in nature nothing exists besides bodies performing purely individual acts according to a law, in investigation this law, the search, discovery, and exposition of it, is the foundation both of knowledge and of operation. And it is this law, with its qualifications, that we mean when speaking of form.

—Francis Bacon, The New Organon (1620), II, §2.
To the modern mind, rationality appears best illustrated by aloof observation coupled with careful calculation. Participation in the destiny of things appears as its opposite, at best an extra-rational romanticism and at worst a rejection of λόγος entirely — irrationalism.

Aloof observation provides the channels through which we obtain data, the facts. Careful calculation assures well-defined patterns of inference enabling us to anticipate with confidence the evolution of facts. As modern intellectuals we then strive to organize our data and to formalize our inferences: through organizational observation (experiment) we search, detect, and formulate inter-relations, and through formalized calculation we infer elaborate reconstructions of the inter-relations.

In Book Four we shall explore the formalization of inference. In Book Five we shall explore the “conditions for the possibility” of the purely rational concepts essential both to data collection and to logical computation. Prior to embarking on these explorations, we may ask how we can organize aloof observation in order to engender empirical concepts systematically.

§7.1 Fact vs. generalization

If men had available the correct Account of Nature and of Experience, and kept to it, and if they could master two things, viz. [a] deposing received opinions and notions and [b] refraining the mind for a while from the highest generalizations and those preparatory to them, they would be able, by their own inborn mind, without any other art, to embark on my form of Interpreting. For Interpretation is the true and natural work of the mind, once freed from impediments. The New Organon, I, §130.

We must learn to look for ourselves. Such looking does battle with second-hand conceptions of what we might see, and also with our own tendency to jump to conclusions. Looking is itself an art, one that can be learned, and one that represents our own “true and natural work” as thinking beings. It provides the copestone to interpretation: to what we today may call concept-formation.

Consider a very elementary example of an empirical claim (drawn from Paul Goodman’s Growing up Absurd, 1960):

At an undergraduate school in Harlem, they used to test the intelligence of all the children at two-year intervals. They found that every two years each advancing class came out ten points lower in “native intelligence.” That is, the combined efforts of home influencing and school education, a powerful combination, succeeded in making the children stupider year by year; if they had a few more years of compulsory home ties and compulsory education, all would end up gibbering idiots.

The author provides no data representing the events to which he refers. Still, we can imagine the form such data would take if they could support the preparatory conclusion that each advancing class in the Harlem school dropped ten points, and then the highest conclusion that the drop is caused by home influencing and school education. We gather data from individual cases into collections. In the present example, we can imagine a case matrix:

<table>
<thead>
<tr>
<th>Case</th>
<th>Facts</th>
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| 1    | a. Determinations regarding school education.  
      | b. Determinations regarding home influencing.  
      | c. Other determinations (age, race, religion,...).  
      | d. Various test-scores at two-year intervals. |
| 2    | a. (In parallel with data in Case 1)  
      | b.  
      | c.  
      | d. |
| 3    | a. (In parallel with data in Case 1)  
      | b.  
      | c.  
      | d.  
| ...  | a. (In parallel with data in Case 1)  
      | etc.  
      | etc. |

Such a matrix records what was actually done, the facts. Each Case will have a proper name — but only for the sake of re-checking the data or expanding the data-base. Each Result must be existential rather than inferential: must formulate a one-time determination devoid of generality. The purpose of the matrix is to extract a pattern of events regarding children: first, a synchronic pattern (of events in the classroom, at home, on tests) and then a diachronic pattern (connections between antecedents
and consequents). The narrative stance is that of a social engineer addressing a school board or a parliamentary committee.

The construction of the case matrix requires that the investigators carefully formulate a methodology: data-gatherers must approach each Case with the same questions. The answers, the data, appear clustered around each other, not around the child: we wish to extract a pattern from the data, we have no interest in fathoming the potentiality of the child — as a parent might, or even a teacher.

Goodman claims that a pattern has been detected: the test-scores, arranged properly for comparison, drop in numerical values at two-year intervals. Investigators then connect two things: exposure to home and school conditions and debilitation of mental performance. Thus is born a complex concept, a temporal inter-relation, one allowing us to anticipate the progression of events. And, more importantly, to modify conditions to alter the progression. For instance, the author describes the efforts of an adventurous principal:

One method he swears by is to invite the free expression of criticism and hostility, e.g. “Write a composition telling why you hate your father — why you hate school — why you hate me.”

A new line of investigation, then: devise a case matrix recording the evolution of test-scores in those children whom teachers had encouraged to express criticism freely.

Both at the heart of modern knowledge, where the investigator operates, and on the surface, where the public takes notice, lie causal concepts: ways of understanding inter-relations across points in time. Yet these complex concepts mask simpler concepts already at work. In our example, the investigator must form a concept of “Harlem school child” in a way that allows one to identify each child for testing. And, much more challenging, we must all become clear about the concept of “intelligence,” especially since we envisage a numeric scale for it, i.e. a way by which we may measure each child against others. And what about the averaging process? Surely not every child’s performance dropped exactly ten points: we have a statistical concept so that we may predicate groups and not only individuals.

Each of the many concepts employed in an investigation deserves careful consideration in the discussion of the methodology. We define them carefully, always in anticipation of further encounters — precisely so that we or our successors can improve the formulation for future investigations. Each definition consists of an inter-relation of factors: of criteria. These definitions set the standards for what we recognize and anticipate, therefore also of what we experience at any given moment. We take our stand with intensional meanings and then venture out to meet their extensional counterparts.

By introducing methodologically determined, strictly singular determinations, one hopes to induce a generality: a law, Bacon says. In this process, we do not learn a form by which we recognize and perhaps help actualize the potentiality of singular beings. Rather, we learn a rule that allows us to enter into the governance of the way bodies behave. Most famously, Bacon formulates the significance of his new organon in a four-part aphorism:

Human knowledge and human power meet in one. For where the cause is not known the effect cannot be produced. Nature to be commanded must be obeyed. And that which in contemplation is as the cause is in operation as the rule. *The New Organon*, I, §3.

But how are we to understand the passage from the case matrix to the generality? Essential to Baconian methodology is the awareness that the passage is precarious. The modern spirit of investigation prescribes a critical, even a skeptical stance. Pursuing Goodman’s example, we would review the data-base with such questions as: How many Cases where there? Were the same children tested at those many intervals? Were the effects of home and school sufficiently powerful to override other environmental and genetic factors? What was the range of test-score variation? What were the deviations from the mean? How could we measure the respective effects of each of the two antecedents, home and school? Are there any further signs in the children that they fail to meet challenges intelligently? — Answers to these questions could easily upset both the preparatory and the highest generalizations the author cites.

Apart from raising the practical question of how we might bolster the passage toward generality, improve and assure it, we may raise the theoretical, the logical and ontological question: How does this kind of induction engage us in inference? Or: What is the structure of the passage from a collection of facts to the formulation of a generalization?
We start by observing individuals, we pass from these to collections of determinations, and from these collections we extract patterns (first diachronic, then synchronic ones). Accuracy of the initial determinations allows us to abandon the individuals. We then concentrate on collections: on particulars. In these arrays of particulars we detect patterns of behavior (first, the drop in test-scores, then the temporal connections). From these patterns we then extract formulations that appear as rules: such-and-such conditions (a., b., and c.) lead to such-and-such results (an average drop in scores, as recorded in d.). And, finally, we envision ways of creating new situations by altering the initial conditions of schoolchildren — presumably with a view to enhancing their average ability to meet challenges requiring intelligence.

But we logicians insist on asking: What grounds do we have for this jump beyond individuals, first into particulars and finally into a generality? No doubt we all jump. Yet this very disposition of ours locates our propensity to err: all too often we jump hastily, especially in dealing with freshly appearing cases — the future for which much modern investigation formulates its rules.

The tradition of British empiricism, from Francis Bacon through John Locke and David Hume to John Stuart Mill and their spiritual successors today, incessantly argues that the grounds ... modern empiricism fully agrees with Plato and Aristotle, who argue that empirical determinations enable us to develop at most a knack, a rule of thumb, for handling passing affairs, never an art for fathoming individuals, aiding their maturation. Yet these same modern thinkers wish to confine art to the interpretation of particularity: to Bacon's "true and natural work of the mind," namely the art of aloof observation supplemented by the arts of number. Such art has no other foundation than the human mind itself: our own energies. The laws we detect may appear at first to be those of the Plan of the Universe, but they end by being products of our own intercourse with particulars.

John Stuart Mill formulates succinctly the ontological commitment following upon Bacon's premiss that "in nature nothing exists besides bodies performing purely individual acts":

Now, all which man can observe are individual cases. From these all general truths must be drawn, and into these they may be again resolved, for a general truth is but an aggregate of particular truths, a comprehensive expression by which an indefinite number of individual facts are affirmed or denied at once. —A System of Logic (1843), Book Two, III, §3.

General truths may be resolved into particular truths, the determinations recorded in a case matrix. The laws Bacon proposes as culminating investigation should replace not only the traditional forms but also the universals needed for valid syllogistic reasoning and supposedly learned by Socratic or Aristotelian induction. Mill understands clearly the logical commitment following upon the ontological one:

All inference is from particular to particular; general propositions are merely registers of such inferences already made, and short formulas for making more; the major premiss of a syllogism, consequently, is a formula of this description, and the conclusion is not an inference drawn from the formula but an inference drawn according to the formula, the real antecedent, or premiss, being the particular facts from which the general proposition was collected by induction. Those facts, and the individual instances which supplied them, may be forgotten; but a record remains, not indeed descriptive of the facts themselves, but showing how these cases may be distinguished.... Book Two, II, §4.

That is, how new cases can be discerned: syllogistic inference masks a simple continuation of the case matrix beyond the last recorded case, we remembering the summary of expectation rather than the particulars themselves, let alone the individuals generating the particulars:

When, therefore, we conclude from the death of John and Thomas, and every other person we hear of in whose case the experiment has been tried, that the Duke of Wellington is mortal like the rest, we may, indeed, pass through the generalization, "All men are mortal," as an intermediate stage, but it is not in the latter half of the process, the descent from all men to the Duke of Wellington, that the inference resides. The inference is finished when we have asserted that all men are mortal. What remains to be performed afterwards is merely deciphering our own notes. Book Two, III, §3.
In Book Four we shall explore in detail the work of formal inference, once we have severed such inference from the concerns of classical logic. Meanwhile, Mill’s account of empirical inference anticipates, in an uncanny way, the modern formal principle of universality: instead of reading a universal truth as talking about all subjects having a named property, Mill and modern logicians read it as saying that if we decide some instance qualifies as that “subject” (now itself a predicate), then we also decide that it will have the named property. We read (II, §4):

... a proposition might be considered in two different lights, as a portion of our knowledge of nature or as a memorandum for our guidance. Under the former or speculative aspect an affirmative general proposition is an assertion of a speculative truth, viz. that whatever has a certain attribute has a certain other attribute. Under the other aspect it is regarded not as a part of our knowledge, but as an aid for our practical exigencies, by enabling us when we see one of the two attributes to infer that it possesses the other, thus employing the first attribute as a mark or evidence of the second. Thus regarded, every syllogism comes within the following general formula:

Attribute A is a mark of attribute B
The given object has the mark of A, therefore
The given object has the mark of attribute B.

In Book Five we shall turn to the consideration of the “conditions of the possibility” of even establishing a case matrix: of gathering and organizing data. For already these acts of determination recall our own condition as artisans of the mind, the modern location of universality. Yet only from contemplation of actual procedures might we gather the evidence for answering this axiomatic question.

§7.2 Experiment and hypothesis: Mill’s methods

There remains [after misty tradition and vertiginous disputation] direct experience which, if simply occurring, we call accident; if queried, experiment. Experience by itself ... is a mere groping, as of men in the night who probe all around for the chance of coming upon the right way; while they had much better wait for the day, or light a candle, before starting on their way. In contrast, the true order of experience first lights a candle and then by means of the candle shows the way, beginning with ordered and digested experience, least distorted and least erratic, from it educing axiomata, and from established axiomata yet new experiments. New Organon, I, §82.

We already, and naturally, surmise generalities — passively, by accident. The result is that we pre-judge events, i.e. remain blind to events unfolding in their freshness. No longer trusting the arts already proving themselves in the performances of our artisans, Bacon urges us to query experience: to form experiences (make experiments, we would say). From such carefully formed experiences we extract axioms (in the Stoic sense: basic assertions such as “these children are performing ever more poorly on tests”). We are lighting candles to show the way, and the cycle of experimentation begins.

Prior even to setting up an experiment we must anticipate what we might find. In human terms, we must have a hunch about the connection between antecedent conditions and subsequent developments. In Goodman’s example, we must foresee the possibility that home and school conditions stand in some relation to poor performance on intelligence tests; otherwise we might simply accept the flow or, more likely, ascribe some more fateful cause to it, a cause over which we have no control and therefore one absolving us from further responsibility as teachers or principals, parents or parliamentarians.

In formal terms, we need an hypothesis: a thesis we tentatively lay over the events, one allowing us to construct more elaborate case-matrices. Each Case must be examined carefully for a large number of possible predicates bearing on exposure to classroom events, exposure to home events, pre-school exposures, activities out on the streets, and other genetic and environmental features. Our hypothesis not only anticipates an overall result, but helps us factor our investigation into a matrix of possible predicates. The investigator then approaches each case with a series of questions, looks carefully for each answer, and records the answer in condensed symbols — computerizable into a data-base. Schematically:
Consider another example drawn from popular science (Heinz Haber’s *Man in Space*, 1953):

A few years ago a small number of people living in various sections of the United States ... developed what the physician calls cataracts—small, irregular, opaque spots on the tissue of the lens. ... It turned out that all the individuals who had developed these cataracts were physicists and that all of them had been connected with nuclear-energy projects during the war. While they worked with cyclotrons in atomic-energy laboratories they had been the targets of stray neutron rays.

In a critical mode, we would ask for a complete list of the physicists developing cataracts, primarily to review their cases to make sure they shared no other occupational hazards or genetic features: we would be looking for a random background against which to measure the agreement between exposure to atomic-energy laboratory conditions and the development of cataracts. But we would also ask for a list of other physicists working under similar conditions, and examine their subsequent medical records: we would wish to confirm the agreement suggested by the author’s popular account. And of course we would ask how the original investigators narrowed down the antecedent conditions from “working with cyclotrons” to being “targets of stray neutron rays”—a research project in itself. Finally, our knowledge would not likely remain at the speculative stage, but translate into a memorandum for our guidance, a rule for our practical exigencies (procedures and technologies for avoiding stray neutron rays).

§7.2.2 The method of difference

The patterns we detect in our data issue in rules: given one set of conditions, something follows. A rule we have formulated will inspire greater confidence if we show that removal of the conditions makes a marked difference: change the home and school environment, and the children cease losing intelligence; protect physicists from stray neutron rays and they don’t contract cataracts. Or: investigate children in other school districts alike except in the conditions accused of debilitating the pupils, and determine whether these others remain clean of the effects. If the rule holds in any practical way, it will formulate necessary antecedent conditions—those, the removal of which prevents the effect.
Let us consider a fresh example, drawn from yet another book in popular science (Kenneth M. Smith's *Beyond the Microscope*, 1943 & 1947):

It was assumed for a long time, by analogy with the mosquito and other blood-sucking vectors, that the virus of typhus was injected by the louse when sucking blood. But apparently this is not so. The infection is not in the saliva of the louse, as it probably is with the mosquito, but in the faeces. The disease is thought to spread through the faeces coming into contact with scratches or abrasions in the skin, and scratching is the usual concomitant of louse infection. This fact was suggested in 1922 by two workers who fed infected lice on a monkey, while taking great care that no faeces from the lice should come into contact with the monkey's skin. They found that the monkey remained healthy.

Lice inject saliva directly into the blood and deposit faeces making their way into the blood through abrasions: two factors otherwise conceived as one. Which of these two accounts for the progression of the effect? In some Case, preferably expanded to many, investigators artificially prevent the operation of the one factor, and the usual effect (evident in many other Cases) does not ensue. We have then narrowed the focus of research: established an “axiom” opening the way to yet further experimentation.

The Method of Difference allows us to light candles in a way that the Method of Agreement does not: we aggressively eliminate what would otherwise take place. In combination, the two methods enable us to toggle back and forth: if our experiments do not destroy the Cases, or morally compromise our own position, we can alternately induce and remove the effect: disempower or empower children, infect or protect monkeys. Knowledge and power meet in one — but the power here is ours, not the δυναμις of the beings we encounter.

The methods of Agreement and Difference both work on a single principle, that of Negation. Bacon speaks of “exclusion and rejection”:

Now what the sciences stand in need of is a form of induction which shall analyze experience and take it to pieces, and by a due process of exclusion and rejection lead to an inevitable conclusion. *The Great Instauration*, Plan, 2.

Mill speaks similarly of Agreement and of Difference:

Both are methods of elimination. This term ... is well suited to express the operation ... which has been understood since the time of Bacon to be the foundation of experimental inquiry: namely, the successive exclusion of the various circumstances which are found to accompany the phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon. The Method of Agreement stands on the ground that whatever can be eliminated, is not connected by any law. The Method of Difference has for its foundation, that whatever cannot be eliminated, is connected with the phenomenon by a law. Of these methods, that of Difference is more particularly a method of artificial experiment; while that of agreement is more especially the resource employed when experimentation is impossible. *A System of Logic*, Book Three, VIII, §3.

The Principle of Negation fits well our condition in aloof observation: our task is to clear out and clear up, so that we may formulate concepts, and especially those of temporal inter-relations. On the modern view of knowledge as power of our own λογος, the clutter of our initial condition holds no promise.

In contrast, Aristotle claims that our task is to heed multiplicity for the sake of discovering the sameness “underlying” it (the subject we encounter): learning is θεσις, positioning (of the thing, and of ourselves in regard to the thing). And negation, non-being, becomes evident primarily as privation: an incitement to pursue that of which the thing is deprived.

§7.2.3 *The method of concomitant variations*

Already on a case matrix displaying the evolution of test scores, we would surely note gradations both in performance and in exposure to those home and school conditions we learn to accuse. And investigators would certainly take notice if they detected a pattern in these variations:
for example, if those children exposed to twice as many hours of TV were to fall in points twice as much. And we could imagine a gruesome experiment in which we test this hypothesis. Indeed, much research into cancer and other diseases consists of establishing various groups of plants, animals or people, exposing them to various dosages of some substance suspected of enhancing or damaging health, and determining the effects over sometimes long intervals. Exactly concomitant variations reinforce our generalization, opening the way to the formulation of reliable rules for practical exigencies. And lack of such variation may weaken the hypothesis (although the methods of Agreement and Difference may still yield “axioms” for tighter research).

Many phenomena of interest to modern science of the earth resist the complete manipulation required for the full employment of Agreement and Difference.

For example, in the case of heat, though we cannot expel it altogether from any body, we can modify it in quantity, we can increase or diminish it; and doing so, we find that such increase or diminution of heat is followed by the expansion or the contraction of the body. In this manner we arrive at the conclusion, otherwise unattainable by us, that one of the effects of heat is to enlarge the dimensions of bodies; or what is the same thing in other words, to widen the distances between their particles. Mill, Book Three, VIII, §6.

Another:

Let us now suppose the question to be, what influence the moon exerts on the surface of the earth. We cannot try an experiment in the absence of the moon, so as to observe what terrestrial phenomena her annihilation would put an end to; but when we find that all the variations in the position of the moon are followed by corresponding variations in the time and place of high water, the place being always either the part of the earth which is nearest to, or that which is most removed from, the moon, we have ample evidence that the moon is, wholly or partly, the cause which determines the tides. *Ibid.*

Even the abstract principles of Newtonian mechanics receive confirmation in this manner:

... let us consider ... the case contemplated in the first law of motion; viz. that all bodies in motion continue to move in a straight line with uniform velocity until acted upon by some new force. This assertion is in open opposition to first appearances; all terrestrial objects, when in motion, gradually abate their velocity and at last stop; which accordingly the ancients, with their *inductio per enumerationem simplicem*, imagined to be the law. Every moving body, however, encounters various obstacles, as friction, the resistance of the atmosphere, &c., which we know by daily experience to be causes capable of destroying motion. It was suggested that the whole of the retardation might be owing to these causes. How was this inquired into? If the obstacles could have been entirely removed, the case would have been amenable to the Method of Difference. They could not be removed, they could only be diminished, and the case, therefore, admitted only of the Method of Concomitant Variations. This accordingly being employed, it was found that every diminution of the obstacles diminished the retardation of the motion.... The simple oscillation of a weight suspended from a fixed point, and moved a little out of the perpendicular, which in ordinary circumstances lasts but a few minutes, was prolonged in Borda's experiments to more than thirty hours... *Ibid.*, §7.

Mill remarks:

Although the most striking applications of the Method of Concomitant Variations take place in the cases in which the Method of Difference, strictly so called, is impossible, its use is not confined to those cases; it may often usefully follow after the Method of Difference, to give additional precision to a solution which that has found. When by the Method of Difference it has been ascertained that a certain object produces a certain effect, the Method of Concomitant Variations may be usefully called in, to determine according to what law the quantity or the different relations of the effect follow those of the cause. *Ibid.*, §6.
§7.2.4 The method of residues

It often happens that we can methodically relate many elements of complex effects (such as the failure of human beings to be alert to their circumstances) to antecedent factors and still note that some portion of the effect remains unaccounted for. In our example of school children we may only surmise that, besides such factors as may be determined in the classroom and in family life, there may be others. In much biological and psychological research these “others” are often hypostatized (with such terms as “instinct”), but in astronomy, chemistry and physics the discrepancy between carefully calculated predictions and the observed phenomena often leads to further observation and to the detection of additional factors, likewise quantifiable. Most famously, astronomers discover that there must be a planet at a certain location—because their calculations of the motions of other celestial bodies needs a planet of that sort. Mill cites Herschel:

Unexpected and peculiarly striking confirmations of inductive laws frequently occur in the form of residual phenomena, in the course of investigations of a widely different nature from those which gave rise to the inductions themselves. A very elegant example may be cited in the unexpected confirmation of the laws of the development of heat in elastic fluids by compression, which is afforded by the phenomena of sound. The inquiry into the cause of sound had led to conclusions respecting its mode of propagation, from which its velocity in the air could be precisely calculated. The calculations were performed; but, when compared with fact, though the agreement was quite sufficient to show the general correctness of the cause and the mode of propagation assigned, yet the whole velocity to be accounted for could not be shown to arise from this theory. There was still a residual velocity to be accounted for, which placed dynamical philosophers for a long time in a great dilemma. At length Laplace struck on the happy idea, that this might arise from the heat developed in the act of condensation which necessarily takes place at every vibration by which sound is conveyed. The matter was subject to exact calculation, and the result was at once the complete explanation of the residual phenomenon, and a striking confirmation of the general law of the development of heat by compression, under circumstances beyond artificial imitation. Ibid., ix, §5.

Mill is quick to point out that the Method of Residues is close cousin to the Method of Difference, and depends for its legitimacy on the certainty of previous inductions—and that we can never be quite certain of the residue’s standing until we can obtain its effect “artificially,” i.e. separately (not easily done for merely hypostatized factors like “instinct” or “gravity”).

Even with these reservations, the Method of Residues is one of the most important among our instruments of discovery. Of all the methods of investigating the laws of nature, this is the most fertile in unexpected results: often informing us of sequences in which neither the cause nor the effect were sufficiently conspicuous to attract of themselves the attention of observers. The agent C may be an obscure circumstance, not likely to have been perceived unless sought for, nor likely to have been sought for until attention had been awakened by the insufficiency of the obvious causes to account for the whole of the effect. And [the effect] c may be so disguised by its intermixture with [other effects] a and b, that it would scarcely have presented itself spontaneously as a subject of separate study. Ibid., viii, §5.

§7.3 Pitfalls of observational induction

Installed now above phenomena, no longer helping beings actualize their own potential, we must both gather data and evaluate them, in alternation. We can structure both processes in rigorous methodological manners. And, in the last decades of the twentieth century, rapid computing machines have come to our aid in the storage and organization of data as well as in the detection of patterns and in the mathematical operations necessary for the development of theories regarding the data.

Yet, in the simplest human ways, errors abound. Let us first consider five very common forms of error. All five occur in the course of pre-scientific life as well as in the course of scientific investigation.

Hasty Generalization: most obviously, one swallow does not make a summer. From an extensive case-matrix we would like to draw at least preliminary generalizations. Yet we are then jumping beyond the givens,
and we may err in doing so. At the very least, careful observation requires of us the simple acknowledgement that we are in fact jumping. We can always ask whether the number of cases is sufficient. Also, whether the data had been collected non-prejudicially. Moreover, perhaps the children deliberately falsified the performance on the tests. Perhaps, too, the tests were ill designed to test intelligence.

Hasty Application: the future may not always resemble the past. Our generalizations reflect concepts of the behavior of phenomena, and we expect these concepts to find new applications tomorrow, or next year, or in other parts of the universe. But perhaps things elsewhen and elsewhere will behave differently. Nature is uniform, we would like to believe. Yet our very looking at the offerings of the next day can prevent what we see: we instill a uniformity in sets of phenomena that could offer diversity instead. At the very least, we should bear in mind that observation remains rational only insofar as we remain willing to observe new phenomena freshly, and not fall back un-self-critically on old generalizations in new applications.

Hasty Identification: we easily mistake what we believe we directly perceive. In a courtroom one attorney may ask a witness, “Did you see blood on the victim's collar?” and the opposing attorney may shout, “Objection! Witness is not a medical expert.” Should the witness then be confined to the question whether he saw “something red” on the collar? At what point can we trust our conceptual interpretation of sensory stimuli? In ancient literatures the question of proper identification arises in dramatic form: Surely that's my husband Amphitryon, thinks Alkmene. Surely that's my son Esau, thinks Isaac. In forensic medicine investigators raise the same questions in reference to the presence of organic and inorganic matter indicating (identifying?) the presence or absence of individual agents.

Post hoc (ergo propter hoc?): one of the questions every researcher must ask is whether the presence or precedence of one factor in the evolution of an effect should count as a cause. The Method of Difference helps us answer this question. Yet it may be that this Method cannot well apply, especially to individual cases where artificial experimentation would violate the cases themselves. Does drinking in the early evening cause the melancholy ensuing later? Even if, in every case under study, we correctly detect the temporal pattern, perhaps the melancholy caused the drinking! Does the moon cause the behavior of tides, or do the tides cause the moon to behave the way it does? In all such cases we must learn to distinguish the temporal order in which we register data from the temporal order essential to the new understanding of causality.

Reification: we may easily fall into the trap of assuming that a thing, an “extension,” must correspond to a concept we have formed. Consider this passage from Vance Packard’s one-time popular Hidden Persuaders (1960):

Housewives consistently report that one of the most pleasurable tasks of the home is making a cake. Psychologists were put to work exploring this phenomenon for merchandising clues. James Vicary made a study of cake symbolism and came up with the conclusion that “baking a cake traditionally is acting out the birth of a child,” so that when a woman bakes a cake for her family she is symbolically presenting the family with a new baby, an idea she likes very much.

From such a half-baked account of the causes of female cake-baking, one could easily slip into hyponstatizing a “factor,” an entity called “instinct for proving female functionality.” The reification becomes complete if a researcher places it on a case matrix as a possible item. Still, the conceptualization has its purposes, as the author indicates.

In The New Organon (I, §§44-68), Francis Bacon organizes the pitfalls of modern induction under four headings: Idols of the Tribe, Idols of the Cave, Idols of the Marketplace, and Idols of the Theater. His account of “idols” intends to parallel the traditional account of paradoxisms. Yet it differs remarkably from Aristotle’s account, in which paradoxisms appear as distorting what contorts itself already, so that contemplation of their untruth leads to a contemplation of the truth itself. In contrast, Idols reveal our own temptation to rely on left-overs, and contemplation of them reveals only human weakness. One breaks through these left-overs by constructing fresh case-matrices and affirming axiomata: the only cure for the disease.

“The Idols of the Tribe have their foundation in human nature itself,” we read. That is, we all distort and discolor what we experience by mingling our own nature with the events.
The human understanding is prone to suppose the existence of more order and regularity in the world than it finds. ... once it has adopted an opinion it draws all other things to support it. ... it is moved by those things which most strike it. ... rejects difficult things from impatience. ... commonly ceases where sight ceases. ... prone to abstractions. ...

Ibid., §§45-51.

"The Idols of the Cave are the idols of the individual man," we read. That is, each has his or her own cave into which the light of nature streams, but only to be distorted by one's education over time and disposition at the moment: "the spirit of man is variable and full of perturbation, and governed as it were by chance."

People become attached to certain particular sciences. ... some are stronger and apter to mark differences of things, others to mark their resemblances. ... some are given to admire antiquity, others to admire novelty. Ibid., §§54-56.

"It is by discourse that men associate, and words are composed according to the comprehension of the vulgar." We naturally discourse in the vulgar tongue, and thereby often trap ourselves in the Idols of the Marketplace: for this tongue supplies us with formidable left-overs leading us "into numberless empty controversies and idle fancies."

Discussions of learned men end often in disputes about words; with which it would be more prudent to begin — by defining them, and by recurring to individual instances to justify the definitions. ... some words name things that do not exist ("fortune," "prime mover"), others name poorly defined things ("humid," "earth") and actions ("generate," "corrupt"). Ibid., §§59-60.

"Lastly, there are Idols which have immigrated into men's minds from the various dogmas of philosophies, and also from wrong laws of demonstration. These I call Idols of the Theater, because in my judgement all the received systems are but so many stage plays, representing worlds of their own creation in an unreal and scenic fashion." Bacon dwells at length on this one Idol (§§61-67). And he names Aristotle as the chief author of the stage plays.

Each of these Idols does in fact parallel something in classical logic. Yet... Does the proper understanding of "true nature" require that we decipher immediacy to discount it? Bacon permits no redemption of αἰσθησις. Does education and disposition simply get in the way? Again, Bacon envisions us as unredeemable in our individuality; only by joining together in team effort can individuals overcome their corrupted nature (I, §139). Is language to be condemned as rooted in vulgar enterprises — or is everyday language vulgarized from a higher vocation, into which we must restitute it? And are the founding literatures of our tradition essentially distractions from our true condition — or do we, as readers, fail to re-enact them in consonance with our own testimony?

We could debate such questions as they cross over our traditions. Yet the historical fact remains that the conceptual component of observational rationality requires a purity of encounter with "bodies performing purely individual acts." On the far side, instances loom. On our side, we must carefully form concepts corresponding to the multiplicity of these instances. And on this our human side our own prior interests crowd in on the act of forming concepts, and easily skew the result. Simple human interests can only obscure the light: they provide no premonitions of any worthwhile development. We must fight them as we would any ruthless enemy. Anticipating Descartes, Bacon writes: "let every student of nature take this as a rule: that whatever his mind seizes and dwells upon with peculiar satisfaction is to be held in suspicion, and that so much the more care is to be taken in dealing with such questions to keep the understanding even and clear."

Modern investigation becomes simply another form of self-deception unless we learn the wisdom of the gap between concept and percept. Precisely to the extent that we aim to observe, aloofly, gather and enter...
interpret carefully, we must uphold a new difference: that between our own conceptual framework and the instances revealed in experience. One price we pay when developing observational rationality is that we must remain forever skeptical of our own results. Openness is a prime virtue. And it is a condition for securing power over what appears.

§7.4 Theory and reality

Today we all know that aloof observation and calculation unfold within elaborate theoretical constructs. There are well established theories of motion in space, of electricity, of optics, of sound, of genetic structure, of nuclear energy, of geological formation, of chemical bonding, of the functioning of plants and animals, of human behavior, and so on — too many to list adequately, even in outline.

Scientific theories transcend case matrices of the sort recommended by Bacon and systematically illustrated by Mill. They have structures of justification that rely more heavily on computation than on observation. And, contrary to Bacon’s claim that the right method will level men’s wits (I, §§61 & 122), modern scientific theories remain inaccessible even to most highly educated people: each theory generally belongs to a handful of specialists in a field much narrower than those listed in a university’s catalogue of courses.

The plethora, the autonomy of well founded theories raises two inter-related questions: Is there any one theory unifying them all? and To what realities do theories refer? In Book Five, on the conditions of truth, we will address these questions “axiomatically.” Yet even now we can pursue these questions historico-philosophically.

During the heyday of the Enlightenment, when intellectuals were first struggling to establish the new rationality of observation and computation, the one theory overreaching all others had a name: mechanism. Thinkers understood that if there was to be an alternative to the classical understanding of “learnables” (the four causes) the new understanding had to suppose that daily realities followed a pre-established Plan: much like clockwork, the universe (i.e., the collection of bodies performing purely individual acts) must operate according to rules formulated in reference to measurable quantities (weight and size and speed — later, force and energy). Such formulas would allow one to argue from presented phenomena to unknown causes (whether in trying to understand the past, the present, or the future). And if reality did not follow a formulable Plan, how could we possibly investigate it aloofly, as distinct from learning it the way artisans do? Each item we investigate must be part of a larger machine.” A machine, in fact, that only God could have made, given its intricacies; all the more reason to investigate it.

Essential to the defense of mechanism was the rejection of final causes. Each thinker in the seventeenth and eighteenth century takes his turn. Bacon: final causes “have relation clearly to the nature of man rather than to the nature of the universe” (I, §48). Descartes: final causes “are useless in natural affairs” (Meditations, iv). Spinoza: “Nature has set no end before herself, and ... all final causes are nothing but human fictions” (Ethics, 1, Appendix); any “final cause ... is nothing but human desire” (iv, Preface). Leibniz: “Souls act according to laws of final causes — by desires, goals, and means. Bodies act according to laws of efficient causes — laws of movement” (Monadology, §9). And Kant, although he recognizes that the life-sciences require a “teleological connection of causes and effects,” also knows that the first commitment

* In Part Six of his Discourse on Method (1637), Descartes details his arguments for publishing his hitherto unpublished works on optics, geometry, and meteorology:

... au lieu de cette philosophie spéculative, qu’on enseigne dans les écoles, on en peut trouver une pratique, par laquelle connaissant la force et les actions du feu, de l’eau, de l’air, des astres, des cieux et de tous les autres corps qui nous environnent, aussi distinctement que nous connaissions les divers métiers de nos artisans, nous les pourrions employer en même façon à tous les usages auxquels ils sont propres, et ainsi nous rendre comme maîtres et possesseurs de la nature.

One generally quotes this passage to underscore the new interest in concrete applications of the theoretical principles of mechanism. But notice also the explicit acknowledgement of the previous venue of philosophy: “... knowing the force and the actions [of environing bodies] just as distinctly as we know the various métiers of our artisans ...” The old (theoretical) knowledge has focussed us on the métiers of our artisans: on their accomplishments, their services, their powers as evident in what they deliver. The new (practical) knowledge, Descartes claims, will focus us directly on the force and actions of natural events. The intellectual vocation now stands along side the productive ones; it ceases to be that of highlighting the dramas of human being engaged within nature.
of scientific investigation is to discover mechanical causation (*Critique of Judgement*, §77).

Two realities, then: human reality and objective reality. On the one side — the near side, our side — looms desire, with its concomitants: purpose, recollection, anticipation, attention to events. On the other side — the far side, the obdurate side — looms nature, now considered as forced and as forcing according to hidden mechanisms governing each item as pushed and pulled, pushing and pulling other items. The theory indigenous to observational rationality posits this division of labor, this dualism of human subject, individual or collective, and a nature comprising objects to be determined and regulated (rather than subjects to be discovered, loved, and cultivated).

Yet look again. The modern theory of mechanism relocates purposes into human subjectivity (nowadays a redundant expression). It does so, however, with a scarcely hidden agenda: desire and purpose, recollection and anticipation, may now congeal into attentive human planning, carefully devised reconstruction of the natural environment — the rationality essential to modern technology, whether in medicine or in space exploration. A will to power, no doubt, and one that some environmentalists seek to restrain in ethical considerations, without always pausing to re-examine the logic embedded in the epistemology of such reconstruction.

The question often comes upon intellectuals: How does theory relate to reality? Most commonly, one raises this question with a view to the accuracy of detailed theoretical accounts of various phenomena, since these phenomena present such a rich array and research always leaves room for improvement. Then, too, one raises the question with a view to the legitimacy of theoretical accounts, since occasions incessantly arise where a heart-felt, a “subjective” response does more justice to circumstances (wilderness parks, spouses, ...). Historically, too, one can wonder at the fact that only a tiny portion of our inherited intellectual ambition endorses the theory of mechanism — and that this portion appears even tinier when we turn to non-western understandings of human involvement with circumstance.

The strictly epistemological question differs from all the others. Once we admit that the understanding of any one mechanism engenders a plan for intervening in the process (for reconstructing events), we begin to lose sight of the original boundary between the objective and the subjective, the dualism initially posited as fundamental. True, we look to phenomena for mechanisms. And we then devise our own machinery of intervention and reconstruction, whether for curing a disease or exploring the moon. But we do all this only because we have inherited the metaphysical supposition that mechanism is the prime reality: we adjust our own planning to the Plan of the Universe — just as Descartes and Leibniz recommended. Yet in research itself we may just as well reverse the temporal order of the story: we plan, we envision how events might better run their course (i.e., better for our own satisfaction), we devise constructs that we then impose on the course of events, trying out our own constructions, modifying them as we go along — just as we would trek through the Himalayas with a tentative map, but adjust both the map and our own itinerary at each moment, according to the givens at that moment, and without insisting that there be one true map for this trek for or any subsequent trek.

Epistemologically, the question has arisen with creative vigor in the twentieth century: Does rigorous investigation of nature unveil or impose mechanism?

Within the epistemological question, we may detect a logical conundrum: What exactly does a research result describe? Scientists generally agree — more likely, however, before they sniff the wind — that their research on the intricacies of animal, vegetable or mineral phenomena culminates only at the point where their understanding allows them to reproduce what they are trying to understand. How else, except in reproduction, can we prove our understanding? Thus modern theory also engages us in the alteration of circumstances — not, of course, according to any purpose internal to them, but according to our own purposes. Yet the criterion of reproduction assures us that the final account of research insight will read much as a recipe for baking bread does: it will describe first and foremost what the researchers have done, and in a manner paving the way for others to do the same. The description of research doings does gather around various determinations of what happens (did and will) on various occasions (empirical data), and it does include abstract accounts of mechanisms, as on a drawing board (rational calculation). Yet already epistemologically we can rightly ask whether the two sides of the description are separable, and how they relate to one another.
As logicians, we ask about the focus of modern investigation. Where is the “logical subject”? Where is the “reality” at issue in the theory of mechanism — the reality that we are indeed discovering, describing, contemplating as the correlate of our own rationality? Formally, the accounts clearly focus on the activities of recollection, anticipation, and careful attention to detail, and above all on the imposition of constructs. Accounts do not in fact focus on the objects that appear in the course of organization, nor simply on the subjective acts in isolation. Rather, they focus on the human organization of objective occurrences, on this event as the unit out of which the two sides can fall whenever the event ceases. As fall-outs, both sides become, if not unreal, then “real” in some derivative sense — even though children will assume that the pictures left behind on the desk each evening somehow replicate the realities.

It may seem strange, then, that the reality at issue in modern theory lies neither in what we confront as objects nor in our own subjectivity, but rather in the ongoing affairs in which these two sides enter as players, like the famous Zen koan of hands shaking hands. Yet it is surely no more strange than that the reality of classical contemplation should lie neither in άισθησις nor in λόγος, but rather in the τέλος at work in any encounter. On both accounts, reality requires of the beholder an ability, and of course a willingness, to participate in its event.

Meanwhile, the very success of observational rationality has in fact led to a total revision of our understanding of deductive inference. The syllogism, as Bacon and Mill already recognized, no longer serves as the paradigm of inference. Mill went so far as to argue that all inference is inductive. Since the 1840s, however, much modern science has blossomed with a strongly deductive component. In the wake of these developments, thinkers, especially mathematicians, have formulated elaborate systems of inference with non-syllogistic foundations. The demands of these systems form the topic of Book Four.
Appendix: Induction in Arithmetic

Tradition has it that over the entrance to his Academy Plato had a warning carved in stone:

\[ \text{μηδεὶς ἀγεωμέτρητος εἰσίτω} \]

**NO ONE UNGEOMETRICAL MAY ENTER**

Whatever Plato intended by this “prerequisite,” it indicates a faith in what we more generally call “mathematics” as preparation for philosophical work — perhaps for intellectual work of any kind (Plato would immediately cite astronomy and music). Moreover, the study of number (\( \text{ἄριθμος} \); quantity, amount) can even be understood as preparation for geometry. For, anciantly, “arithmetic” (as distinct from “logistic,” calculation) shared with geometry its focus on configurations (“numbers of things”), whereas geometry goes further to the study of points, lines, planes and perhaps solids. While arithmetic calls upon intellection (\( \text{νοησις} \)) confined to “how many,” geometry builds on this intellection when passing to the study of “where”: perhaps it helps draw us out of the “hollows of the earth” and to place us on the earth, where we can “envision it whole” (*Theaetetus*, 174E) and discover “its many wondrous places” (*Phaedo*, 108C).

Be all that as it may, we can turn to more contemporary kinds of arithmetic study to illustrate a kind of “induction of the universal from the individual case” that may indeed serve as preparation for participating in the Socratic induction that Aristotle understood to be the forerunner of his own less dialectic (elenctic) and more natural induction.

For instance, there are easy *formulas* available to students for calculating Permutations and Combinations of items, and Combinations over Kinds. And nowadays pocket calculators even have buttons marked \( nP_r \) and \( nC_r \). But how might a student become a “primary name-giver”
of these (Cratylus, 436B)? How might we “search and discover” the matter itself? How might we recover the “workings” (πραγματικά) the names have left behind (Critias, 109D)? Let us consider…

§1. Permutations

Permutations are the possible ways we may produce ordered arrangements by drawing upon a collection of items. For example, if there are three people in a waiting room (Alice, Betty, and Carol), we can imagine a number of ways (indeed, exactly six) in which they could line up to see the doctor: ABC, ACB, BAC, etc.

The question of permutations is always a question of the quantity of projected sets: how many ordered arrangements are possible, given various conditions on an original set of so many items. In the case of very small collections, we can determine the number by fairly simple experimentation: if there are two individuals in the waiting room, it is clear that there are only two possible orders of visitation; with three we could list six. But soon we would be hard pressed for space and, moreover, we could easily lose track.

If there are five people in the waiting room, we could reason out the possible orders in the following way. The doctor has five choices for the first visitation, but only four for the second, three for the third, two for the fourth, and no choice for the fifth. For the first two choices alone, there are 20 possible orders, for the first three 60, for the first four 120, and for the last there is no new possible order. Thus there are 120 possible sets of ordered visitations for the original set of five people.

This one example illustrates the multiplication principle of permutations: if the first position can be filled in m different ways, and if the second position can be filled in n different ways, then the number of different “fillings” (permutations) = m × n.

Yet we sometimes want to know how many permutations are possible over a portion of a collection. For instance, we might wish to know how many sets of different three-letter initials are possible given twenty-six paper letters in a box (so each projection uses them up). The multiplication principle supplies the answer: 26 × 25 × 24 = 15,600.

From these examples, we may generalize and symbolize: the number of permutations (possible for a set) of n elements taken (in subsets of) r at a time will be the factorial of n limited by r:

\[ nPr = n(n - 1)(n - 2) \ldots (n - r + 1) \]

In effect, the r tells us how many factors of the factorial to take into account. To simplify the calculations of permutations, we may prove that the formula is identical to:

\[ nPr = \frac{n!}{(n - r)!} \]

The proof proceeds by way of two identities. First:

\[ \frac{n!}{(n - r)!} = \frac{n(n - 1)(n - r + 1)(n - r)(n - r - 1)(n - r - 2) \ldots}{(n - r)(n - r - 1)(n - r - 2) \ldots} \]

which is obtained by expanding the factorials (and recalling that r will lie somewhere between n and 1). Finally:

\[ = n(n - 1)(n - 2) \ldots (n - r + 1) \]

which we obtain by dividing out the common factors.

In the special case where r = n (the doctor will see all five patients in one set) we had best recur to the first—the “base”—formula:

\[ nPn = n! \]

Otherwise, we would have to decide that 0! = 1 (which makes no intuitive sense).

§2. Combinations

While permutations are sets of ordered elements possibly generated from an original set, combinations are sets of elements possibly generated from an original set regardless of order.

For instance, if a cowboy can herd only three steers at a time, drawing upon an original set of five steers, an onlooker might wonder how many different selections he can make. One way of deciding would be to list the five over and over again, and find a way of eliminating two at time so that all possible combinations are assured and none are repeated.

Again, as the numbers involved increase, such experimentation soon becomes tedious and fraught with the possibility of error. We need to develop a generalized procedure allowing us to compute combinations a priori. Such a procedure we may develop along the following lines, drawing upon the earlier formula for permutations:
The number of permutations of five steers taken three at a time is:

\[ s P_3 = 5 \times 4 \times 3 = 60 \]

Yet any three of the five steers may be selected in six different ways:

\[ s P_3 = 3! = 3 \times 2 \times 1 = 6 \]

These latter are all “duplicate” combinations, so the number of different combinations is 60 ÷ 6 = 10.

Carefully examining what we have already done, we may generalize: the number of combinations possible from a set of \( n \) elements taken into subsets of \( r \) elements (“taken \( r \) at a time”) is equal to the number of permutations of \( n \) elements taken \( r \) at a time divided by the number of permutations of \( r \) elements taken \( r \) at a time. Or:

\[ n C_r = \frac{n P_r}{r P_r} \]

Recalling the compacted formula for permutations, and that \( r P_r = r! \), simple algebra leads to the compacted formula for combinations:

\[ n C_r = \frac{n!}{r!(n-r)!} \]

This version allows us to compute the number of combinations directly.

§3. Permutations and combinations over kinds

So far, we have considered sets having individuals as elements, with the consequence that each formation of a permutation or a combination can draw upon each element only once.

How about sets having kinds as members? For instance, license plates can take on various permutations with the alphabet of 26 elements and the ten integers 0-9, making repeated use of each letter and integer: in jurisdictions prescribing three letters followed by three numbers, there will be \( 26^3 \times 10^3 \) possible and distinct license plates.

A simpler example: a class of two students might receive either one of three marks — Honors, Pass, and Fail — and there will be \( 3 \times 3 = 9 \) possible assignments of marks. Here again, the multiplicative principle holds and, presumably, no mark is “used up” by being assigned.

Now, in this last example: How many combinations of marks are possible? We might count: both could get H’s, both Ps, or both Fs = 3 combinations; but there might be an H and a P, an H and an F, as well as a P and an F — 3 more, for a total of 6. Yet such counting is feasible only when we have a very small number of kinds and very few sets-to-be-formed.

To break the habit of counting, consider how many combinations of grades (A, B, C, D, F) are possible in a class of seven students, where the possibilities range from straight A’s through straight F’s. Here we have two givens: the sets-to-be-formed are seven, i.e. \( r = 7 \), and the kinds of marks are five, i.e. \( k = 5 \). How might we now proceed? How might we determine the quantity for \( k C_r \)?

One way to proceed is to devise an analogy to the original, but one allowing us to abandon the notion of kinds. The beginnings of such an analogy is pictured by the following parallel tables:

```
<table>
<thead>
<tr>
<th>Tom</th>
<th>Kim</th>
<th>Joe</th>
<th>Sal</th>
<th>Tim</th>
<th>Eta</th>
<th>Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>B</td>
<td>F</td>
<td>A</td>
<td>D</td>
<td>F</td>
<td>B</td>
</tr>
<tr>
<td>D</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>F</td>
<td>D</td>
<td>F</td>
<td>D</td>
<td>F</td>
</tr>
</tbody>
</table>
```

Analogy

```
X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw}
\times X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw}
\times X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw} * X_{aw}
```

Each row in the table in the original represents one possible assignment of grades. The corresponding row in the analogy represents the same assignment, but in reference simply to elements — in such a way that the number of A’s corresponds to the number of X’s (if any) to the left of the first star, the number of B’s to the number of X’s (if any) between the first and the second star, etc., ending with the number of F’s (if any) corresponding to the number of X’s following the last star. With the names attached to the X’s, there is a one-to-one correspondence between the original and analogy: we could construct either one from the other. And each has \( 7^2 \) rows.

Now we may treat each of the 11 elements as items — i.e. we may count the dividers as items too, while keeping the four distinct: we thereby generate a variation on the analogy, a third table, one having 11! rows (permutations of the 11 items). But, for discerning the possible combinations, we must disregard entirely the order of the items. We set ourselves up to do this by imaginatively reshuffling each row so that all the X’s huddle together to the left side and that all the dividers, each labeled according to its function, huddle on the right side. We thereby allow for a ghostly version of this third table, one arbitrarily chosen row of which will be:
Now the question has been relocated: How many repetitions of the left-hand portion (of the ghostly version) are there throughout the table? This we determine by asking: How many permutations are possible for any one re-huddled row? Well, we may look at the one above: there are 7! permutations of the X’s, and for each one of these there are 4! permutations of stars. This one arbitrarily considered row (each and every row has 7!×4! repetitions in the table of 11! rows. So we need only divide the total number of rows by the number of these repetitions for any one row: 11!/(7!×4!) = 320. We can now imagine a final table: it will contain 320 rows, each now displaying one combination (of 11 items) distinct from all others.

Looking back upon the manner in which we obtained this one result, we can generalize. The number of permutations (in the analogy) over k kinds taken r at a time is \((r+k-1)\)! — and to obtain the number of combinations we divided this by the product of \(r!\) and \((k-1)!\). Thus:

\[ kCr = \frac{(r+k-1)!}{r!(k-1)!} \]

It should be noticed, though, that the formula reflects what I have called the third table. Here, each row consists of r slots to the left plus k-less-one-1 slots to the right. And each row of the final table contains a unique combination (unordered set of items) drawn from the third table. But we are only interested in the combinations on the left side of the virtual version. So we must ask, first, whether the sets on the left capture all possible combinations: yes, because if there was one missing there would be an entire row missing. And, second, whether they capture too many: no, because if any duplication in the left set would imply a duplication of the entire row (since for every combination on the left is matched with every combination on the right). So the rows we have selected for the final table represent all the possible combinations, and only these.

Most importantly, the formula above represents what we did (or at least I did)—intellectually, constructively. Pictures it, even—or rather its form.

And now we can re-calculate the formula, infer a more manageable version—just as we did for \(nP\) and \(nC\). In this case we can factor out the \(r!\) by reformulating \((r+k-1)!\) as \(r! \times (r+k-1)!\):

\[ kCr = \frac{(r+k-1)!P(k-1)}{P(k-1)!} \]

Especially with this second formula, we can easily proceed to the business of generating answers: there are 6 combinations of 3 kinds of marks assigned to 2 students at a time; there are 320 combinations of 5 kinds of marks assigned to 7 students at a time; and so on. But when generating answers with formulas we assume the formula: we become what Socrates might call “secondary name-givers” — and defeat, or at least abandon, the purpose of the present considerations. For the task has been to illustrate how we might “search and discover” the origin of any formulas, any “generalization,” by careful consideration of one example from which the formula might itself be generated.

We must distinguish, then, not only between two kinds of generation (of answers and of formulas for answers) but also between direct (intuitive) apprehension of an example and the (discursive) modifications of the formula generated by inferences external to the example. This latter distinction, too, is difficult, since we seem to be “discoursing” even when directly apprehending the procedures by which we extracted the formula. The task is not to separate but to distinguish—and to keep the intuition in the forefront guiding every move we make.

§4. Birth of intellection out of sensation

The derivation of \(nP\), \(nC\), and \(kC\) illustrates the nature of intuitive (vs. axiomatic) proofs. It proceeds by examining one empirical example carefully (just as Euclid examines one drawn triangle); then re-examining what one actually did (“thought thinking itself”); and, finally, seeing intuitively (with “the eye of the soul”) that the construction and decision reflects a form that holds independently of the one example: this form we then symbolize as a rule—a formula with variables.

In his Seventh Letter, Plato speaks at length about how anyone is deluding himself who thinks he can pick up insight from anyone’s written works. Only learning directly from the matter itself can lead to insight. He then structures the development of such insight:

For each being there are three things necessary if there is to be cognition of it. The cognition comes fourth; and as fifth one must posit the thing itself, what we aim to make sense of, what truly is. Of these, the first is a name, the second an account (λογος), the third an image.
Now, the image (ειδωλον, eidolon) is essentially sensory. Thus we begin by examining one example in sensation (imagined or recalled—usually written). At the same time, we name what we are searching for: permutations, combinations, or whatever. Simultaneously with the account (λογος) of what we mean by the name, we now account for the example itself. We are thereby exercising the intellection (νοησις) already native to us. Once satisfied that we have fully seen what we set out to see, we ask ourselves how we arrived at this full vision. We are now no longer looking at the eidolon, but rather at how we would proceed in any such case: different as different cases may be, they lend themselves to the same intellection. From one example we then attain to the universal. If we turn to fresh examples, it is only to refresh our intellection—or to help others exercise their own.

Thinkers have always noticed this peculiar “induction” of the universal. George Boole remarks in his *Investigations of the Laws of Thought* (1854: Chapter One, §4) that, while the laws of nature, which are “either inductive inferences from a large body of facts” or “physical hypotheses of a causal nature serving to explain phenomena,” and in either case only “probable conclusions,” we learn that . . . knowledge of the laws of the mind does not require at its basis any extensive observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances.

Notable in Boole’s passing formulation is that it would ask us to consider our three formulas as “laws of the mind”—in sharp constrast to the laws of nature.

In the same modern vein, but in a more systematic context, Kant asks in his *Prolegomena to Any Future Metaphysics* (1783), §7, whether our ability to do mathematical work does not “presuppose some a priori basis of cognition that lies deeply concealed—and yet might become manifest by way of its effects if only we could energetically ferret out their first beginnings?” He answers:

But we find that all mathematical cognition has this peculiarity: it must present its concept in intuition, doing

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* In religious controversies the word gets translated as *idol*: the charge is leveled at some for worshiping a sensory image rather than the super-sensory at issue in true worship.
Appendix: Induction in Arithmetic

The term “intellectual intuition” (intellektuelle Anschauung) surfaces in Kant’s work mainly after the first edition of his Critique of Pure Reason. See, e.g., the 2nd edition at 308B; then also his Critique of Judgement, §77 (also §62 on Plato’s “intellectual communion”); and his Prolegomena to Any Future Metaphysics, Appendix—where he remarks, in a footnote:

…all swooning idealism collapses — the idealism that (we can already detect in Plato) inferred, from our cognitions a priori (even those of geometry), another intuition, different from that of the senses (namely, intellectual intuition) — all because it never occurred to anyone that the senses themselves might intuit a priori.

Kant’s insistence on space and time as the origins of intuitive induction in mathematics fits well with our own commitment to modern science generally, which aims precisely to formulate phenomena discursively rather than to contemplate noumena intuitively. While developments in mathematical work (formal science) since 1900 have retreated from the Kantian and Newtonian insistence on the reference to space and time, “material” science necessarily retains it in some way if it is not to fall back once again into superstition.

However, Plato’s account has its foothold in another venue. In his Republic (521C) he has Socrates ask how “guardians of the city” can “become” — how we can “become” in such a way that we devote ourselves to (meld with) “what-is” and thereby pull ourselves out of “what-becomes,” namely a kind of lostness in the generation and destruction of the things arising for encounter (things in αἰσθησις). What discipline (what µαθησια) introduces the draw (δικαίων) pulling the soul toward what-is (τὸ ἔνω) toward being (our true estate: ἡ οὐσια)?

Socrates proposes the discipline of ἀριθμος: “arithmetic,” the study of amounts (numbers of things), the soil in which “logistic” (calculation) will also take root. Now, Glaucon can see that, for example, Agamemnon — or, rather, any good guardian — must have learned to count; but he wonders (as we might) what this has to do with the draw away from what-becomes toward what-is — or what role this draw plays in the “becoming” of guardians. To reply, to locate the draw, Socrates holds up three of his own fingers: whereas, he says, we recognize each of the three (middle, ring, and pinkie) as the same (as one and all fingers) even though numerically distinct, the senses report this “same thing” to be both large and small, to the left and to the right, thick and thin, hard and soft. It is this “second” matter — finger-in-sensation — that “summons or awakens intellection” (523D), namely to distinguish between finger-itsel and finger-in-sensation, and thus to decide in what sense there is one thing, in what sense two, in what sense three, and so on.

Plato’s argument here deserves more careful attention. It introduces “from scratch” the now-familiar distinction between subject and predicate, and we could trace out parallel accounts both in Plato’s own works and then in Aristotle’s. But all along the attentive reader will no doubt pose another question: Is this example of the fingers likely to summon the intellection of potential guardians, draw their souls out of the mire of becoming into the light of being? Is it likely even to convince a young soul that sensation is the root of the confusion? The example seems rather impoverished, and in need of Socrates to confuse anyone. Couldn’t most anyone — or at least anyone with leadership potential — provide other examples, ones much better suited to illustrate the confusion and the draw?

To answer these questions, we must raise another: What is the function of examples in the education of leaders — and in the literature designed for such educational purposes (such as Plato’s)?

Agamemnon must count in order to lead others. He must recognize arrangements on the battlefield and behind the lines, along with their multiple “beings” — each as singulars, in sensation, and containing multiplicities ever in flux: one giving way to the next. Without release (λύσις) from sensory arrangements, he will not be able to lead his soldiers through the flux. Education in leadership must help souls understand “the difference between the one, the two, and the three” — not for commercial use in the pursuit of private benefit (525C), but for distinguishing sameness and difference in the city. This is the proper use of the discipline (523A): it trains the soul in the detection and interrelation of unity and multiplicity in the preservation of the city.

* The term “intellectual intuition” (intellektuelle Anschauung) surfaces in Kant’s work mainly after the first edition of his Critique of Pure Reason. See, e.g., the 2nd edition at 308B; then also his Critique of Judgement, §77 (also §62 on Plato’s “intellectual communion”); and his Prolegomena to Any Future Metaphysics, Appendix—where he remarks, in a footnote:
In both spoken and written discourse, examples remind us of where we are and what intellecction must deal with. Yet the New Literature proposed by Plato contrasts with the Homeric Literature of the time in that it asks us to acknowledge the intellection essential to coherent response to these “same” examples (and especially the intellection essential to coherent leadership of the “same” city). Thus one essential demand of the New Literature is that readers provide their own examples—first, imaginary ones parallel to those in the text itself and, second, to those they meet when they re-descend into the cave. Homer, in contrast, invites us to dwell on the examples recited.

Kant’s account of cognition entails a much different assessment of the power of examples. In the formal sciences (mathematics), we must indeed “present the concept in intuition,” but this means “in a spatio-temporal configuration.” Here, the sensation plays no essential role (not even the chalk on the board, let alone any story about patients waiting in a doctor’s office; it only serves to supply refutations by counter-example). However, in the material sciences we do need examples: “I cannot represent in intuition the concept of a cause in general except in an example supplied by experience; and similarly with other [“material”] concepts” (A715/B743). But in the sciences of guardianship—including that of philosophy—“examples are the go-cart of judgement”:

A physician, a judge, a politicist may have at his command many excellent pathological, legal, or political rules, even to the degree that he may become a thorough teacher of them—and yet easily stumble in their application. For, although admirable in his understanding, he may be wanting in natural power of judgement. It may be that he comprehends the universal in abstracto and yet remains unable to distinguish whether a case in concreto comes under it; or it may be that he has not received, through examples and actual practice, adequate training for this particular act of judgement. Indeed, the one and only real benefit of examples is that they may sharpen the judgement, yet they more usually impair correctness and precision of the insight at issue in the understanding; for only very seldom do examples fulfil the condition of the rule (as casus in terminus), and they often weaken the effort to comprehend the rules in their universality, in independence of the particular circumstances of experience, and so accustom us to use rules as formulas rather than as principles. Examples are the go-cart of judgement, and those who lack natural talent can never dispense with them. (A134/B173)

Whatever else may be detected in this account, we should not miss Kant’s insistence that rules owe nothing to the examples: sensation does not itself summon intellection, and intellection lords it over examples. How can this be the case, especially since Kant’s critical project claims to account for the way things happen and to avoid the “swooning idealism” of Plato? As we shall see in Book Five, Kant’s account of phenomena stems from the assumption that “objects must conform to our cognition” rather than that our cognition must conform to things in their being (B.xvi). There is no way that Kant—modern philosophy in general—could understand intellectual competence as defined by “helping complete what nature is unable to finish, and following her.”

We might consider the possibility that, while Plato and Aristotle may obstruct the appreciation of the processes and the achievements of modern science, Kant and modern thinkers generally obstruct the appreciation of the conditions and tasks of guardianship.

However, all traditions help us understand the tense marriage of sensation and intellection, and the various births possible from this intercourse. The basic event may be considered neutral: from one example we must achieve insight into the universal. Traditions differ in their respective accounts of how this is possible."

* Cf. Aristotle’s Rhetoric, ii, 20 (and p. 151 above). The Greek for “example” is παράδειγµα — our “paradigm”: what looms to the side and sets the standard (Socrates asks for an “example” of piety in reference to which anyone’s action might “fit”: Euthyphro, 6E). Thus in Greek literature an “example” occasions the standard—whereas in modern literature the word suggests something looming to the side of the standard (universal).
But, so far, only the ancient account addresses the question philosophically (rather than psychologically) why most people have difficulty reading reflective works even in their own tradition. Typically, each such work requires of its readers that they apply themselves to an example and yet also think the universal in it. Precisely sacred texts do this, as thinkers of the Third Interpretation insist when distinguishing between the literal, the allegorical, the tropological, and the anagogical meanings of the mini-stories in the Bible. Descartes then asks us to consider the various perceptions possible for a single piece of wax — much as geometers ask us to consider the various properties of a triangle drawn in sand. Berkeley asks us to consider the peculiarities of trying to think what happens when a tree falls in the forest, out of hearing. Kant asks us to analyze what it means to experience at all — reader's choice, but (as he said in regard to “cause in general”) we can only proceed with one experience “in view.” In the 20th century, Heidegger broached head-on a new question: What does it mean to be in a world — any world you choose, so long as it is one engulfing you — and to elicit this meaning in all its generality?

In all these cases, readers who cannot concentrate on one example, and detect the universal in it, will find the most powerful texts unintelligible.

*I detect a pedagogical parallel in Confucius' Analects, Book Seven, Chapter 8, dating from around 500 BC.:

The Philosopher says:
For one not keen, there is no unveiling.
For one not stumped, there is no developing.
If you have one corner shown to you
and you do not search for the other three
then there is indeed no return.
Prelude

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Appendix I: Axiomatic deduction
Appendix II: Axiomatic induction
Appendix III: Axiomatic set formation
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Mathematical Logic
The Workings of Formalism

The original effort, evident in the writings of Plato and Aristotle, aimed to understand thinking (νοέων) as the penetration of predicates, an ecstasy into subjects, and talking (λόγος) as consummated in such penetration. Indeed, our own nature, our λόγος-bound condition, appeared as enshrining our potentiality for moving into such intimacy with things. And this intimacy provided the criterion by which we could distinguish knowledge from familiarity, a knowledgeable intimacy restoring to names their function of naming—and to us our work of calling things by their names.

Nowadays we are confronted on all sides—in academic, commercial, and government institutions—by a highly developed thinking apparently contrary to the original: a kind of thinking born of the effort to organize things, their predicates, so that we may know them without becoming intimate with them. Intimate with an organization, perhaps, but not with the things organized—a knowledge allowing us to operate on things at a distance.

The venue for contemplating our λόγος-bound condition has shifted from the hands-on to the hands-off, from the being-centered to the response-centered, from the artisanal to the scientific form of engagement with circumstances. This shift already entails enormous consequences for the ways we read and talk about the literature of logic. In the writings of Plato and Aristotle, the analogue for what we today more comfortably call “logic” remains part of an effort to restore thinking and talking to the truth of things, to re-attune language with circumstances as they are in themselves—to overcome the illusion, already engendered in artisanal
work, that we actualize our rational nature by controlling circumstances. After the shift, logic has clearly become part of the now familiar effort to reconstruct circumstances, to rediscover, in the image of our own inventions, what we encounter.

Reconstruction rather than restoration. Thus we invent artificial languages. For instance, water may serve as the model to which chemists look in order to draw their picture of H\textsubscript{2}O, but this artificial vocabulary represents a special version of water, one literally and formally purified, and the version employed in all subsequent work within the discipline, whether theoretical or laboratorial. The artificial language of modern logical work serves as the prototype for reasoning with such purifications.

As Descartes and Leibniz recognized, an artificial language facilitates intellectual work enormously. For one thing, it allows us to withdraw from direct dealings with problems at the interface: in this way we can concentrate on formulating our own responses to circumstances. For another, it encourages us to withdraw together: we can readily agree on the permissible forms of our responses. And it already establishes a clear path on which we may progress together in our effort to understand connections: a method allowing us to backtrack to review, to correct, and to repeat previous steps. In these two thinkers, and a host of others, we find the guiding hope that a single artificial language expresses not things we encounter, but the thoughts we develop.

In retrospect, at least, we can see that the modern ambition to construct an autonomous language has installed into our thinking a dualism of formal articulation and actual encounter. This dualism is foreign to our ancient traditions, which generally held that thoughtful talking, λόγος, grew out of our encounter with affairs at hand, πράγματα, as a plant grows out of the soil. The modern dualism of language and thing is the likely source of the more familiar dualism of abstract and concrete that very much defines our educational system.

Now that the hope to establish the new logic in our institutional affairs has been fulfilled, we can wonder whether the dualism has been entirely benign. One question is whether human articulation that is adequate to itself is thereby also adequate to circumstances. Already at the time of Leibniz’s efforts, there were critics who raised this question, and Leibniz replied:

No one should fear that the contemplation of characters will lead us away from things themselves (à rebus abducat); on the contrary, it leads us into the interior of things (ad intima rerum ducet).\footnote{From a letter to Walter von Tschirnhaus, dated May, 1678. Item 19 in \textit{Leibniz: Philosophical Papers and Letters}, edited by Leroy E. Loemker (D. Reidel Publishing Company, 1969 & 1976). See also Items 28 and 68 in this collection.}

Whether indeed modern thinking leads us into intimacy with things is something we can decide only after understanding what it means to engage in the contemplation of characters (\textit{characterem contemplatio}), what we might mean by a thing (\textit{res}), and what counts as getting into a thing, becoming intimate with it.

\footnote{Preface to a projected \textit{Science génér\'e\i le}, contained in \textit{Opuscules et fragments inédits de Leibniz}, edited by Louis Couturat, reprinted by Georg Olms, 1961, pp. 153-57.}
§1. Stoic beginnings

Tellingly, the early modern thinkers who formulated the original ambition of the “new logic” were very much schooled in the Stoic literature of late antiquity: Epictetus, Seneca, Marcus Aurelius, and critics-at-large such as Sextus Empiricus and Diogenes Laertius, who commented on the earlier, strictly logical (but lost) writings of Zeno (300 B.C.), Chrysippus (250 B.C.) and the like. The Stoics accepted the earlier Socratic model of the good man as the one who takes his stand in what he does rather than in what he undergoes. But they generally ignored the Platonic and Aristotelian principle on which such goodness was based: the promotion of the good of what one encounters. Consistently with their ethical aloofness, they insisted that the basic unit of logical considerations be the unanalyzed proposition.

As distinct from commands, questions, and various avowals (promises, agreements, acknowledgements), a proposition sets us up to decide whether it is true or not. Aristotle analyzed this setup, asking us to detect the subject of the proposition, something we can face, and then to assess whether or not, and how, the proposed predicate pertains to it: whether and how what we say about the subject is... those who walked around while talking, in contrast to the Stoics, those who originally lodged themselves on a porch.

The Stoics objected to the Peripatetics. They rightly pointed out that much of our thinking and talking does not require that we analyze our proposals at all. For instance, we think and talk in the form:

If \( p \) then \( q \)

But \( p \)

\[ \therefore q \]

“If Fido is fed he stays quiet. But he was fed this morning right before the mailman came. Therefore he was quiet at that time.” We can formalize this argument, purifying it while also introducing our first logical character:

\[ F \rightarrow S \]  
F is short for the full proposition “Fido is fed.”

\[ S \]  
S is short for the full proposition “Fido is silent.”

\[ \therefore \]

Again, this form of thinking and talking is so common that it receives its own name: modus ponens, the mode of thinking that derives a conclusion by positing a proposition (here, “Fido is in fact fed”). The purification consists in dropping the temporal reference (this morning when the mailman came): this is understood in the context of the talk. The logical character, the arrow, intends to capture the sense of “if ... then.” In more recent work, this and the other signs or symbols are called logical constants. Indeed, we shall restrict the meaning of each so that it stays the same, in contrast to the variables \( p, q, r \) and the rest, which are place-holders for ever-varying propositions.

A second form of thinking and talking parallels the first, but introduces negation:

If \( p \) then \( q \)

But it is not so that \( q \)

\[ \therefore \]

It is not so that \( p \)

“If it had rained yesterday, someone would have extended the awning over the porch. But the awning was never extended over the porch. Therefore it did not rain yesterday.” This argument we may formalize much as the one about Fido:

\[ R \rightarrow A \]  
R is short for “It rains” and

\[ \sim A \]  
A is short for “The awning is extended.”

\[ \therefore \]  
\[ \sim R \]

A third form of thinking and talking introduces yet another logical character:

Either \( p \) or \( q \)

But it is not so that \( p \)

\[ \therefore q \]
The character $\lor$ stems from, and corresponds to the Latin *vel*. Stoic disjunction actually corresponds to the Latin *aut*: one is affirmed and one is negated. We may devise a special character for this: $\lor$; however, it is not very useful to include it among our primitive symbols.

"This spruce tree is either badly affected by the spruce bud worm, or it lacks sufficient water. There are no signs of the worm. Therefore it must be under watered." This argument we may both purify and formalize:

B $\lor$ ~W B for "This tree is affected by the bud worm"
~B W for "This tree has enough water."
∴ ~W

Again, the form of argument here is so common that it has its own name: Disjunctive Syllogism. The new character, our modern $\lor$, symbolizes inclusive disjunction: the sense of (two) possibilities, where both may in fact hold; and we may pronounce it as "or."

A fourth form of argument has no commonly accepted name, but we may call it the not-both form. And it permits the introduction of yet another logical constant:

Not both $p$ and $q$
But $p$
∴ It is not so that $q$

"You cannot both drink and drive safely. But you are drunk. So you cannot drive safely." In logical form:

~(D & S) D for "You are drunk,” and
D S for “You can drive safely."
∴ ~S

The new character, the ampersand, reads as “and.” We often conjoin propositions, asserting them together rather than one at a time, especially in order to negate the result, as when a physicist says we cannot determine both the location and velocity of a particle.

A fifth form plays on the fact that the Stoics understood disjunction in the exclusive sense: either one but not both:

$p$ or $q$ and not both:
But $p$
∴ It is not so that $q$

* The character $\lor$ stems from, and corresponds to the Latin *vel*. Stoic disjunction actually corresponds to the Latin *aut*: one is affirmed and one is negated. We may devise a special character for this: $\lor$; however, it is not very useful to include it among our primitive symbols.
\[
p \lor q \lor r \\
\sim p \\
\sim q \\
\therefore r
\]

About this last form, Benson Mates reports:

> According to Chrysippus, even dogs make use of this sort of argument. For when a dog is chasing some animal and comes to a junction of three roads, if he sniffs first at the two roads down which the animal did not run, he will rush off down the third road without stopping to smell.*

By all accounts, the Stoic logicians believed that their five undemonstrated argument forms, plus the rules of inference, sufficed to demonstrate the validity of all other valid forms. Yet the simple rule of compounded employment of the five does not suffice for this purpose. It has proved to be the task of modern logicians to provide adequate sets of such undemonstrables (primitives, we say today).

§1.1 Stoic ontology

The Stoics and the Peripatetics opposed one another, often bitterly. But certainly not because each believed the other to be formally wrong. Rather, each told a different story about the occasions on which the formal considerations could take on flesh.

What counts as an instance of those place-holders, \( p, q \), and the rest? First of all, atomic (rather than compound) propositions: utterances that make a claim to be true (and which may therefore be false). As the Peripatetics, so too the Stoics quickly leave to one side the often much more important and difficult utterances, among which are questions, admonitions, commands, prayers, policy-statements, and promises.

But to what can we really give utterance claiming truth? Only that to which we can actually testify. And to what can we actually testify?

This is an ontological question, the answer to which determines what sorts of propositions rightly serve as instances of the place-holders.

The Stoics distinguished definite, indefinite, and intermediate propositions. A definite proposition is deictic: “That-there is making noise,” “This-here is brown,” “This is a parrot,” and “This [parrot] is talking.” In such cases, the speaker indicates and the listener heeds the source of the determination. An indefinite proposition is particular: “Some [thing] is making noise,” “Some [one] extended the awning,” and perhaps “It’s snowing.” In such cases the determination comes first, while that to which the determination applies, that to which we would ultimately have to look, floats on the horizon, out of focus. And an intermediate proposition is singular: “The Eiffel Tower is made of iron,” “Venus travels around the sun,” “The Prime Minister is fat,” and “Jean is lean.” In each case a source of the subsequent determination is named, not directly focused on — listened to, looked at, touched, smelled, or tasted. Yet one possible significance of naming something consists in deferral: when I name something absent I promise a presence, a focus, on some other occasion.

Not included among the eligible instances are universal propositions: “Human beings are conditioned by λόγος,” “Spruce trees are conifers,” and “Water extinguishes fire.” Such talk invites Peripatetic analysis. For, on the Aristotelian account, utterances about any of something can satisfy the necessary condition for testimony only if the speaker or the listener can claim at least potential insight into the essence of the thing, an essence that both transcends and governs the individual instances encountered. Here, truth is an event of transparency to the universal within or above the individual instance, an event conditioning the possibility of recognizing that a predicate does indeed belong to the instance. Thus the subject, what is addressed in an utterance, is lurkingly dual, both universal and individual, and the truth of an utterance depends on its (our) participation in this duality.

Like mainstream modernity, the Stoics besieged the Peripatetic version of truth, the kind of universality on which such truth depends. The conventional way of remarking on the difference between Peripatetic and Stoic on this question of proper subjects of discourse highlights the Stoic doctrine, shared by several schools in ancient times (Epicureans as well as Stoics), that, as Francis Bacon put it much later, “... in nature,

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nothing truly exists besides individual bodies, performing pure individual acts according to a law ...” (New Organon, II, §2).

Actually testifying, as I might to my wife, I give voice to something: “Look, there’s Stanley again pecking at the seeds we laid out yesterday.” Until my wife joins me at the window to look and listen, my utterance is intermediate rather than definite — while an indefinite utterance would say only that something is tearing into the seed. In any of the three cases, my voice bears upon, calls attention to something encountered. And it calls into being a determination about what is encountered. A threefold event, then: I speak up (correlative with my wife perking up), something emerges to which I address myself (and my wife), and determinations (decisions: right or wrong) hover in the air.

Speech is not always testimonial. The parroting of answers on an examination, the gossiping of unengaged people, the niceties we utter to pass over awkward moments in uncommitted social gatherings: these and other sound-alikes lack one or more of the three folds of genuine (whether true or false) utterance. Especially in our respective native tongues, we easily devise ways of talking (speaking or listening) as though we were not the agents of the talk (someone else really said what I now pass on): we then fail to bring the talk to bear on encounterables, and our talk consists of free-floating determinations, mere gas (*flatus vocis*) that is neither right nor wrong until someone takes it up and brings it to bear on something that actually presents itself.

Each fold of genuine utterance has several names, and various translations into English: the voice (ἡ фωνη), what has chanced to present itself (το τυγχανον), and what is said about it (το λεκτον). For “voice” one sometimes reads “sound,” but this is misleading. Voicing is a sounding, but one that stems from our own willingness to bear testimony (writing is then a deferred voicing, a marking down of what we are willing to voice, or an inviting of others to lend a voice — as a musician prepares a score that must then be played). To give voice is to mean something, so that another name for the first fold is our own act of meaning, bearing down (το σημαινον). As Socrates insists in Plato’s *Protagoras* (347), what we need above all is to cease adopting extraneous voices: to grow up into our own voices and λόγοι.

For “what has chanced to present itself” we might wish to read “object,” but this too is misleading, at least to historically sensitive ears. Only after such thinkers as Descartes and Kant can we significantly rename encounterables as “aims,” as in the expression “What is the object of your attending philosophy classes?” or Bergson’s easily misinterpreted question, *Quel est l’objet de l’art?* — What is the purpose, the intent, the obstacle to be overcome? As soon as we call something an object, we invite the kind of aloof investigation only modernity inaugurated. The Stoics, for all their ethical aloofness, still understood encounterables as *subjects*, as indeed given and other than us and requiring that we address ourselves to them in their terms. And the Stoics in fact retained the Peripatetic name, emphasizing only that the subject is on the other side, “outside”: το εκτος υποκειμενον. The generic Stoic name for the subject, το τυγχανον, reflects the ontological assumption that voicing requires acknowledging that what happens at the other end is a matter of chance, τυχη, rather than an instance of a built-in, overriding necessity of development toward a fulfillment (a universal).

For “what is said,” the lekton (to transliterate the Greek word), I have offered “determination” as a reading. Both our own voicing and the subject presenting itself are embodiments (bodies), unlike this third fold of a testimony. When we ask what was said on an earlier occasion, we hesitate between repeating the words (the voicing of the speaker) and reintroducing the reference (what the speaker was drawing the listeners into). The lekton is the latter: outside is Stanley, Stanley is pecking at the seed (either of which may be wrong, as my wife might testify). Incomplete lekta are either (1) proper names (a name we claim applies to only one presentation, even at different times and places), (2) general names (“raven” applies to Stanley among other presentations), or (3) properties (verbs, adjectives): each of these as applying to a subject. Complete lekta are what we would call complete sentences, basic units of speech having a meaning exposing them to criticism as being either true or false. Indeed, another name for this third fold is το σημαινον, what is meant.

All three folds are essential to genuine semantic considerations. Words as voicings only mean something, and are genuine words, as someone’s, and as articulating either a present, a recollected, or an

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* Stanley is the name we gave to a raven that had somehow injured his right wing and could only hop around, unable to fly; his fellows would alight next to him, squawking, evidently urging him to rejoin them.
anticipated subject. Sounded out simply to fill the air, or to offer an occasion for a schoolroom exercise in analysis, words are not really (no longer or not yet) words, any more than a hand severed from the body is really a hand.

Yet it is clear that even grammatically complete lekta permit repetition lacking a genuine voice, as when we insist that we are only repeating what we have heard (a disposition often encouraged in our school systems). It is also clear that a speaker or writer may vehemently defend lekta and yet fail utterly to bring attention to bear on any presentations, anything encountered. And these deficiencies may prevail even when the lekta can be successfully voiced and focussed by another. Thus it is essential to distinguish between truth and true determinations: truth names the event in which we stand up boldly to what stands forth, and therefore enact our ability to decide first-hand what can be truly said; we then concentrate on what is going on, the affair itself (τὸ πρᾶγμα, another Stoic name for “what is said”).

Logic of either ilk, Peripatetic or Stoic, aims to help us distinguish between genuine and unguenuine talk. It stems, perhaps, from a horror at the prospect or retrospect of talk degenerating into babble, a kind of sound-screen concealing what deserves or even commands attention and response. Each version grows out of and often silently promotes a sense of what is real: an ontology. The Stoic sense of reality differs significantly — indeed, epochally — from the Peripatetic, and paved the way for developments in Scholastic and finally in modern logic.

In the works of Plato and Aristotle, the sense of reality is that of artisanal acknowledgement of a universal defining the fulfillment of the being one helps to be. In the works of the Stoics the sense is that of self-contained testimony about what in fact presents itself: individual bodies performing pure individual acts.

The difference in ontology is at least faintly evident in the different categories each school proposes. The Aristotelian checklist for genuine talk requires that we talk about things that are (both as individuals and as universals), about their qualities, their quantities, where and when they are, what they are doing and undergoing, their bearing on others, their

conditions and their postures. The Stoic checklist is much more laconic: genuine talk must elicit a subject and its qualities, and then the condition it happens to be in and the bearings it happens to have on other things. While the Peripatetic account reflects our involvement in the actualization of beings, the Stoic account fits our efforts to report on things: its venue is that of public lectures and the like.

If I were to deliver a Stoic talk on “The Ravens of Sackville” I would begin by commenting on what I take to be the subject of my talk, and proceed to give something of a description of them (colors and sounds, as well as various names by which they are called): here I would be sure to report on Stanley, and make it clear that what I am saying is based on a multiplicity of experiences with individual ravens: the very word “raven” would refer to this set, the collection of individual ravens, and not to the inner nature of ravens, their ravenhood. Under what their condition happens to be I would include their various habits I have noticed (or, citing others, what might be noticed): what they do for a living, and what they suffer in the end. Most of all, perhaps, I would talk about how my ravens tie in with the other things around Sackville, animals, plants, and minerals—in our recent spirit of ecology. And those of my listeners who genuinely listen will remain alert to the indefinite and intermediate standing of my propositions, requesting perhaps that I take them on a field trip so that I can point to, get them to point to the individual bodies grounding the talk: Stanley in the flesh, and fresh ones as well.*

§1.2 Logical vs. prosaic form

Just as Platonic-Aristotelian logic asks us to understand presented instances of a being by way of the form they aspire to fulfill, so modern logic asks us to understand factual instances of talk by way of the form they confusedly represent.

* Benson Mates compares Stoic logic with the logic formalized in the works of Frege, Carnap, and Quine (noting differences as well as similarities). In the works of Ludwig Wittgenstein I find a concern for genuine talk that lies even closer to that of the Stoics; see, e.g., The Blue and Brown Books (1958) on language games as a kind of therapy to cure us of our “craving for generality” and our “contemptuous attitude toward the particular case.”
Consider the character for conjunction, &. We pronounce it as we would the English “and.” Thus “Stanley is a raven with a broken wing” reads logically and fluently as R & B: “Stanley is a raven” and “Stanley has a broken wing”—two conjoined claims. Here the logical formulation seems to emerge flawlessly from the English prose.

But “and” often fuses other meanings together with conjunction. Consider “Stanley flew into a power line and broke his right wing”: in this case the prosaic “and” likely means “and then, as a consequence.” The prose then includes both a conjunction, F & B, and a third thing, a temporal differential ($t_1 ... t_2$) and perhaps also causal connection (impact of soft object on hard object resulted in a break of soft one). Newton most fully, but Leibniz already in part, developed ways of formalizing both temporality and causality, ways that undergraduates in physics routinely apply to more contrived circumstances. In the study of logic, however, we confine ourselves to pure logical form, leaving aside consideration of time and force; it is an easy matter to add these latter considerations, especially once we understand the priority of form. We cannot look to the prose to assure ourselves absolutely of the logical form, nor can we look to what happens in the way of objects colliding with one another to assure ourselves absolutely of the natural form: in both cases we must engage in the form to clarify the instances.

Consider the character for negation, ~. We pronounce it as we do the English “not.” Thus “Stanley isn’t out there” we would symbolize as ~S, where S = “That out there is Stanley.” The logical thought here is that we approach encounterables with a positive claim and seek confirmation of this claim; if we find what we encounter warranting the claim, we affirm that “That out there is Stanley.” If, on the other hand, we fail to find any warrant we eventually deny it, recording our denial in the form ~S, which we would pronounced most accurately and most succinctly as “Not: that out there is Stanley.”

The confusions of prosaic negation are immense, and they spill over into strictly logical work. The question arises: Under what conditions can we testify to something not being the case—as distinct from not being able to testify that it is the case? For failure to find something to be the case is hardly the same as success in finding something to be not the case. Lawyers often drive witnesses to change their testimony from claiming to have seen that X was not present to claiming only that they did not see X present. On the other hand, there are conditions that seem to warrant the inference of a negation with more power: From “if that is Stanley it cannot fly, but it can fly” we want to infer ~S in the stronger sense, and not just as a failure to see Stanley.

Consider again the character for disjunction, ∨. We pronounce it as we do “or,” and have already noted that the prose is inherently ambiguous. Unlike the Stoics, logicians now choose the weaker sense—for the sole reason that, bearing the inclusive sense carefully in mind, we can easily build up the stronger, the exclusive sense. “Stanley has broken either his left or his right wing, but not both of them” we may fluently translate as (R ∨ L) & ~(R & L).

Finally, consider the character for implication, →. If “Stanley has a broken wing,” then “Stanley cannot fly.” We also pronounce this logical constant as “implies,” especially when we have only place-holders: $p$ implies $q$. Just as in the case of conjunction, prose instances of implication often suggest a causal connection, and one we easily bring to the surface in our discussions: birds fly by flapping their wings, and flapping requires intact bones (universal considerations). Another kind of prosaic implication is the volitional: if “you pass twenty courses” then “the university will grant you a degree.” And of central interest in logical study is formal implication: we settle upon conditions allowing us to assert absolutely that some formulations imply some other formulations, that premisses imply conclusions, that $(p & q) → p$ absolutely.

Not only do prosaic instances of “and,” “not,” “or,” and “if ... then” often mean more than the corresponding logical characters we devise for our artificial language: our prose language contains many different words easily brought into correspondence with single logical characters.

The ampersand, &, may translate not only “and” but also other prosaic conjunctives. Stanley is a raven but he cannot fly; although Stanley cannot fly, he can find food for himself, and so on.

The curl, ~, may translate not only “not” but also other grammatical negations: Stanley eats the sunflower seeds rather than the corn kernels; he is incapable of flying. Also, many grammatically positive adjectives conceal a negation: “dead” might better read “not alive;” “empty” might better read “not containing fuel.”
The wedge, $\lor$, may translate not only “or” but also “unless”: I shall attend class unless I am sick, $A \lor S$. Also simple lists of possibilities, where no disjunction is mentioned at all.

And the arrow, $\rightarrow$, may translate not only “if ... then” and “implies” but also “entails.” The compound “only if” often tricks the unwary: Stanley will survive the winter only if we feed him reads: if we don’t feed him he will not survive, or if he survives we feed him.

Several hundred years after the heyday of Stoic logic, Galen, himself a follower of Aristotle, complained that the followers of Chrysippus focussed attention more on “manners of speaking” ($\lambda \varepsilon \xi \varepsilon \upsilon$) than on “matters of urgency” ($\pi \rho \alpha \chi \varsigma \mu \alpha \alpha$): more on diction (how we are talking) than on affairs (what is going on). In his rebuke Galen is echoing Aristotle, who often remarked that shallow-pates construct their arguments (their $\lambda \delta \gamma \omicron \omicron$) with a view to names rather than matters ($\delta \nu \omicron \mu \alpha \omicron \tau\alpha$ rather than $\pi \rho \alpha \chi \varsigma \mu \alpha \alpha$). Indeed, intellectuals always run the risk of falling into pedantry. Yet no competent logician concentrates on diction, but rather on form. And the question forever arises: What do we count as form? What is its source? Its status?

Stoics as well as we Moderns look for form in linguistic responses to circumstances: in this realm one detects different instances that more or less exemplify a single form. In contrast, the Peripatetics looked for form in our circumstances: here—in what we encounter, in nature—one also detects different instances sharing a common destiny, a unique form in which we may knowingly participate. Both traditions agree that the form overreaches the instances, so that we must be prepared to transcend what immediately offers itself. Yet, in profound acknowledgement of this overreach, no competent logician will commit the Procrustean Fallacy of assuming that legitimate instances must be hewn to their respective forms. The purpose of detecting the form of instances is to enable fruitful work with them. It is those who fail to detect form who generally fail to work well with instances, and end by discrediting them. Logical study aspires to disclose ways of responsibly and creatively crediting affairs.

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$^*$ *Galen’s Institutio Logica* (translated by J. S. Kieffer, The Johns Hopkins Press, 1964), iv, 6. Galen was the 2nd-century physician whose “Galenic medicine” was bitterly attacked and finally overturned by modern thinkers in their effort to establish a new, the now familiar mechanistic medicine.

§1.3 Possible worlds: truth-functionality

One way of formally defining each of the logical characters is to state the conditions under which we would hold the compound to be affirmed or denied once the actual propositions filling the place-holders are affirmed or denied. For instance, we may define the ampersand as yielding an affirmed compound when, and only when, the two propositions being conjoined are both affirmed. Thus the affirmation of a conjunction is a *function* of its two conjuncts (how they happen to be affirmed or denied).

Now, as logicians, we do not determine whether any atomic proposition is to be affirmed or denied. Indeed, the logical disposition insists that any atomic utterance that means something at all is a *proposal* rather than immediately a statement of fact: we have not yet decided whether it is true or false. The disposition required is that of openness, momentary suspension of judgement. For us, a properly formulated atomic proposal is always born in the modality of possibility (of contingency: whether it is to be affirmed or denied depends on how things present themselves “out there”). This disposition correlates with the acknowledgement that a proposal may indeed lose its innocence, that someone may look to its subject-out-there and voice a determination about it: and then the proposal graduates, for those who look and voice, to the modality of facticity (Aristotle says: “simply so”). Finally, we shall soon discover that some compounds graduate almost immediately, by virtue of form alone, to the modality of necessity: they must always be affirmed.

The peculiarly logical insistence that proposals are born in the mode of possibility expresses itself in the notion of a *variable*—what I have hitherto called a “place-holder.” If any given atomic proposal, e.g. “Stanley is out there pecking at the seed,” may be either true or false but—in the end, upon our making a decision—not both, then we can generalize, i.e. speak of $p$ as standing for any such proposal.

And with the notion of a variable we may define each of our symbols truth-functionally. We naturally start with the curl, stipulating the decision regarding the status of $\neg p$ for each of the two possibilities of $p$:

\[
\begin{array}{c|c|c}
& p & \neg p \\
1 & T & F \\
2 & F & T \\
\end{array}
\]
We here record our decision to call any negated true proposition false, and any negated false proposition true.

The remaining logical operators connect two propositions, each of which may be either true or false. Thus “things out there” may turn out in one of four different ways in regard to any effort to connect the two. Each operator is then defined truth-functionally in reference to the same basic matrix of four worlds:

**CONJUNCTION:**

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p &amp; q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**DISJUNCTION:**

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \lor q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**IMPLICATION:**

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

These tables reflect semantic decisions: we shall mean by conjunction that we judge the compound to be true when and only when we judge both the components to be true; we shall mean by disjunction that we judge the compound to be true so long as we judge at least one of the components to be true; and we shall mean by implication that we judge the compound to be false only when we judge the antecedent to be true and the consequent to be false (material implication, with no criteria for judging how or why the consequent is related to the antecedent).

We may extend the use of the tables to confirm the validity of our basic rules of inference: that, so long as the premisses are true, the conclusion will be true. Consider, for instance, the one Stoic indemonstrable (which we can now demonstrate):

Not both \( p \) and \( q \)
But \( p \)
\[ \therefore \text{It is not so that } q \]

For this argument form we may construct the following table:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p &amp; q )</th>
<th>( \sim(p &amp; q) )</th>
<th>( \sim q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
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<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

It will be seen that in all worlds where the premisses are both affirmed (here, only one world, the second), so too is the conclusion. Thus, without even knowing what \( p \) and \( q \) might stand for, let alone whether we would judge them to be true or false, we can see that it is impossible that premisses in this form could be true while the conclusion in this form be false. And this judgement satisfies the sufficient condition, set by Aristotle and accepted by all logicians ever since, for judging the argument form to be valid.

Tables are even more usefully employed to become clear about an inference that is invalid. For instance, the “fallacy of denying the antecedent,” as it is called: If I drink I shall surely get into an accident, but I haven’t drunk, therefore I shall not get into an accident. We recognize immediately that something is wrong. The wrongness may be displayed in reference to possible worlds:

<table>
<thead>
<tr>
<th>D</th>
<th>A</th>
<th>D \rightarrow A</th>
<th>\sim D</th>
<th>\sim A</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

The final column represents the possibility of the conclusion being false in worlds 1 and 3, but world 3 represents both premisses as true. Formally, then, and apart from the fact that there are many ways to get into an accident, we can see that the inference is not valid.

Defining logical characters truth-functionally, we open the door to a vast array of possible characters, one for each conceivable function. Even for a single variable we can devise four functions:

\[
\begin{array}{c|c|c|c|c}
\hline
p & \tilde{f}_1(p) & \tilde{f}_2(p) & \tilde{f}_3(p) & \tilde{f}_4(p) \\
\hline
T & T & T & F & F \\
F & T & F & T & F \\
\hline
\end{array}
\]
Of these, the third is the only one deserving a symbol of its own. Yet the others are not meaningless. The second we might call the re-assert function, the first the verum function, and the fourth the falsum function.

It is one thing assert something: to give voice to a determination about what one encounters ("Hey, there's Stanley again!"). It is another thing to back off to defend the assertion ("It's true that Stanley is out there!"). We normally have no need of a symbol for this re-assertion, one corresponding to the curl; it is enough simply to assert: enough, because the confirmation is the same. As a matter of fact, though, we do have a symbol to indicate something like re-assertion: when we wish to signal our claim that we can prove a formula as a theorem, we prefix the formula with ⊢ (perhaps better called pre-assertion).

It may seem silly to devise an operator that, placed before a variable, would invite us to judge the assertion of anything to be true: if it happens to be true, now that we consider it, it is true; and if it happens to be false, now that we consider it, it is true. Yet Plato and Aristotle, at least, take Parmenides and Protagoras very seriously: the claims, namely, that testimony can only be positive and must be true.

And it may also seem silly to devise an operator that, placed before an assertion, would invite us to judge the new assertion false, no matter whether the original is true or false. Yet many thinkers, our own skeptics in the West as well as some renowned sages of the East, have remarked that as soon as we back off from direct testimony and wish to evaluate the result, now the left-over assertion, we create an assertion that, even if originally true, is now necessarily false.

There is a difference, increasingly problematic in logical study, between asserting something, i.e. directly testifying to what one encounters, and re-considering the assertion. Already to deny an assertion, we rise into reflection: we notice, from above, that the assertion does not embody a viable testimony. Similarly, when we assess the testimony of others: we suspend judgement and then decide: deciding affirmatively, we in effect, although silently and invisibly, make use of the second function.

The remaining logical characters are functions of two variables, and recall the possibility of sixteen different functions:

\[
\begin{array}{c|cccc}
P & Q & P \lor Q & P \rightarrow Q & P \land Q \\
\hline
T & T & T & T & T \\
T & F & T & T & F \\
F & T & T & T & F \\
F & F & F & T & F \\
\end{array}
\]

We can here survey the familiar functions, plus some others that do have some use and therefore conventional symbolizations: the tenth for exclusive disjunction, the seventh for equivalence, and the third for the reversed-order of implication.

In addition, the first and the last perform much as the verum and falsum functions do: put any two assertions together, and we may wish to judge the result true for that very reason (an oratorical trick) or false for that very reason (a philosophical thought: we have no business testifying in any non-atomic manner). The fourth and the sixth give priority to the order of the assertions, as the eleventh and thirteenth negate by priority. The fifteenth is called the dagger, and has some artificial uses.

Of the unusual functions, the ninth represents the chief premiss of the Stoic form we demonstrated with a table: the notion of "not both." It has its own name: the stroke. Bertrand Russell credits H. M. Sheffer with having advanced the project of *Principia Mathematica* by showing that the symbols for negation and disjunction could be defined in terms of the stroke, thus reducing the number of necessary primitive symbols:*

\[
\begin{align*}
\neg P & \overset{\text{def}}{=} P \downarrow P \\
P \lor Q & \overset{\text{def}}{=} (P \downarrow P) \downarrow (Q \downarrow Q).
\end{align*}
\]

Defining one symbol in reference to others requires that we discover a complex definiens that in fact bears the same array of values (in the

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* Introduction to the Second Edition (1925): The most definite improvement resulting from work in mathematical logic during the past fourteen years is the substitution ... of the one indefinable "p and q are incompatible" [the stroke: \(f_9\)] (or, alternatively, "p and q are both false" [the dagger: \(f_{15}\)] for the two indefinables "not-p" and "p or q." This is due to Dr. H. M. Sheffer. *Trans. Amer. Math. Soc.* Vol. XIV, pp. 481-488.]
possible worlds) as does the definiendum: one may quickly calculate the 
four possible values of \((p \land q)(q \land q)\) and discover that this complex has the 
same array of values as does \(p \lor q\).

In what follows under the heading of Natural Deduction (as opposed to 
Axiomatic Deduction) we shall employ only one defined function: the 
seventh on the table, the double arrow. This symbol, \(\iff\), we define as the 
conjunction of the third and the fifth:

\[
p \iff q =_{\text{def}} (p \to q) \& (q \to p).
\]

We shall call this new symbol the sign for a “biconditional”: we may 
pronounce it as “if and only if” or as “is equivalent to” (in the strict 
sense of “will have the same value as”). We often employ equivalence 
in statements of policy (volition): I shall travel to Montreal \(\text{if and only if} \)
I pass my logic exam. Also in statements of empirical determination 
even when not entirely reliable: this solution is acid \(\text{if and only if} \)
this litmus paper dipped into it will turn red. In each case one claims that the 
pair is equivalent: certainly not in content (meaning) but in affirmation 
or denial. The equivalence here is called material: one claims that two 
propositions happen to stand or fall together, not that the essence at issue 
in either one entails the other. The \(\iff\) in this complex form depends on what we happen to substitute into the 
variables.

The equivalences of special logical interest hold no matter what 
propositions happen to fill the place-holders: logical equivalences. For 
instance, we will soon be able to prove

\[
[(p \to q) \& (q \to p)] \to [(p \& q) \lor (\neg p \& \neg q)]
\]
as well as its contrary

\[
[(p \& q) \lor (\neg p \& \neg q)] \to [(p \to q) \& (q \to p)]
\]

whereupon we may conjoin these two and, by definition, conclude

\[
[(p \to q) \& (q \to p)] \iff [(p \& q) \lor (\neg p \& \neg q)].
\]

Traced out through all four possible worlds, this formula holds: having 
the one side we have the other, rejecting either side we reject the other. 
Any empirical statement in one form will be equivalent to the modified 
statement in the other form.

Unlike a merely material equivalence, a logical equivalence 
expresses a connection that will hold \textit{a priori} for any instance of our 

meaning something empirically. Logical proofs formally reveal such \textit{a priori} connections. And make extensive use of them as well.

§2. Natural deduction: a sentential calculus

Tables showing possible combinations of affirmation and denial 
serve to display what we \textit{mean} by our logical constants, viz. an under-
ground act of rationality, an act of creating constants as functions of 
variables. But tables themselves do not show inference at work. We 
ourselves infer formulas by way of rules. Both the formulating and the 
applying of rules manifest an above-ground rationality, an act of devising 
a handful of procedures to prove an infinite number of inferences, as well 
as an act of agreeing to confine ourselves to these procedures.

§2.1 Sentential rules of inference

Directly analogous to the five Stoic undemonstrated forms, but fully 
workable, are the eight rules of inference proposed by the German 
mathematician Gerhard Gentzen.\footnote{Originally published in 1934; English translation: “Investigations into Logical 
Deduction” in \textit{The Collected Papers of Gerhard Gentzen}, North-Holland 
Publishing Company (Amsterdam & London, 1969).} They take the form of introduction and 
elimination rules for each of the four symbols taken now as primitive (as 
defined in the way the rules authorize us to shuffle variables around them).

\textit{Rules for conjunction}

Given any two propositions \(p\) and \(q\), atomic or compound, we may 
infer the conjunction of the two:

\[
p \quad q \\
\therefore p \& q \quad \text{[or: } \therefore q \& p]\]

The formal name of this rule is Ampersand-Introduction, usually called 
Conjunction. Its twin authorizes us to extract from such a conjunction 
either one of its conjuncts:

\[
p \quad q \\
\therefore p \quad \text{[or: } \therefore q]\]
The formal name of this rule is Ampersand-Elimination, usually called Simplification. Taken together, these two rules embody the intuitive sense of the logical character.

**Rules for disjunction**

Given any proposition \( p \), atomic or compound, we infer the disjunction of this proposition with any other, given or not:

\[
\therefore \quad p \vee q \quad [\text{or}: \quad \therefore \quad q \vee p]
\]

The formal name of this rule is Wedge-Introduction, usually called Addition. A parent may know that the Easter egg is hidden among the roses, and quite truthfully tell the children that it is hidden either among the petunias or among the roses. Policies often take the form “either \( A \) or \( B \) will get you \( C \);” formalized, this premiss resists employment if you only have one of the antecedents, say \( A \): you need the disjunction, \( A \) or \( B \), in order to obtain \( C \).

The twin: How can we begin with a disjunction, \( p \vee q \), and obtain a formula with one wedge less? Well, if both \( p \) and \( q \) imply a third proposition, \( r \), we may certainly infer \( r \):

\[
\begin{align*}
p \vee q \\
(p \rightarrow r) \& (q \rightarrow r)
\end{align*}
\]

\[
\therefore \quad r
\]

The formal name of this rule is Wedge-Elimination, and there is no commonly accepted nickname (let’s call it Positive Dilemma). It not only shows operationally the meaning of the wedge, it also makes use of a logical character not yet operationally defined (the arrow).

Strictly, rules work on at most two lines; such is the nature of binary digital reasoning: a greater complex emerges as a mutation of one element or as a combination of two. However, in proofs employing Positive Dilemma, we typically establish three propositions, the disjunction and two implications, from which we will indulge the inference directly (bypassing the Conjunction).

**Rules for implication**

Both in life and in logic, we introduce an implication by trying out the antecedent proposition and discovering that, assuming its truth, we find ourselves driven, empirically or logically, to a consequent. Since the antecedent was only assumed, not itself proved, i.e. since its status was hypothetical rather than assertoric, we can only conclude that, \( \text{IF } p \text{ THEN } q \). There must be a context of other propositions and rules that provides the drive; call it the “Gamma” of the argument, \( \Gamma \):

\[
\begin{align*}
\Gamma \\
[p] \quad \text{(i.e., assume } p) \\
q \quad \text{(managed to derive } q \text{ from } \Gamma \text{ and } p)
\end{align*}
\]

\[
\therefore \quad p \rightarrow q
\]

This procedure is called Conditional Proof. It represents a form of reasoning that not only permeates mathematical work, but also formalizes the kind of reasoning most natural to ordinary affairs. For these latter require imaginative explorations of the sort: Let’s figure out what would be the case \( q \) if such-and-such were the case \( p \)—a figuring-out taking place within a \( \Gamma \). One can engage in a series of such explorations, and line up the results, calling them conditionals or hypotheticals. Reasoning here is a matter of imagination. Gentzen and others call a system of deduction based on this rule natural, as distinct from some of the more contrived systems logicians have devised under the general heading of axiomatic deduction.

The twin of Conditional Proof is the ubiquitous modus ponens:

\[
\begin{align*}
p \rightarrow q \\
p
\end{align*}
\]

\[
\therefore \quad q
\]

The formal name of this rule is Arrow-Elimination, but it seldom goes by that name. Its very ubiquity renders it rather unimaginative, the very opposite of its twin.

**Rules for negation**

How might we legitimately decide that a proposition deserves denial — deserves, that is to be recorded in our repertoire with a negation-sign preceding it, indicating the assertion (the affirmation) of its negation? Withdrawn into intellectual activity, depriving ourselves of sensory encounters (in more or less Stoic, nowadays in mathematical fashion), we have little reason to affirm any proposition of an intellectual sort, no reason at all to affirm any proposition about things actually encountered. And what reason could we ever have to deny that some proposition holds of empirical givens? Here, the most to which we could ever testify is that we do not recall having anything given that would warrant the proposition.
Yet, within our own intellectual domain, there is one proposition, compound, that must be denied: \( p \land \neg p \). In the domain of empirical judgements, we do in fact change our minds and entertain contradictories of this sort; indeed, it is essential to intellectual integrity that we be able to consider, at once, both a proposition and its contradictory: juries and judges conduct their deliberations with both in view, even if lawyers and clients come down immediately on one as against the other. When the judgement falls, however, one will exclude the other; otherwise, there has been no decision, no judgement. And within our intellectual domain we mime the final judgement, not the deliberation. We thus assert, affirm, \( \neg(p \land \neg p) \) for any proposition inserted into the place-holder \( p \).

And the rule is this: any proposition that leads to such contradiction deserves, itself, to be contradicted, denied, negated:

\[
p \rightarrow \text{a contradiction} \\
\therefore \neg p
\]

The formal name for this rule is Negation-Introduction, and is often called simply reductio ad absurdum. We commonly employ this rule by assuming \( p \) and deriving, within the context \( \Gamma \) and with the rule of conditional proof, both a formula \( q \) and its contradiction. What we have shown, then, is that the introduction of \( p \) destroys the integrity of our intellectual apparatus, the one possibility even a Stoic cannot bear.

And the twin of Negation-Introduction? The rule looks much the same: if the negation of some proposition leads to a contradiction, then we authorize ourselves to affirm the proposition:

\[
\neg p \rightarrow \text{a contradiction} \\
\therefore p
\]

Of course, the formal name of this rule is Negation-Elimination, and it often goes by the name of Indirect Proof.

The two negation rules differ significantly. With the first, we cast out of our system a proposition that causes the irreparable damage that a self-contradiction wreaks. With the second, we bestow full membership on a proposition on the grounds that expelling it from our system breaks the system down. The difference, the question, is cognitive in purport. Knowing \( \neg p \) (on the grounds that the affirmation of \( p \) leads to a contradiction) amounts to denying \( p \). But in what sense can we claim knowledge of \( p \), affirm \( p \), simply because we have shown that the effort to deny it leads us into serious trouble?

Analogies abound. I may find that if I consume a certain substance, my body breaks down; I have good reason to deny that it contributes to my health. But if I find that eliminating a certain substance from my diet leads to a breakdown in my health, I most honestly attest to my inability to get along without it: I falsify this double negative if I proceed directly to affirm that the substance is good for my health, since I have yet to understand the relation between health and that substance. A good deal of empirical induction takes the form of discovering that asserting \( \neg p \) (empirically removing the state of affairs described by \( p \)) leads to a breakdown; what one then concludes is \( \neg\neg p \). And one must then get down to the work of understanding the conditions under which one can genuinely assert \( p \).

In mathematical work since Euclid, knowledge of a formula has required a process called proof. A proof is constructive (or direct) if, out of principles laid down at the outset, one can build up the formula. A proof is negative if it shows that the formula is inconsistent with the system (Negation-Introduction). And a proof is indirect if it shows that the effort to deny the formula leads to an inconsistency (Negation-Elimination): in this case, however, we would be more honest if we admitted that what we have proved is a double-negative (by Negation-Introduction). Of the three kinds, the constructive embodies knowledge, the negative shows what forever lies outside the system (no matter what we insert into the variables), and the indirect laisse à désirer.

Gentzen proposed his system of Natural Deduction largely in order to accommodate the acknowledgement of the difference between Negation-Introduction and Negation-Elimination. We may exfoliate the system as a purely abstract creation in complete indifference to the distinction.

§2.2 Arguments and derived rules

In Aristotle's logical works, the interplay of form and instance begins as the interplay of a being's de facto presence and its de jure fulfillment, and the interplay remains within finite bounds: there are only six categorial forms and only a handful of valid argument forms. In contrast, mathematical logic has no inherent bounds: we can always devise new forms from the fixed number with which we begin.
modus tollens

We should be able to formalize, and prove valid, an argument such as “If our gardener had come yesterday afternoon, the lawn would be cut, but the lawn is not cut, therefore the gardener did not come”:

\[ G \rightarrow L \]
\[ \neg L \]
\[ \therefore \neg G \]

The form of this argument corresponds to one of the Stoic forms, the one we now call modus tollens. But it does not appear among our eight primitive rules if inference. In the style we might call Egyptian, we could simply add this form to our collection, as a ninth rule. In the Euclidean style, however, we would like to derive it. And not just for this empirical configuration, but for the configuration per se. First, we rise into the form of the argument, noting the premisses and what we would like to squeeze out of them:

1. \( p \rightarrow q \) Premiss
2. \( \neg q \) Premiss / \( \therefore \neg p \)

Next we devise a strategy: if we insert the contradictory of the foreseen conclusion into our \( \Gamma \) (here, the collection of the two premisses above and the eight rules of inference), we should be able to derive a contradiction:

3. \( p \) Assumption (for \( \neg \)-intro.)
4. \( q \) 3,1 modus ponens
5. \( q \& \neg q \) 4,2 Conjunction
6. \( p \rightarrow \text{contr.} \) 3—5 RCP
7. \( \neg p \) 6 Negation-Introduction

The procedure here exactly parallels Aristotle's method per impossibile for proving his Baroco and Bocardo; both procedures employ the Rule of Conditional Proof. Beginning with the Assumption at line 3, each line has a hypothetical status until the entire excursion is closed off with an RCP; thus while lines 6 and 7 are established within \( \Gamma \), lines 3, 4 and 5 are not established, and therefore cannot be used again. To distinguish between established and unestablished formulas we could invent two more symbols. Since this would be rather tedious, let us simply remember that lines within an excursion hold only within that one excursion. We conclude an excursion (discharge the assumption and all its intervening progeny) by an Arrow-Introduction, as in line 6.

Meanwhile, we have derived modus tollens as a rule: every time we wish to employ this pattern, we could imagine repeating the proof above. But one purpose of developing a logical system is to avoid such repetition.

Hypothetical syllogism

Analogous to Barbara is the form of an argument such as this:

Whenever John comes for visit, I talk about books.
Whenever the weather is good, John comes for a visit.
\[ \therefore \text{Whenever the weather is good, I talk about books.} \]

The form differs from Aristotle's Barbara in that propositions take the place of terms. Inverting the order of the premisses, the form of this argument is:

\[ p \rightarrow q \]
\[ q \rightarrow r \]
\[ \therefore p \rightarrow r \]

The strategy for proving the validity of this argument form is extremely simple: assume \( p \) and obtain \( r \) with two applications of modus ponens. Out of respect for its ancestor, we call the resulting derived rule Hypothetical Syllogism.

An argument is valid if and only if its form is valid. As in Aristotelian logic, this distinction between argument and form not only bids us to scrutinize any given argument to extract its form, but also permits us to invent arguments to fit the form. Both acts, but especially the two in tandem, embody what we mean by “rationality”: our essential engagement in \( \lambda \delta \rho \gamma \delta \zeta \). An actual inference often results from this interplay, and thereby illustrates (shows, proves) rationality at work—much as an athletic performance illustrates (shows, proves) the competence that has developed prior to the performance. Perceiving the competence at work in the performance (participating in the competence), one can testify to the role of imagination in the rational extraction of form, and the role of creative imagination in the invention of arguments to fit a form. Logical work and poetic work are kindred sports.

Constructive dilemma

Arguments often take the form: either A is the case or B is the case, but if A is the case then X is the case, and if B is the case, then Y is the case, so either X is the case or Y is the case. For example: either the
thieves entered through the front door or they entered through the side window; if they entered through the front door they had a key, and if they entered through the side window it was not latched; so either they had a key or someone forgot to latch the window. The argument appears valid, and we would like to prove it so — prove the form to be valid, no matter what we substitute into its variables.

1. \( p \lor q \) Premiss
2. \( p \rightarrow r \) Premiss
3. \( q \rightarrow s \) Premiss /∴ \( r \lor s \)

The strategy is to show that each of the disjuncts of Premiss 1 implies the desired conclusion, and then to employ Positive Dilemma:

4. \( p \) Assumption
5. \( r \) 4,2 modus ponens
6. \( r \lor s \) 5 Addition
7. \( p \rightarrow (r \lor s) \) 4—6 RCP
8. \( q \) Assumption
9. \( s \) 8,3 modus ponens
10. \( r \lor s \) 9 Addition
11. \( q \rightarrow (r \lor s) \) 8—10 RCP
12. \( r \lor s \) 7,11,1 Positive Dilemma

The formulas designated as Premisses are imaginative place-holders for subsequent arguments: when we happen to have Premisses 1, 2, and 3, we may proceed directly to the conclusion designated on line 12, employing this derived rule called Constructive Dilemma.

**Disjunctive syllogism**

One of the most common forms of argument consists in affirming a disjunction and negating one of the disjuncts to conclude the other disjunct. For instance, either I take the bus to Moncton or I take the train to Moncton, but the trains are not running, so I'll take the bus. Formally, we would like to prove the derived rule:

1. \( p \lor q \) Premiss
2. \( \neg p \) Premiss /∴ \( q \)

The strategy again is to show that each disjunct of the first premiss implies the desired conclusion. That \( q \) implies \( q \) we may prove in a very boring manner:

3. \( q \) Assumption
4. \( q \rightarrow q \) 3—3 RCP

That \( p \) implies \( q \) we may prove by a rather thought-provoking procedure: our \( \Gamma \) already includes \( \neg p \), so assume \( p \) (a contradiction) and we can prove anything we wish.

5. \( p \) Assumption
6. \( \neg q \) Assumption
7. \( p \land \neg p \) 5,2 Conjunction
8. \( \neg q \rightarrow (p \land \neg p) \) 6—7 RCP
9. \( q \) 8 Negation-Elimination
10. \( p \rightarrow q \) 5—9 RCP
11. \( q \) 10,4,1 Positive Dilemma

Notice that this is the first proof making use of Negation-Elimination. Disjunctive Syllogism formalizes the common assumption that, having a disjunction, we can know one disjunct, count it among our warranted propositions, by satisfying ourselves that we must deny the other. Formally, i.e. with no concern for situations of direct testimony, the assumption itself seems warranted. But the proof is only indirect.

§2.3 Theorems: inference by replacement

A theorem most obviously appears as a formula that we have already proved to hold no matter what we substitute into the variables (so long each substitution instance is a genuine proposition: propositions comprise the domain of sentential theorems).

In some more originary sense, a theorem is a formula, the meaning of which we have successfully contemplated: we know that it holds, and we understand how it formulates a truth within our system, a truth about the way we have assembled our logical constants and inferential patterns (our primordial \( \Gamma \)).

We prove a theorem much the way we prove a derived rule, or prove that an argument is valid — except that we have no premisses. A premiss is proposition we assume to be true for reasons outside the proof itself: we “keep” it afterwards. We may construe the proof of a theorem as a proof in which all assumptions have been discharged.

**Six basic theorems**

Consider the following three theorems:

Theorem 1: \( p \rightarrow p \)
Theorem 2: \( \neg(p \land \neg p) \)
Theorem 3: \( p \lor \neg p \)
These three parallel, in a strictly formal way, Aristotle’s ontological axioms of Identity, Contradiction, and Excluded Middle: that each being has an ultimate identity (rather than being only its manner of relating to other beings), that ultimately a being is unable both to be and not to be (or both to have a property and not to have the property), and that ultimately we must decide one way or the other (tertium non datur).

The first two are easily proved: the first requires a one-line employment of RCP, and the second reveals the basis of the two Negation-rules. The third deserves special contemplation:

\[
\begin{align*}
1. & \quad \neg(p \lor \neg p) & \text{Assumption (for \neg-elim.)} \\
2. & \quad p & \text{Assumption (for \neg-intro.)} \\
3. & \quad p \lor \neg p & \text{Addition} \\
4. & \quad \neg p & \text{Assumption (for \neg-intro.)} \\
5. & \quad p & \text{Addition} \\
6. & \quad p \lor \neg p & \text{Addition} \\
7. & \quad \neg p & \text{Negation-Introduction} \\
8. & \quad (7) \lor (1) & \text{Conjunction} \\
9. & \quad (1) \lor \neg p & \text{Addition} \\
10. & \quad p & \text{Negation-Elimination} \\
\end{align*}
\]

The spirit of this proof is obviously indirect. It shows what happens when we try to deny the disjunction: the one disjunct must then be denied (by \neg-intro.), and then we may construct a formula that contradicts the original denial. What we have actually shown is that we cannot self-consistently dispense with Excluded Middle. Confined to Negation-Introduction, we could only conclude \neg(p \lor \neg p).

Since Aristotle, and as recent as Brouwer and his students, Excluded Middle has occasioned much controversy. If we defined our symbols, textbook fashion, in reference to binary options (truth-tables), and confined our work to the formal exfoliation of theorems confined to variables, nothing questionable would arise. But Aristotle, Brouwer, and Gentzen engage us in fully logical, i.e. contemplative considerations: in reference not to variables, but to arguments: Aristotle asks whether it makes sense to claim to know, in advance of the event, that things will either be or not be in a suggested way; Brouwer asks whether it makes sense to claim even to imagine, in advance, that every item in an infinite collection has or does not have a given property.

Consider another triad of theorems:

Theorem 4: \[ p \rightarrow (q \rightarrow p) \]
Theorem 5: \[ \neg p \rightarrow (p \rightarrow q) \]
Theorem 6: \[ (p \rightarrow \neg p) \rightarrow \neg p \]

Theorems 4 and 5 reveal what our primitive rules of inference permit once a proposition is established within or without the system: \( p \) (a complex) within permits a marriage with any proposition by way of the arrow, while \( p \) successfully ostracized entails that its re-introduction would allow us to affirm any proposition we choose.

Theorem 6 has an application worthy of some comment. The Peripatetics had shown invalid forms to be invalid by noting the incongruence of the accused form with the way the world appears at the interface. For instance, the AEE-1 form would allow for the following inference:

All dogs are mammals
No cats are dogs
\[ \therefore \quad \text{No cats are mammals} \]

The success of this procedure requires that one know the premisses to be true of the world, and the conclusion to be false of the world. The procedure then relies on extra-logical insights. Such reliance does not bother Aristotelians, since for them logical form is born out of such insight. But it does bother Stoics: for they aim to settle questions strictly in-house, independently of what one happens to know empirically about how matters stand “out in the world.” Now, we can make use of Theorem 6 to show that if a given form were valid, then it would be invalid (therefore it is invalid); Cardan (1501-1576) remarked that this is “the most wonderful thing that has been discovered since the beginning of the world, namely to prove something from its opposite.” E.g.:

All 1st-figure forms with universal majors and affirmative minors are valid.

No AEE-1 forms are 1st-figure forms with universal majors and affirmative minors.
\[ \therefore \quad \text{No AEE-1 forms are valid.} \]

The first premiss we may show to be correct by examining all possible first-figure forms with a universal major and an affirmative minor. The second premiss is correct by definition of E-form propositions and affirmative propositions. If the argument itself is valid, then the conclusion is also correct. And if the conclusion is correct, then the
argument, itself in the AEE-1 form, is invalid. By Theorem 6, then, the argument (its form) is invalid.

One peculiarity of this last employment of Theorem 6: The variables take as instances not propositions about the world (definite, indefinite, intermediate) but propositions about our own (in-house) propositions, our own acts of assertion. In the present instance, \( p \) becomes “Arguments in the AEE-1 form are valid,” an assertion about our own logical setup. Not until we can take the subject, τὸ ἔξτος ἔξως, as the human spirit itself (Descartes, Hegel) can propositions having our own logical acts as logical subjects serve as substitution instances of the variables. The modern objection to “extra-logical” reliances belongs essentially to the development of a logic in which the only real subject is thinking itself.

Equivalence theorems

By definition, two expressions are logically equivalent if and only if (we can prove that) each implies the other. Thus:

Theorem 7: \( p \leftrightarrow \neg p \) Double Negation

A sentential equivalence also has this property: each side will have the same value, no matter what the values of the propositions instantiating the variables. As a consequence of this consideration, we see that replacing one side with the other in a complex expression will yield yet another complex (another line in a proof) having the same overall value as its predecessor. Thus, from

\[ r \lor [p \to (q \to \neg p)] \]

we can obtain another,

\[ r \lor [p \to (q \to p)]. \]

In general, it should be intuitively evident that such replacement is a legitimate mode of inference, and we shall adopt it as an additional rule of inference. We shall have the opportunity to show, inductively, not only that it is legitimate, but also how one can effect such replacement relying only on the other eight rules. Meanwhile, many equivalences reveal the grammar of logical formulations:

Theorem 8: \( (p \to q) \leftrightarrow (\neg q \to \neg p) \) Contraposition
Theorem 9: \( (p \lor q) \leftrightarrow (\neg (\neg p \land \neg q)) \) De Morgan’s Th. I
Theorem 10: \( (p \land q) \leftrightarrow (\neg (\neg p \land \neg q)) \) De Morgan’s Th. II
Theorem 11: \( (p \to q) \leftrightarrow (\neg p \lor q) \) Implication
Theorem 12: \( (p \lor (q \land r)) \leftrightarrow ((p \lor q) \land (p \lor r)) \) Distrib. I

Theorem 13: \( [p \land (q \lor r)] \leftrightarrow [(p \land q) \lor (p \land r)] \) Distrib. II
Theorem 14: \( (p \leftrightarrow q) \leftrightarrow [(p \land q) \lor (\neg p \land \neg q)] \) Equivalence

Other theorems are more useful than revelatory:

Theorem 15: \( (p \leftrightarrow q) \to [(r \lor p) \leftrightarrow (r \lor q)] \) Summation
Theorem 16: \( (p \leftrightarrow q) \to [(r \land p) \leftrightarrow (r \land q)] \) Multiplication

With these last two in mind, we may begin to suspect how we might effect Replacements without invoking a separate rule.

Theorem 17: \( [p \to (q \to r)] \leftrightarrow [(p \to q) \to (p \to r)] \) This last is a very powerful formula: along with Theorem 4, it would allow us to dispense with the Rule of Conditional Proof.

Theorem 18: \( [p \to (q \to r)] \leftrightarrow [q \to (p \to r)] \) Commutative Pr.
Theorem 19: \( [(p \land q) \to r] \leftrightarrow [p \to (q \to r)] \) Ex-/Importation
Theorem 20: \( (p \to q) \leftrightarrow [p \to (p \land q)] \) Absorption

These theorems, along with the primitive and the derived rules, comprise an apparently self-sufficient realm of thinking, a pure transcendence. This realm proves itself in its application to arguments—just as Euclid’s geometry proves itself in the measurement of lands.

Consider an argument by William James, from his 1884 lecture “The Dilemma of Determinism” (now available in his Essays in Pragmatism). Throughout the lecture, James underscores the peculiarities of believing that everything, both the events we face and our own response to these events, is pre-determined within an infinite chain of causes. Chief among these peculiarities is our own capacity to regret: What sense can it have to feel bad about something that has happened, or anything we have done, if everything is pre-determined? James sums up his argument:

Our determinism leads us to call our judgments of regret wrong, because they are pessimistic in implying that what is impossible yet ought to be. But how then about the judgments of regret themselves? If they are wrong, other judgments, judgments of approval presumably, ought to be in their place. But as they are necessitated, nothing can be in their place; and the universe is just what it was before—namely, a place in which what ought to be appears impossible.

A careful reading of the argument might lead to the following paraphrase: If we endorse determinism, we commit ourselves to believing that regret is foolish, since feeling bad about things implies we believe things could
have been otherwise, contrary to the doctrine we endorse; still, if we do in fact feel bad about something, this too we must consider to be ineluctably caused, so that calling it foolish (recommending an alternative attitude) would again be foolish. Thus the human response of regret, our own or another’s, contradicts the endorsement of the doctrine. — This paraphrase may be formalized:

1. \( D \rightarrow N \)  \( D = “I (am wise to) endorse Determinism.” \)
2. \( N \rightarrow (R \rightarrow F) \)  \( N = “I hold that Nothing can be otherwise.” \)
3. \( F \rightarrow B \)  \( R = “I Regret an incident.” \)
4. \( N \rightarrow \neg B \)  \( F = “I consider myself Foolish.” \)
∴ \( R \rightarrow \neg D \)  \( B = “I can respond in a Better way.” \)

“And if anyone doubts what I have proposed, I can say to him: Let us calculate— with pen and paper we can settle the question.” So we read in Leibniz’ fragment “Preface to General Science.” Once we have the premisses formulated in our special language, we can indeed prove the argument to be valid.

The moment I in fact regret something (and who has not?) I must abandon the doctrine of determinism. If; that is, I affirm each of the four premisses. James considers these to be necessary. What we mean by endorsing determinism is believing that nothing can be otherwise than it happens to be. What we mean by considering oneself (or another) foolish is that we recognize one could have behaved differently. One thing we mean by things being unable to be otherwise is that one who regrets cannot help regretting.

James’ premisses then express semantic tautologies of the sort: if that’s an oak, then its a tree; if she’s my mother, then she had at least one child; if I own a house, then I own some real estate. The argument as a whole then consists of unravelling what we mean rather than of discovering how things are: a suitable distinction, once we have embarked on the Stoic project of understanding our own thinking in separation from the demands that animals, plants and minerals, friends, enemies and communities exert upon us.

Our rationality finds expression not only in the working out of a transcendental realm of thinking, but also in the crossover into the immanent realm of argument. For here we must extract premisses and conclusions and take responsibility for the result.

**Linearity**

Modern logical work engages us in a linear interpretation of reason in at least one sense: we insist on exfoliating, from a handful of primitives, an entire system, each new member of which must clearly derive from a finite number of steps, each justified by one of the primitives as applied to what has already been established within the system. The system is infinite in the obvious sense that there is no foreseen limit to its expansion. And our own activity of creating the system is purely transcendental: it owes nothing to the domain of arguments to which it might be applied.

In contrast, Aristotle reduced valid forms of inference to a handful of primitive forms to create a completed system applicable to extra-logical conditions (mainly artisanal and oratorical, with illustrations drawn from geometrical reasoning).

One consequence of the modern project is that we must prove the legitimacy of even the most obvious moves. For instance, the following theorems:

**Theorem 21:** \( (p \& q) \leftrightarrow (q \& p) \)  \( \text{Commutation} \)
**Theorem 22:** \( (p \lor q) \leftrightarrow (q \lor p) \)  \( \text{Commutation} \)
**Theorem 23:** \( [p \& (q \& r)] \leftrightarrow [(p \& q) \& r] \)  \( \text{Association} \)
**Theorem 24:** \( [p \lor (q \lor r)] \leftrightarrow [(p \lor q) \lor r] \)  \( \text{Association} \)
**Theorem 25:** \( p \leftrightarrow (p \& p) \)  \( \text{Indempotence} \)
**Theorem 26:** \( p \leftrightarrow (p \lor p) \)  \( \text{Indempotence} \)

Not that these are entirely trivial. Having begun with the undefined symbols of Conjunction and Disjunction, we need to express their operational meaning within the system. Furthermore, proofs of more engaging theorems often require the employment of these simpler theorems precisely to retain the principle of linearity.

One interest of linearity is independence of primitives. We allow ourselves eight sentential rules of inference, and insist that all else be derived with these rules only. It is conceivable, however, that one or more of these eight be derivable from the others. For instance, we may think we can derive the Rule of Addition:

1. \( p \)  \( \text{Premiss} /.* \) \( p \lor q \)
2. \( \neg p \)  \( \text{Double Negation (Theorem 7)} \)
3. \( \neg p \rightarrow q \)  \( \text{Theorem 5 & modus ponens} \)
4. \( \neg p \lor q \)  \( \text{Implication (Theorem 11)} \)
5. \( p \lor q \)  \( \text{DN (Theorem 7)} \)
Yet the argument is circular. For the proof of Implication, employed in
line 4, makes use of Addition, what we are trying to prove.

Obviously, when we draw upon established theorems to prove further
theorems we easily lose track of the line of development and commit the
fallacy of circular proof. The employment of a theorem is an invocation
of the proof we have developed for it: we imagine running through the
whole development again, this time in the exact shape required. In our
imagination, then, the only justifications allowed are those provided by
our eight rules of inference; in the more complicated “proof” above, we
would detect the employment of Addition in the course of trying to
derive it.

Besides allowing us to formalize the concern to avoid circularity, a
system of thinking allows us to raise the question of completeness: Does
the handful of primitive rules of inference suffice for proving all that we
otherwise know should be provable? In the case of the sentential
calculus, we have an effective tool for determining, in advance, what we
should be able to prove: the construction of a table showing all possible
combinations (worlds) of affirmations and denials.

One alternative to linear reasoning seems to have been the Egyptian
thinking that apparently influenced the Pythagoreans and against which,
it is said, Euclid developed his system of geometry. On the Egyptian
model, one may come to a decisive insight upon a direct consideration of
the form, and such insight may even result in, or allow one to formulate,
a mathematical theorem; the Pythagorean Theorem is one such example,
Pythagorean Harmonics another. The breakthroughs on which poets,
painters, philosophers, physicists, mathematicians — yes, even logicians
— build their works, however systematic these works may appear as we
first approach them, likely have an Egyptian core.

But another alternative to linear reasoning is the hodge-podge
thinking that always threatens situations: one absorbs opinions from
parents, teachers, acquaintances, books, TV programs, then applies them
haphazardly in living-room conversation, in parliaments, in jury
deliberations, even on site, only to discover that they do not hang
together, that one judgement (application of the opinion to a given case)
contradicts another, that any effort to think through one opinion runs in
circles with other opinions. Insistence on formal linearity underscores the
need to draw a thread through the multiplicity.

§3. Natural deduction: a predicate calculus

Sentential forms allow us to formalize inference without drawing
upon any supposed universality in nature. However, we still engage in
universality, that of the variables: we say of any argument in a given
form, of any inference according to one of the eight rules, that it is valid.
As we unfold the patterns of natural deduction, and move on to other
kinds of inference, we shall repeatedly discern what always holds, what
never holds, and what only sometimes holds. But the grounds of such
universality are now transcendent, not transcendent.

Moreover, many concrete arguments evidently draw upon a
universality grammatically analogous to the way Aristotelian forms do.
Coupled with my thought that Fido engages in Laughter, Aristotle’s claim
that only humans can engage in Laughter would yield the conclusion that
Fido is human:

\[
\begin{align*}
\text{All L are H} \\
\text{Fido is L} \\
\therefore \text{Fido is H}
\end{align*}
\]

According to some ancient reports, the Stoics interpreted the universal
premiss much as modern thinkers such as John Stuart Mill and Francis
Bradley do: as hypothetical rather than as categorical:

\[
\begin{align*}
\text{If } x \text{ engages in laughter then } x \text{ is human.} \\
\text{This } x \text{ (called Fido) engages in laughter.} \\
\therefore \text{This } x \text{ (called Fido) is human.}
\end{align*}
\]

This interpretation brings Aristotelian-like universality close to the grips
of Stoic formulation: \( p \rightarrow r \) and \( p \), therefore \( r \). Except that now we have
another variable inside the propositional one: the \( x \).

To be able to handle many concrete arguments, we must delve into
some propositions in order to formalize what they assert. And we must
then add to the Stoic list of three a fourth kind of legitimate propositional
form:

\[
\varphi x \rightarrow \psi x, \text{ for any given } x.
\]

The Stoic “indefinite” we may interpret analogously as a conjunction of
two predications of an object:

\[
\varphi x \& \psi x, \text{ for at least one findable (not directly given) } x.
\]

Whereupon we may interpret the “definite”:

\[
\varphi t, \text{ where } t = \text{this (an individual given at the moment).}
\]

And the “intermediate”:

\[
\varphi n, \text{ where } n = \text{a named (but not given) individual.}
\]
Already in the notion of a "given" we may note an aspiration peculiar to modern thinking, an aspiration become second nature to us today. Constructing a system, our efforts direct themselves not to what we encounter (so that we may fully encounter it, fathom its nature) but to our own formulations. Accordingly, we moderns experience difficulty when talking about what a given may demand of us (the way our spouses, our animals, our gardens, even earth, air, fire, and water make demands on us). We may in fact discover that things withhold themselves from us, that we might have to honor their absence in order to earn their presence. Even in scientific research we may wonder whether something supposedly "given" is not really a function of how we conceptualize the conditions of its appearance. And in logical study itself we will eventually have to consider whether our own formulations can count as "given": we shall soon discover that our system teeters into self-contradiction if we disregard the difference between different kinds of "givens."

§3.1 Quantifiers

An actual given is, by definition, individual: a this-here (τόδε τι, Aristotle called it), something on which one must focus as other than oneself, and to which one must respond. One kind of response, the one of interest in logical study, takes the form of an attribution: this-here is ϕ. However, we often allow a proxy to stand in for an individual: its (proper) name. Referring to an individual by name allows us to talk about it despite its absence: we thereby remember and anticipate encounters with individuals, for ourselves as well as for others.

The phrase "given any x..." expresses a pure anticipation, even a challenge to an interlocutor: give me what you like (but one thing at a time!), and I will tell you something about it. Until otherwise restricted, an x governed by such a consideration is restricted: some item, but not just any item, will satisfy the formula. The sense of this phrase we may capture in a logical symbol: \(\forall x\). “Some birds laugh” reads

There exists a being that is a Bird and that Laughs

This reading translates into our special language as

\[ \exists x(Bx & Lx) \]

In contrast to Aristotle, logicians call this new character the existential quantifier.

Both quantifiers govern variable givens. The universal governs promiscuously: it allows for any instance (subject to reservations, as we shall see). The existential governs demurely: it requires an actual instance whose identity remains concealed. The distinction is not that of whole vs. part. It is rather that between indifferent and restricted application. While Aristotle could say that what belongs to all belongs to some (that from All S are P we may immediately infer Some S are P), our modern analogs do not permit such inference: from \(\forall x(\phi x \to \psi x)\) alone we cannot obtain \(\exists x(\phi x \& \psi x)\). Modern existential statements, unlike Ancient particular ones, have a powerful standing all their own, and render services beyond the ken of their universal counterparts.

However necessary quantifiers prove to be, inferences depending on the configurations inside the governed formula require that we strip off the symbols to obtain the bare configurations themselves. After this stripping we may proceed as in the Sentential Calculus, and then reclothe the result.

Following Gentzen again, we must spell out the rules by which we may Introduce and Eliminate each of the quantifiers.

Rules for the universal quantifier

Any unrestricted object variable appearing in a legitimately derived formula may, by definition, take any object as an instance while retaining the validity of the formula. For instance, after deriving \(Dx \to Dx\) ("if x is a dog then x is a dog") from the sentential theorem \(p \to p\), we may record the “any-ness”: \(\forall x(Dx \to Dx)\).
The rule of \( \forall \)-Introduction reads: from any established formula \( \varphi x \) containing a free object variable we may infer another formula in which the universal quantifier precedes; or: from \( \varphi x \) we may infer \( \forall x \varphi x \).

What does this rule effect? A free variable already stands for anything, any “object.” Instead of reading “we may infer” the rule might just as well read “we may re-shape”—with a new logical character. But this new character, the upside-down A (for “any”), serves a purpose: it binds the variable. One effect of such binding is that the variable is now governed by this quantifier, and no other quantifier may intrude on it. We shall soon consider formulas with multiple quantifiers (along with multiple variables, free and bound), and we shall have to exercise care that the respective governorships remain distinct.

One rule-of-caution: we rightly introduce a quantifier only on the whole of the original line. For the quantifier symbolizes what the occurrence of a variable means: given any \( x \) substituted throughout the formula, the formula retains its truth. By way of a counter-example: from \( D x \rightarrow D x \) we would not wish to infer \( D x \rightarrow \forall x D x \), since this latter would read “for any \( x \), if it is a dog then everything is a dog.”

In reverse: a formula governed entirely by a universal quantifier may be stripped of this quantifier. This stripping may occur in one of three ways, the simplest of which renders precisely the formula obtained by \( \forall \)-Introduction: the variable \( x \) is then once again free. However, we may also strip the universal by inserting a named or pointed-to individual wherever the variable occurs: if Fido is a dog, then Fido is a dog; if this lamppost is a dog then this lamppost is a dog. And, corresponding to the Stoic indefinite proposition, we may insert a supposed individual existence into each instance of the variable. This third kind of \( \forall \)-stripping has several uses, most notably in the course of conditional proofs.

The rule of \( \forall \)-Elimination reads: from any established formula \( \forall x \varphi x \) we may infer another formula without the universal quantifier; or: from \( \forall x \varphi x \) we may infer \( \varphi x \) — with the \( x \) having one of three different bearings.

What does this rule effect? It asserts a relative superfluity of the quantifier. So long as this quantifier precedes (has as its scope) the entire formula, it is not needed; indeed, mathematicians often formulate their theorems without it. However, it is needed whenever it occurs within a formula. If we wish to assert that “everybody voting for the Liberals implies that democracy does not flourish,” \( \forall x L x \rightarrow \neg F \), we would not wish to reformulate this straightaway without the quantifier: \( L x \rightarrow \neg F \) means that “democracy does not flourish if anybody votes for the Liberals.”

These two rules can occasion some reflection on the notion an “object variable,” a notion that was first worked out fully during the modern era (in the course of the Fourth Interpretation).

Any character in a formula that can take multiple instantiations we may call a variable. In Peripatetic formulations, we then have subject variables and predicate variables, and in Stoic formulations we have sentential variables. In contrast, logical constants (Leibnizian “characters”) we hope will have the same meaning throughout the variable instantiations: Aristotle and his followers understood “being” (whether as substance, \( \varsigma \omega \), or as predication, \( \varepsilon \nu \nu \) or \( \nu \nu \)) as sharing in sameness, while we moderns establish sameness in the symbols defined operationally.

(The \( \varphi \) in our formulas is also a variable. It simulates the notion of a predicate, and we therefore borrow this ancient term to describe it. However, we shall see that the \( \varphi \) of \( \varphi x \) fits most comfortably with the notion of a function — another notion worked out only in the Fourth Interpretation).

But the \( x \) in our present formulas opens a new vista of contemplation. Peripatetic variables require one to focus on a being and thereby to decide (testify) what can be said about it: logical organization here issues from ontological concentration. Stoic variables require one to focus on one’s assertions and thereby to organize these with a minimum of ontological concentration. Now, however, our logical effort to account for universals within a sentential framework leads to the effort to organize what we say (our \( \varphi \)'s, \( \psi \)'s, and \( \xi \)'s) around what we encounter (\( x \)'s, \( y \)'s, and \( z \)'s) so that our organization can take on all comers (one at a time) without our having to know them holistically. In Stoic fashion, our knowledge is self- rather than other-based.

**Rules for the existential quantifier**

Any individual to which we attribute a property \( \varphi \) serves as a justification for asserting that there exists an \( x \) such that \( \varphi x \). After all,
having an individual “in view,” either directly, indirectly, or by name, means that we are testifying to an existence.

Moreover, if we have established a formula $\varphi x$ that holds no matter what individual stands in for $x$, we may assert that there exists something such that $\varphi x$. For the indifference opens the door to any existence whatsoever (with some reservations to come); the existential assertion would be illegitimate only if there did not exist anything at all.

The rule of $\exists$-Introduction reads: from any established formula $\varphi x$ (including cases where an indicated or named individual stands in for the variable) we may infer a formula in which the existential quantifier precedes the original or: from $\varphi x$ we may infer $\exists x \varphi x$.

Some proofs require the existential version, even where the universal is available (just as, having $p$, we often employ Addition to obtain a less definite but needed disjunction). And many proofs require the “indefinite” existential version when we have in fact a given individual.

However, in conditional proofs where we only suppose $\varphi x$, e.g. that there is an isosceles triangle or that there is a superman, the $x$ remains restricted to the existential version until we have completed the proof; here is a case where we must remember the status of a variable as introduced in a proof; remember, that is, to employ at most $\exists$-Introduction. A concrete indication of the need for this restriction would be the “deduction” of “$x$ is a dog implies given anything it is a dog”:

1. $Dx$ Assumption
2. $\forall x Dx$ $\forall$-Introduction (wrong!)
3. $Dx \rightarrow \forall x Dx$ RCP

In predicate logic the fact that a procedure allows us to arrive at this kind of conclusion (existence implying universality) suffices to discredit the procedure. Such easy passage into universality is a Holy Terror, much as in Sentential Logic contradiction is a Holy Terror.

And how, by what rule, may we strip a formula already governed by an existential quantifier? The difficulty is this: the variable does not allow for just any object, so that we cannot here simply set $\varphi x$ loose. The significance of $\exists x$ is precisely that $\varphi x$ may hold for some candidates only.

Yet if we assume, suppose, the bare function $\varphi x$ and derive something, $p$, from this assumption (and whatever else we have in our system, our $\Gamma$) we will have shown that the bare (undifferentiated) existence of something with property $\varphi$ implies $p$.

The rule of $\exists$-Elimination reads: from any established formula $\exists x \varphi x$ and the conditional $\varphi x \rightarrow p$ (where $p$ contains no free occurrence of $x$) we may infer $p$.

For instance, from “some dogs are honest” we may infer “there exists something honest”:

1. $\exists x (Dx & Hx)$ Premiss /.: $\exists x Hx$
2. $Dx & Hx$ Assumption
3. $Hx$ Simplification
4. $\exists x Hx$ $\exists$-Introduction
5. $(Dx & Hx) \rightarrow \exists x Hx$ RCP
6. $\exists x Hx$ 1,5 $\exists$-Elimination

Again, the word of caution: we must make sure that the $p$ has no free occurrence of the variable at issue. If we had taken line 3 as our $p$ we would then derive, in place of line 6, $Hx$ (with no restriction on $x$), whereupon the perception of one instance of honesty would lead to the assertion that everything is honest.

Also, one must remember that the variable $x$ remains restricted during the conditional proof. Only after line 5 could we introduce the universal quantifier, obtaining $\forall x [(Dx & Hx) \rightarrow \exists x Hx]$, the sense of which intuitively justifies the rule of $\exists$-Elimination.

Another indication of the need to remember the restricted status of the variable $x$: in proofs requiring a second existential maneuver nested within the first, e.g. the assumption that something has the property $\psi$, we must designate this existence by another variable, say $y$ (we must assume $\psi y$ rather than $\psi x$). For there is no way of knowing straightaway that the two existences are the same. Ignoring this restriction, we could easily derive from “some people are virtuous” and “some buildings are ten meters high” the assertion that “some people are ten meters high.”

The notion of a restricted variable gives rise to a simpler version of $\exists$-Elimination: since each existential formula guarantees only one existing individual, we may designate the minimal meaning of $\exists x \varphi x$ as $\varphi a$, where the $a$ stands in for whatever individual satisfies the original; and if there is a second existential formula, $\exists x \psi x$, we must designate its minimal meaning as $\psi b$, where $b$ stands in for whatever (possibly different) individual satisfies this second formula; for any third existential
formula, we would have to designate as $c$ the existing individual. Furthermore, the presence of one of these indefinite (particular) designations would suffice to justify re-introducing the existential quantifier to proceed with the proof. These procedures exactly parallel Gentzen’s $\exists$-Elimination and $\exists$-Introduction, and have the advantage of allowing us to dispense with the conditional proof otherwise necessary. Yet when we come to proofs involving relations this alternative version becomes cumbersome.

### §3.2 Predicate theorems

Theorems offer the opportunity to contemplate the workings of our system; the very word says as much. Consider the following three:

- **Theorem 1:** $\forall x \phi x \rightarrow \exists x \phi x$
- **Theorem 2:** $[\forall x (\xi x \rightarrow \psi x) \& \forall x (\phi x \rightarrow \xi x)] \rightarrow \forall x (\phi x \rightarrow \psi x)$
- **Theorem 3:** $[\forall x (\xi x \rightarrow \psi x) \& \exists x (\phi x \& \xi x)] \rightarrow \exists x (\phi x \& \psi x)$

The second two of these correspond to the traditional Barbara and Darii forms, and show no more than the fact that our formal system can provide such analogies.

Theorem 1, in contrast, suggests an analogy with traditional subalternation: from *all*, we may derive *some*. Yet the analogy here is especially misleading. For instance, from “All laughing beings are human,” $\forall x (Lx \rightarrow Hx)$, we derive $\exists x (Lx \rightarrow Hx)$, there exists something such that if it laughs it is human: *we do not* derive, as the Peripatetics did, the statement that “Some laughing beings are human.” The mathematical logician differs from the Peripatetic on the status of the $\phi$: $\phi x$ now stands for a function, an internalized formula regarding givens rather than as a predicate *indebted* to a subject.

Consider now two more theorems, these in bi-conditional form:

- **Theorem 4:** $\forall x \phi x \leftrightarrow \exists x \neg \phi x$
- **Theorem 5:** $\exists x \phi x \leftrightarrow \forall x \neg \phi x$

Henceforth we may refer to *both* these equivalences indifferently as Quantifier Negation. In the proofs of both theorems, the left-to-right proofs proceed by Negation-Introduction: the antecedent together with the assumed affirmative version of the consequent leads to a contradiction, so we end with the desired negative version. But the proofs of the right-to-left portions both depend on Negation-Elimination; that of Theorem 4:

1. $\neg \exists x \neg \phi x \quad /.: \quad \forall x \phi x$
2. $\neg \phi x \quad \text{Assumption}$
3. $\exists x \neg \phi x \quad \exists$-Introduction
4. $(3) \& (1) \quad \text{Conjunction}$
5. $\neg \phi x \rightarrow \text{contrad.} \quad 2 \rightarrow 4 \text{ RCP}$
6. $\phi x \quad \neg$-Elimination
7. $\forall x \phi x \quad \forall$-Introduction

Confining ourselves to Negation-Introduction, we would include only $\forall x \neg \phi x$: having shown in some concrete instance the impossibility that something exist that does not satisfy the function $\phi x$, we may infer that, given any existence, we “cannot afford to deny” that it satisfies that function. Similarly with the right-to-left portion of Theorem 5: confining ourselves to Negation-Introduction, we will be able to derive from $\neg \forall x \phi x$ only $\neg \exists x \phi x$.

The question here again bears on the status of indirect proofs. Having previously shown that existence (Theorem 4) or universality (Theorem 5) cannot stand in our system, we must ask whether we may rightly claim to know that, respectively, universality or existence *does*.

We may raise the same question regarding the quantified versions of the principle of Excluded Middle:

$$\exists x \phi x \lor \forall x \neg \phi x$$

$$\forall x \phi x \lor \exists x \neg \phi x$$

These formulas may themselves be proved as theorems, again relying on Negation-Elimination. But consider the employment of the first version above: in some actual proof in, say, mathematics, we might be able to show directly that the assumption of $\forall x \phi x$ leads to a contradiction; with Disjunctive Syllogism, then, we would like to conclude $\exists x \phi x$. The question is now whether we can rightly claim to *know* that there exists something satisfying the function $\phi x$—or whether such existence must still be *found*. What sense can it have to say that we know something exists if we have not actually found it? Indirect proofs might best be understood as justifying faith rather than knowledge, a preliminary confidence rather than a genuine insight.

Now consider the following six theorems, where $p$ stands for any proposition or function in which no variable occurs that could be governed by the evident quantifier:

- **Theorem 6:** $[\forall x \phi x \& p] \leftrightarrow \forall x (\phi x \& p)$
- **Theorem 7:** $[\exists x \phi x \& p] \leftrightarrow \exists x (\phi x \& p)$
Theorem 8: \([\forall x \phi x \vee p] \leftrightarrow \forall x (\phi x \vee p)\)

Theorem 9: \([\exists x \phi x \vee p] \leftrightarrow \exists x (\phi x \vee p)\)

Theorem 10: \([\forall x \phi x \rightarrow p] \leftrightarrow \exists x (\phi x \rightarrow p)\)

Theorem 11: \([\exists x \phi x \rightarrow p] \leftrightarrow \forall x (\phi x \rightarrow p)\)

The first four of these, easy to prove, show that any such \(p\) (one containing no free occurrence of \(x\)) can slip in and out of the scope of a quantifier without affecting the affirmation of the formula—so long as the logical constant is either the ampersand or the wedge. The last two, where the arrow reigns, show a change in the quantifier as the \(p\) slips in or out of range. The proofs show why: by Implication, the arrow becomes a wedge; then, with Theorem 8 or 9 and Implication again, Quantifier Negation yields the desired result.

These six theorems also show how we can shift quantifiers out onto the periphery of formulas. Some logical considerations require that all quantifiers precede the entire formula; and we could prove inductively (with mathematical induction) that every formula in which this is not the case can be transformed into an equivalent, into what logicians call a Normal Form, in which this is the case. Meanwhile, we may list a four theorems to this effect:

Theorem 12: \([\exists x \phi x \rightarrow \forall y \psi y] \leftrightarrow \forall x \forall y (\phi x \rightarrow \psi y)\)

Theorem 13: \([\forall x \phi x \rightarrow \forall y \psi y] \leftrightarrow \exists x \forall y (\phi x \rightarrow \psi y)\)

Theorem 14: \([\exists x \phi x \rightarrow \exists y \psi y] \leftrightarrow \forall x \exists y (\phi x \rightarrow \psi y)\)

Theorem 15: \([\forall x \phi x \rightarrow \exists y \psi y] \leftrightarrow \exists x \exists y (\phi x \rightarrow \psi y)\)

And, once all quantifiers have as their scope the entire formula, we may wonder whether their order makes any difference. We shall have occasion to see that the order can be essential in the case of relations.

Theorem 16: \(\forall x \exists y (\phi x \& \psi y) \leftrightarrow \exists y \forall x (\phi x \& \psi y)\)

Theorem 17: \(\forall x \exists y (\phi x \& \psi y) \leftrightarrow \exists y \forall x (\phi x \& \psi y)\)

Given this last theorem, it would be easy to show that quantifiers over the arrow are also switchable.

### §3.3 Domains

Variables of any kind have a domain, a pool of possible instances. Candidates for the variables of our sentential calculus are propositions—neither perceived objects (Stoic “chanced-upons”) nor conceived objects (e.g., numbers), but whole sentences.

Our predicate logic we design precisely to permit analysis of sentences into predicate variables and object variables. The \(\phi\)'s, \(\psi\)'s, and \(\xi\)'s take predicates as instances—our own conceptual imputations. The \(x\)'s, \(y\)'s, and \(z\)'s take ... what? Objects, we say.

What counts as an object? Very roughly: something we confront in our doings, as distinct from the doing itself. Both propositions and predicates we ourselves form: in these two cases, the domains belong to, express our own doings. The analysis into predicate and object requires then that there be a difference: that the object stand opposite the predicate (better: that it refer to something different from the predicate—more on this later).

In concrete discourse, the domain is often silently presupposed. “There's some laughter,” \(\exists x L x\), said during a discussion of a class hour or a movie showing: we likely assume that the domain comprises people; we would be surprised to discover that the laughter stemmed from an electronic device, a parrot, or a hyena. Similarly in figurative speech: “All genuine passion thinks of nothing but itself,” \(\forall x (P x \rightarrow ...\)).

Naming the domain also shortens both natural and artificial formulations: whatever we are talking about already has this name (number, human, student, ...) and we need not repeat it every time (\(\exists x L x\) suffices, we need not add the determination of \(x\) as a Person).

* In “first-order quantification logic,” another name for the predicate logic under consideration, the place-holders for predicates are open to all comers: implicitly, they are governed by the universal quantifier. When quantifying a predicate existentially it becomes necessary to make the quantification explicit; e.g., we might wish to formalize something like “given any object, there exists at least one predicate attributable to it”: \(\forall x \exists \phi x\). Such considerations belong to what is commonly called “second-order quantification logic” (to be discussed in §4.4).
§3.3.1 The necessity of naming a domain

Concentrating on a given example, we might conclude that we could simply include within our formulas a predication limiting the range of the discourse \((x \text{ is a number, } x \text{ is a human, ...})\). We would thereby retain the open-door policy. But several considerations weigh decisively against this possibility.

For one: the expression \(\varphi x\) must make logical sense. To make sense logically, an expression must arrive in the mode of possibility: it must pose the possibility of being either affirmed or denied, one or the other, one against the other. As in proper judicial proceedings, where the question "guilty or not" must arrive as such, without prejudice.

Now, depending on the sort of object, a predication may make no sense. For instance, what would it mean to consider, without prejudice, whether a number, in the mathematical sense of the word, is green? Or whether a logical constant is noble? Only things perceivable with the eyes can be green or not. Only beings capable of moral actions can be noble or not. Similarly, what sense can it have to ask whether my dog is prime (in the mathematical sense of the word)? Or whether my house is valid (in the logical sense of the word)? Only numbers can be prime or not. Only inferences can be valid or not. In such cases we might be tempted to deny that the chosen \(\varphi\) belongs to the chosen \(x\) — to deny it because such attribution is nonsense from the start and not because we have taken the time to look freshly at the \(x\). Such denial is a denial of relevance, of pertinence. The denial is rightly prejudiced, therefore not routinely logical, and therefore not equivalent to affirming \(\neg \varphi x\).

The limitation of formulas to a domain appears essential to the preservation of the truth-functionality of compound formulas: our logical interest in understanding the truth of a formula depends on the alternatives embedded, non-prejudicially, in smallest units. These units are now broken into two parts, a predicate and an object. Each part must fit the other, prior even to the decision whether the fit deserves affirmation or denial. Thus at the basis of predicate logic lies an anterior tension. Concrete talk, even abstract formulation, unfolds against the background of this tension. Cut free from such tension, talk degenerates into babble, a shuffling of leftovers, in sound or writing, from better days.

One way of retaining the essential tension is to acknowledge the necessity of a domain of the discourse.⁶

Yet another consideration: failure to limit the domain of possible objects leads to outright contradictions. Whereas the first consideration might appear rather metaphysical, a matter of diet, this second consideration recalls the battle of logical life vs. logical death.

For instance, it certainly makes sense to point to someone and say: everything he or she is now proclaiming is false, \(\forall x(Sx \rightarrowFx)\). But now consider the case where I tell you that everything I myself am now saying is false: one \(x\) that seems to qualify for instantiation into \(\forall x(Ix \rightarrow Fx)\) is precisely this formula itself, call it \(P\). Now, \(P\), being something I am now saying, I decree to be false: if the original is true, you and I will both judge it to be false (\(P \rightarrow \neg P\)); but if the original is false, then I have lived up to my promise (so far I am telling the truth: \(\neg P \rightarrow P\)). Assuming that a formula can be an instance of itself, we may derive \(P \& \neg P\).⁷ It seems that the domain of instantiation of any formula must be limited to things other than the formula.

In the same vein: it can be inductively proved that the introduction of \(\exists \varphi x \rightarrow \forall \varphi x\) into our system does not generate a contradiction. To be sure, the formula is not always true; in fact, it represents the human error of asserting a universal where the situation warrants only the existential. But it has true instances, as well: take any universally true formula and place before it its existential cousin, e.g. \(\exists x(\varphi x \rightarrow \varphi x) \rightarrow \forall x(\varphi x \rightarrow \varphi x)\). Yet if we let \(\varphi\) be the predicate "valid" and allow the formula to pick up instances from our own domain of well formed formulas (if, that is, we let various formulas, invalid as well as valid, stand in for the object-variable \(x\)), we need only have one formula that is valid to prove that all are valid, and then only one invalid formula to

⁶ Note that Russell's axiomatic deduction, unlike our present natural deduction, does aspire to dispense with the need for a domain. Principia Mathematica is more Stoic and, despite its name, less mathematical than Gentzen's work.

⁷ Notice that the argument here depends upon treating the basic logical notion "false" as a predicate — or rather "true" as a predicate that is now being denied. But this notion, like that of naming a domain, antedates predication: we engage it in order to understand, and even to formalize predication. There is something involved, circular, even incestuous, about asking whether truth (or falsehood) is truly (or falsely) predicated of a proposition.
derive a flat-out contradiction. Again, then, it seems that we must limit our object-variables to things other than our own formulations.

In sum: not only various phenomenological considerations of rational discourse, but also some strictly logical considerations of formal inference lead toward the necessity of confining instances so that, despite the universal quantifier, not just any object will do. Yet both requirements (that the $\phi$ and the $x$ make sense in unison, and that the $x$ be other than the formulas working on it) are essentially negative and therefore vague: we may be able to object to a given case, but we cannot formalize exhaustively the criteria either for “making sense” or for “otherness of instance.” Moreover, some sorts of rational discourse do require that we admit a pronouncement as an instance of itself ("only kindly given advice is helpful" reminds us to proffer this advice in a kindly way). And mathematical induction requires sequentially reflexive instantiation.

Still, $\phi x$ represents, in a highly abstract way, attention to something. A towardness with two sides, most immediately an $x$ on the far side (a presentation) and a $\phi$ on the near side (our attribution). So long as we operate our system routinely (as in the derivation of theorems of the usual sort) we need not recall the intricacy of this attention. Naming a domain, we establish a contract marriage assuring a compatibility both at the initial level of atomic predication and at the developed level of inference.

The Stoics apparently understood the primary domain of predication (for their sentential logic) to be bodily perceivables. By analogy, a Stoic account of our modern predication might restrict the domain of object variables to such things as minerals, vegetables, animals, or humans. But the modern interest in logic stems from the concerns of modern mathematics and physics, where the domains of relevance are such things as numbers, points on a line, points on a plane, points in three-dimensional space, points in a temporal sequence.

Having named the domain, we may derive a predication: $\forall x \delta x$, where the delta now represents the property mentioned in the naming of a domain (mineral, human, number, point on a plane). But the naming precedes this predicking. We may not, without falling into a vicious circularity, define the domain predicatively. The act of naming is an act of acknowledging a ‘towardness’ of attention: minerals are, numbers are, and are what we make use of. The acknowledgement here is essentially open: we cannot rightly quantify the domain (saying: all numbers, all
d

points), but only insist that any one proffered item carry this qualifying credential.

The status of a domain in quantifier logic parallels that of a genus in Aristotle's logic, where the definiteness of a species presupposes an indefinite background out of which we eke the definiteness. Similarly, it parallels the Axiom of Separation in Zermelo's set theory (Appendix III), where any new set must be extracted from an already existing set (so that the notion of a set cannot be extended absolutely: there can be no set of everything at once). In all three cases, we are driven to admit that we cannot formalize the conditions of legitimate formalization: an acknowledgement comes first, then the formalization.*

§3.3.2 Invalidity

How can we prove that an inference is not justified? Or that, from $\exists x \phi x$ and $\exists x \psi x$, we may not rightly infer $\exists x(\phi x \& \psi x)$? That, from $\forall x(\phi x \rightarrow \psi x)$ and $\forall x(\phi x \rightarrow \xi x)$, we may not infer $\forall x(\psi x \rightarrow \xi x)$?

One way, the traditional Aristotelian: devise a counter-example. Here, however, we must draw upon our beliefs about how things are ... outside our logical constructs: we believe that there is something living and that there is something on Mars, but we do not believe that there is something that is both; therefore we judge the form of this argument to be invalid. Invalid because a valid form must hold for any (genuine) domain.

In Stoic and Modern fashion, we prefer that the basis of logical decisions remain firmly accessible within our own constructs, that we need not recur to any consideration of how things happen to be empirically outside our own rational constructs.

* Bertrand Russell devised an alternative contract of compatibility by proposing a hierarchy of predication within the system. His Theory of Types posits the necessity of an internal organization preventing formulas from becoming instances of themselves. Critics of this theory have shown it to require much repair and several ad hoc hypotheses. Russell's purpose was to show how a system of logic could represent human reason as both totally independent of domains and complete in its conceptual expendability. Ever since Gödel showed such completeness to be impossible, the Theory of Types has come to serve primarily as a kind of minority report highlighting the importance of the question. We shall consider Russell's project again in §4.4.2.
About how things are *a priori*, however, there is a governing assumption: that, namely, general (Stoic indefinite as well as outright universal) propositions express sums or products of individual predications. Thus the existential form:

$$\exists x \varphi x$$

is short for $$[\varphi a \lor \varphi b \lor \varphi c \lor \ldots]$$, and the universal form:

$$\forall x \varphi x$$

is short for $$[\varphi a \land \varphi b \land \varphi c \land \ldots]$$, where the individuals $$a, b, c, \ldots$$ are drawn, in any given case, from a viable domain — so that each predication poses a genuine question as to whether it holds or not.

Now, we can *imagine* a domain of only two items, $$a$$ and $$b$$. On this finite domain the first test-form, $$\exists x \varphi x \land \exists x \psi x \therefore \exists x (\varphi x \land \psi x)$$, reads:

$$\varphi a \lor \varphi b$$

$$\psi a \lor \psi b$$

$$\therefore (\varphi a \land \psi a) \lor (\varphi b \land \psi b)$$

The form is now sentential, and it is clear that the premisses may be true and the conclusion false. We then judge the form to be invalid — on the assumption that failure to hold in a finite domain means that it will fail to hold in an infinite domain (an assumption intuitively evident for monadic predications, but easily shown to be illegitimate for polyadic predications).

But notice that for a domain of *one* individual the test-form would not yet show its invalidity. In contrast, the analogue to the traditional AAA-2 requires only that we imagine a domain of one:

All P are M = [P \rightarrow Ma]

All S are M = [Sa \rightarrow Ma]

$$\therefore$$ All S are P = [Sa \rightarrow Pa]

And the form of the following argument requires *three*:

All oaks are animals

Some rats are animals

Some rats are not animals

$$\therefore$$ All oaks are rats

Establishing invalidity by imagining finite domains, we avoid the Aristotelian task of discovering truths and falsehoods regarding items outside our own conceptual apparatus. However, we still assume that any crossover into factual conditions will commit us to detecting objects, formulating predicates, and deciding in each case, one at a time, whether the predicate applies to the object. In the routine business of establishing validity vs. invalidity, this assumption remains unproblematic. In contemplation, however, it leaves us with some residual questions.

Given any Frenchman there is at least one Chinaman: $$\forall x (Fx \rightarrow \exists y Cy)$$; an obvious domain is “human beings” — a domain covering both object variables. But sometimes we need two domains. Given any Frenchman there is at least one vision of perfect government: $$\forall x (Fx \rightarrow \exists yVy)$$. In this latter case, the variables cannot range over both human beings and visions (ideas, projections), since these “objects” take different kinds of predicates: it makes no sense to ask both whether something has French citizenship and whether it is a plan for a system of government. Thus a single proposition can require us to seek references in two different directions at once.

Since the method for establishing invalidity requires a decision about the cardinality of the finite domain, we might wonder whether we can make use of the whole range of possible cardinalities. Not just one or two (the most common), but zero to *?*

A formula holding for a domain of *no* individuals, for an *empty* domain? If there are no objects, then a predication cannot take place. Thus every conditional universal formula holds for an empty domain and no existential formula does: there is no trial. In short, if we refuse to try our formulas out in any genuine crossover, we lose the primary thought of logical study, viz. that our forms reflect the possibility of being either true or false *when tried*.

Rather than according ourselves the liberty of postulating an empty domain for formulas, we might *define* the empty domain as the one generated by any formula that holds for no domain. For example, $$\varphi x \land \lnot \varphi x$$, no matter what the quantifier, can find no instance, no foothold, even in our imagination. Similarly, $$Fx \land \forall x$$ (with F as Frenchman and V as envisaged political system) defies any illustration (since the same individual would have to be both a human being and a plan). In these two cases (self-contradiction and category incompatibility) we can continue to assert the formula only so long as there are no instantiations for the object variable: we generate, imaginatively, the notion of an empty set.

Thus it is significant to ask whether a formula has a *model*, an instance in which it makes sense: whether it applies to a non-empty domain. By producing a model for a formula we show that the formula
is consistent. However, we might also think of the sense of “model” as painters use the word: a model is then a possible source on which the form is patterned. A concrete model for $\exists x (\varphi x \land \psi x)$ is “something (my Uncle Jake) has both the property of being human and weighing three hundred pounds.” A concrete model for $\forall x (\varphi x \rightarrow \psi x)$ is “if I weigh three hundred pounds then I am overweight.” The Stoic forms seem to be patterned on concrete models of these sorts.

If the premiss-forms, taken together in a conjunctive string, have no abstract model (no possible array of values in which all are affirmed), the argument-form is vacuously valid. A form is shown to be invalid if and only if the conjoined premisses have a model, one whose array of values also renders the conclusion false.

Since models have cardinalities, we may raise another question (the decision-problem, as it is called). Suspecting a form to be invalid, we try it out on a finite domain of one individual; unable to discover any abstract array proving the form invalid, we proceed to a domain of two individuals, and so on. How can we know we may stop trying, or tell our computer to stop expending the effort?

It can be proved (with mathematical induction) that, if a domain of $2^n$ individuals (where $n =$ the number of distinct predications $\varphi$, $\psi$, $\xi$, ... in the argument form) fails to reveal the needed array of values, then the argument form must be valid — must, if our deductive apparatus is adequate, allow us to derive the conclusion from the premisses.

§4. Natural deduction: the category of relation

We naturally compare one thing with another. Two individuals: Mary is more affectionate than Eliza, Joe runs faster than Terry. Two kinds: Elms are more susceptible to disease than maples, bears run faster than horses. We are here employing the category of relation, one of the ten on Aristotle's list. Instead of focussing on one being, primary or secondary, we somehow consider two, and straddle these for the sake of something else, the way they relate to each other. In the examples cited, we are also measuring one against the other, and might envision a scale of more or less, and even calibrate this scale numerically.

Developing techniques for formalizing the category of relation, modern logic distinguishes itself once more from the logic of antiquity.

It also puts itself in valuable service to the development of modern science, where relations form the backbone of intelligibility.

The Aristotelian tradition down-played the category of relation. The art of measuring one being against another is the art of counting and ranking. Any art (such as carpentry) requires one to measure things this way. But the full development of the art ultimately requires one to measure each being against its own potentiality, its own being (Statesman, 283C). Without this latter art, our disposition to compare two things leads us to neglect each for what it is: a mother applauds her son because he behaves the way the neighbor's child does, an enthusiast demeanes a Belgian work horse on the grounds that it cannot race the way an Arabian does. Unable to know the subject before us, we tend to assess it, handle it, modify it by changing the subject: by recalling other examples of the same kind, or other kinds of the same genus, and hoping to eke out an understanding of one from the juxtaposition of two.

Indeed, we begin learning something new by comparing it with something to us old. Activities, most likely: from skating to skiing, from piano playing to organ playing. Events, too: from snow storm to hail storm, from dog fight to cock fight. And kinds of beings: from spruces to larches, from trout to bass. We learn an activity by comparing our own gestures with those of others: we imitate. We learn events and beings by detecting similarities and differences. And, closer to overcoming the external art of measurement, we learn to rank cases: some performances are better than others of the same kind, some beings of the same kind are healthier than others.

Plato, Aristotle, and the Scholastics understood comparison as a process of detecting sameness among differences. The sameness is ultimately that of what we encounter, the destiny hovering over what appears in its sometimes wonderful, sometimes distressing multiplicity. The sameness governs our own competent performances, and it explains apparently complicated events. Above all, it justifies the judgement that houses and spruces and trout differ in quality: one instance comes closer to its destiny than another. Aristotle repeatedly claims that the value of literary figures of speech is that they incite us to search out the sameness camouflaged by the glaring differences.

Modern physics developed as a new understanding of comparison, an understanding in which the category of relation becomes the primary focus of attention. Most obviously, the categories of time and place,
together with the category of quantity, come to express the modern idea of motion, then the modern idea of force: ideas that are essentially relative. One thing lies to the left of another thing, one thing occurs after another thing, one thing moves faster than another thing relative to a third thing, a thing weighs more now than it did previously. Modern investigation of nature takes the form of the detection, formulation, and calculation of inter-relations among x’s, y’s, and z’s.

As mere information and classification, science still smacks of the traditional interest in hovering species. In fact, however, such (old-fashioned) science now serves only as data-banking. Rock, plant, and animal formations now appear as processes of ever-evolving relations, and our understanding of nature as a whole requires us to consider events on a par with our understanding of economic forces. About any given rock, plant, animal, or human being by itself, all we can say is that it bears down on other things: that it wants to impose itself on other things, just as these things impose themselves on it. While Plato already suggested this determination (ability to act or be acted on) as the initial criterion for attributing legitimate being to something we talk about, Nietzsche was the first to detect in this determination a resounding principle: by itself, each thing is its drive to power. And instead of trying to detect an overriding destiny for each individual, we had better try to detect patterns of mutation reducible to the power relations that envelop complex events.

A responsible understanding of modern science (indeed, of our modern condition) requires an appreciation of, even a dexterity in the articulation of relations.*

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* In his 1897 article “The Logic of Relatives,” C. S. Peirce first organized the manner of presenting relations as we are familiar with it today (The Collected Papers of Charles Sanders Peirce, Harvard, 1933 & 1960, Volume III, especially §§332-356). Augustus De Morgan had already called attention to the centrality of relations in modern thinking; see his 1860 article “On the Syllogism, No. IV, and on the “Logic of Relations” (Transactions of the Cambridge Philosophical Society, Volume 10, 1864). For an overview reviewing the shifting terminology, see Chapter VII, Section 5, of William and Martha Kneale’s The Development of Logic.

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§4.1 Kinds of relations

Some are necessarily transitive. If x is Larger than y and y is Larger than z, then x is Larger than z: so, in general, with numerical measurements of size (one, two, or three dimensional space), of time (age), weight, or cardinality (items on a list). Similarly “to the Right of” and other relations of locative orientation (Above, Below, West of). And “Earlier than” and “Later than”: relations of temporal sequence. In general, for any relation T that is transitive:

$$\forall x \forall y \forall z (T xy \land T yz \rightarrow T xz).$$

Some are necessarily symmetrical. If x is Next to y then y is Next to x; if x is in Line of sight of y, then y is in Line of sight of x; if x is Two miles from y, then y is Two miles from x. Newton proposed as a necessary truth that “to every action there is always opposed an equal reaction”: this principle could be interpreted as a symmetrical relation: if x Hits y with force F, then y Hits x with force F. In general, for any relation S that is symmetrical:

$$\forall x \forall y (S xy \rightarrow S yx).$$

And some are necessarily reflexive. The prime example of a necessarily reflexive relation is that of Identity, often expressed in mathematical formulas with the equals sign, “=”: 2 + 3 is identical to, has the same reference as 1 + 4, and vice versa. And for any number x, Ixx. In general, for any relation R that is reflexive (most notably Identity):

$$\forall x R xx.$$
points in space; and in mathematics the most frequently employed domain of the relation of identity is that of numbers. These various domains arise as issues in mathematical science, as developed by early modern thinkers such as Descartes, Leibniz and Newton. Kant would say that the relations here are not only a priori, but also synthetic: they illustrate the way we constructively conceive of things in space and time.

Some relations are anti-transitive. For instance, “is exactly 10 inches taller than”: If I am exactly 10 inches taller than my wife, and she is exactly 10 inches taller than our daughter, then it is impossible that I be exactly 10 inches taller than my daughter.

Some are anti-symmetrical. For instance, “is to the West of”: if Toronto is to the West of Moncton, then it is impossible that Moncton be to the West of Toronto (domain: locations on the standard flat map of Canada!).

And some are anti-reflexive: For anything, it is impossible that it be to the North of itself or Heavier than itself or Later than itself (domain: things measured in space at a given moment of time!).

Some relations are neither necessary nor impossible, but contingent. If I differ in weight from my wife, and my wife differs in weight from our daughter, I may or may not differ in weight from my daughter: “differ in weight from” is neither necessarily nor impossibly transitive, but contingently so; similarly with “is two units distant from” (domain: points on a plane). Many negative relations are only contingently symmetrical: “being no taller than” and “occurring no later than.” Contingently reflexive relations are relations of sameness in regard to a category other than the category of being: “having the same height as, the same velocity as, the same age as” — all these are necessarily reflexive for a single entity at a single time and place, but become contingent when the x and y of the expression Rxy are themselves not identical.

The relations cited so far bear upon numeric measurements: size, weight, temporal sequence and spatial location. Their numerical component explains perhaps the a priori necessity, impossibility, or contingency we impute to them.

Once in the swing of algebraic relations, we can turn to human relations: “I love my wife.” L iw; “barbers shave other people,” $\exists x (Bx \rightarrow \exists y ((y \neq x) \& Sxy))$; but “some barbers shave themselves,” $\exists x(Bx \& Sxx)$; “anyone who genuinely gives something to another person, also gives himself to that person,” $\forall x \forall y \forall z (Gxyz \rightarrow Gxxz)$; “everyone has a mother,” $\forall x \exists y Myx$.

Both Aristotle and Descartes argue that to understand anything at all, we also have to understand ourselves: the one claims that we must put ourselves into what we understand, the other that we must assert our own conditions for what we understand. Positing for x the domain of human beings and for y the domain things, we might formalize both these claims as expressing a transcendental reflexivity, $\forall x \forall y (uxy \rightarrow uxx)$.

In his Lettres du voyant, Rimbaud remarks: “It is false to say I think (je pense), one should say I get thought (on me pense).” Similarly, in his Beyond Good and Evil ($\S 17$), Nietzsche remarks: “a thought comes when ‘it’ wants, and not when ‘I’ want, so that it is a falsification of the event to say that the ‘I’ is a condition of the predicate ‘think.’” Positing for x the domain of human beings and for y the domain thinkables, we might formalize both these claims as expressing a transcendental symmetry, $\forall x \forall y (Txy \rightarrow Tyx)$.

Aristotle distinguishes two kinds of beings, Art-made and Nature-made. Hammers, chairs, and automobiles are Caused by Human agency: $\forall x [Ax \rightarrow \exists y (Hy \& Cxy)]$, where “y causes x” refers only to the efficient cause. In contrast, lice, carrots, trees, and horses (Natural things) somehow cause themselves: $\forall x (Nx \rightarrow Cxx)$, where “x causes itself” refers to the divine within each natural being.

Kierkegaard remarks (Concept of Dread) that “it is one thing to understand something that is said [by oneself or by someone else]; it is quite another to understand oneself in what is said”: when fulfilled, reading and writing, listening and speaking are triadic rather than dyadic relations. Specifically, for instance, one reads a book about something, but ultimately also about oneself: $\forall x \forall y \forall z (Rxyz \rightarrow Rxyz) — R$ for “genuinely reads”; domain of x: human beings; of y: works; of z: foci of attention.

The modern transcription of relations follows the precepts of modern (extensional) logic. Thus the relations that come under consideration are strictly external: two (or more) points of reference (items in a domain), points either distinct from one another or identical with one another, come under a concept that relates them. This modern restriction to external relations accords with Aristotle's understanding of relation: the
Hegel aspired both to overcome and to re-incorporate the modern passion for reducing reflexivities to external relations: “everything depends on grasping and expressing anything true not [only] as substance but equally as subject” (Phenomenology of Spirit, Preface). Søren Kierkegaard extends this project; e.g., consider the opening lines of his Sickness unto Death: “Man is spirit. What is spirit? Spirit is the self. What is the self? The self is a relation which relates itself to its own self.” In a similar vein, Francis Bradley introduced the notion of an internal relation to account for “relations” between predicates and their subject, individuals and their kind, parts and their whole (windows of a house, hooves of a horse).

But other reflexive relations defy such easy resolution. To shave oneself might mean that a hand shaves a face that happens to be continuous in flesh with the hand. But what does it mean for one to love oneself? To deceive oneself? To doubt oneself? To understand oneself? The English formulation seems to suggest two things, an agent and a patient. As events, however, reflexive loving, deceiving, doubting, and understanding retain a unity: they are not external relations at all, but rather redoubled insertions of the human spirit. Indeed, Aristotle employs reflexive phrases to name the very unity of whatever can become an issue for us, whether animal, plant, or mineral: “…not everything is in relation (πρὸς τί), but some are also themselves by themselves (αὐτή καθ αὐτήν)” (Philosophy, 1011a17); similarly in Plato’s Phaedo (e.g., 65C) we read that the soul must come to be herself by herself (αὐτή καθ αὐτήν) in order to reach out to what is (its sameness). The question, however, is whether reflexivity is a relation on a par with the others.

* Hegel aspired both to overcome and to re-incorporate the modern passion for reducing reflexivities to external relations: “everything depends on grasping and expressing anything true not [only] as substance but equally as subject” (Phenomenology of Spirit, Preface). Søren Kierkegaard extends this project; e.g., consider the opening lines of his Sickness unto Death: “Man is spirit. What is spirit? Spirit is the self. What is the self? The self is a relation which relates itself to its own self.” In a similar vein, Francis Bradley introduced the notion of an internal relation to account for “relations” between predicates and their subject, individuals and their kind, parts and their whole (windows of a house, hooves of a horse).

§4.2 A medley of relational arguments

The formalization of the category of relation permits us to transcribe into external relations what otherwise might be called internal relations, and to employ our familiar techniques of natural deduction to inferences that would otherwise defy analysis.

Consider the argument of a Christian animal-rights activist:

Lizards are creatures of God.

∴ Anyone who abuses a lizard abuses a creature of God.

Or an argument taking a negative premiss:

Dogs are not trees.

∴ A leash only on a dog is not a leash on any tree.

Or one with a singular premiss, e.g. this sorites (a take-off on “love me, love my dog”):

Fido is close to my heart.

One not loving something close to me does not love me.

∴ One who does not love Fido does not love me.

In general, if P belongs to S, whatever relates to S relates to P, whether a doing, an undergoing, a placing, a timing.

While considering the “philosopher of arithmetic” who wishes to keep his field “pure” by reducing all computations to counting on fingers, Augustus De Morgan writes:

And, all unreasonable as he is, he would be more reasonable than the logician. For it is the truth that all arithmetical result can be obtained by counters: it is not the truth that all inference can be obtained by ordinary syllogism, in which the terms of the conclusion must be terms of the premises. If any one will by such syllogism prove that because every man is an animal, therefore every head of a man is a head of an animal, I shall be ready to —

set him another question.”

We may now formalize De Morgan’s “unordinary” syllogism:

All humans are animals.

∴ A head Belonging to a human is a head Belonging to an animal.

Let us formalize the argument about lizards more fully. The transcription of the conclusion introduces the relation Axy: x Abuses y.

The challenge is to discover the appropriate transcription of the
conclusion. Try this: Given any $x$ (domain: persons), if there exists a $y$ (domain: animals) that is predicated as a lizard and is abused by $x$, then there exists a $z$ (same domain as $y$) that is predicated as a creature of God and is abused by $x$. The full argument then reads:

$$\forall y (L_y \rightarrow G_y)$$

$$\therefore \forall x [\exists y (L_y \land A_{xy}) \rightarrow \exists z (G_z \land A_{xz})]$$

And the derivation of the conclusion from the premiss is routine.

The transcription of the conclusions in these cases will always appear awkward compared with the original English. The originals recall intensional operations: whatever is belongs to the subject (lizard, dog, Fido, horse) carries over into the predicate of the subject. But these intensional considerations are not amenable to Aristotelian syllogistic analysis. Formalizing the intensional belongings as external relations, we may analyze the original propositions into one-at-a-time references to individuals (Stoic indefinites). However, we must then introduce individuals where none may exist; e.g., heads of horses are now named as though they were separate things.

Extensional analysis helps to clarify why a small elephant is not necessarily a small animal, although a sick elephant is indeed a sick animal: “small” falls under the category of relation. A small elephant is small compared to the average size (height, weight) of a mature elephant. Thus, extensionally formulated:

Anything that is an elephant is also an animal.

$$\therefore \text{Any elephant small compared to M is an animal small compared to M,}$$

where M could be (if need be) named as a measurement on a stick, and “small” be defined as a range along the stick. The adolescent tendency to propagandize such generalities as “everything’s relative” stems, perhaps, from awareness of such examples.

It is easy to show that when several quantifiers precede a relational formula their order makes no difference so long as they are all existential or all universal. However, when there is a mix, the order does make a difference. Consider the following argument:

There exists something such that everything is caused by it.

$$\therefore \text{Given anything, there exists something that causes it.}$$

According to some traditional doctrines, the premiss is true for the domain of “substances” (whereby God, or the “prime mover,” causes itself). Formally, the conclusion may be derived as follows:

1. $\exists x \forall y (C_{xy})$
2. $\forall y (C_{xy})$
3. $C_{xy}$
4. etc.

The following theorem holds: $\exists x \forall y R_{xy} \rightarrow \forall y \exists x R_{xy}$. But we may devise a counter-example to show that the reverse does not hold:

Given any number, there exists a number greater than it.

$$\therefore \text{There exists a number that is greater than any given number.}$$

The effort to validate this argument in this form runs into an impasse:

1. $\forall y \exists x R_{xy}$
2. $\exists x R_{xy}$
3. $R_{xy}$
4. ?

The same form we may prove invalid with a non-arithmetic model. Consider a finite domain of two individuals, $a$ and $b$, for the argument

$$\forall y \exists x (x \text{ Loves } y) \therefore \exists x \forall y (x \text{ Loves } y).$$

In this domain, the premiss and conclusion read:

$$(L_{aa} \lor L_{ba}) \land (L_{ab} \lor L_{bb})$$

$$\therefore (L_{a\land b} \lor (L_{ba} \land L_{bb})).$$

The premiss is true and the conclusion false either when each individual loves the other while not loving itself, or when each loves itself but does not love the other.

When Chaerephon went to Delphi to ask whether anyone was wiser than Socrates, the Oracle answered: “No one is wiser” — $\forall x \sim W_{xs}$. If the Oracle had answered that Socrates was wiser than everyone else, we would formulate the claim as $\forall x W_{sx}$. It is easy to show that the former follows from the latter — and that the latter does not follow from the former.*

It is revelatory that the incorporation of formalized relations into our logical system of deduction requires us to abandon one principle that holds in the case of the logic of monadic predication: we can no longer insist that for any invalid argument form we can devise a finite model in which the premisses are true and the conclusion false.

* Apology, 21A. In Xenophon’s Apology the Oracle says, “No one is more free, more just, more moderate.” On either case, the principle of trichotomy holds: “$x$ is not greater than $y$” allows for two further possibilities.
Instead of the relation “is in Love with,” let us return to the relation “is Greater than,” substituting G for L in the last argument above. Since Gxy is necessarily anti-reflexive, both Laa and Lbb are a priori false. And since Gxy is necessarily transitive, Lab and Lba together lead to the forbidden reflexivity. Thus the premiss leads to a contradiction, and an argument with inconsistent premisses is formally invalid. One can also see that it makes no difference what the size of the domain is: the following argument, so clearly invalid by reference to an infinite domain, is valid for any finite domain:

\[
\forall y \exists x Gxy \\
\forall x \neg Gxx \\
\forall x \forall y \forall z [(Gxy \land Gyz) \rightarrow Gxz] \\
\vdots \exists x \forall y Gxy
\]

Valid by default, as it were: the premisses express the fundamental properties of the natural-number series, essentially transfinite and thus false for any finite domain.

This one example reveals a commitment to quantitatively infinite domains, a commitment eschewed by traditional logic and not obviously necessary in elementary developments of monadic predicate logic. The prime relation of interest in modern thinking is precisely the relation of Greater: whether Later (in time), Larger (in spatial dimensions), Heavier (in mass), or Stronger (in force). Such relations beg for numeric measurements: they fall under the category of quantity, and so the object-variables ultimately take numbers as arguments. The general domain of such relations is then the natural number series with its decimal expansions: an ordered domain. The logic of relations then brings us closer and closer to mathematical considerations.

The relation of transitivity reveals yet another peculiarity. Arguments depending on a transitive relation and with only singular premisses (e.g. “I am taller than my wife, and my wife is taller than my daughter, therefore I am taller than my daughter”) require only one additional premiss, viz. one formulating the a priori transitivity. But consider an argument with universal premisses: “Mature oaks are taller than mature alders, and mature alders are taller than mature petunias, therefore mature oaks are taller than mature petunias”:

\[
\forall x \forall y [(Ox \land Ay) \rightarrow Txy] \\
\forall x \forall y [(Ax \land Py) \rightarrow Txy] \\
\vdots \forall x \forall y [(Ox \land Py) \rightarrow Txy]
\]

Even after inserting the premiss assuring the transitivity of the relation “Taller, we will discover that this argument is formally invalid! The proof of invalidity is simple: imagine any domain, finite or otherwise, in which Ax is always false; both premisses are then true in that domain, while the conclusion may easily be false.

Intuitively, it makes sense to assume that, when we are comparing two things to a third, we know the third actually exists. But his intuition must be explicitly formalized and inserted if we wish to prove such an argument valid. To render the above argument formally valid, we must insert not only the a priori evident transitivity, but also the a posteriori existence of a point of comparison, \( \exists x A x \).

While a formal proof involving universalized comparisons in fact requires a premiss assuring an existence, it would seem to make no difference whether the point of comparison be commonly existent alders, extinct species such as dodos, or imaginary beings like Martians: what we have in mind when thinking through such arguments is a numerical scale rather than the thing itself. If dodos are reported to have measured about two-feet in height, then it is this measurement that counts in the argument: we draw upon an intensional meaning of “dodo,” the same kind of meaning that governs the reference to “oaks” and “petunias.” But our logic is extensional, and here the possible extension must be asserted as factual.

There is a close analogy in traditional logic: the form called Darapti (e.g., All Swiss men must bear arms, All Swiss men must vote, therefore Some people who must vote are people who must bear arms) is traditionally deemed valid, but in our mathematical logic we soon discover we must insert an additional premiss, viz. that there exists something named by “Swiss men.”

The question of validity then becomes also a question of domain: some arguments are valid depending on whether the domain is finite or infinite, others depending on whether something exists in the domain. But restriction of validity to certain domains violates the principle that a form must be valid for any domain (so that we need only cite one counter-example to prove a form invalid). There have been at least three logical responses to this paradox, ones summarily labelled as conceptualist, formalist, and intuitionist.

A conceptualist (like Bertrand Russell) argues that we must acknowledge the primacy of our ability to conceive over our ability to
experience. Running into difficulties of the two sorts reviewed, the conceptualist rejoices at the discovery that we must either introduce the experience separately (of the infinite, of alders) or, better, reconceive the argument as bearing not on empirical existences at all but on numbers.

A formalist (like David Hilbert) argues that we must first devise a set of uninterpreted (meaningless) “marks on paper” and a limited number of operations allowing us to evolve (“deduce”) further marks. Running into the difficulties so far reviewed, the formalist rejoices at the prospect of having to complement intuitive procedures with mechanical ones: to reassess the role of the infinite, or simply to insert the marks necessary to obtain the desired results. (Axiomatic deduction proceeds in this manner, and leads to proofs regarding our system as a whole: topics extending beyond the scope of this treatise.)

An intuitionist (like L. E. J. Brouwer) argues that formal representation on paper stems from a free construction of the mind. Negatively: only that representation enjoys legitimacy that the thinker has in fact evolved directly (so that indirect proofs do not represent knowledge). Positively: such formulated arguments as those above about oaks, alders, and petunias, or the one about the Swiss, would legitimately arise only for one who is in fact experiencing something that can be called “alders” or “Swiss men”: the essential work has been done, and it remains (optionally) to adjust the visual and audible representations to express this work to some extent (never total). As for the notion of the infinite, intuitionists and formalists tend to agree that formal work is confined to finite configurations, so that talk of the infinite means exactly that we are interested in constructing something holding at point \( n + 1 \) on the basis of what we construct at point \( n \) (a construction essential to mathematical induction, and to considering the difference between the potentially and the actually infinite).

The debates among these three—conceptualist, formalist, intuitionist—raged during the first three decades of the 20th century, and contributed greatly to the development of, and to clarity about, mathematical logic. In retrospect, one might appreciate how logic as it has in fact developed invites us to participate in all three as styles: actual work in, actual learning of logic takes us into highly conceptual modes of thinking, requires us to formalize our thinking visibly, audibly, and (most recently) electronically, and retains its legitimacy for the individual

inasmuch as the individual in fact experiences the development as one of free construction.*

§4.3 The relation of identity

In what sense are Mark Twain and Samuel Clemens identical? The Morning Star and the Evening Star? The truck involved in the accident yesterday at the corner of Bridge and Main Streets, and the truck now parked by the door of the Police Station?

In the case of a proper name, we would like to argue, e.g.:

\[
\begin{align*}
\text{Mark Twain wrote books.} & \quad \text{Wt} \\
\text{Mark Twain is Samuel Clemens.} & \quad t = c \\
\therefore \text{Samuel Clemens wrote books.} & \quad \therefore \text{Wc}
\end{align*}
\]

The “is” of the second premiss is that of identity, not that of predication. As words, however, Mark Twain and Samuel Clemens differ in meaning: “Mark Twain” recalls a literary figure, a style, a voice still alive, a number of books, whereas “Samuel Clemens” recalls a private life, a person now dead, the subject of a biography. As names, though, both have, we would like to think, a common reference: a source to which “Samuel Clemens” was applied at the time of its first appearance and which adopted “Mark Twain” for purposes associated with the vocation of authorship. The sameness of reference is the sameness of the \( x \) in our domain of discourse that we happen to have spotted twice and named differently each time.

The same individuals (localities and ages as well as people and horses) we may name differently and yet recognize, sometimes, the sameness despite the difference. Students each have one name on my class list, and often another name by which they expect to be called. Professors may ask students to call them each by their last names, while their spouses call them each by their first names and their peers call them each by their nicknames: in a given case, \( l = f = n \). A mountain may receive one name from the villagers on one side, and another name from the villagers on the other side: what the Germanic Swiss call

* For the original debate, see Philosophy of Mathematics: Selected Readings, edited by Benacerraf and Putnam (Cambridge University Press, 1983), especially Part I on the Foundations of Mathematics. All three parties to the debate concentrate on the development of mathematics: only the conceptualists (“logicists”) understand the questions to pertain directly to logic.
“Matterhorn” the Italians on the other side call “Cervino.” And if we take temporal spans as our domain of discourse, we must learn that Italians have two names for each recent century: one like our “The Eighteenth Century” and another that would translate literally as “The Seven Hundreds.”

There is also an identity of names for kinds: in many regions, “larch” and “tamarack” name the same species of tree; after carefully noting variations, dendrologists devise a Latin or Greek name that serves like the name given to a child at birth, and relegate the popular names to the status of nicknames: in the Canadian Maritimes, Larch = Tamarack = Larix laricina. However, naming a species differs from naming an individual, and the difference becomes pronounced in the question of identity. To the modern mind, at least, two names of the same species are identical by convention, whereas two names of the same individual are identical in reference. That is, in the case of a traditionally understood species, there is no way we can look to something in our domain of discourse and discover it to be the same for two different names. Only if we devise a scheme in which we can treat a species as an individual can we apply the logical notion of identity to the relation between two names. This we shall in fact do when we come to the theory of set formation. For instance, at a given time and place, “The students from Labrador” and “The students who play chess” might be the same, the identical eight, say; but these names are not only descriptive of individuals, but finite in reference — they do not bear on kinds, on universals, in the traditional sense. In the spirit of nominalism, then, we may try to construe a species as a finite but indeterminate set of individuals, and note that two different names cover the same collection, the same individuals, and are identical in this sense.

Identity of the sort formalizable extensionally bears on the relation \( I x y \) (more commonly, \( x = y \)) holding between two instances of naming. Thus our extensional formalization of identity also commits us to interpreting naming extensionally: we label a “subject out there” \( \tau \omicron \epsilon k\tau \nu \omicron \delta \omicron \varepsilon \omicron \nu \varphi \omicron \kappa \epsilon \mu \nu \nu \nu \) the way we may label anything we claim is unique (“Hart Hall” for a building on campus; a Social Security Number for a person in Canada). Such a name serves to identify the individual to which our talk refers: it introduces the subject of a Stoic intermediate proposition, and points the way toward responsible predication of it.

Plato and Aristotle understood identity (sameness) as the basic issue for each individual: the basic issue for itself (each individual larch is trying to fulfill its destiny, what it must be) and also the basic issue for the knower (the tree doctor). The identity of the individual looms as its \( \epsilon \iota \delta \omicron \omicron \varepsilon \) its form, its glowing possibility. Two different individuals may have the same fulfillment, the same form, and in this sense be the same — just as we say more casually that some soldiers are wearing the same uniform, meaning the identical type of shirts, pants, shoes, and caps. Identity of the sort discussed by Plato and Aristotle bears on the dynamism of individual and universal, a dynamism in which both the encountered and the one encountering (substance and knower) participate. Such participation provokes a corresponding interpretation of naming: to name something, we must address it as it is — in its own dynamism, i.e. in its nature. Because a genuine name reflects an act of addressing something both as individual and as kind, i.e. the thing as itself caught in the middle and as catching us in the middle, naming can be better or worse, even true or false. On this interpretation (which we call “intensional” to mark the distinction), a name ultimately reflects nature — where “ultimately” means “when we have finally learned to call a thing by its own name.” In contrast, a name that remains or reverts to being a mere label no longer functions, and is therefore either a pre- or an ex-name, not yet or no longer really a name but reminiscent or anticipative of one — like a hammer that has a broken handle. Indeed, our own condition is often one of having labels only, so that our innermost task is to learn again to name things properly.

The difference between the two interpretations of identity rests on the difference between epistemology and ontology. In our modern logic we shall declare that \( \forall x (x = x) \), whereas in our ancient logic no given being is ever quite identical with its destined being. The apparently self-evident formula \( \forall x (x = x) \) takes as instances not beings but names of something “out there” \( \epsilon \iota \delta \omicron \omicron \varepsilon \omicron \omicron \nu \varepsilon \omicron \nu \nu \ )\), and the claim reads epistemologically: reiteration of a name retains the same reference. In contrast, the Platonic and Aristotelian claim that each real thing has a sameness transcending the multiplicity of its properties bears not on the name but on the thing itself, and recalls both its destiny in nature and our own as knowers (as performers of a craft and namers of what is revealed in the performance).
Since the identity at issue in epistemological inference has its roots on our side rather than on the other side of the encounter, we may formalize it directly.

4.3.1 Two rules for theorems with identity

One rule allowing us to introduce identity, and another allowing us to make use of it, suffice for the task of proving the symmetricity and transitivity of identity, as well as further theorems:

Identity-Introduction: \( \forall x(x = x) \)

Identity-Exchange: \( \phi x, x = y \vdash \phi y \)

The first of these rules is the only one in our collection, now grown to fourteen, that requires no premiss: appropriately, perhaps, since it represents the most self-contained act on our part. The second rule requires that we understand \( \phi x \) as reading “\( \phi \) is a function of \( x \)” e.g., we can formulate “\( x \) is a product of \( y \) and \( z \)” as \( (Pyz)x \), and “\( x \) is identical to \( y \)” as \( (Iy)x \).

The proof of the symmetricity of identity requires careful attention to the function-formulation:

**Theorem 1:** \( \forall x \forall y(x = y \rightarrow y = x) \) read: \( \forall x \forall y[(Iy)x \rightarrow (Ix)y] \)
1. \( x = y \) Assumption
2. \( (Ix)x \) \( = \)-Introduction
3. \( (Ix)y \) \( 2,1 = \)-Exchange
4. \( (Ix)y \) \( \rightarrow \) \( (Ix)x \) RCP
5. \( \forall x \forall y[(Iy)x \rightarrow (Ix)y] \) \( \forall \)-Introductions

One ambition of our (extensional) logic of identity is to undergird the mathematical notion of equality, i.e. to understand formally the significance of the familiar principle that \((3 + 2) = (2 + 3)\). This principle holds because the names on either side refer to the same thing.

Similarly, we can prove in logical terms the transitivity of identity, therefore also the familiar mathematical principle that two quantities equal to a third are equal to each other:

**Theorem 2:** \( \forall x \forall y \forall z [(x = y \land y = z) \rightarrow x = z] \)

To facilitate proofs involving the negation of an identity, we introduce the following definition:

\[ x \neq y =_{\text{def}} \neg(x = y) \]

**Theorem 3:** \( \forall x \forall y \forall z [(x = y \land y \neq z) \rightarrow x \neq z] \)

**Theorem 4:** \( \forall x \forall y (x \neq y \rightarrow y \neq x) \)

4.3.2 Numeric quantification

Without the relation of identity we are unable to formalize statements about how many things have a property. We may formalize “All students have enrolled” and “At least one student has enrolled,” but how might we formalize the number of enrollees? We have yet to do justice to one of the most important categories of modern thinking: quantity.

With the relation of identity we may now capture numeric determinations in our formalisms. We do so in a squeeze-play between an expanded version of the notion of “at least one” and a new notion, that of “at most.” (We then employ the principle of trichotomy: of any two numbers one is either less than, or more than, or identical to the other, exclusively.)

We easily construe the statement that there are at least two as saying that there is an \( x \) that enrolled, there is a \( y \) that enrolled, and that the \( x \) and the \( y \) are not identical:

\[ \exists x \exists y (Ex \land Ey \land x \neq y) . \]

Similarly with “at least three,” and the rest.

The general notion of “at most” we may construe as “given any one more, it will be identical to one already given.” For example, we may transcribe “at most two students enrolled” as saying that, dipping into our domain three times and discovering \( Ex \), \( Ey \), and \( Ez \), one of \( x \), \( y \), and \( z \) must be identical to one of the others:

\[ \forall x \forall y \forall z ((Ex \land Ey \land Ez) \rightarrow (x = y \lor x = z \lor y = z)) . \]

And if there are at least two and at least two, there are exactly two. This exact number we may formalize by conjoining the previous ones in a number-generating squeeze:

\[ \exists x \exists y (Ex \land Ey \land x \neq y) \land Ey \land z ((Ex \land Ey \land Ez) \rightarrow (x = y \lor x = z \lor y = z)). \]

Similar conjunctions allow us to formulate a numeric range: “there are at least two and at most three enrollees” becomes:

\[ \exists x \exists y (Ex \land Ey \land x \neq y) \land \forall x \forall y \forall z ((Ex \land Ey \land Ez) \rightarrow (x = y \lor x = z \lor y = z \lor z = w)). \]
More importantly, we may formulate the notion of “exactly one” ("one and only one"): there is one who is enrolled, $\exists x E x$, and any two positive determinations of enrollment require that the references be the same, $\forall x \forall y ((E x & E y) \rightarrow x = y)$. At this point, however, let us note that the conjunction of these two is equivalent to a more succinct construal, viz. “there is one $E$, and if anything happens to be $E$ it will be identical to that first one”:

$$\{ \exists x E x & \forall x \forall y ((E x & E y) \rightarrow x = y) \} \leftrightarrow \exists x [E x & \forall y (E y \rightarrow y = x)].$$

This equivalence could be proved as a theorem, and expanded to allow for more succinct formulas capturing “exactly two,” and the rest.

The formalization of “one and only one” allows us to formulate a uniqueness of reference while neither encountering the $x$ referred to nor having a proper name for it. Consider such phrases as “the student who stole my book” and “the building on campus housing ... of the description. As Bertrand Russell famously argued, these phrases are especially necessary for modern scientific work.

Let us begin with a simple example. How shall we understand a complete statement, in context, such as “The apple on the kitchen table is rotten.” I tell you this in the living room, and you go to look for yourself. The statement is either true or false, we would like to assume. But what does that mean in this case? If the apple is rotten, I was right. If it is not rotten, I was mistaken. But what if you find no apple at all on the table? Or two apples? Or no table? In either case, your mission to determine whether something is rotten, or not, has been frustrated: you cannot deliver any judgement at all — on the ... you will be calling upon the analysis of the grammatically incomplete sentence, “The apple on the table,” reading this as $\exists x \exists y [A x & T y & O x y & \forall z ((A z & O z y) \rightarrow z = x)].$

The original proposition then includes, under the governorship of the existential quantifier, the additional sub-formula, $R x$, $x$ is rotten.

The analysis of “definite descriptions” (Russell's designation) recalls the existential presupposition engendered by much of our talk. I tell you that I must leave you to walk my dog, and you quite rightly suppose that there exists a dog I can call my own, and that I have only one (unless I meant: one of my dogs); if you subsequently learn that there does not exist a dog I can call my own, or that I have two dogs, you likely judge that I spoke falsely. Children, too, talk convincingly about an imaginary friend, fully endowed with a proper name — and so do novelists. In general, much of our talk invokes indirectly one-and-only-one existence, while inviting is a focus directly on determinations about such existence.

In the Stoic senses of the two terms, “definite descriptions” are certainly neither definite nor intermediate: they stand in place of encounters and proper names. Despite Russell's designation, a proposal having a description as its grammatical focus comes closest to the status of a Stoic indefinite proposition: there exists somewhere and sometime a subject with certain pre-properties, and we shall proceed to record further determinations without having this subject in view. Such formulations represent the spirit of modern investigation, whether detective or scientific: we must learn to think and talk about “the monk who is murdering other monks in the Monastery” or about “the center of the Earth’s orbit” as though there were one ... and only one. In general, the grand thoughts of modern investigation have a life of their own, independent of direct grounding in encounters.

Knowledge by description, as Russell calls it, would appear to Aristotle as opinion — as lacking footing in singularity, the birchplace of universality. He would point out that the quantifier governing the formalized version of a definite description is particular rather than singular or universal. Finally, he would ask us to note that Russell's name for the contrasting possibility (“knowledge by acquaintance” vs. “knowledge by description”) itself contrasts with his own name, $\tau e x n$, a knowledge that helps complete while dwelling with singulars.

The basis of a Stoic definite proposition is actual encounter at a time and a place: a singular of sorts. In the modern spirit, we extract from such encounters only a collection of data, i.e. indefinite particulars.

The basis of a Stoic intermediate proposition is a name of the individual considered in absentia. In the modern spirit, we find ourselves
again invited to reduce such naming to indefinite particularity. For instance, A. N. Whitehead says of “Socrates is mortal”:

The proposition may mean ‘It is Socratic and mortal’; where ‘Socratic’ is an additional element in the predicative pattern. …The word ‘Socratic’ means ‘realizing the Socratic predicate in Athenian society.’ …”

On this scheme, “Fido is beautiful” becomes \( \exists x \{ x \text{ is called Fido } \land \forall y \{ (Ty \land Iy) \rightarrow y = x \} \land Bx \} \).

Similarly, both “Mark Twain” and “Samuel Clemens” become shorthand expressions for existential descriptions, so that the formulation of their identity makes no reference to singulars, but only to particulars: there exists ... and there exists ..., and these two existences are identical—in exact parallel to the formulation of “The Truck Involved in the accident in such-and-such way yesterday is the Truck parked now by the door of the Police station”:

\[
\exists x \{ \{ Tx \land Ix \land \forall y \{ (Ty \land Iy) \rightarrow y = x \} \} \land \exists z \{ Tz \land Pz \land \forall y \{ (Ty \land Py) \rightarrow y = z \} \land z = x \} \}.
\]

Such formalizations require, of course, that we discover and formulate the relevant descriptors.

In short, when we wish to formalize our talk about singulars, we moderns find ourselves able, if not driven, to dispense with the singularity entirely. This possibility may shock us, but it represents, in the abstract, a propensity already found in our daily dealings. Even a necessity, as in much aloof investigation. And recognition of this formal possibility can serve to open the question of the various possible ways we might understand singulars in the concrete.

* Process and Reality (1929), “Propositions and Feelings.” Similarly in Whitehead’s 1920 Concept of Nature (Chapter One): only in events do we find something we can rightly call “substances of nature”; for a “logical subject” all we need is an “it,” i.e. a “bare objective for consideration” —everything else, including proper names, amounts to a predicate. Whitehead’s account has a certain phenomenological cogency, to which I myself have attested in The Sense of Language (The Hague, 1973). For a strictly mathematical statement, see F. P. Ramsey’s 1925 essay “Universals,” reprinted in his collected works, The Foundations of Mathematics (in “Socrates is wise” the proper name has no primacy as naming a “subject”).

§4.4 Second-order quantification

The narrative stance of first-order quantification places us, Stoic fashion, “inside” a gamma, over against which arise instances in an “outside” domain. To these instances we then apply our concepts.

There have been some important, although perhaps only apparent exceptions: we determine whether an argument is valid, we imagine deciding whether an instance of \( \varphi x \) is true, we distinguish some relations as having properties that hold independently of empirical content. Validity, truth, and properties of relations appear to be predicates operating on items within our own gamma. However, in each case the real event lies in the activity of deriving or asserting or relating, and we rightly assign these various “inner-gamma attributes” only as a loose way of recalling or anticipating the performance (we may interpret the turnstile, \( \vdash \), this way).

Still, there are cases of intelligent discourse where we actually quantify our predicates. Doing so, we turn toward concepts in our own gamma, treating these much as we treat things “outside.” For instance, I might assert that “Some of Jake’s students have some of his good qualities”:

\[
\exists x \{ Sx \land \exists \varphi \{ G\varphi \land \varphi j \land \varphi x \} \}.
\]

And we often assert that one thing has all (or none) of the qualities that count as good (or bad) — e.g., that “Jake has all the good qualities of any professor”:

\[
\forall \varphi \forall x \{ (G\varphi \land Px \land \varphi x) \rightarrow \varphi j \}.
\]

Finally, our earlier traditions ask us to assess each individual with a view to a standard. The strongest positive version of such assessment might be: “Jake has all the qualities of a good professor”—meaning all the properties essential to being a good professor (baldness not counting among these: Republic, 554C). Letting \( Px \) now stand for “\( x \) is a good professor,” we obtain:

\[
\forall \varphi [ \forall x \{ Px \rightarrow \varphi x \} \rightarrow \varphi j] \).
\]

We here attempt to formulate extensionally a thought traditionally formulated intensionally. Yet \( Px \) now stands for a double concept, the analysis of which presents difficulties that are probably insurmountable. For what can “good” mean extensionally — except that we (individually or collectively) happen to approve of the quality that an individual or a group of professors happens to have? Even apart from this contemplative
question, there is a strictly formal consequence of our formula: if either there is no \( x \) in our domain that we agree to call “good professor,” or there is only one that happens to be Jake himself, it follows that Jake has every property (in the domain of \( \phi \)), including contradictory ones! Leibniz might accept this result — so long as “Jake” names a substance (“each substance expresses the whole universe” — letter to Arnauld, 23 March 1690). And Kant relocates this thought as the Transcendental Ideal for Jake (any one thing stands under the “principle of thorough determination,” i.e. we can, even must think the possibility of determining Jake in regard to every possible pair of contraries; Critique of Pure Reason, A572, B600). — Still, the formalism at most occasions these thoughts as questions.

Hitherto, our formalisms have left the \( \varphi, \psi, \) and \( \xi \) free for instantiations at our will, so that we could envisage (imaginatively) an arbitrary predicate in the place of each. Such is the case throughout our (first-order) predicate theorems: only the necessity of naming a domain for the object-variables reminds us that our choice of predicate is of any consequence. However, in these new examples we talk about predicates as themselves forming collections about which we may say something and over which a quantifier reigns.

With second-order quantification we introduce a new narrative stance: we stand above both a sub-gamma of concepts and a world of instances. And we compare: first, as logicians, we recall that each item chosen from the sub-gamma must combine sensibly, i.e. truth-functionally, with each item chosen from the domain of instances; and, second, in application, we would ask whether each is correct or not. Already to pose the first of these questions we have established a triadic semantics: as namers and comparers, we look to a domain of concepts within our gamma and then to a domain outside our gamma. And any actual judgement will relate the one to the other: concept to percept, predicate to object, attribute to thing. This judgement issues from a pinnacle overlooking both domains. In Kantian language, our own narrative stance is that of a transcendent ego.

Quantifying predicates, we make explicit the structure already lurking in the extensional interpretation of judgement (e.g., Russell's elimination of the “denoting” apparently at work in the classical phrases “all,” “some one,” “exactly one,” and “this-one-here”). Although we continue to formulate the innards of judgement as \( \varphi x \), we must henceforth understand this formulation as shorthand for a relation: \( B\varphi x \), “\( \varphi \) bears on \( x \).” The ancient belonging, \( \dot{v}παρχειν \), the primary event of judgement in classical logic, becomes a relating, a \( προς \tauι \).

The logical reconstrual of predication as a relation corresponds to the modern understanding of scientific demonstration. First, Newtonian mechanics requires that we formulate the relations of points on a space-time grid: science no longer means contemplation of subjects to which predicates belong, i.e. of subjects revealing themselves inasmuch as we become clear about what belongs to them as they are in themselves. Second, and closer to logical work itself, the reconstrual of \( \varphi x \) as \( B\varphi x \) (as \( \varphi \) bears on \( x \)) provides the opening into set theory. For, we may ask, what sense does this bearing have — if not that \( x \) is a member of a collection called \( \varphi \)? Already Leonard Euler, and then more clearly John Venn, understood the classical belonging of \( P \) to \( S \) as the collection \( P \) including the collection of items in \( S \). Rather than understanding the \( B \) in \( B\varphi x \) as primitive, in a fruitless imitation of Aristotle, we will eventually posit Membership as primitive: \( M\varphi \) (or, in the more conventional symbolization, \( x \in \varphi \)). The \( \varphi \) now becomes a proxy name for a kind, just as the \( x \) initially names a proxy individual (object). And, finally, this reconstrual allows us to drop the restriction of object-variables to individuals “outside”: either variable can take concepts (predicates, attributes), so that the innards of judgement can be formulated uniformly as \( Bx \).

Once we allow concepts to serve as instances of variables we open ourselves to new questions of legitimacy. Especially in regard to the properties of this newly conceived relation: e.g., whether we might legitimately predicate a predicate of itself—whether \( Bx \) can be reflexive. Just as we might agree that the predicate “concrete” is a rather abstract notion, so that “abstract” belongs to “concrete” (Bac), so we might wish to argue that the predicate “abstract” is itself abstract (Baa). And if I have a concept of what it means to have a concept, must we not assert that, for this special concept, Bcc? Yet we might still wonder whether the extensional, external relation of belonging permits this sort of self-predication. In set theory we ask a very similar question: Can we legitimately decide that a set belongs to itself, \( Mxx \) (or \( x \in x \))— as when we try to form a set of all sets?

Most instances of \( Bx \) are in fact not reflexive. The question is whether we may conceive of the possibility. Reflexivity will prove
impossible only if we discover a contradiction entailed by the supposition of its possibility.

Russell seems to have found one (Principles of Mathematics, 1905, §101): Let “predicable” be an attribute of a predicate if and only if it can be predicated of itself (so that “abstract” and “concept” are both predicable). Then a predicate will be “impredicable” if and only if it cannot be predicated of itself (so that “green,” not being green, is impredicable). And now we ask: Is “impredicable” predicable, or not? If it is not, then we can indeed rightly predicate it of itself; and if it is, then we cannot rightly do so—a contradiction.

Formally: we define the property “impredicable” as follows:

\[B \phi \equiv -B \phi \phi\]

and from this definition we easily derive:

\[\forall \phi (B \phi \leftrightarrow -B \phi \phi)\]

whereupon now we may employ \(\forall\)-elimination to instantiate every instance of \(\phi\) with “impredicable”:

\[B I \leftrightarrow -B I\]

which leads directly to a contradiction: \(B I \& -B I\).

Contradictions in perceptions already invite us to draw distinctions (Republic, 436B). But contradictions within our own gamma serve especially as exhortations to thought (παρακλητικά της διανοιας), as awakeners to insight (εγερτικα της νοησεως; 524D). For a contradiction demands that we backtrack—here, that we think back to the most elemental of logical acts, the assertion that something is ... something else. In Book Five we shall resume the task of backward thinking. Still in a formal mode, we may note the three chief ways of resolving the contradiction: the semantic, the conceptual, and the mathematical.

§4.4.1 The semantic resolution

In any reflexive narrative stance it becomes necessary to distinguish the mention from the use of a term: between talking about a term and talking about something else with the term. In several of William of Sherwood’s “nine modes of predication” (cited in Book One, §7.) the grammatical subjects illustrate the mentioning of the term: (1) man is monosyllable, (2) man is a noun, and (3) man is a species. From the modern perspective, each such remark mentions man; each predicates not the being man but the name man. The name might in turn refer to something, but in each of these three assertions we are referring to the name, not to the being: to the name’s material sound and material grammar, and to the status of its reference (to a species).

In prose, we customarily distinguish mentioning from using by quotation marks: “green” (the word) is not itself green (if we print it in green, then the print is green, not the word). Then, too, “abstract” may be an abstract word, but now there are two to consider, each differing from the other semantically. In formulas, we may distinguish the mention of a term by bolding it. We can then formalize “‘green’ is not green” to read \(-B gg\). 

The distinction between use and mention gives rise to a logically more important need to distinguish the status of a predicate that may look or sound the same in application but differ according to how the subject-term enters into use. In the sentence, “The insulation of my house is good” the predicate applies to the insulation, and likely has a meaning that I could paraphrase as “effective in preventing heat transfer.” If, then, I say that certain qualities (properties, attributes, predicates) are themselves good, I am shifting attention from the original subject: having good insulation is good, and the meaning of this latter use of good likewise shifts (I might paraphrase its meaning as “desirable”: the quality “having good, i.e. effective insulation” is good, viz. for resale). According to the rule requiring compatible domains, the two uses of the word “good” differ semantically. Thus in a single sentence such as “there are good people who do not have every good quality,” we again need to distinguish the two uses of good, this time with a view to the differing domains of the two object-variables; e.g., we may underline the second-order use:

\[\exists x (P xx \& G x \& \exists \phi (G \phi \& -\phi x))\].

Where, now, can we intervene to prevent the derivation of Russell’s contradiction? Evidently, already the definitions of “predicable” and its complement, “impredicable” confuse use and mention. In both the definens and the definiendum the final items are mentioned while the other items are used. As one result, \(B \phi \phi\) does not illustrate reflexivity of the relation \(B x y\). As another result, the universalized formula

\[\forall \phi (B \phi \leftrightarrow -B \phi \phi)\]

* Or: “green does not bear on ‘green’.” Notice that the reconstrual of \(\phi x\) as a relation of “bearing on” edg us into thinking of the first instance as mentioned as well: only the relation remains unequivocally “in use.”
would have to read: for any mentioned predicate, it will be Impredicable if and only if it (mentioned again) cannot be predicated of itself. So long as we hold to the distinction, we cannot universalize across the board: each variable draws upon a different domain.

The semantic resolution of the paradox evolves within a larger consideration of language levels. Throughout logical work we implicitly acknowledge at least one difference of levels: we work out our formal language in use, deriving formulas from previously postulated rules and definitions, then too from theorems already derived. But we also talk about what we are doing: we employ such-and-such rules, we let \( \varphi, \psi \), and \( \xi \) function as place-holders for predicate-variables, and so on. There are two levels of language here: in its innocence, the first level simply unfolds; we in our innocence simply help it unfold. But then we also look at, think about what we are doing: we then turn what we have been doing into an object-language, and we engage in a meta-language. Each language has its own range, and confusions arise from crossing mindlessly from one to the other. The use-mention distinction illustrates one important employment of the distinction of levels. Alfred Tarski first worked out the larger distinction in formal detail, with Russell's paradox and others in mind:

The main source of the difficulties met with seems to lie in the following: it has not always been kept in mind that the semantical concepts have a relative character, that they must always be related to a particular language. People have not been aware that the language about which we speak need by no means coincide with the language in which we speak. They have carried out the semantics of a language in that language itself and, generally speaking, they have proceeded as though there were only one language in the world.\(^*\)

So, for instance, if we wish to speak of “impredicable terms,” the terms to which this attribute might apply will belong to an object-language, and “impredicable” has no business in that other language. We destroy our own narrative stance by insisting that there be only one language.


§4.4.2 The conceptual resolution

While the semantic resolution asks us to heed carefully the way terms differ in meaning, the conceptual resolution asks us to analyze carefully the basic acts embedded within the notion formalized as \( \varphi x \).

This notion has two senses. On the one hand, \( \varphi x \) represents a concept \( \psi \); in recognition that a concept occurs not as an isolated entity but essentially intends to range over entities of some sort, we represent the full concept in the familiar form, \( \varphi x \) (Russell writes \( \varphi \psi x \)). The concept might be that of “being hurt”; we then write it as \( H x, H y, \) or whatever: the variable here does not indicate difference. On the other hand, \( \varphi x \) represents an ambiguous instantiation of the concept. When Euclid says, “Take a triangle...,” he asks us to consider \( T x \) as one triangle, yet as the proof unfolds it becomes clear that any one triangle will do. In our own logical proofs, the assumption of \( \varphi x \) clearly illustrates this second kind of representation. In such examples, \( \varphi x \) is at work, it functions: it is no longer a pure concept. And, as in our familiar conditional proofs, the introduction of another functioning but ambiguous instantiation (“Take another triangle...”) requires us to retain the possibility of difference, \( T x \) vs. \( T y \), for the duration of the conditional proof.

Now, of special interest both in formal proofs and in concrete knowledge, we must eventually universalize: for some complex formula (e.g., \( \varphi x \rightarrow \psi x \)) we wish to add: “for all values of \( x \)” — in short, we wish to write \( \forall x (\varphi x \rightarrow \psi x) \). For only upon such universalization do our efforts give birth to a proposition (a theorem, a claim to knowledge). The question is how to understand this additional (conceptual) act.

Let us slowly rehearse it. From an ambiguous entity \( x \) satisfying the concept \( \varphi x \) (a Stoic indefinite), we derive: for that ambiguous entity \( \psi x \) also holds. Since, or if, it makes no difference during the proof which entity served in place of the \( x \), we conclude that \( \varphi x \rightarrow \psi x \) “always” holds, holds for any entity. Or can we? The ambiguity permits us to do so, providing only that the next instance is “like” the one we started with. This likeness, now, is understood conceptually: each \( x \) for which the formula holds must not differ significantly from the others. In short, the original concept \( \varphi x \) already signifies a type of entity satisfying \( \varphi \).

Thus the conceptual resolution introduces a metaphysical notion. Entities themselves (conceptual or otherwise) fall into different orders of types: the lowest is that of entities “out there,” perceived individuals
“destitute of complexity,” the next type is that of entities (concepts) bearing on individuals, and the next above is the type that bears on the types below; and so on.

Types are not to be confused with domains: types are hierarchical and conceptually differentiated, while we may name domains freely and ad hoc.

Crucial to Russell's argument developing his Theory of Types is the difference between “all” and “any”: the first creates a totality, and consequently a new entity (all taken together to form a unity of reference); they second does not create a totality, but reads “any one, taken one at a time.” Russell applies this distinction to the question of the traditional principles of Excluded Middle and Contradiction:

The distinction between all and any is ... necessary to deductive reasoning, and occurs throughout mathematics; though, so far as I know, its importance remained unnoticed until Frege pointed it out.

For our purposes it has a different utility, which is very great. In the case of such variables as propositions and properties, 'any value' is legitimate, though 'all values' is not. Thus we may say: 'p is true or false, where p is any proposition', though we can not say 'all propositions are true or false'. The reason is that, in the former, we merely affirm an undetermined one of the propositions of the form 'p is true or false', whereas in the latter we affirm (if anything) a new proposition, different from all the propositions of the form 'p is true or false'. Thus we may admit 'any value' of a variable in cases where 'all values' would lead to reflexive fallacies; for the admission of 'any value' does not in the same way create new values. Hence the fundamental laws of logic can be stated concerning any proposition, though we can not significantly say that they hold of all propositions. These laws have, so to speak, a particular enunciation but no general enunciation. There is no one proposition which is the law of contradiction (say); there are only the various instances of the law.5

Thus Bϕφ commits, already prior to the introduction of any quantifier, what Russell calls a “reflexive fallacy.” And the proposed definition of “impredicable” must take its place among the nonsense verses devised for children.

Or, more slowly. How can the conceptual resolution prevent the contradiction apparently engendered by the concept of “impredicable”? Russell answers: because in every predication (conceptualization) I must recur to a type of entity, pick an instance, and bring it back to a tribunal of a higher type. Every legitimate “all” means “any of those in that lower type.” Thus the “all” cannot include the elements by which we recur to those lower-type instances. The proposed concept impredicable in Bϕφ aspires to do just that, as the definiens, Bϕφ, shows.

For the conceptual understanding of Bxy Russell proposes a rule: “no totality can contain members defined in terms of itself.” There can be no totality of predicates defined with the concept “predicate.” If we ask what formally justifies this rule, we may answer as we usually do in formal logic: violations of it lead to the much-dreaded condition of self-contradiction. Or we may reply in a more thoughtful vein: a concept only comes into being as I encounter something possibly comprehended by the concept — so that the concept “impredicable” fails to come into being except in regard to something different from itself; or: Bxy represents an encounter, a courtship or a battle, a marriage or a victory, requiring two distinct participants — unlike Ixy.

Yet the Theory of Types entails some unfortunate consequences, as Russell was the first to admit. The most painful, perhaps: it follows from the theory that the concept of number differs from type to type: that one kind of 3 apply to the apples on my kitchen counter, and another kind of 3 apply to the traditional concepts of quantifiers (All, Some, This). In a more practical vein, consider the translation of the following example calling for a type-1 and a type-2 of the predicate “usual”: “It’s an unusual person who has no unusual qualities”:

\[ \forall x[\forall \phi(U,\phi \rightarrow \neg \phi x) \rightarrow U,x]. \]

Now, the predicate “unusual” may be relative and vague, but it is clearly numeric. We could conceivably quantify it exactly: x having as arguments groups of individuals and Unusual signifying “less than c” (c being some fixed cardinal number for each group). With some such contrivance, we could assign a definite numeric meaning to both instances of “unusual.” Must we then say that this number differs in meaning

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according to whether it refers to (groups of) people or to (frequencies) of predicates? Numbers, especially, seem to function ubiquitously across Types; indeed, we generate the Hierarchy of Types with ordinal numbers, and we count them with cardinal numbers.

To assure the needed trans-Typicality of much conceptual work, Russell proposes an Axiom of Reducibility. The Axiom in effect states that there is some equivalent way of expressing higher-order functions in lower-order ways; for example, that a quality’s being unusual can be expressed as an affair of individuals’ being rare who have this quality, that a quality’s being good (desirable) could be expressed as an affair of houses’ being desirable that have this quality. So, too, we assume that there being three sets of three things can be expressed as there being nine things.

Whatever its merits (and its subtleties: the sketch has been rough to a fault), the Axiom is neither purely mathematical nor intrinsically necessary. And it is hardly revealing, as any principle should be: rather, we introduce it ad hoc to compensate for difficulties arising in our efforts to formalize realizations in a manner free of contradiction.

§4.4.3 The mathematical resolution

While both the semantic and the conceptual resolutions require that we distinguish orders of existences outside the actual formalism, we may also prevent contradictions (of the sort engendered by “impredicable”) within our formalism. We do this by requiring that any defined concept be proved to exist before we permit ourselves to employ it in a proof. Guilty and excluded until proved innocent!

Consider again the definition necessary for deriving our test-contradiction:

$$B\phi =_{\text{def}} \neg B\phi$$

This definition assumes that there can exist a concept encompassing a negation absolutely. Like assuming we have a concept of non-man, or of non-white. In traditional Aristotelian logic, we can only define a term on the basis of a pre-affirmed genus, from which we differentiate a species: e.g., from the genus “living beings” we may separate out “those who have λογος.” As modern mathematicians, we can imagine a negative variation on such positive definition: we define a “complementary class,” i.e. “those living beings who do not have λογος.” Yet even here we first focus on the positive and then exclude from the focus those failing to have an additional property.

In mathematics since Zermelo’s development of set theory, the Axiom of Separation has served to formalize the ritual by which a concept may prove itself to be a legitimate. Adapted, the Axiom reads:

$$\exists \psi \forall \chi \exists B \psi \chi \leftrightarrow (\exists \psi B \psi \chi \land \xi \chi)$$

The formula serves as a kind of defining machine, a Principle of Progeniture evident in one of its halves:

$$\exists \psi B \psi \chi \land \xi \chi \rightarrow \exists \phi B \phi \chi.$$  

That is, once we have a concept-in-function we can imagine any other function (ξχ: where ξ might be negative) to produce yet another concept within our system. The reverse half, notice, serves as a Principle of Paternity: once we prove that the issue is unique (that one and only one concept has been conceived), the Axiom assures us that it must satisfy the formula in reverse.

* For a perspicacious account of Russell's project, see F. P. Ramsey's *The Foundations of Mathematics* (London, 1931), especially the title essay (e.g., pp. 28 ff.). This essay also differentiates the paradoxes, classifying some as strictly mathematical or logical, and others as external or merely syntactic, i.e. containing “some reference to thought, language, or symbolism, which are not formal but empirical terms” (p. 20).
Appendix I: Axiomatic Deduction

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Axiomatic Deduction

The first volume of *Principia Mathematica* appeared in 1910, wrought chiefly by Bertrand Russell, with some editorial assistance from Alfred North Whitehead (who took direct responsibility for the subsequent two volumes). The task of this monumental work paralleled that of Isaac Newton's similarly entitled work of 1687: *Philosophiae Naturalis Principia Mathematica*, “natural philosophy's mathematical principles.” Russell proposed to derive from a handful of initially only logical formulas the entire body of mathematics, employing a very limited number of inference rules and supplying definitions as needed.

The motivation for this massive project stemmed from the recognition, by mathematicians, that certain styles of reasoning appeared to generate contradictory results. The hope was that, by starting with obviously consistent formulas and rules, and by proving theorems in ordered sequence, one could create a unified and complete system. After having shown in a 1930 article that such a project was indeed feasible for general logic, Kurt Gödel showed in a 1931 article that, for any finite set of primitives, valid mathematical formulas could be devised that could never be either proved or disproved from that set.

Russell's project ended by shifting attention from the proofs of theorems to the proofs of meta-theorems: theorems about axiomatic systems themselves. That is, the more astute readers of *Principia Mathematica* developed techniques for showing that a given system can never lead to a contradiction (consistency), can in fact lead to all the theorems within its purview (completeness), has been stripped of all redundancy (independence of primitive formulas and rules), and more. Indeed, what logicians now call “axiomatic deduction” soon passed out of the hands of Russell (who hoped to derive mathematics from logic) and into the hands of the formalists (who aspire to create for mathematics
§1. *Russell's sentential calculus*

We here allow for only two primitive symbols: the wedge and the curl, understood in the conventional manner. The symbol for implication we then define (D1):
\[(p \rightarrow q) = \text{def} \neg(p \vee q)\]
(Instead of the arrow, Russell employs the horseshoe: \(\supset\).) The symbol for conjunction we define (D2):
\[(p \& q) = \text{def} \neg(\neg p \vee \neg q)\]
(Instead of the ampersand, Russell employs the dot: \(\cdot\).) Finally, we define the symbol for the bi-conditional exactly as we do in natural deduction (D3):
\[(p \leftrightarrow q) = \text{def} [(p \rightarrow q) \& (q \rightarrow p)]\]
(Instead of the double arrow, Russell employs equivalence: \(\equiv\).)

Whenever there appears within a line an instance of one side of the definitional equation, we may generate a new line with the other side in its place. Russell notes:

... definitions are concerned solely with the symbols, not with what is symbolised; they are included for practical convenience, and are theoretically unnecessary.

In axiomatic deduction we make frequent use of such definitional transformation. In natural deduction, remember, we had to deduce the theorems traditionally called Implication and De Morgan's.

In place of the powerful Rule of Conditional Proof, axiomatic deduction makes use of primitive formulas. Russell adopts five such "primitive propositions" (as he calls them), each with a name of its own, as well as an identifying number (\(\ast 1.1\) is the rule of *modus ponens*):

1. \((p \vee p) \rightarrow p\) \hspace{1cm} \text{Principle of Tautology} \hspace{0.5cm} \ast 1.2
2. \((q \rightarrow (p \vee q))\) \hspace{1cm} \text{Principle of Addition} \hspace{0.5cm} \ast 1.3
3. \((p \vee q) \rightarrow (q \vee p)\) \hspace{1cm} \text{Principle of Permutation} \hspace{0.5cm} \ast 1.4
4. \((p \vee (q \vee r)) \rightarrow (q \vee (p \vee r))\) \hspace{1cm} \text{Associative Principle} \hspace{0.5cm} \ast 1.5
5. \((q \rightarrow r) \rightarrow ((p \vee q) \rightarrow (p \vee r))\) \hspace{1cm} \text{Principle of Summation} \hspace{0.5cm} \ast 1.6

It will be noticed that each of the five appears already definitionally transformed; for instance, the first reads, primitively formulated:
\[\neg(p \vee p) \vee p.\]

Only by applying rules from outside onto the formulas inside can we generate genuinely new lines (inferences: discounting definitional transformations). Instead of relying on the eight sentential ones of natural deduction, we now manage with only two: *modus ponens* and Substitution.

The Rule of Substitution requires some comment. Already in natural deduction we make use of a theorem such as \(p \rightarrow p\) to assert, e.g., \((p \& \neg p) \rightarrow (p \& \neg p)\) — perhaps to prove the principle of non-contradiction. We thereby substitute a complex formula, here \((p \& \neg p)\), for each instance of the simple variable. To justify such substitution of single variables with compound formulas, we simply recall the way we naturally derived the original theorem, and imagine repeating the procedure with the compound.

In axiomatic deduction, now, every line has the same status: each has been derived as assertoric (unlike lines within a conditional proof, whose modality is that of possibility). As a result, there is no way of imagining a re-run of a theorem with the compound instead of the single variable. Rather, we must admit, axiomatically, that we are making use of the Principle of Substitution. This is indeed a fundamental admission, as Russell points out:

... definitions are concerned solely with the symbols, not with what is symbolised; they are included for practical convenience, and are theoretically unnecessary.

... definitions are concerned solely with the symbols, not with what is symbolised; they are included for practical convenience, and are theoretically unnecessary.

But what counts as a legitimate substitution-instance? We today answer: any well formed formula (wff), defined (inductively, notice) as follows:

1. Any simple variable \((p, q, r, ...)\) is a wff.
2. Any two wffs connected by the wedge \((\vee)\) and surrounded by parentheses is a wff.
3. Any wff with a curl \((\neg)\) placed before it is a wff.
4. Only formulas generable as above are wffs.
Appendix I: Axiomatic Deduction

In addition, those formulas are wffs that meet these criteria when definitionally re-transformed.

Russell himself employs a system of dots in place of parentheses (his system appears to today as cumbersome). Whenever, they must be paired off, and no more than two variables or complexes should be included within any given pair. The computer program I use for generating formulas insists on parentheses being placed outside to embrace the entire formula on a line.

From each of the five primitive formulas we may derive a familiar theorem. The procedure is that employed by Russell. To each theorem he assigns a number, and to many a proper name.

6. \((\neg p \lor \neg p) \rightarrow \neg p\)  [1 pl\neg p  [for the original \(p\) substitute \(\neg p\)]
7. \((p \rightarrow \neg p) \rightarrow \neg p\)  [6 D1 reductio ad absurdum *2.01]
8. \((p \rightarrow \neg q) \rightarrow (\neg p \lor q))\)  [2 pl\neg p]
9. \((p \rightarrow q)\)  [8 D1 Simplification *2.02]
10. \((p \rightarrow \neg q) \rightarrow (q \lor \neg p))\)  [3 pl\neg p; q\lor\neg q]
11. \((p \rightarrow \neg q) \rightarrow (q \lor \neg p))\)  [10 D1]
12. \((p \rightarrow \neg q) \rightarrow (q \lor \neg p))\)  [11 D1 Transposition *2.03]
13. \((p \lor \neg q) \rightarrow (q \lor \neg p))\)  [4 pl\neg p; q\lor\neg q]
14. \((p \lor (q \rightarrow r)) \rightarrow (\neg q \lor (\neg p \lor r))\)  [13 D1]
15. \((p \lor (q \rightarrow r)) \rightarrow (\neg q \lor (\neg p \lor r))\)  [14 D1]
16. \((p \lor (q \rightarrow r)) \rightarrow (\neg q \lor (\neg p \lor r))\)  [15 D1]
17. \((p \lor (q \rightarrow r)) \rightarrow (q \lor \neg p))\)  [16 D1 Commutation *2.04]
18. \((q \lor r) \rightarrow ((\neg p \lor q) \rightarrow (\neg p \lor r))\)  [5 pl\neg p]
19. \((q \lor r) \rightarrow ((\neg p \lor q) \rightarrow (\neg p \lor r))\)  [18 D1]
20. \((q \lor r) \rightarrow ((\neg p \lor q) \rightarrow (\neg p \lor r))\)  [19 D1 Syllogism *2.05]
21. \(((q \lor r) \rightarrow ((\neg p \lor q) \rightarrow (\neg p \lor r))) \rightarrow (((p \rightarrow q) \rightarrow (p \rightarrow r)) \rightarrow (((q \lor r) \rightarrow (p \rightarrow q) \rightarrow (p \rightarrow r))))\)  [17 pl\neg p; q\lor\neg p; r\lor\neg p]
22. \(((q \lor r) \rightarrow ((q \lor r) \rightarrow (p \rightarrow r)))\)  [20,21 MP Syllogism *2.06]

These two versions of Hypothetical Syllogism recurrently serve, often indifferently, to enable the generation of theorems. The following is an entire list of the “propositions” (as Russell calls them) derived in *2 (Immediate Consequences of the Primitive Propositions):

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. ((p \rightarrow (p \lor p)))</td>
<td>Indempotence *2.07</td>
</tr>
<tr>
<td>26. ((p \rightarrow p))</td>
<td>[from 2]</td>
</tr>
<tr>
<td>27. ((\neg p \lor p))</td>
<td>Identity *2.08</td>
</tr>
<tr>
<td>29. ((p \lor \neg p))</td>
<td>[from 26]</td>
</tr>
<tr>
<td>31. ((p \rightarrow \neg p))</td>
<td>[from 29]</td>
</tr>
<tr>
<td>35. ((p \lor \neg p))</td>
<td>[from 5, 31, 29; lemma for next] *2.13</td>
</tr>
<tr>
<td>38. ((\neg p \rightarrow p))</td>
<td>[from 3, 35]</td>
</tr>
<tr>
<td>50. ((\neg p \rightarrow q) \rightarrow (\neg q \rightarrow p))</td>
<td>Double Negation *2.14</td>
</tr>
<tr>
<td>56. ((p \rightarrow q) \rightarrow (\neg q \rightarrow p))</td>
<td>[20, 31, 12, 20] Transposition *2.15</td>
</tr>
<tr>
<td>63. ((\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q))</td>
<td>[12, 20, 38, 22] Transposition *2.16</td>
</tr>
<tr>
<td>72. ((p \lor q) \rightarrow (p \lor q))</td>
<td>[20, 7, 2] Transposition *2.17</td>
</tr>
<tr>
<td>77. ((p \rightarrow (p \lor q)))</td>
<td>[2, 3, 22] Addition *2.2</td>
</tr>
<tr>
<td>79. ((\neg p \rightarrow (p \lor q)))</td>
<td>[2, 21] Addition *2.2</td>
</tr>
<tr>
<td>81. ((p \lor (p \rightarrow q)))</td>
<td>[2, 24] Addition *2.2</td>
</tr>
<tr>
<td>85. ((p \lor (p \rightarrow q)))</td>
<td>[2, 25] Addition *2.2</td>
</tr>
<tr>
<td>87. ((\neg p \lor (p \rightarrow q)))</td>
<td>[2, 26] Addition *2.2</td>
</tr>
<tr>
<td>88. ((p \lor (p \rightarrow q)))</td>
<td>modus ponens *2.27</td>
</tr>
<tr>
<td>91. ((p \lor (q \rightarrow r)) \rightarrow (p \lor (r \lor q))))</td>
<td>Association *2.3</td>
</tr>
<tr>
<td>99. ((p \lor (q \rightarrow r)) \rightarrow (p \lor (r \lor q))))</td>
<td>Association *2.31</td>
</tr>
<tr>
<td>108. ((p \lor (q \lor r)) \rightarrow (p \lor (q \lor r))))</td>
<td>Association *2.32</td>
</tr>
<tr>
<td>114. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>Summation *2.36</td>
</tr>
<tr>
<td>119. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>Summation *2.37</td>
</tr>
<tr>
<td>124. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>Summation *2.38</td>
</tr>
<tr>
<td>137. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,41]</td>
</tr>
<tr>
<td>140. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,42]</td>
</tr>
<tr>
<td>141. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,43]</td>
</tr>
<tr>
<td>143. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,44]</td>
</tr>
<tr>
<td>145. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,45]</td>
</tr>
<tr>
<td>149. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,46]</td>
</tr>
<tr>
<td>153. ((q \lor (q \rightarrow r)) \rightarrow (p \lor (q \rightarrow r))))</td>
<td>[2,47]</td>
</tr>
</tbody>
</table>

* James B. Gerrie's *Logical Theorist* 5.1. This program greatly enhances one's ability to generate formulas in consonance with the rigid requirements of logical syntax. The unforgiving nature of the program reminds us incessantly of the exactitude of syntax required in axiomatic deduction. Then, too, there are the conveniences regarding the insertion of brackets, the recording of justifications, the construction of tables, and more.
Russell’s Sentential Calculus

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>340. (((p ∨ q) → (p ∨ r)) → (p ∨ (q → r)))</td>
<td>De Morgan’s</td>
</tr>
<tr>
<td>344. ((p → (p ∨ r)) → (p → (q → r)))</td>
<td>De Morgan’s</td>
</tr>
</tbody>
</table>

Now on to *3 (The Logical Product of Two Propositions):

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>346. ((p &amp; q) → (~p ∨ ~q))</td>
<td>De Morgan’s</td>
</tr>
<tr>
<td>351. (~p → q) → (~p ∨ ~q)</td>
<td>De Morgan’s</td>
</tr>
<tr>
<td>353. ((~p ∨ ~q) → (~p &amp; q))</td>
<td>De Morgan’s</td>
</tr>
<tr>
<td>357. (p → (q → (p &amp; q)))</td>
<td>Conjunction</td>
</tr>
<tr>
<td>359. (~q → (p → (p &amp; q)))</td>
<td>Conjunction</td>
</tr>
<tr>
<td>367. ((p &amp; q) → (q &amp; p))</td>
<td>Commutation</td>
</tr>
<tr>
<td>368. (~p ∨ (~p → ~p)) → (p ∨ ~p)</td>
<td>353 q~p</td>
</tr>
<tr>
<td>369. (~p ∨ ~p)</td>
<td>29 pl/p</td>
</tr>
<tr>
<td>370. ~p &amp; ~p</td>
<td>Non-Contradiction</td>
</tr>
</tbody>
</table>

Russell remarks of this last theorem that “in spite of its fame we have found few occasions for its use.” Indeed, the principle of non-contradiction is ubiquitous only in natural deduction.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>378. (p &amp; q) → p</td>
<td>Simplification</td>
</tr>
<tr>
<td>382. (p &amp; q) → q</td>
<td>Simplification</td>
</tr>
<tr>
<td>399. (((p &amp; q) → r) → (p → (q → r)))</td>
<td>Exportation</td>
</tr>
<tr>
<td>411. ((p → (q → r)) → ((p &amp; q) → r))</td>
<td>Importation</td>
</tr>
<tr>
<td>413. (((p → q) &amp; (q → r)) → (p → r))</td>
<td>Syllogism</td>
</tr>
<tr>
<td>415. (((p &amp; q) → (p → q)) → (p → r))</td>
<td>Syllogism</td>
</tr>
<tr>
<td>417. ((p &amp; (p → q)) → q)</td>
<td>modus ponens</td>
</tr>
<tr>
<td>422. (((p → q) &amp; p) → q)</td>
<td>[not listed as a theorem] modus ponens</td>
</tr>
<tr>
<td>435. (((p &amp; q) → r) → (p &amp; ~r) → ~q))</td>
<td>modus tollens</td>
</tr>
<tr>
<td>438. ((p &amp; q) → (p → q))</td>
<td>3.4</td>
</tr>
<tr>
<td>440. (p → r) → ((p &amp; q) → r)</td>
<td>3.41</td>
</tr>
<tr>
<td>442. (q → r) → ((p &amp; q) → r)</td>
<td>3.42</td>
</tr>
<tr>
<td>451. (((p → q) &amp; (p → r)) → (p → (q &amp; r)))</td>
<td>Composition</td>
</tr>
<tr>
<td>474. (((p → q) &amp; (r → p)) → ((q &amp; r) → p))</td>
<td>3.44</td>
</tr>
</tbody>
</table>

This last expresses the Wedge-elimination basis to natural deduction.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>483. (p → q) → ((p &amp; r) → (q &amp; r))</td>
<td>Factor</td>
</tr>
</tbody>
</table>

Russell remarks: “This principle shows that we may multiply both sides of an implication by a common factor; hence it is called by Peano the ‘principle of factor.’ ... It is the analogue, for multiplication, of the
primitive proposition *1.6 [Summation].” The proofs of the next two theorems can be compacted as follows:

484. \(((p \to r) \& (q \to s)) \to (p \to r)\)  \hspace{1cm} \text{Simplification}
485. \((p \to r) \to ((p \& q) \to (r \& q))\)  \hspace{1cm} \text{Factor}
488. \(((p \to r) \& (q \to s)) \to ((p \& q) \to (r \& q))\)  \hspace{1cm} 484,485 H.S.
491. \(((p \& q) \to (r \& q)) \to ((p \& q) \to (q \& r))\)  \hspace{1cm} \text{Commutation \& H.S.}
494. \(((p \to r) \& (q \to s)) \to ((p \& q) \to (q \& r))\)  \hspace{1cm} 488,491 H.S.
495. \(((p \to r) \& (q \to s)) \to (q \to s)\)  \hspace{1cm} \text{Simplification}
496. \((q \to s) \to ((q \& r) \to (s \& r))\)  \hspace{1cm} \text{Factor}
502. \((q \to s) \to ((q \& r) \to (r \& s))\)  \hspace{1cm} \text{Commutation \& H.S.}
505. \(((p \to r) \& (q \to s)) \to ((q \& r) \to (r \& s))\)  \hspace{1cm} 495,502 H.S.
508. \(((p \to r) \& (q \to s)) \to ((p \& q) \to (r \& s))\)  \hspace{1cm} 494,505,322 \#3.47

Russell remarks: “This proposition, or rather its analogue for classes, was proved by Leibniz, and evidently pleased him, since he calls it *praecelarum theorema* [most beautiful theorem].” The final theorem in this section parallels the last: Constructive Dilemma.

509. \(((p \to r) \& (q \to s)) \to (p \to r)\)  \hspace{1cm} \text{Simplification}
510. \((p \to r) \to ((p \& q) \to (q \& r))\)  \hspace{1cm} \text{Summation}
513. \(((p \to r) \& (q \to s)) \to ((p \& q) \to (q \& r))\)  \hspace{1cm} H.S.
514. \(((p \to r) \& (q \to s)) \to (q \to s)\)  \hspace{1cm} \text{Simplification}
515. \((q \to s) \to ((q \& r) \to (r \& s))\)  \hspace{1cm} \text{Summation}
518. \(((p \to r) \& (q \to s)) \to ((q \& r) \to (r \& s))\)  \hspace{1cm} H.S.
521. \(((p \to r) \& (q \to s)) \to ((p \& q) \to (r \& s))\)  \hspace{1cm} 514,518, 322 \#3.48

At the opening of \#4 (Equivalence and Formal Rules), Russell reveals the way he understands what subsequent commentators have called his “logicism”:

Symbolic logic considered as a calculus has undoubtedly much interest on its own account; but in our opinion this aspect has hitherto been too much emphasized, at the expense of the aspect in which symbolic logic is merely the most elementary part of mathematics, and the logical prerequisite of all the rest.

In keeping with the project of *Principia Mathematica*, Russell more and more asks us to appreciate theorems as precursors of algebraic formulas — to wish “to assimilate symbolic logic as far as possible to ordinary algebra” (comment on \#4.71).

Also at the opening of this new section, Russell in effect introduces the Rule of Replacement (which henceforth he tacitly uses as his proofs become increasingly schematic):

It should be observed that, if \((p \leftrightarrow q, q)\) may be substituted for \(p\) without altering the truth-value of any function of \(p\) which involves no primitive ideas except those enumerated in \#1. This can be proved in each separate case, but not generally, because we have no means of specifying (with our apparatus of primitive ideas) that a function is one which can be built up out of these ideas alone.

\((p \leftrightarrow q) \to [f(p) \leftrightarrow f(q)]\).

Once we introduce the Principle of Mathematical Induction, we shall in fact be able to prove (inductively) the Rule of Replacement to hold throughout our system.

Of special interest in \#4 are the following theorems:

554. \(((p \& (q \lor r)) \leftrightarrow ((p \& q) \lor (p \& r)))\)  \hspace{1cm} \text{Distribution \#4.4}
583. \(((p \lor (q \& r)) \leftrightarrow ((p \lor q) \& (p \lor r)))\)  \hspace{1cm} \text{Distribution \#4.41}
593. \(((p \to q) \leftrightarrow (p \to (p \& q)))\)  \hspace{1cm} \text{Absorption \#4.7}
597. \(((p \to (q \& r)) \leftrightarrow ((p \to q) \& (p \to r)))\) \hspace{1cm} \text{Decomposition \#4.76}

\%(p \to q) \to (p \to (r \leftrightarrow (r \to q)))\) \hspace{1cm} [for proof of \#13.101 below] \#4.84
\%(p \leftrightarrow q) \to (p \leftrightarrow (r \to q))\) \hspace{1cm} [for proof of \#13.101 below] \#4.85

\$\S.2. Russell’s predicate calculus\$

One of the achievements of mathematical logic has been to show how we can reconstrue the edifice of Aristotelian logic (a logic of universals and particulars) as a superstructure built upon the edifice of Stoic logic. Russell calls this project his “Theory of Apparent Variables,” and at the opening of \#9 he formulates it:

The purpose and interest of this number are purely philosophical, namely to show how, by means of certain primitive propositions, we can deduce the theory of deduction for propositions containing apparent variables from the theory of deduction for elementary propositions.
The primitive propositions comprise two primitive formulas and one primitive rule:

598. ($\forall x (\phi \rightarrow \psi)$)  \textit{\textbf{$\forall$-Elimination \textasteriskcentered10.1}}

599. ($\forall x (p \lor \psi) \rightarrow (p \lor \forall x \psi)$)  \textit{\textbf{$\forall$-Mobility \textasteriskcentered10.12}}

(Russell employs ($x$) for $\forall x$.) The new primitive rule reads:

\[ \phi, \therefore \forall x \psi \]

\textit{\textbf{$\forall$-Introduction \textasteriskcentered10.11}}

In addition, there is one new definition:

\[ \exists x =_{\text{df}} \neg \forall x \neg \]

\textbf{D4 \textasteriskcentered10.01}

(Russell places $\exists x$ in parentheses; he also places a $\phi x$ at the end of each expression.)

To our inductively defined set of well-formed formulas we must then add:

any $\forall$ which is appended ($\forall$) and (an appropriate designation of variable to be governed) is a $\forall$; as is any formula then definitionally transformed by D4.

To display the workings of these new elements in the system, I reconstruct (in compacted form) the first few proofs:

600. ($\forall x \phi x \rightarrow \psi x$)  \textit{\textbf{$\forall$-Elim. $\phi x/\psi x$}}

603. ($((\forall x \phi x \rightarrow \phi x) \land (\forall x \psi x \rightarrow \psi x))$)  \textit{\textbf{$\forall$-Elim & Conjunction (357)}}

604. ($((\forall x \phi x \rightarrow \phi x) \land (\forall x \psi x \rightarrow \psi x)) \rightarrow ((\forall x \phi x \land \forall x \psi x) \rightarrow (\phi x \land \psi x)$)

\textit{\textbf{praeclarum theorema (508) $p\forall x \forall y$; $\forall x \phi x$; $\phi x \forall y$; $\forall x \forall y$; $\forall x \forall y$}}

605. ($((\forall x \phi x \land \forall x \psi x) \rightarrow (\phi x \land \psi x))$)  \textit{\textbf{modus ponens \textasteriskcentered10.14}}

Russell points out the difficulty into which this last theorem plunges us. If we in fact endorse two such propositions as “All primes are odd” and “All dogs are mammal” we might conjoint them to obtain the consequent of the formula. With $\forall$-Introduction we then obtain: “Given any $x$, if it is a prime it is odd and if it is a dog it is mammal.” Since no single $x$ will satisfy both antecedents, the result is vacuous. Russell remarks that, for $\textasteriskcentered10.14$ to be significant, the functions named by $\phi x$ and $\psi x$ must take arguments (instances) of the same type. The same domain, mathematicians say today; but there is a difference, as we shall have occasion to note.

Russell’s proof of $\textasteriskcentered10.2$ appears to be circular. So far as I can see, it requires line 608 as a lemma—in anticipation of $\textasteriskcentered10.21$. Line 608 is much used in the sequel.

606. ($\forall x (\neg p \lor \phi x) \rightarrow (\neg p \lor \forall x \phi x)$)  \textit{\textbf{$\forall$-Mobility $p\lor \neg p$}}

* The names are my own. Readers of Principia Mathematica will find $\phi$ and $\times$ much more diffuse than what I here present. For one thing, Russell offers two versions of the derivation of quantifier from sentential logic ($\phi$ posits many more primitives). For another, he intermingles the formal development with consideration of his “Theory of Types.”
Appendix I: Axiomatic Deduction

563. \((-p \to \forall x \neg \phi x) \to \forall x(-p \to \neg \phi x)\) 652 pl/\neg p; \phi \neg \phi x

566. \((\exists x \phi x \to p) \to \forall x(-p \to \neg \phi x)\)

567. \((\forall x(-p \to \neg \phi x) \to (-p \to \neg \phi x))\) \forall\text{-Elim.}

568. \((\exists x \phi x \to p) \to (\phi x \to p)\)

569. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

570. \((\neg x \to \forall x \neg \phi x) \to (-p \to \neg \phi x)\) H.S.

571. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

572. \((\forall x(\phi x \to p) \to (-p \to \neg \phi x))\) Transposition

573. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

574. \((\forall x(\phi x \to p) \to (\phi x \to p))\)

575. \((\exists x \phi x \to p) \to (\phi x \to p)\)

576. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

577. \((\exists x \phi x \to p) \to (\phi x \to p)\)

578. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

579. \((\forall x(\phi x \to p) \to (\phi x \to p))\)

580. \((\exists x \phi x \to p) \to (\phi x \to p)\)

581. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

582. \((\forall x(\phi x \to p) \to (\phi x \to p))\)

583. \((\exists x \phi x \to p) \to (\phi x \to p)\)

584. \((\forall x(-p \to \neg \phi x)) \to (-p \to \neg \phi x)\)

585. \((\forall x(\phi x \to p) \to (\phi x \to p))\)

586. \((\exists x \phi x \to p) \to (\phi x \to p)\)

Without the proofs:

692. \((\phi x \to \exists x \phi x)\)

695. \((\forall x \phi x \to \exists x \phi x)\)

704. \((\neg \exists x \phi x \leftrightarrow \forall x \neg \phi x)\)

729. \((\neg \exists x \phi x \leftrightarrow \forall x \neg \phi x)\)

742. \((\forall x(\phi x \to \psi x) \to (\forall x \phi x \to \forall x \psi x))\)

767. \((\forall x(\phi x \leftrightarrow \psi x) \to (\forall x \phi x \leftrightarrow \forall x \psi x))\)

785. \((\forall x(\phi x \to \psi x) \to (\exists x \phi x \to \exists x \psi x))\)

790. \((\forall x(\phi x \to \psi x) \to (\exists x \phi x \to \exists x \psi x))\)

796. \((\forall x(\phi x \to \psi x) \to (\exists x \phi x \to \exists x \psi x))\)

801. \((\forall x(\phi x \to \psi x) \to (\exists x \phi x \to \exists x \psi x))\)

816. \((\forall x(\phi x \to \psi x) \to (\exists x \phi x \to \exists x \psi x))\)

834. \(\neg x \to \neg \phi x\)

Russell’s Predicate Calculus

It is perhaps fitting to terminate this excursion in Russell’s *10 with his proof of the traditional Barbara, or rather a proof of its extensional rendition:

835. \(((\forall x(\phi x \to \psi x) \& \forall x(\psi x \to \xi x)) \to ((\psi x \to \psi x) \& (\psi x \to \xi x)))\)

836. \(((\forall x(\phi x \to \psi x) \& (\psi x \to \xi x)) \to (\phi x \to \xi x))\)

839. \(((\forall x(\phi x \to \psi x) \& (\psi x \to \xi x)) \to (\phi x \to \xi x))\)

842. \(((\forall x(\phi x \to \psi x) \& (\psi x \to \xi x)) \to (\forall x(\phi x \to \xi x)))\)

And, finally, one proof from Russell’s *11 (The Theory of Two Apparent Variables): a relational theorem. It will be noted that we need no new “primitive propositions” internal to our system.

843. \(((\forall y R x y) \to R x y)\)

844. \(((\forall y R x y) \to R x y)\)

845. \(((\forall y R x y) \to R x y)\)

846. \(((\exists y R x y) \to R x y)\)

848. \(((\exists y R x y) \to R x y)\)

849. \(((\exists y R x y) \to R x y)\)

\$3\$. Identity, hierarchy and reducibility

Rather than introducing new primitives to permit the deduction of theorems involving identity, Russell introduces what appears to be a notational definition:

\(x = y \equiv \forall \phi x(\phi x \to \phi y)\).

Russell explains: “This definition states that \(x\) and \(y\) are to be called identical when every predicative function satisfied by \(x\) is also satisfied by \(y\)” *(PM*, p. 168). Thus the student of *Principia Mathematica* must return to philosophical considerations in which the symbolism of “predicative function,” namely \(\phi x\), finds its meaning.

In our passion for performing deductions we easily forget the original philosophical motive, the specifically logical motif: the study of, the whole notion of, asserting. Some of what we say or hear (write or read) appears in the form of a claim, one stemming from and calling for a judgement. And a judgement is about something: it recalls an “object” and, saying something about it, either hits or misses it. Thus a judgement raises the question of truth. And although, “personally,” we may already have reasons convincing us that a judgement is true or that it is false — either that it hits upon or passes by how things are “empirically” (a
contingent assertion) or that it presents a formal articulation always true or always false “noetically” (logical truths, logical self-contradictions)—the judgement itself first appears in the modality of possibility (yet to be confirmed one way or the other, yet to be proved one way or another).

In strictly logical study we consider the minimal conditions for the possibility of assertion. These will be the conditions for the health of an assertion: what does not meet the conditions is not quite, or not at all, an assertion—and so cannot, in this logical guise, either hit or miss, cannot send us to anything, fails to be about something. And yet might continue to sound or to look as if it presented a claim that deserved confirmation, demonstration or refutation.*

Now, in modern logic, the first two minimal conditions are that there be a concept and an instance at issue in an assertion. A simple example (PM, pp. 14-15): in “x is hurt” we can distinguish the concept (call it \( \varphi(x) \)) and an instance (ambiguously, the \( x \), which we may then imagine as \( a \) or \( b \) or \( c \), these being names for individuals: Albert is hurt, Betty is hurt . . . ). In his effort to assimilate mathematics to logic, Russell calls \( \varphi \) a “propositional function” (a relation, which may be monadic, as in “\( x \) is hurt”; dyadic, as in “\( x \) hurts \( y \)”); triadic, as in “\( x \) hurts \( y \) with \( z \)” etc.). Such a function becomes an elementary proposition when it assumes a definite signification (or value): “this-here is hurt,” “Socrates is hurt,” “Meletus hurts Socrates,” etc. It becomes a generalized proposition when we assert that the function holds either “in all cases” or “in some case(s)”; our familiar quantifiers remove the variability of the variable, making it merely apparent, for what is now asserted is the all or the some, and neither a definite nor an ambiguous value.†

But where does a function obtain its values? Evidently, there must be a “collection” (a set) in which we dip to obtain our values. This collection must be available (in imagination) already:

* In a strictly philosophical study, we would raise the question about the conditions for language of any sort, in which assertion figures as a derivative variation. In Plato and Aristotle, Kant and Hegel, Heidegger and Wittgenstein we find philosophical considerations that in fact call into question the assumptions regarding the minimal conditions of strictly logical assertion.

† Russell calls the quantified versions “incomplete symbols, having no meaning except in use” (PM, p. 162): the “real” assertion takes the form of a function, i.e. a working (only imagined in our logical study).
theorems of identity to hold across the whole hierarchy of types. If, that is, “every propositional function is equivalent, for all its values, to some predicative function of the same argument or arguments” (PM, p. 166). This supposition (for ψ’s of one argument) Russell formulates as the Axiom of Reducibility:

\[ \exists \forall \varphi \exists z (\exists z \leftrightarrow \varphi !z) \]

*12.1

Consider, e.g., the function “x has all the predicates any pig has”:

\[ \forall y [\forall y (Py \rightarrow \psi y) \rightarrow \psi x] \]

or, with Bωy for ψ Belongs to y:

\[ \forall y [\forall y (Py \rightarrow B\psi y) \rightarrow B\psi x] \]

What the Axiom claims in this cases — where the ψ (the property) is introduced with reference to quantified expressions — is that there must be a collection (a class, a set) of all those properties that pigs have, and that therefore there is a strictly predicative function, namely the one describing that collection directly. The Axiom states that we can dispense with the quantification on ψ and proceed with a predicative function: “x is ϕ” — where ϕ now describes the class (collection, set) posited by the Axiom, and ϕx has the name “predicative function.”

And with this Axiom we can then proceed to prove our first theorem:

\[ \vdash (x = y) \rightarrow (\psi x \rightarrow \psi y) \]

*13.101

First to obtain an expression of Reducibility for x and y together:

1. \((\forall z (\psi z \leftrightarrow \varphi !z) \rightarrow (\psi z \leftrightarrow \varphi !x))\) *10.1 \(\forall\)-elimination
2. \((\forall z (\psi z \leftrightarrow \varphi !z) \rightarrow (\psi z \leftrightarrow \varphi !y))\) *10.1 \(\forall\)-elimination
3. \(((\forall z (\psi z \leftrightarrow \varphi !z) \rightarrow (\psi z \leftrightarrow \varphi !x)) \& ((\forall z (\psi z \leftrightarrow \varphi !z) \rightarrow (\psi z \leftrightarrow \varphi !y)))\) *3.21
4. \((\forall z (\psi z \leftrightarrow \varphi !z) \rightarrow (\psi z \leftrightarrow \varphi !x) \& (\psi z \leftrightarrow \varphi !y)))\) *3.43 Composition
5. \(\neg \neg (\forall z (\psi z \leftrightarrow \varphi !x) \& \neg \forall z (\psi z \leftrightarrow \varphi !y))\) *2.16 Transposition
6. \((\forall \forall (\psi z \leftrightarrow \varphi !x) \& (\psi z \leftrightarrow \varphi !y)) \rightarrow \neg (\forall \forall (\psi z \leftrightarrow \varphi !x) \& (\psi z \leftrightarrow \varphi !y))\) *10.1

Then to obtain the necessary series of identities:

13. \((\forall z (\varphi !z \rightarrow \varphi !x))\) *2.05 Syll.
14. \((\forall z (\varphi !z \rightarrow \varphi !y))\) *10.11 \(\forall\)-intro.
15. \((\forall \neg \forall (\varphi !z \rightarrow \varphi !x) \& (\varphi !z \rightarrow \varphi !y)))\) *10.21 QM
16. \((\neg \forall \neg \forall (\varphi !z \rightarrow \varphi !x) \rightarrow \neg \forall \neg \forall (\varphi !z \rightarrow \varphi !x))\) *2.16 Tr.
17. \((\exists \forall \forall (\varphi !z \rightarrow \varphi !x) \rightarrow \exists \forall \forall (\varphi !z \rightarrow \varphi !x) \& (\varphi !z \rightarrow \varphi !y)))\) *10.01 Def. of \(\exists\)
18. \((\exists \forall (\varphi !z \rightarrow \varphi !x) \& (\varphi !z \rightarrow \varphi !y)))\) *12.1 \(\text{modus ponens}\)

At this point in Principia Mathematica the proofs are becoming extremely sketchy if not sloppy: that of *13.101 is only five lines in length. With Dr. Gerrie’s Logical Theorist, I needed thirty-nine lines, employing line 17 above as a premiss.

Russell proceeds to sketch out proofs for other theorems, e.g.:

\[
(x = y) \leftrightarrow \forall \forall (\varphi !x \leftrightarrow \varphi !y)
\]

*13.11

\[(x = y) \leftrightarrow (\psi x \leftrightarrow \psi y)
\]

*13.12

\[(\psi x \& (x = y)) \rightarrow \psi y
\]

*13.13

\[(\psi x \& \neg \psi y) \rightarrow x \neq y
\]

*13.14

\[
(x = y) \leftrightarrow (y = x)
\]

*13.15

\[
((x = y) \& (y = z)) \rightarrow (x = z)
\]

*13.16

\[
((x = y) \& (y \neq z)) \rightarrow (x \neq z)
\]

*13.17

\[
\forall \exists (\exists (y = x)
\]

*13.18

\[
\exists \forall (y = x)
\]

*13.19

\[
\neg \forall (y = x)
\]

*13.191

\[
\exists \forall \exists (((x = b) \leftrightarrow (x = c)) \& (\psi c \leftrightarrow \psi b))
\]

*13.192

\[
\exists \forall (y = x) \& \forall (y = x)
\]

*13.195

\[
\exists \forall (((z = x) \& (w = y)) \& \varphi x w) \leftrightarrow \varphi x y
\]

*13.22

§4. Reducing the number of primitives

After the publication of Principia Mathematica, creative interest in axiomatic deduction became that of David Hilbert and his students (the formalists): to devise systems of deduction, reliant only on mechanical procedures, in order to assure the prevention of self-contradictions in advanced mathematical work.

A subsidiary interest, rather sportsman-like: to devise ways of slimming down the number of primitive symbols, primitive formulas, and primitive rules.
The number of primitive symbols is easily reduced to two: one for the sentential calculus (the Stoic "not both," or the Sheffer stroke): 

\[ \neg p \equiv \neg p \]

\[ p \lor q \equiv (p|p)\{q|q\} \]

and one for the predicate calculus (for universality). This reduction depends heavily on our willingness to understand other operators definitionally—a questionable willingness, as we shall see in the following section.

As for the number of primitive formulas: Paul Bernays soon showed, already in 1918, how one could derive Russell's fourth, the Associative Principle, from the first three and the fifth. Starting with \( r \rightarrow (p \lor r) \), with two applications of Summation and then Permutation, one can obtain (with repeated employment of H.S.):

\[ [p \lor (q \lor r)] \rightarrow \{(q \lor (p \lor r)) \lor p\}. \]

Obtaining \( p \rightarrow (p \lor r) \) by Addition and Permutation, and proceeding with Addition, Summation, and Tautology, one can then obtain:

\[ \{q \lor (p \lor r) \lor p\} \rightarrow q \lor (p \lor r). \]

And H.S. provides the desired result.

In the July 1953 issue of the short-lived Journal of Computing Systems, Carew A. Meredith showed how to derive Russell's four independent primitives from single ones, e.g. from

\[ \{(p \rightarrow q) \rightarrow (r \lor (s \lor r))\} \rightarrow \{(s \rightarrow p) \rightarrow (r \lor (t \lor p))\}. \]

With the help of Dr. Gerrie's Logical Theorist and following Meredith's Substitutions and applications of modus ponens, I obtained Addition (at line 38), Identity (63), Permutation (70), Summation (81), and Tautology (92). (See the derivation at the end of this appendix.)

For the predicate calculus we need only the shorter primitive formula, \( \forall \)-Elimination— if we employ DR-\( \forall \) in place of Russell's simpler Rule of \( \forall \)-Introduction. With the one primitive formula and this alternative rule for introducing the universal quantifier, we may obtain Russell's simpler version as a Derived Rule:

0. \( T \) any derived theorem, the shortest being \( p \rightarrow p \)
1. \( \forall \phi \) a formula derived within the system
2. \( T \rightarrow \forall \phi \) Russell's \( +2.02 \) (line 9) and MP
3. \( T \rightarrow \forall \forall \phi \) New \( \forall \)-Introduction
4. \( \forall \forall \phi \) \( 0 \), MP

And in the following way we may obtain the usable version of Russell's other primitive formula (the key lemma at line 608, with no free occurrence of \( x \) in the antecedent \( p \)):

1. \( \forall x(p \rightarrow \phi x) \rightarrow (p \rightarrow \phi x) \quad \forall \)-Elim.
2. \( \forall x(p \rightarrow \phi x) \rightarrow \phi x \quad \text{Importation} \)
3. \( \forall x(p \rightarrow \phi x) \rightarrow \forall \forall \phi x \quad \text{New } \forall \)-Introduction
4. \( \forall x(p \rightarrow \phi x) \rightarrow (p \rightarrow \forall \forall \phi x) \quad \text{Exportation} \)

Finally, with Implication, Double Negation, and the like, we could obtain the original version of Russell's \( \forall \)-Mobility.

Thus, aside from Substitution, we need only one primitive symbol, one primitive formula, and one primitive rule for each of the sentential and predicate calculi—and, following PM +13, one more primitive (the Axiom of Reducibility) and a definition formalizing Leibniz's Principle of the Identity of Indiscernibles:

\[ (x = y) =_{\text{def}} \forall \phi !x \rightarrow \phi !y. \]

Can we discard Substitution as a primitive rule? In 1927, J. von Neumann proposed that we posit not primitive formulas but primitive schemata: each of Russell's five formulas would then stand for the infinite number of formulas in that form. We suddenly have (or imagine having) an infinite number of primitives. However, to discover which one to use, we must proceed exactly as Russell advises: substitute into the original one! Practically, then, we have gained nothing. And theoretically we would be obscuring the basis of all logical work: the interplay of instance and form.

§5. Axiomatic vs. natural deduction

Axiomatic deduction proceeds from presumed primitive formulas, and relies on very few rules. In contrast, natural deduction, first developed nearly a quarter century after Russell’s work, presumes no such formulas, and relies on many rules. Because it proscribes Conditional Proof, axiomatic deduction requires much abstract ingenuity to discover substitution instances. Natural deduction, making incessant use of assumptions, requires that we imagine possibly true antecedents.

But to what are we directing ourselves during logical study? Since its beginnings in antiquity, logic has been a study in which we extrapolate rules of inference, rules allowing us to formalize inferences having content other than the language we ourselves are formalizing. That is,
logicians originally contemplated situations in which premisses are imagined as posited elsewhere, e.g. in geometry or in oratory, more recently in modern science. On this traditional understanding of logic, logicians have no formulas (premisses) of their own, only rules — along with philosophical ruminations on the significance of rule-based operations in themselves as well as in application.\(^*\)

By introducing primitive formulas in order to exfoliate a self-sufficient logical system, Russell hoped to do for logic what Peano had done for arithmetic — and to incorporate mathematics into logic.

Yet the opposite has in fact resulted. After Russell's *Principia Mathematica*, logic has in fact been assimilated to mathematical thinking. Once we focus on our own formulas, hoping thereby to achieve the kind of self-containedness so admirable in mathematical work, we have forsaken the traditional vocation of logical study and lose, by this marriage, our original family name.

The question of whether and how logical study can be self-contained rather than reference-based arises in close alliance with the question whether and how we logicians can dispense with naming domains for the inferences we study. On this question axiomatic and natural deduction again differ.

In any actual or imaginary application of natural inference, we begin with a premiss someone already takes to be true; e.g.: “There exists someone such that, given any second person, the first loves this second.”

\[\exists x \forall y (y \neq x \rightarrow Lxy)\]  
**Premiss**  
**Domain:** persons

It is natural, at least Stoically, to name what we take to be available “out there” (in this case, beyond the formula serving as a premiss, even if still within the mind’s work). As consultants we might remind our clients of what they already know, and then we might, as Gentzen did, propose a set of rules, by close adherence to which anyone may infer the familiar conclusion:

\[\forall y \exists x (y \neq x \rightarrow Lxy)\]  
**\(\varphi\)-assumption, \(\forall\)-elim., \(\exists\)-intro., \(\forall\)-intro, etc.

As consultants, we then withdraw, making note of the pattern: we record our ability to derive the latter formula from the former . . . for any domain — in case another customer solicits our help with a premiss in the same form. We may even record the pattern as a formula, and display it among other such formulas, to show our wares; these we then call theorems. But what we as logicians now have, actually, is a derived rule, one among many that we accumulate in the course of practicing our vocation. As philosophers, however, we contemplate the significance of such application. And one such significance is that someone is assuming the original premiss to be true, and that such an assumption is warranted only if it says something that can be checked out against a domain of instantiations.

Russell’s system, on the other hand, displays formulas first of all; derived rules consist only of sub-routines reliant on derived formulas (e.g., Syllogism, Conjunction, DR-\(\forall\)). Thus, when he derives \(*10.14, (\forall x \varphi x \& \forall y \psi y) \rightarrow (\varphi x \& \psi y)\), he must remark that, for the formula to be significant, the functions named by \(\varphi x\) and \(\psi y\) must take arguments (instances) of the same type. Type 0, we might say, comprises individuals that we can encounter sensorially (e.g., a pen, a chair, a person); Type 1 will then comprise predicates we apply to some one individual of the first type (its time, ... its species, its relation to other individuals); Type 2 will then comprise whatever predicates apply to Type 1 instances (a long time, a big place, a bright color, a quick action, a dyadic relation, a tautologous predication, a universal proposition). And so on: a hierarchy of types defined internally to the system. Thus a type differs from a domain, although the service rendered, and the examples easily proffered, run parallel to one another, and serve much the same purpose. The Theory of Types, for the exposition of which *Principia Mathematica* was largely written, is metaphysical in intent; vaguely metaphysical since, as Russell remarks:

\[\text{It is unnecessary, in practice, to know what objects belong to the lowest type, or even whether the lowest type of variable occurring in a given context is that of individuals or some other. For in practice only the relative types of variables are relevant; thus the lowest type occurring in a given context may}\]
be called that of individuals, so far as that context is concerned. (p. 161)

For example, if our formulas and deductions intend to interrelate numbers, we likely take real numbers as our Type 0, even though in another context we might desire to consider numbers not as things we talk about, but as things with which we talk about something else.

And on what criterion may we infer that two predications (functions) pertain to the same type? Russell offers the test:

... if there is any one argument \( a \) for which both “\( \phi a \)” and “\( \psi a \)” are significant, then the range of arguments for which “\( \phi a \)” is significant is the same as the range of arguments for which “\( \psi a \)” is significant. Hence, by our principle, the values of \( x \) for which “\( \phi x \)” is significant are the same as those for which \( \psi x \) is significant. (p. 95)

More simply: by “significant” we mean possibly true or false.

These considerations intend to confine legitimate substitution instances to totalities that do not contain formulas defining or commenting on the totalities themselves (PM, p. 161). We might indeed hear a proposition about the totality of what Cretans say, but this proposition cannot sense-fully be counted among the items within this totality: if a Cretan happens to say something about all Cretans, he has stepped out of his role as Cretan, or we take him out of that role, in order to stand on the sidelines with a possibly true, possibly false proposition about that totality (i.e., about the items within the totality).

In concrete examples such as that of the infamous Cretan, we could just as easily talk of domains. However, Russell's Theory of Types aspires, in Stoic fashion, to construct a hierarchy within the system. Properly understood, a domain contains items standing over against, outside the system: only in this way can we preserve the truth-functionality of our formulas and operations. Russell's efforts to trade hierarchy-on-the-inside for primitive over-against-ness serve well to prod us into an acknowledging this difference.

And one more difference.

Once we dispense with the necessity to name a domain, we can easily follow Russell's advice and introduce the majority of the needed logical constants by way of definition: Conjunction and Implication henceforth appear only as truth-functional re-arrangements of Disjunction and Negation, Existence appears as a negation of Universality, and Identity appears (definitionally) as Indiscernibility.

In contrast to Russell's axiomatic, Gentzen's natural deduction defines each of the logical constants operationally: one for introducing each, and one for making use of it. Here we must prove what Russell defines: Implication, \((\neg p \lor q) \rightarrow (p \rightarrow q)\), De Morgan's Theorem, \((\neg p \land \neg q) \rightarrow (p \land q)\), and Existence Discovery, \(\neg \forall x \neg \phi x \rightarrow \exists x \phi x\). As consultants, we will receive premisses in the form of each of the antecedents, and we should be able to derive, by honest work rather than by definition, conclusions in the form of the consequents.

But the question is not one of industry vs. sloth. Rather, it is a question of legitimate vs. illegitimate conceptions of proof. Or at least one of acknowledging different conceptions of inference.

In natural deduction, we derive each of those formulas by making crucial use of Negation-Elimination: by assuming the negation of the consequent (for De Morgan's, the negation of each conjunct), deriving a contradiction, and then undoing the negation of what was assumed. In all honesty, we should inform our client that the effort to dispense with the consequent (i.e., the desired conclusion) destroys the integrity of the system in which the antecedent (i.e., the premiss) has been accepted as a member. What we can indeed prove, by Negation-Introduction, is the negation of a negation: \(\neg \neg C\). We would mislead our client if we claimed to have delivered C itself: we have not derived it, for we have not constructed it. Our advice might read: “You cannot get along without C”—just as nutritionists might advise us that the human organism cannot get along without a certain substance (vitamin, mineral) while they remain in the dark how the substance in fact serves the vitality of the organism.

Thus we may locate the difference in the question whether we should accept the one version of Double Negation: \(\neg \neg p \rightarrow p\).

We may also locate it in the formula for Excluded Middle: \(p \lor \neg p\). This Russell derives immediately, by definition, from Identity. Considered truth-functionally, \(p \lor \neg p\) can never be proved false. However, to prove it true syntactically, without relying on definition, we must employ Negation-Elimination. And this employment might just beg the question. We can ask, as contemplative logicians and mathematicians
from Aristotle to Brouwer have asked, what it can mean, in a consultative context, to know any instance of the formula.

The question heats up when we consider quantified formulas. For instance, from Identity and Russell's definition of the Existential Quantifier, we may easily "prove" that the negation of a universal leads to the assertion of an existence: \( \neg \forall x \phi x \rightarrow \exists \phi x \). In Gentzen's natural deduction we obtain the consequent from the antecedent only by invoking Negation-Elimination. As consultants, we should inform our client that we have not actually discovered the existence; we have only shown that, once ostracizing \( \forall x \phi x \) from our system (on the grounds of a contradiction) we must also deny that the existence can be denied—on pain of engendering yet again a contradiction. Existence must be discovered, not created by fiat: in logical work, we must construct existence out of the premisses given and the theorems already derived.

This last consideration we can raise again in reference to the quantified version of Excluded Middle: \( \forall x \neg \phi x \lor \exists \phi x \). Again, we can obtain this formula axiomatically, i.e. definitionally, from the sentential theorem \((\neg p \rightarrow q) \rightarrow (p \lor q)\). But consider what it can mean: either All (are not) or There exists (at least one that is). Both claims are rather powerful. Not having established either one, how can we claim to know them even disjunctively? To be sure, the knowledge here in question is a relatively uncommitted one: either-or. Yet can we rightly rise above the question of universality and existence, and proclaim a knowledge of what we see below? The formula simply draws up an agenda of considerations. In consultative practice, we might proceed to show that one or another of the disjuncts must be expelled from our system: we are then left with one more item on the agenda. To employ Disjunctive Syllogism, the proof of which invokes Negation-Elimination, to settle the second item of consideration appears to shortcut the whole affair of responsible inference.

The question is finally whether definitions can be creative, as against being merely notational. Our definition of bi-conditionality is clearly a notational affair: instead of writing out each of the implications, we tie them together in the double-arrow.

The fact that the formulas we have been considering require (in natural deduction) proofs employing Negation-Elimination suggests that the passage from the premiss to the conclusion introduces knowledge where there was none: they would be creative in a vicious sense.

Gentzen proposed his system of natural deduction partly to isolate the employment of Negation-Elimination, as Brouwer and his students (the intuitionists) insisted was necessary to retain inference as an affair of knowledge. It is then possible to remain with the not-able-to-deny version of theorems otherwise affirmative in form.

Still, it is possible, and helpful, to organize even the limited version of natural deduction into axiomatic form. The following set of primitive formulas covers all the rules of (sentential) natural deduction, including RCP, while excluding Negation-Elimination:

1. \( (p \rightarrow (q \rightarrow p)) \) 
2. \( ((p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))) \) for RCP
3. \( (p \& q) \rightarrow p \) 
4. \( (p \& q) \rightarrow q \) 
5. \( (p \rightarrow (q \rightarrow (p \& q))) \) 
6. \( (p \rightarrow (p \lor q)) \) 
7. \( (q \rightarrow (p \lor q)) \) 
8. \( (p \rightarrow r) \rightarrow ((q \rightarrow r) \rightarrow ((p \lor q) \rightarrow r)) \) Positive Dilemma
9. \( (p \rightarrow q) \rightarrow ((p \rightarrow \neg q) \rightarrow \neg p) \) RAA
10. \( (\neg p \rightarrow (p \rightarrow q)) \)

We could now pause to derive a number of sentential theorems:

15. \( (p \rightarrow p) \) 
16. \( \neg (p \& \neg p) \) 
17. \( (p \rightarrow (q \& \neg q)) \rightarrow \neg p \) 
18. \( (p \rightarrow (q \rightarrow r)) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q)) \) Positive Dilemma
19. \( (p \rightarrow q) \rightarrow ((p \rightarrow q) \rightarrow \neg p) \) RAA
20. \( (p \rightarrow (p \rightarrow q)) \rightarrow q \) 
21. \( (p \rightarrow (p \rightarrow q)) \rightarrow (p \rightarrow (p \rightarrow q)) \)

We could derive many more theorems. But it is possible to show (inductively) that three formulas may never be derived: Excluded Middle, the one Double Negation, and \( \neg \)-Elimination.

§6. The truth of axiomatization: pure reason

Originally, pure reason (\( \nu \nu \theta \)) appeared not as a human faculty (power). It named divine actuality—or, from the human standpoint,
Appendix I: Axiomatic Deduction

Also, "love of self" has two meanings: one perverse, namely enclosure within strictly human interests, and one sublime, namely release into the divine in nature herself (the topic of Book Nine, Chapter 8, of Aristotle's work).

Thus our own rational nature, our own destiny to reason, appeared fraught with an essential ambiguity turning on the word "own": on just who, or what, "has" reason — or, on "where" reason has its proper location. Whatever the difficulties this ambiguity posed to Plato and Aristotle and their progeny, its effect on our own condition seemed clear to them: our ultimate task is to become more than human—to become as divine as possible. Yet the paradox of the ambiguity remained: we become our true selves by becoming divine—not in ourselves but precisely, or rather strangely and vaguely, out of ourselves.

Thus in Aristotle's *Metaphysics* we find an essential duality in the name "pure reason." On the one hand, such reason is indeed human (1075 a 7); it is "of the soul" (993 b 11)—so that difficulties may be not in matters themselves but in ourselves: "for as the eyes of bats to the blaze of day, so is the soul's pure reason to the things that are by nature most illuminated of all." On the other hand, it (νοῦς) names a "nature" that is "in" whatever is; it names "what gets things moving" (1025 b 22), what is "unmoving" in itself, "always" at work, and "apart" from things as we initially deal with them (1064 a 29-b 3). Thus our contemplation "of" (i.e., within) pure reason receives the name "theology" to keep it distinct from two other names, neither of which corresponds very much to their modern homonyms: "physics" (the contemplation of things having their own origin of motion) and "mathematics" (which contemplates things "unmoving" but not "apart").

And in his *Nicomachean Ethics*, Book Ten, Chapter 7, we find an account of the consequences of this understanding of pure reason for understanding the best life, that of contemplation: its actualization is "like" the activity of a god (1178 b 25).*

The very early modern thinkers retain some vestiges of the ancient understanding of pure reason, if only in their insistence on an increasingly distant Source of the Order at issue in intellectual work. Clearly, though, the modern project consists centrally in the enactment of reason as human. Reason is "ours"—even if as a "gift," since the Giver has now withdrawn. In contemplation, there are still two tasks: one is to extract reason in its purity from its mixture in worldly dealings, and the other is to adjust it precisely to its complement in the mixture. Neither of these tasks permits us to understand reason as itself divine (except, of course, in metaphorical or hyperbolic defense of the project of rationalism). The extraction of pure reason reveals precisely ourselves in our "human nature." Pure reason (νοῦς, Vernunft) locates the pure self—what Kant calls the *transcendental self.*

The axiomatization of logical calculi mimes, in a strikingly appropriate manner, the modern understanding of the activity of pure reason. Natural deduction follows the model of reasoning as mixed in with tasks other than itself: it shows reasoning at work with, and in service to, what's "outside"—with the "materials" provided by other disciplines. In this regard, the natural deduction developed by Gentzen and others, subsequent to Russell's axiomatic deduction, differs from the "natural deduction" developed by Aristotle "only" (yet very importantly!) inasmuch as the "natural" complement of the one is mathematical science and that of the other is public speaking (oratory) and intimate pedagogy (dialectic).

At the outset of axiomatic formalization, we discern (and then posit) exactly two rules, each of which says something basic about pure reason. First, substitution mimes the basic principle, the very origin of pure reason as "intuition": the interplay of form and instance, each "side" in the play having its sense only in relation to the other. Second, *modus ponens* mimes the same basic principle, the same origin of pure reason, now as "analysis": the detachment of part of a formulation (a λογος), given its complementary part (formally, the detachment of the "consequent" once the "antecedent" is given).

Moreover, the two rules are mutually dependent: substitution works on the assumption that any one form implies any other that is an instance of the first, and *modus ponens* works on the assumption that the basic

* Michel Foucault's *Les mots et les choses* (1966) reviews in great detail the gradual evolution of the notion of "man," of "human nature" as the (virtual) focal point of intellectual investigation into life, labor and language, which then, together with the mathematical sciences on the one side and Kantian philosophy on the other (the "analytic of finitude"), gave birth to such "human sciences" as psychology, sociology, and literary studies. This evolution depends entirely on more radical transformations in the understanding of rationality, most centrally that of pure reason.

*Also, "love of self" has two meanings: one perverse, namely enclosure within strictly human interests, and one sublime, namely release into the divine in nature herself (the topic of Book Nine, Chapter 8, of Aristotle's work).
form of the rule \((p \rightarrow q, p :. q)\) works on any substitution instance. All by itself, each rule is like the sound of the clap of one hand.

Now, these two rules may reflect reason in its purity, but they still have no power to generate any formulations all by themselves—no power to mime the movement—the getting things moving—essential to reason, pure or mixed. Understood as endowed with these two rules, reason passively awaits “materials” for its actualization. By itself, the form-instance interplay only allows us to discern analogies: the hierarchy of the family is like that of the city, the structure of an atom is like that of a planetary system, the physiology of a pig is like that of a human being. By itself, modus ponens only allows us to take apart conditional formulations somehow handed over to us from the “outside.” And in both cases, reason is tempted to lose its purity.

Thus the need for—the role of—what Russell calls “primitive propositions.” These, too, mimic pure reason: their truth lies in both pure intuition (as forms of instances) and pure analysis (detachment of one part, given the other part). At the basis, namely in the sentential calculus, the truth of the axioms may be laid out for inspection in a “truth table”—given the intuitive principles of Identity (of variables and operators in the forms), Non-contradiction and Excluded Middle (for the “possible worlds”).

And now we—the transcendental self—may generate each new propositional form; indeed, with the proper choice of axioms (Russell’s will do), we can generate all the sentential and predicate forms we could ever consistently hope for (as can be proved inductively). That this generation is immaculate is clear from the fact that there is no recourse to anything “outside”: reasoning here is a strictly in-house affair. That it is also what Kant calls “synthetic” is evident in this: its progeny depends in every case upon an interaction of rule (“above”) with a form (“within”: whether primitive or derived). The “primitive propositions” themselves allow for no movement of their own: they are not already pregnant, but require constant re-insemination, always from the same source, namely the rules “above” that only “we” can “apply”—i.e., that only pure reason enacts.

Thus axiomatic deduction provides us with a perfect formal imitation, a well-constructed mime, of pure reason, with results clearly visible on paper or on screen. Bertrand Russell had hoped to inscribe all of mathematics within this construction, and David Hilbert had hoped the mime could parallel the unfolding of mathematics and thereby provide proofs of completeness and consistency. As is well known, within twenty years of the inception of such hopes Kurt Gödel proved the impossibility of such all-inclusive self-containedness. Yet his proofs only insist upon a multiplicity of self-contained systems; they in no way attenuate the basic modern understanding of pure reason itself. Axiomatic deduction continues to incarnate the basic structures of pure reason understood transcendentally (“humanly”) rather than transcendentally (“divinely”).
Appendix I: Axiomatic Deduction

A Curiosity

Meredith's derivation of Russell's four sentential primitives (§4) employs Polish notation and proceeds compactly, not accounting for the need for definitional transformation. I here translate it into Gentzen's notation and condense it as Meredith did. The following substitutions suffice in each instance for generating the subsequent line with *modus ponens* and D1:

M1  \[((p \to q) \to (r \vee (s \to t))) \to ((s \to p) \to (r \vee (t \to p)))\]
    \[1 p((p \to q); \quad q((r \vee (s \to t)); \quad rl\sim(s \to p); \quad slt; \quad tl(t \to p)\]

M2  \[((r \to (p \to q)) \to (~s \to (p \vee ((t \to p) \vee (p \to q))))\]
    \[1 p|t; \quad q(p \to q); \quad rl\sim(s \to p); \quad slt; \quad tl(t \to p)\]

M3  \[((t \vee p) \to r) \to (~s \to (p \vee ((p \to q) \vee r)))\]
    \[1 p((t \to p \vee p); \quad q|r; \quad rl\sim(s \to p); \quad slt; \quad tl\]

M4  \[((p \to q) \to (t \vee p)) \to ((s \to p) \to (r \vee (t \vee p)))\]
    \[4 q|q(p \vee p); \quad tlq; \quad rl\sim\]

M5  \[((p \to (q \to p)) \to (q \to p)) \to ((s \to p) \to (p \to (q \to p)))\]
    \[4 p((p \to (q \to p)); \quad q(q \vee p); \quad tl\sim(p \to s); \quad sli\]

M6  \(((t \to (p \to (q \vee p))) \to (r \vee ((s \to p) \to (p \to (q \vee p))))\]
    \[4 p((s \to p) \to (p \to (q \vee p))); \quad q(p \to (q \vee p)); \quad tl\]
    \[\quad sl((p \to (q \to q)) \to (q \to q \to (p \to (q \to p))))\]
    \[\quad tl\]

M7  \((t \vee (r \vee ((s \to p) \to (p \to (q \to p))))\]
    \[7 tl\sim(t \vee (r \vee ((s \to p) \to (p \to (q \to p))))\]
    \[\quad rl\sim(t \vee (r \vee ((s \to p) \to (p \to (q \to p))))\]

M8  \(((s \to p) \to (p \to (q \to p)))\]
    \[4 p|q(q \to p); \quad q|p; \quad tl\sim\]
    \[8 s|q(q \to p)\]

M9  \(((s \to (q \to p)) \to (r \vee (p \to (q \vee p))))\]
    \[4 p(p \to (q \vee p)); \quad q|lq \vee p); \quad tl\]
    \[\quad sl(s \to p); \quad rl\]

M10 \((t \vee (r \vee (p \to (q \vee p))))\]
    \[10 tl\sim(t \vee (r \vee (p \to (q \vee p)))); \quad rl\sim(t \vee (r \vee (p \to (q \vee p))))\]

M11 \((p \to (q \vee p))\]
    \[Addition\]
    \[4 p((p \to q) \to p); \quad q|q(p); \quad tl\sim(s \to p); \quad sli\]
    \[3 tl(p \to q); \quad rl\]

M12 \(((t \to ((p \to q) \to (p \to q)) \to (r \vee ((s \to p) \to (p \to q))))\]
    \[12 tl\sim((s \to p) \to (p \to q)); \quad rl\sim((s \to p) \to (p \to q)); \quad slt\]
    \[11 q(p \to q)\]

M13 \(((s \to p) \to ((p \to q) \to p))\]
    \[4 q|p; \quad tl\]
    \[13 s|p\]

M14 \(((s \to p) \to (r \vee ((p \to q) \to p)))\]
    \[1 p|s; \quad q|p; \quad s|p \to q); \quad tl\]

M15 \(((p \to q) \to (s \to ((p \to q) \to p)))\]
    \[1 p(p \to q); \quad qls; \quad s|p; \quad ts\]

M16 \(((p \to (p \to q)) \to (r \vee ((s \to p) \to (p \to q))))\]
    \[16 q|q(p); \quad rl\sim((p \to q) \to p); \quad rl\sim(p \to q))\]
    \[11 q|p\]

M17 \((s \vee (p \to p))\]
    \[17 sl\sim(s \vee (p \to p))\]

M18 \((p \to p)\]
    \[Identity\]
    \[1 rl\sim(p \vee p); \quad slp; \quad tlq\]
    \[17 s\sim(p \to q); \quad pl(p \vee q)\]

M19 \(((p \to q) \to (q \to q))\]
    \[Permutation\]
    \[11 p\sim((p \to q) \to (q \to q)); \quad qlr\]

M20 \((r \vee ((p \to q) \to (q \to p)))\]
    \[1 p|r; \quad rl\sim(p \to q); \quad sl|q; \quad tl\]
    \[20 rl\sim(r \to q)\]

M21 \(((q \to r) \to ((p \to q) \to (p \to r)))\]
    \[Summation\]
    \[1 q|q; \quad rl\sim(p \to q); \quad slp; \quad tl\]
    \[21 q|p\]

M22 \(((p \to p) \to (r \to p))\]
    \[22 rl\sim(p \to p)\]

M23 \(((p \to p) \to (p \to p))\]
    \[22 rl\sim(p \to p)\]
    \[Tautology\]
Appendix II: Axiomatic Induction

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Axiomatic Induction

Mathematical induction is a form of inference regarding properties occurring at successive points in a series; it allows us to conclude something about what happens at any point. The series here must run parallel with the ordered sequence of positive integers; even if the series consists of what happens every 3½ hours, the first point after 0 will be the ordinal 1, marking the cardinality 3½, the next will be 2, marking the cardinality of 7, and so on.

The inference itself is logical inasmuch as it enacts a rule regarding predication of some property. It is mathematical inasmuch as the subject of the predication is formulated as an ordinal number $n$ marking some point in the series.

Thus, instead of our usual logical formulation $\phi x$, we now begin to talk about $\phi n$: $\phi$ “happens” at point $n$. Yet we soon discover that the analogy falters. Formulations describing what happens in the series will more naturally take the form of a function: on the $n$th day of Christmas my true love will give me a number of turtle doves very possibly determined in reference to the cardinality $n$ — in any case, by a polyadic predication.

§1. The first version of RMI

In its simplest form, the Rule of Mathematical Induction reads:

$\phi_b$ Base Case (determined manually)

$\forall n[\phi n \rightarrow \phi(n + 1)]$ Induction Step (proved by RCP)

$\therefore \forall n(b \leq n) \rightarrow \phi n$ RMI

Imagine the ordered sequence of points such that the Quantity at each point $n$ is doubled at point $n + 1$. A natural exemplification of this
schema could be this: the population of a species doubles every so many hours or years. A playful exemplification could be: King Karl agrees to give Beggar Bob one grain of wheat on January 1st and on each successive day double the number of grains given on the previous day. Taking the Quantity at the first point as the unit of measurement, we can easily calculate the magnitudes (populations, gifts) for the first few points in the series:

<table>
<thead>
<tr>
<th>Point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
<td>256</td>
</tr>
</tbody>
</table>

But, let us say, we would like to have a formula by which we could calculate the Quantity arising at any arbitrarily chosen point, without having to calculate the entire sequence leading to it.

Examining the chart already constructed, we notice that the Quantities are all powers of 2. Glancing at the cardinal number directly above each Quantity, we notice that we need only subtract 1 from each (e.g., 6 less 1) and the result (here: 5), taken as the power of 2, yields the Quantity at that point. Empirically, the same pattern works for all nine cases: \( \forall n[Q_n = 2^{n-1}] \)

But how can we rest assured that the empirically justified formula holds beyond the empirical construction—out into an infinity of points?

The pattern detected \textit{a posteriori} has the status of an “hypothesis”—a guess. Yet we can show that if the guess works at any arbitrarily chosen point \( n \) it works at point \( n + 1 \) as well. The proof requires that we explicitly formulate the “game rule” as a premiss:

1. \( \forall n \forall N(Q_n = N) \rightarrow (Q_{n+1} = 2N) \) Premiss (game rule)
2. \( Q_1 = 2^{1-1} \) Assume this \( N \) at some arbitrary \( n \)
3. \( (2) \rightarrow (Q_{n+1} = 2 \times 2^{n-1}) \) 1 \( \forall \)-elims. with \( n \) and \( N \) of line 2
4. \( Q_{n+1} = 2 \times 2^{n-1} \) 2,3 MP
5. \( Q_{n+1} = 2^{n+1-1} \) 4 definition of powers
6. \( (2) \rightarrow (Q_{n+1} = 2^{n+1-1}) \) 2—5 RCP
7. \( \forall n[Q_n = 2^{n-1}] \rightarrow (Q_{n+1} = 2^{n+1-1}) \) 6 \( \forall \)-intro.

Having now the Base Case (nine of them, in fact) and the Induction Step, we may apply RMI to conclude that at any point \( n \) the Quantity will be equal to \( 2^{n-1} \).

Let us consider carefully how the inference works. First, we decide on the series at issue: for convenience, the days of the calendar (in the case of population growth the intervals between each point may have been anything, e.g. 3 hours, 27 minutes, and 6 seconds). Then we note a game rule describing what happens any one point to the next: the game rule here allows for any Quantity, so that if Beggar Bob had received \( 3\frac{1}{4} \) grains at some point, he would have received \( 6\frac{1}{2} \) at the next point. We then examine the slow manual development of the events at each point and notice a pattern: we now posit this pattern as an hypothesis in order to \textit{deduce} from the game rule, with our rules of natural deduction \textit{plus} an understanding of the arithmetical notion of power, that, if our guess works at one point it works at the next as well. Taking our manual calculation at point 9 as the base case and the last line of the proof above, we could construct an argument:

1. \( \varphi_9 \) manually determined \( (2^9 - 1 = 256) \)
2. \( \forall n[\varphi_n \rightarrow \varphi_{n+1}] \) Induction Step \( (\varphi = 2^n - 1) \)
3. \( \varphi_9 \rightarrow \varphi_{10} \) 2 \( \forall \)-elimination
4. \( \varphi_{10} \) 1.5 \textit{modus ponens}

Again, we could instantiate the second premiss (the Induction Step) with \( n = 10 \) to obtain \( \varphi_{11} \). And so on, recurring each time to line 2, with no other limit than that of our own time, energy, or desire. Surveying this possible process of instantiation and \textit{modus ponens}, we intuit that \( \varphi_n \) will hold no matter where we land. RMI sums up the intuition of this process: we \textit{infer}, according to a rule that we have formulated and that corresponds to our intuition, \( \forall n \varphi_n \).

It was apparently Augustus De Morgan who, as late in mathematical history as 1838, first enunciated the principle as a principle:

There is ... one particular method of proceeding which is extremely common in mathematical reasoning, and to which we propose to give the name \textit{successive induction}. It has the main character of induction in physics, because it is really the collection of a general truth from a demonstration which implies the examination of every particular case; but it differs from the process of physics inasmuch as each case depends on one which precedes.*

One of De Morgan’s examples reads: “The formula \( x^n - a^n \), \( n \) being a whole number, is always algebraically divisible by \( x - a \).” Changing the

---

* \textit{Penny Cyclopaedia of the Society for the Diffusion of Useful Knowledge} (1838), pp. 465-6. In his \textit{Formal Logic} of 1848 (Chapter XI), he reiterates the importance of induction “in every work which professes to treat of the matter of arguments,” and further exemplifies it.
variables into $\forall n \forall a \forall b \exists x[(a^n - b^n) = x(a - b)]$, we may formalize the proof as follows:

1. $\exists x[(a^n - b^n) = x(a - b)]$ Assumption
2. $(a^n - b^n) = m(a - b)$ $\exists$-elim.
3. $(a^{n+1} - b^{n+1}) = [(a \times a^n) - (b \times b^n)]$ Definition of powers
4. $= [(a \times a^n) - (a \times b^n) + (a \times b^n) - (b \times b^n)]$ Sub/Add
5. $= [a(a^n - b^n) + b^n(a - b)]$ Factoring
6. $= [a[m(a - b)] + b^n(a - b)]$ Letting $y = [(a \times m) + b^n]$
7. $= [(a \times m) + b^n] \times (a - b)]$ $\exists$-intro.
8. $y(a - b)$ Factoring
9. $\exists y[(a^{n+1} - b^{n+1}) = y(a - b)]$ $\exists$-intro.
10. $(1) \rightarrow (9)$ RCP
11. $\forall n \forall a \forall b \exists x[(a^n - b^n) = x(a - b)] \rightarrow \exists y[(a^{n+1} - b^{n+1}) = y(a - b))]$ $\forall$-intr.
12. $\exists x[(a^n - b^n) = x(a - b)]$ By manual calculation: $x = 1$
13. $\forall n \forall a \forall b \exists x[(a^n - b^n) = x(a - b)]$ 12,11 RMI

The purpose, of course, is to show that $x$ will always be a whole number, a qualification evident in the assumptions we make in the abbreviation introduced to obtain line 8: if $a$, $b$, and $n$ are whole numbers, and if multiplying, powering, and adding with whole numbers yields a whole number, then “wholeness” is bequeathed from point to point in the series.

This last proof differs from the earlier one in several respects. For one, it has no premiss, no game rule stated at the outset and drawn upon in the course of the proof. For another, the predicate $\varphi$ proved to apply at each point (viz. “$\varphi$ is cleanly divisible by $a - b^n$) remains the same (even though the $x$ proving such clean divisibility varies from point to point). The $\varphi$ is then applied to a Quantity generated at each point: it is a predicate of a predicate.

So far, the examples of induction have all been arithmetic: questions of cardinality from one point to the next along the ordinality of the series of positive integers. Kant and others have understood such examples as engaging us in the intuition of time. For an example engaging us in the intuition of space as well, we may consider an induction analogous to one proposed by Pascal, nearly two hundred years before De Morgan.

Let us construct a pyramid by following two “game rules”: (1) positing at the top a “generating” block containing the cardinal 1, we construct successive lower levels of blocks by staggering each new level by half a block both left and right (allowing for one more block than on the previous level) and (2) we assign to each new block a cardinal equal to the sum of the staggered blocks directly above it. Thus:

Now, two patterns immediately leap to the eye: every outside block of the pyramid has the cardinality of the generating block at the top, and the blocks contiguous with these outside blocks exhibit the progression of the natural-number series as our eyes move downward. Let us prove axiomatically these first two as theorems.

**Theorem 1:** Given any $r$ (= row), the cardinality of the left-most block will be 1 (or: identical to that of the generating block).

**Inductive Proof:** (1) By following the rule that the cardinal of any one block, except the topmost, is the sum of the blocks directly above it, we have displayed for inspection a series of base-cases, in each instance discovering that there is only one block above it, and that this block has the cardinality of 1.

(2) Assuming that the cardinality of the outermost block of some arbitrary row $r$ is 1, we construct the
The First Version of RMI

The cardinality of the first block in the next row, \( r + 1 \), by adding together the cardinalities above it; but by the other rule of construction (the staggering of the blocks), we find only the one block, this one having (by hypothesis) the cardinality of 1; so that the “sum” once again (at row \( r + 1 \)) is identical to 1. Q.E.D.

With the base cases and the Induction Step, we conclude the theorem as holding for \( r \geq 2 \).

**Theorem 2:** The cardinality of the block adjacent to each outside block is identical to \( r - 1 \). (This accounts for the progression of the natural numbers as our eyes descend just inside the crust of the pyramid.)

**Inductive Proof:**

1. By following the rules of construction, we have displayed for inspection that the theorem holds for the first few cases.
2. Assuming now that the theorem holds for an arbitrary row \( r \), we proceed to construct the cardinality of the second block in row \( r + 1 \): it will be the sum of \( 1 \) (Theorem 1) and \( r - 1 \); so the sum = \( r \). Q.E.D.

With the base cases and the Induction Step, we conclude that the theorem holds for \( r \geq 2 \).

Now, a more careful eye might notice in the pyramid a proportion holding between any two adjacent blocks in each of the constructed rows:

**Theorem 3:** The cardinal in the left block is to the cardinal in the right block as the number of blocks to the left is to the number of blocks to the right (inclusively).

For instance, in the fifth row \( 4 : 6 \) (the two cardinals left and right) equals \( 2 : 3 \) (the number of cells left and right, inclusively). But how can we be sure that this proportion will hold throughout the future construction of the pyramid?

**Lemma:** The proportion holds for the pairs to the extreme right and left in each row. (We consider only the pairs on the left side.)

**Proof:** Each left member of the pair has the cardinality of 1. Also, the number of blocks to its left, inclusively, is 1. Now, by Theorem 2 the cardinality of the companion block is \( r - 1 \). Moreover, the number of blocks in each row is identical to the (ordinal) number of the row itself (this is “more comprehended than demonstrated,” as Pascal says; i.e., it belongs to the intuition enacted in the construction). So the number of blocks to the right, inclusively, is \( r - 1 \) \( (r \) less the one block to its left, its companion). And \( (r - 1):(r - 1) = 1:1 \). Q.E.D.

The proof, notice, is only indirectly inductive (in that it makes use of Theorem 2).

But how about the massive number of blocks further into the pyramid? Here we must make use of induction:

Let \( x, y, z \) stand for the cardinals in three successive blocks in some row \( r \). Let \( m, n \) stand for the cardinals of the two blocks in row \( r + 1 \) defined by the three blocks above them. Then, too, let \( QLx \) stand for the Quantity of blocks to the left of the block with cardinal \( x \) (inclusive), and \( QRy \) stand for the Quantity of blocks to the right of the block with cardinal \( y \) (inclusive):

\[
\begin{array}{ccc}
\ldots & x & y & z & \ldots \\
\ldots & m & n & \ldots \\
\end{array}
\]

1. \( QLx + QRy = QLy + QRz \) \hspace{1cm} Intuition of line formation
2. \( QLy = QLm \)
   \( QRy = QRn \) \hspace{1cm} \{ Intuition of staggered spaces
3. \( \frac{x}{y} = \frac{QLx}{QRy} \)
   \( \frac{y}{z} = \frac{QLy}{QRz} \) \hspace{1cm} \{ Assume /:. \( \frac{m}{n} = \frac{QLm}{QRn} \)
4. \( x + y = m \)
   \( y + z = n \) \hspace{1cm} \{ Game Rule of construction
5. \( x = \frac{y \times QLx}{QRy} \)
   \( z = \frac{y \times QRz}{QLy} \) \hspace{1cm} \{ 3 Algebraic transformations
6. \( \frac{m}{n} = \frac{x + y}{y + z} \) \hspace{1cm} \{ 4 Algebraic combination
Thus we have proved the Induction Step. Together with the base cases constructed for inspection, and the Lemma accounting for the pairs forming the outer crust of the pyramid, we conclude that Theorem 3 holds for \( r \geq 2 \).

\[ \text{§2. The second version of RMI} \]

Let us consider another game: King Karl must give one grain of wheat to Beggar Bob at point 0 (December 31), another grain at point 1, and henceforth a number of grains equal to \( 2Q_{n-1} + 3Q_{n-2} \). The task now is to show that at any point \( n \) (starting with \( n = 2 \)), \( Q_n \) will be greater than \( 3^{n-1} \) (the amount, say, that Beggar Bob's child will receive). Manual calculations show that this relation holds for the first few points (days):

- Point (day): 0 1 2 3 4 5 6 \( \ldots \) \( \infty 
- Child's gift \([= 3^{n-1}]\): n/a n/a 3 9 27 81 243 \( \ldots \) \( \infty 
- Beggar Bob's gift: 1 1 5 13 41 120 363 \( \ldots \) \( \infty 

The peculiarity of this example is that the game formula, viz. \( \forall n [Q_n = (2Q_{n-1} + 3Q_{n-2})] \), does not take the form of a recursive definition forwards from one single point. Rather, it leaps further back to previous quantities. In order to accommodate this leap, we must employ another version of RMI ("strong induction"). In this version the Induction Step to be proved reads:

\[ \forall n [\forall m (m < n \rightarrow \varphi_m) \rightarrow \varphi_n] \]

Or, we would like to prove that if at every point less than \( n \) the relation holds, then it holds for the next case, namely point \( n \), as well. Before contemplating this variation (in order to contemplate it), let us proceed with the proof:

1. \( \forall n [Q_n = (2Q_{n-1} + 3Q_{n-2})] \) \hspace{1cm} Premiss (game rule)
2. \( \forall m (m < n \rightarrow (3^{m-1} < Q_m)) \) \hspace{1cm} Assumption \( \vdash 3^{n-1} < Q_n \)
3. \( Q_n = (2Q_{n-1} + 3Q_{n-2}) \) \hspace{1cm} \( \forall \)-elim.
4. \( (n-1) < \) \hspace{1cm} Intuition of series
5. \( (n-2) < \) \hspace{1cm} Intuition of series
6. \( (4) \rightarrow (7) \) \hspace{1cm} \( \forall \)-elim. \( m \) as \( n-1 \)
7. \( 3^{n-2} < Q_{n-1} \) \hspace{1cm} 4.6 mods ponens & arithmetic
8. \( (5) \rightarrow (9) \) \hspace{1cm} \( \forall \)-elim. \( m \) as \( n-2 \)
9. \( 3^{n-3} < Q_{n-2} \) \hspace{1cm} 5.8 mods ponens & arithmetic
10. \( 2 \times 3^{n-2} < 2Q_{n-1} \) \hspace{1cm} 7 algebraic Principle 1
11. \( 3 \times 3^{n-3} < 3Q_{n-2} \) \hspace{1cm} 9 algebraic Principle 1
12. \( [2 \times 3^{n-2} + 3 \times 3^{n-3}] \in [2Q_{n-1} + 3Q_{n-2}] \) \hspace{1cm} 10.11 Principle 2
13. \( [2 \times 3^{n-2} + 3 \times 3^{n-3}] < Q_n \) \hspace{1cm} 3.12 Identity-exchange
14. \( [2 \times 3^{n-2} + 3 \times 3^{n-3}] = 3^{n-1} \) \hspace{1cm} Arithmetic of powers
15. \( 3^{n-1} < Q_n \) \hspace{1cm} 13.14 Identity-exchange
16. \( (5) \rightarrow (15) \) \hspace{1cm} 2\( \rightarrow\) 15 RCP
17. \( \forall n [\forall m (m < n \rightarrow (3^{m-1} < Q_m)) \rightarrow (3^{n-1} < Q_n)] \) \hspace{1cm} 16 \( \forall \)-intro.

Looking now to our table of Base Cases, we notice that the desired conclusion \textit{in fact} holds starting with \( n = 2 \). Thus the conclusion reads...
more fully: \( \forall n(2 \leq n \to (3^{n-1} < Q_n)) \). One must be careful when
determining the factual beginning — the first point at which the
predication at issue becomes correct. The first few cases may elude the
formula, have a life of their own, before the hypothesized order sets in.

Our task, in contemplation, is to highlight the suppositions allowing
one to conclude such lines as 17 above. Two such suppositions I have
called Principles (lines 10-12):

1. \( \forall x \forall y \forall z \exists x \leq y \to (x \leq y) \) — for \( x \) as a positive number
2. \( \forall w \forall x \forall y \forall z \exists w \leq x \wedge (y < z) \to (w < (x + z)) \)

To these principles of multiplication and addition, we could add those of
division and subtraction. Similarly with the source of lines 4 and 5.
These lines we may derive from what one might call “ordinal intuition.”
Or we could postulate \( 0 < 1 \) and employ the principle of substraction.

§3. The redundancy of the first version

Each version stipulates a recursive march forward from a starting
point, proceeding always one point beyond the last. The weak Induction
Step focusses attention on the juncture of any two adjacent points.
The strong Induction Step insists on gathering all previous points to back up
the passage to the next.

Reliance on only one previous point would seem to be a special case of
relying on all. Thus we should be able to manage with only the stronger
version of the rule — and thereby to lighten our load of basic
Rules of Inference. Let us assume, then, that only this second were at
our disposal, but that we have (in the solution of some problem) obtained
the conditions for the application of the first version. We would then like
to prove, from the two premisses:

1. \( \forall n(\phi_b \to \phi_{n+1}) \) Schematized Base Case
2. \( \forall n(\phi_n \to \phi_{n+1}) \) Schematized Induction Step (weak version)

the Induction Step suitable for the strong version of RMI. We may
considerably shorten the proof by introducing in advance two lemmas,
which we could prove separately:

- **Lemma A:** \( [(p \lor q) \land (r \to q)] \leftrightarrow [(p \to r) \to q] \)
- **Lemma B:** \( \exists x(\phi x \to p) \leftrightarrow [\forall x \phi x \to p] \)

3. \( (n - 1) < n \) Intuition of serial relations
4. \( \phi_{n-1} \to \phi_n \) \( \forall \)-elim. \( n \) as \((n - 1)\)
5. \( (\neg (n - 1) < n) \lor \phi_n \) Addition
6. \( (\neg (n - 1) < n) \lor \phi_n \) \& \( (\phi_{n-1} \to \phi_n) \) 5,4 Conjunction

Thus to any problem we have solved by using the first version of RMI
we could adjoin lines 3 to 10 above and come to the desired conclusion
employing the second version instead.

Yet there is something peculiar about the reference of “\( n - 1 \)” in the
above proof. When allowing it to pass into \( m \) at line 8 we must assume
that what we have called “\( n - 1 \)” exists in the series structuring the
problem to which the premisses 1 and 2 abstractly refer. For, assuming
that an ordinal series begins at some point (usually at point 1 or point 0),
it will definitely not be so that, given any \( n, n - 1 \) exists in the series.

The question then arises, How can we justify the needed assumption of
existence?

The answer makes a somewhat ghostly use of premiss 1. Recall the
form of the conclusion, which reads in full:

\[ \forall n(\phi_b \to \phi_n) \]

Of relevance, then, is only whether “\( n - 1 \)” is greater than or equal to the
number of the Base Case: whether \( b \leq (n - 1) \). True, when \( b = n, n - 1 \)
may not exist (namely, if \( b \) happens to occupy the least position in the
series). But clearly in the series beginning with “\( b + 1 \) every “\( n - 1 \)” will
name an existing point. However, in this case the initial conclusion
should read \( \forall n(\phi_{b+1}) \). This conclusion may then be
extended to cover \( b \leq n \) as well — by pointing to premiss 1 as covering
the one extra case.

§4. Seven meta-theorems in logic

With the aid, now, of induction, we may formally prove a number of
theorems bearing on our logical system as a whole.

§4.1 Conjunctive normal form

Once we have at our disposal a specifiable set of primitives or
theorems, we can show by RMI that any formula in sentential or
predicate logic has a logically equivalent counterpart called a Normal
Form. One useful way of construing this “normality” is as follows:
Normal Form is that shape containing, apart from any quantifiers, only
a string of conjuncts, inside of which are only units (like \( p \) or \( \phi x \)), each
with or without a single curl, and these connected only with the wedge (thus making parentheses inside the conjuncts intuitively superfluous); and, in formulas involving quantifiers, these appear outside the series of conjuncts, governing the entire subsequent string. For example:

\[
\neg(p \lor q) \quad \text{has as its Normal Form} \quad (\neg p) \land (\neg q)
\]

\[
\neg(p \land \neg(q \lor r)) \quad \text{has as its Normal Form} \quad (\neg p) \lor (q \land r)
\]

\[(p \land \neg\forall \varphi x) \quad \text{has as its Normal Form} \quad \exists \forall \varphi x \land (\neg \varphi x)
\]

\[(\exists \forall \varphi x \lor \neg\neg p) \quad \text{has as its Normal Form} \quad \exists \forall \varphi x \lor (p \land r)
\]

\[\exists \forall \varphi x \rightarrow p \quad \text{has as its Normal Form} \quad \forall \forall \forall \varphi x \lor p
\]

\[\exists \forall \varphi x \lor \exists \forall \varphi y \quad \text{has as its Normal Form} \quad \exists \forall \varphi x \lor \exists \forall \varphi y
\]

Now, an inductive proof ranging over any formula must begin with a definition of “formula.” In the present instance, we may define our formulas (well formed ones: wffs) inductively. That is, we stipulate the recursive conditions for constructing the set of wffs. This set will be formed from an infinite series, the “points” of which are in turn defined by the number of primitive logical operators in the wffs (these operators are \(-, \lor, \land; \) the other logical operators, \(- \rightarrow, \land, \lor, \land \), \(\exists \), \(\forall \) are understood as introduced by definition).

1. any atomic formula (e.g., \(p, \forall x\)) is a wff: seed members of the set).
2. any unitized wff with \(- \) in front of it is a wff.
3. any two unitized wffs coupled with a \(\lor\) or \(\land\) is a wff.
4. any wff will be generated in one of the ways listed above.

By “unitized” I mean sufficiently bracketed on the outside to prevent confusion (some systems stipulate that every wff, even the seed member and sequences of curls, must be fully bracketed).

Now we may display the construction of the set (its members) at the first few points:

- **point 0** \(p\)
- **point 1 by (1):** \(\neg p\)
  - by (2): \(p \lor q\)
  - by (3): \(\forall p\) [understood: \(\forall\) is followed by \(x\) or \(y\) or ...]

Note that the \(p\)'s and \(q\)'s here and throughout are place-holders only. For instance, \(p\) might be \(q\) or \(r\), \(\forall x\) or \(\forall y\), \(\forall z\); only the number of logical operators counts. Notice too that, if we wished, we could here and throughout allow for vacuous generalizations: in \(\forall p\), the \(\forall\) may have no variables to govern. To continue:

- **point 2 by (1):** \(\neg\neg p, \neg(p \lor q), \neg\forall p\)
- by (2): \(\neg p \lor (p \lor q) \lor r, \neg\forall p \lor q\)
- by (3): \(\forall \neg p, \forall(p \lor q), \forall\forall p\)

Note that, for simplicity's sake, we assume commutability in formulas generated by (2): it makes no difference whether we have \((\neg\forall p \lor q)\) or \((\neg p \lor q)\). — And by now the pattern has become clear:

- **point 3 by (1):** negate each formula at each previous point.
- by (2): combine each wff from any previous line with each and every other wff, such that the total number of logical operators equals 2, i.e. the sum of the ordinal-numbers, less one (to allow for the new \(\lor\)).
- by (3): place \(\forall\) before each wff at previous point.

And so on at point 4 and beyond.

Proof that we can always find (construct) a Normal Form proceeds in reference to this recursively defined set of wffs. Inspecting the formulas at points 0, 1, and 2, and a sampling from points 3 and 4, we easily see that the first few formulas already appear in Normal Form, while the others allow passage to Normal Forms with the use of DN, DeM, Distribution over \(\lor\), and predicate theorems permitting us to relocate any internal quantifiers out front (their order is indifferent).

To obtain the Induction Step, we hypothesize the derivability of Normal Forms for all formulas at points \(m\) prior to point \(n\), and ask how we can know that all formulas at this next point \(n\) will allow us to derive equivalents in Normal Form.

In accordance with the rules for constructing the series, there are only three ways by which formulas may arrive at point \(n\):

- by annexing a curl to a formula at point \(n - 1\)
- by adding any two formulas from previous lines, so long as the sum of their ordinals equals \(n - 1\), and
- by annexing a universal quantifier to a formula at point \(n - 1\).

All three ways require operations on formulas hypothesized to have Normal Forms.

When the formula born at line \(n\) has as its parentage a formula at line \(n - 1\), plus now a curl preceding that parent, we may apply De Morgan's and/or a QN to break up the original conjunctions into a new series of
negated disjunctions. Each of these disjuncts has an equivalent Normal Form (by the hypothesis); we then transform each of these by De Morgan’s to get the curl inside: indeed, we proceed until curls are attached only to atomic formulas (using Double Negation to reduce the plethora to one or zero for each atom).

When the formula born at line \( n \) has as its parentage two formulas drawn from previous lines (so the number of operators totals, with the new wedge, \( n \)), we have a formula in the schema \((\text{NF}_1) \lor (\text{NF}_2)\)—since we are supposing that each of the components has a Normal Form. Now, each of these components consists of a conjunction, so we may repeatedly perform a distribution to re-arrange the new-born as a series of conjunctions, each of which will contain only a string of disjunctions of atoms (each with at most one curl). Tedious if either both of the original NFs are lengthy; but imaginable as a finite string of our own operations.

And when the formula born at line \( n \) has as its parentage a formula at line \( n - 1 \), plus now a universal quantifier placed before it, the new formula is immediately in Normal Form.

Thus we can imagine effective ways of showing how we could pass from any one line \( n - 1 \) to its immediate successor. An imaginative exercise, notice.

With Base Case(s), the Induction Step, and RMI we may conclude that any formula will allow us to derive an equivalent in Normal Form. Again, however, remember that the proof is context-bound; it reads: within a system having the formulas specified (theorems or primitive formulas), any \( \text{wff} \) can be transformed into an equivalent Normal Form: in that context, we have the wherewithal to construct a derivation of an equivalent Normal Form.

### §4.2 Consistency and soundness

A logical system is inconsistent if one line of inference establishes a formula \( A \), while another line of inference establishes \( \neg A \).

Consistency is a negative property of a system: its inability to authorize the generation of both a formula and its contradiction. Strictly, what we would like to prove is that the system does not have the property of being inconsistent. The demand for such consistency echoes the Aristotelian axiom of non-contradiction. In modern thinking, it also expresses the passion to form a reliable \( a \ priori \) counterpart to the vicissitudes, uncertainties, and contradictions that arise in our time- and place-bound efforts to work with and to make sense out of our actual circumstances. Echoes of Stoicism: even if our \( a \ posteriori \) conditions are fraught with inconsistencies, we would like to rest assured that our own responses remain rational, i.e. that we ourselves can retain integrity within the general disintegration.

A sentential calculus quickly leads to an inconsistency if we allow it to contain even a contingent formula as a theorem. For with the rule of substitution we may quickly deduce its own contradiction. A predicate calculus, in contrast, can contain a contingent formula, e.g.

\[ \exists \phi x \rightarrow \forall \chi \psi x \lor (\exists \phi x \land \exists \psi x) \rightarrow \exists (\phi x \land \psi x), \]

and remain consistent.*

But how can we know (prove) that a contradiction can never be found in the never-ending workings of even just the sentential calculus? To answer this question we need (1) an infinite series, (2) a proposed property of each member of the series, (3) a basis where the property holds, and (4) the Induction Step showing that if the property applies at all previous points it also applies at the next point.

1. The series can be any deductive sequence, although we shall consider only Russell’s. (Unlike the series employed in the inductive definition of \( \text{wffs} \) for the first meta-theorem, the series here can start all over again from scratch for any deductive sequence. The march through the series will differ somewhat from system to system, most notably as we switch from axiomatic to natural deduction.)

2. The proposed property we shall call “0”: we would like to show that this property holds at every point in the series, starting with the points at which the primitives are located.

3. Each of Russell’s five primitives has this property 0 on the following interpretation of the two logical constants:

\[
\begin{array}{c|c|c|c}
\text{p} & \neg p & \text{q} & \text{p} \lor \text{q} \\
\hline
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
& 1 & 0 & 0 \\
& 1 & 1 & 1
\end{array}
\]

* One (ultimately illegitimate) way of proving a contradiction even here would be to allow our own formulas to serve as the domain over which the variables range; at least one (any theorem) has the property \( \phi = \text{“logically true”} \) and at least one (\( e.g., p \land \neg p \)) does not have that property.
Appendix II: Axiomatic Induction

Of course, 0 and 1 represent in parallel the truth-functionality at the heart of all logical development: the principle that judgements, whether empirical or logical, are binary, either pro or contra and not both. We often enough forget that logical contemplation unfolds this principle. Yet in the present case it is wise to forget it. Rather than searching for the truth of formulas we must search simply for a line not having the one property, here called 0. If no line in a deduction can have it, then no two lines can ever conflict, all lines will stand together on this score—and we may revert to the natural, truth-functional interpretation of the connectives.

(4) We prove the Induction Step by supposing that every formula \( m \) (where \( m < n \)) has the property 0 and then proceeding to show that, on this supposition, each of the two rules (Substitution and modus ponens) for generating the next formula (now at point \( n \)) will also have it: that the previous ones necessarily bequeath this property to each and every one of their progeny. We then obtain the Induction Step by RCP.

Finally, then, with the Basis and the Induction Step, we infer, by RMI, that every formula in the series will have the property 0; therefore that no formula will have the property 1; and therefore that there can be no two lines, one of which has 0 and the other of which has 1 as their respective properties. Now we can understand 0 as indicating our warrant to judge a formula logically true, and its counterpart as our warrant to judge it logically false: the infinite series contains no two lines that would force us to judge the same formula both true and false.

But what about domains of two, three, four, ... \( n \) individuals? The question of consistency is this: whether we can have in our system two formulas identical in every respect save that one of the two has a curl placed before it. And this question is decidable in reference to the notion of “satisfiability”: a formula is satisfiable if and only if there is at least one assignment of values (properties) such that, on that assignment, the formula has the preferred value, here the property 0. Now, if a formula is not satisfiable in a domain of one individual, it is not satisfiable in a domain of \( n \) individuals, where \( n > 1 \).” Now, to discover a contradiction, we would have to derive a formula and then also a contradictory of that formula. The one will have the property 0 in a domain of one individual, and the other one would have to have the property 1 in that domain: it will not be satisfiable. Now imagine expanding the domain to whatever size one wishes: the second formula will never be satisfiable, i.e. never have the property 0, no matter how large the domain. Thus if a formula such as the second (the contradictory of the first) cannot be derived for a domain of one individual, it cannot be derived for any domain.

Still, it will be noticed that a contingent predicate formula such as that of Holy Terror II, \( \exists x \phi x \rightarrow \forall x \phi x \), also resists contradiction in a domain of one individual—or in any finite domain. Thus the contemplation of consistency based on the notion of satisfiability leaves our system open to the possibility of inflation by contingency. It is important, then, that we prove our system not only to be consistent, but also (syntactically) sound: i.e. that every formula derived from the system's primitives and rules enjoys the modality of necessity.

For the sentential calculus the proof of consistency is already a proof of its syntactic soundness: every formula generated will have the property 0, understood as the warrant for judging it logically necessary. As we have just seen, however, proving that the predicate calculus can yield no contradiction does not in itself exclude the possibility of contingent formulas appearing as theorems.

Adding these extras to our starting collection for the sentential calculus, we can see immediately that the inductive proof already employed will not be affected: we can easily show that the extra primitives also have the property 0, and that the extra rule allows us to reiterate a formula.

Meta-theorems: Consistency and Soundness

This principle of satisfiability holds for monadic predication only; one can find polyadic predications for which it does not hold, namely ones requiring infinite domains.
The task is to envision a truth-table demonstration that the two extra primitives also have the property 0, and that any application of the extra rule will also assure that the line thereby generated has the property 0.

But how may we do this? Consider again the one primitive,
\[ \forall x(p \lor \phi x) \rightarrow (p \lor \forall x \phi x). \]

Now, \( \phi \) can be interpreted as some (arbitrarily chosen) predicate like “white” or “three-feet tall,” a monadic predicate somehow belonging to our own conceptual framework. And \( x \) may be interpreted as an individual \( a \) (likewise chosen arbitrarily, this time from a domain of things outside the first conceptual framework). Of course, the variable \( p \) will naturally take assertions as instances. Now, for \( p \) we may assume that it will either have or not have, exclusively, the special in-house property 0. And similarly for every application of the conceptual property \( \phi \) to individuals. The peculiarity of this latter, however, is that they are double-edged in reference, i.e. partly inside and partly outside the conceptual scheme. And, of course, there are now infinite propositions to account for in their doubleness: \( \phi a? \ \phi b? \ \phi c? \ldots \). Still, we may define as a class the collection of all possible combinations for a given \( \phi \). And it seems plausible that we might in turn divide this class cleanly to form two sub-classes: one containing all those instances to which, for whatever reason, we would ascribe the property 0 (most naturally, of course, “true”) and the other containing all those to which we deny this property, assigning rather the property 1. Having made this division (imaginatively), we may then imagine selecting one delegate from each, \( \alpha \) from the sub-class of items \( a_0, b_0, \ldots \) having the property 0, and \( \beta \) from the sub-class of items \( a_1, a_1, \ldots \) having the property 1. These two delegates we now send back to our primitive for instantiation, and establish in both cases that the formula has the property 0.

We may then do the same for the second extra formula:
\[ \forall x \phi x \rightarrow \phi x. \]

Here, however, we notice a difference: the formula is open. Indeed, it intends to allow us to disburden ourselves of the universal quantifier. Again, though, we may imagine interpreting \( \phi \) as a monadic predicate, along with an understanding that there is a domain of individuals over which the formula ranges. We may then think this open formula distributively: the antecedent tells us to dip into the domain to pick any individual; the consequent tells us (tautologically) that whatever the \( \phi \) and whatever the individual, \( \phi \) will belong to the individual. Now, if we happen to pick an individual to which \( \phi \) does not apply, then the antecedent itself does not have the property 0, but neither does the consequent, and therefore the primitive as a whole has the special property 0. If, on the other hand, the individual does have \( \phi \), then the formula already has the property 0. So the primitive always has the property 0 — by proxy vote (the original class being cleanly divided and delegates \( \alpha \) and \( \beta \) sent in for instantiation).

As for the extra rule, \( \forall x \phi x \rightarrow \forall x \phi x \): it certainly cannot be applied distributively. That some arbitrarily chosen \( a \) happens to have, by our judgement (whether empirical or formal), the property \( \phi \), certainly does not justify the inference that every individual has that property. Yet the rule in question applies only to lines in our system — lines which, although taking the schematic form \( \forall x \), we assume to have the property 0 for any instance already. Applying \( \forall x \) to \( \forall x \phi \) then serves two purposes. First: it announces to the world that the \( \forall \phi \) has already arrived safely with the special property; if the insides (i.e., \( \forall \phi \)) at the newly generated line has the property 0, then the announcement that it does so also has that special property (if something is materially correct, then saying so is also logically correct). And second: it seals off the variable so that subsequent applications of a quantifier do not intrude upon the variable at issue. There might be some new thoughts here, but not ones that formally allow us to gainsay the introduction of the quantifier.

So it seems that the predicate calculus is also sound. Yet the induction assumes that we are able to sort out the infinite into sub-classes defined in reference to whether or not, exclusively, a property \( \phi \) would

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* With regard to something being correct and then asserting it to be correct, we may think of the difference between “my acting virtuously no matter what” and quantifying this in the assertion that I always so act. Or the difference between discovering something in the woods (the way deer behave) or in a logical deduction (that a certain pattern of inference is valid), and defending or explaining the discovery. Humanly, there is a big difference. The logical question of the difference has been worked out in great detail by Alfred Tarski: “The Concept of Truth in Formal Languages” (1931), in his collection *Logic, Semantics, Metamathematics* (1956), pp. 152-278.
The legitimacy of this imaginary sorting depends upon our ontology. Cf. S. Shapiro’s *Philosophy of Mathematics* (Oxford, 1997), pp. 22-23:

Let $P$ be a property of numbers. For an intuitionist, the content of the expression $\neg \forall x P x$ is that it is refutable that one can find a construction that shows that $P$ holds of each number; the content of $\exists x \neg P x$ is that one can construct a number $x$ and show that $P$ does not hold of $x$. The latter expression cannot be inferred from the former because, clearly, it is possible to show that a property cannot hold ... of the mathematician, will accept the law of excluded middle ... From the realist’s perspective, the content of $\neg \forall x P x$ is simply that it is false that $P$ holds universally, and $\exists x \neg P x$ means that there is a number for which $P$ fails. Both formulas refer to numbers themselves; neither has anything to do with the knowledge-gathering abilities of mathematicians, or any other mental feature of them. From the realist’s point of view, the two formulas are equivalent.

The dispute pertains to the status of the conclusion: Does a proof employing EM signal hypothetical or established knowledge on our part? It does not affect the mechanical operations of the proof. Rather, it adds to the question of an infinite series another question, that of the role of imagination in logical operations, and in our contemplation of them.

4.3 Completeness of the sentential calculus

A calculus is (syntactically) complete if and only if for every formula well formed in the manner prescribed by the calculus we may either provide, with the rules of the calculus, a derivation of the formula, or devise an appropriate domain in reference to which we may show that the formula does not hold (that it is contingent or self-contradictory).

For an example of an incomplete calculus, consider one comprising only three of Russell’s independent primitives: there are formulas, assertions when applied to items in certain domains (e.g., moral qualities, arithmetic quantities, times, logical formulas).

Logical study of any sort entails self-criticism, the central tool of which is truth-functionality, or the principle of non-contradiction: we work out patterns of human response on the basis of this exigency of judgement: that one must decide between two incompatible patterns. The disputes between anti-EM and pro-EM logicians may remind us of the question stemming from the Stoics of late antiquity and haunting all modern intellectual work: the question of the difference and the relation between in-house and extra-house determinations (rather than the classical question of the difference and the relation between a being and its essence).

Insofar as the principle of Excluded Middle simply reflects a decision at home, there need be no real dispute: one of the perks of logical study is that we may include in our work any assumption that does not lead to a self-contradictory (or an unwanted contingent) formula. So long as the assumption is acknowledged as such.

The dispute resumes when our logical work bids us to travel abroad. And when introducing the predicate calculus we imagine ourselves packing our bags for just such a journey. The dispute here is not whether we may introduce EM, but whether we may consider it to be tautological in form: either *a priori* analytic or *a priori* synthetic (or, in contrast, simply arbitrary and pretentious) when instantiated, imaginatively, over an infinite set of essentially foreign elements.

The dispute pertains to the status of the conclusion: Does a proof employing EM signal hypothetical or established knowledge on our part? It does not affect the mechanical operations of the proof. Rather, it adds to the question of an infinite series another question, that of the role of imagination in logical operations, and in our contemplation of them.
starting with the fourth primitive, that are logically true (as shown by a truth table) and that will forever defy proof.

It is conceivable that, even with the four primitives, there still be some logically true formula, and thus an infinite number of such formulas, that defy proof. Should this be the case, we would have to add as a fifth primitive either that formula or some ancestor of it.

Completeness is a major issue for any system. For the challenge of system-building is precisely to form a finite number of basics (rules, and perhaps primitives) from which we may deduce the rest.

Once again, the challenge poses a question bearing on the infinite. There are infinitely many formulas that are logically true, yet we want to know, in advance and over all, whether our system is complete. In the case of the sentential calculus, the “deserving” has an immediate standard: a truth table.

We may accept this challenge in the spirit of Descartes: we may imagine that some all-powerful but nonetheless logical demon presents us with a lengthy formula and demands that we actually derive it in our system.

The first thing we will do is to create a truth table for the formula to determine whether it is a tautology or not. If not, we shall not try to derive it, but rather refer to our earlier consistency proof that such a formula can never be rightly derived, even by the all-powerful demon. Our demon must then provide us with one that passes this first test.

Having, finally, a truth-table tautology, we reduce it to Conjunctive Normal Form (we have already proved, inductively, that this will always be possible: now we actually do it — since a given formula is finite). This derivation we place in a scratch file.

Now, the new formula must consist of conjuncts, each one of which will be a tautology. Why? Because every formula derived from a tautology will be a tautology (something we have already proved, inductively). Furthermore, a string of conjoined disjunctions can only be a tautology if in each conjunct there is at least one atomic formula appearing a second time with a curl in front of it.

This last condition allows us to begin reconstructing the original formula. For we can now invoke for each conjunct one of our theorems, viz. $p \lor \neg p$. With the primitive we call Addition we can continue to build up each one of the conjuncts evident in the derived CNF. Permutations and Associations may be necessary if we perform this construction on a machine (which will insist on a purely effective procedure). With each conjunct appearing on some line in our proof, we may employ Conjunction to reproduce exactly the CNF in our scratch file.

And, finally, we look to the way we reduced the demon’s formula to CNF: all formulas employed in this reduction take the form of bi-conditionals. So we need only re-trace our original steps to procure that formula. And we have met the challenge.

This proof of completeness, although meta-theoretical, is not directly inductive. It does, however, borrow the results of the two previous inductive proofs regarding reducibility to CNF and the requirements of consistency.

Indeed, we could integrate into our deductive system this proof of completeness, namely that any Tautology is Provable:

$$\forall x_1 (T x_1 \rightarrow P x_1),$$

by introducing the following Axioms:

Axiom A: every Tautologous WFF has an equivalent CNF, or

$$\forall x_1 (T x_1 \rightarrow \exists y_1 (D x_1 x_1 & D y_1 x_1 & C N F y_1)),$$

Axiom B: anything Derivable from a Tautology is also a Tautology, or

$$\forall x_1 (T x_1 \rightarrow \forall y_1 (D y_1 x_1 \rightarrow T y_1)),$$

Axiom C: each Conjunct of a Tautologous CNF is a Tautology, or

$$\forall x_1 \{ (C N F x_1 & T x_1) \rightarrow \forall y_1 (C y_1 x_1 \rightarrow T y_1) \},$$

Axiom D: in each Conjunct of a Tautologous CNF a Letter Occurs that is at least once Affirmed and once Negated, or

$$\forall x_1 \forall y_1 \{ (C N F x_1 & C y_1 x_1 & T y_1) \rightarrow \exists z_1 (L z_1 & O z_1 y_1 & A z_1 & N z_1) \},$$

Axiom E: any Conjunct of a CNF in which a Letter Occurs that is once Affirmed and once Negated is Provable in the system, or

$$\forall x_1 \forall y_1 \{ (C N F x_1 & C y_1 x_1 & \exists z_1 (L z_1 & O z_1 y_1 & A z_1 & N z_1)) \rightarrow P y_1 \},$$

Axiom F: given any CNF in which each Conjunct is Provable in the system, anything Derivable from it is Provable in the system, or

$$\forall x_1 \{ (C N F x_1 & \forall y_1 (C y_1 x_1 \rightarrow P y_1)) \rightarrow \forall z_1 (D z_1 x_1 \rightarrow P z_1) \},$$

where all the variables are subscripted to indicate that their possible instances are confined to the “type below.”
We then have a sufficient condition for judging any sentential system complete: it must posit as primitive, or allow us to derive, in an effective manner, such formulas as Distributions, De Morgans, Double Negations, Excluded Middle, Summations, Contraposition, Permutations and Associations.

§4.4 The rule of replacement

In neither axiomatic nor natural deduction do we make provision for Replacement except by way of definition. If we then introduced Replacement as a another rule (calling it RR) we would confuse inferential with definitional development.

Yet we do derive equivalences within our systems. And we know, intuitively, that having \( P \leftrightarrow Q \) and a formula \( f \) containing \( P \) we should be able to infer another formula exactly like the first except that \( Q \) takes the place of \( M \). In the sentential part of our calculus we can even envision a truth-table justification: \( f(P) \) and \( f(Q) \) will necessarily have identical patterns of values. But to prove our intuition (especially in regard to the transfinite introduced by quantifiers) we must somehow consider the replacement of \( P \) with \( Q \) as applicable to any formula containing \( P \): \( (f)[f(P) \rightarrow f(Q)] \)—whatever the modality of the formula. And whatever its length.

The proof requires that we make use of the series of wffs constructed for our first inductive proof— the one regarding CNFs.

Suppose that we have indeed an equivalence \( P \leftrightarrow Q \). We would like to prove that any formula \( f \) in which \( P \) figures as an element can validly lead to a formula of the same form except that \( Q \) will appear in place of \( P \). At point 0 the formula must take the form of \( P \) itself, and we can manually derive its counterpart (equivalent), namely \( Q \):

1. \( P \rightarrow Q \) The relevant half of the assumed equivalence
2. \( P \) The first case: \( p = P \)
3. \( Q \) \( \forall \)-intro.

At point 1 we have three wffs, each representing a type that must be dealt with separately:

\( \neg p \):
1. \( Q \rightarrow P \) The relevant half (the converse, notice)
2. \( \neg P \) Case at issue: \( P \) is embedded in \( \neg p \)
3. \( \neg P \rightarrow \neg Q \) Contraposition
4. \( \neg Q \) \( \forall \)-elim.

\( \lor q \):
1. \( P \rightarrow Q \) Case at issue: commutability assumed
2. \( P \lor q \) Summation & \( \lor \)-elim.
3. \( (P \lor q) \rightarrow (Q \lor q) \) Summation & \( \lor \)-elim.
4. \( Q \lor q \) \( \forall \)-intro.

\( \forall p \):
1. \( P \rightarrow Q \) The relevant half
2. \( \forall P \) Case at issue: \( P \) embedded in \( \forall p \)
3. \( Q \) \( \forall \)-intro.
4. \( \forall Q \) \( \forall \)-intro.

So far we have effected, manually (albeit schematically), replacement at points 0 and 1 in the infinite series of wffs. When doing so, we used all three of our rules (Substitution implicitly), a number of theorems (note especially Contraposition and Summation), the one primitive (\( \forall \)-elimination), and implicitly also the definition of Equivalence (but notice that in our system we arrive at an equivalence by first deriving each of the halves).

The purpose of constructing the proofs at the first few points of a series is to make palpable the theoretic basis of the meta-theoretical induction to follow. Here, manual construction reveals not only the main theorems that will come into play, but also a recurrent pattern: when proving \( f(Q) \rightarrow f(P) \), we could also prove the reverse: \( f(P) \rightarrow f(Q) \).

So we have the Base Case(s). Now for the Induction Step. Picking up on the original sequence defining the three game rules for the generation of wffs, we could imagine first making explicit the fourth condition (that the inductive definition excludes all other possibilities), and then hypothesizing for Strong Induction:

1. Given any formula, if it is a wff it is generated in one of the three ways specified.
2. \( \forall m[\neg P(m < n) \rightarrow \text{there exists a manual proof showing that } f(P) \leftrightarrow f(Q) \text{ at } m] \).

Hypothesis

3. If the formula at \( n \) is generated in the first of the three ways, we may effect a proof of “type 1”: the \( f(P) \) at \( m \) is assumed to yield \( f(Q) \) with manual effort, and vice versa; we use the vice versa, Contraposition, and \( \lor \)-elimination to obtain the negated version at \( n \).

See type 1

4. If the formula at \( n \) is generated in the second of the three ways, we may effect a proof of “type 2”: each of the disjuncts coupled at point \( n \) is
assumed to yield its counterpart; and we now Summation to effect the derivation of the formula with \( Q \) in place of \( P \) at point \( n \). See type 2

5. If the formula at \( n \) is generated in the third of the three ways, we may effect a proof of “type 3”: after employing \( \forall \)-elimination, we are left with a formula from the line preceding line \( n \); we have hypothesized that this line yields its counterpart, so we need only obtain the counterpart and apply \( \forall \)-introduction.

See type 3

6. (2) \( \rightarrow \) there exists a manual proof showing that \( f(P) \leftrightarrow f(Q) \) at \( n \). RCP

7. \( \forall n \exists \text{Proof}[f(P_n) \leftrightarrow f(Q_n)] \) \( \forall \)-intro., Base Cases, RMI

But what exactly have we proved? We have not proved a Derived Rule within the available system. In effect, though, we have a scheme, a recipe for constructing the proof required in a given instance; e.g.:

Constructive Proof of a Replacement

\[ P \leftrightarrow Q \text{ and } r \rightarrow \exists x[(s \& P) \rightarrow \varphi x] \quad \therefore \quad r \rightarrow \exists x[(s \& Q) \rightarrow \varphi x] \]

1. \( Q \rightarrow P \) Relevant half of equivalence
2. \( \neg P \rightarrow \neg Q \) Contraposition
3. \( (s \vee \neg P) \rightarrow (s \vee \neg Q) \) Summation
4. \( (s \vee \neg Q) \rightarrow (s \vee \neg P) \) Contraposition
5. \( (s \vee \neg r)(s \vee \neg P) \rightarrow (s \vee \neg r)(s \vee \neg Q) \) Contraposition
6. \( (s \vee \neg r)(s \vee \neg P) \rightarrow (s \vee \neg r)(s \vee \neg Q) \) Contraposition
7. \( (s \vee \neg r)(s \vee \neg Q) \rightarrow (s \vee \neg r)(s \vee \neg P) \) Contraposition
8. \( \forall x(s \vee \neg r)(s \vee \neg Q) \rightarrow \forall x(s \vee \neg r)(s \vee \neg P) \) \( \forall \)-intro., Base Cases
9. \( \forall x(s \vee \neg r)(s \vee \neg Q) \rightarrow \forall x(s \vee \neg r)(s \vee \neg P) \) \( \forall \)-intro., Base Cases
10. \( \forall x(s \vee \neg r)(s \vee \neg Q) \rightarrow \forall x(s \vee \neg r)(s \vee \neg P) \) \( \forall \)-intro., Base Cases
11. \( \forall x(s \vee \neg r)(s \vee \neg Q) \rightarrow \forall x(s \vee \neg r)(s \vee \neg P) \) \( \forall \)-intro., Base Cases
12. \( (r \vee \neg r)(s \vee \neg r)(s \vee \neg P) \rightarrow (r \vee \neg r)(s \vee \neg r)(s \vee \neg Q) \) Contrap.
13. \( (r \vee \neg r)(s \vee \neg r)(s \vee \neg P) \rightarrow (r \vee \neg r)(s \vee \neg r)(s \vee \neg Q) \) Contrap.
14. \( (r \rightarrow \exists x(s \& P) \rightarrow \varphi x) \rightarrow (r \rightarrow \exists x(s \& Q) \rightarrow \varphi x) \) Def. of \( \rightarrow, \& \), and \( \exists \)

At which point we may employ the second of the premisses to obtain the desired conclusion.

The meta-theoretical proof of the Rule of Replacement signals the existence of finite procedures for constructing proofs of the sort above—on the condition that the theorems employed have been proved (or taken as primitive) in the given system. If the proof above illustrates all the maneuvers needed for any such replacement exercise, then the theorems necessary to obtain in advance of the application of our meta-theorem are Contraposition, Summation (two forms: one for adding left, another for adding right), Hypothetical Syllogism (immediately derivable from the preceding), and the two predicate primitives (Quantifier Mobility is immediately derivable from \( \forall x[p \vee \varphi x] \rightarrow [p \vee \forall x\varphi x] \)).

Although we have paved a way into a slice of infinity, we never quite arrive. Within the inductively defined set of all well formed formulas (a sub-set of which is the set, also infinite, of the formulas in our own system), we have established (1) that Replacement can be manually effected for the smallest wffs and (2) that, once we have started, we can always proceed to effect a manual proof for the next size of wff. Foreseeing the possibility of recurrent applications of Substitution and modus ponens, without limit, we conclude that such Replacement will be feasible wherever we arrive in the series.

Concluding some \( \varphi \) about any anticipated point of growth on the grounds that \( \varphi \) belongs to a beginning point and that we have devised a way of showing that \( \varphi \) will always belong to the next point: this conclusion from two highly restricted premisses manifests our own ability to range freely; as Kant might say, it manifests our ability to synthesize a priori. Yet, he conclusion drawn in the name of RMI formulates what we may prove at any point where we arrest the growth. In the present instance, RMI reveals that there exists a constructive proof of Replacement. As already remarked, the infinity at issue is potential, and the actuality lies in the finite construction.\(^{6}\)

\section*{4.5 The independence of primitives}

In order to prove that we could never derive an otherwise logically true formula from other such formulas we must resume, in a radical way, the axiomatic thought that a system consists only of “marks on paper”

\( ^{6} \) In his dissertation on proof theory (1930), Jacques Herbrand remarks on the nature of the meta-theoretic employment of induction (\textit{Logical Writings}, Dordrecht, 1971, p. 51):

It must be carefully noted that there is a certain difference between the manner in which we employ reasoning by recursion here and the manner in which it is sometimes employed in mathematics. Here, it is never anything but an indication, in a single formula, of a procedure which must be employed a certain number of times in each particular case. In mathematics, on the other hand, it can happen that this reasoning is used in the case of concepts for which a material representation is not possible, as opposed to our signs; as, for example, for the set of integers or of real numbers.
and mechanical rules of construction. We must abandon the reference to meaning (and so of truth) and resort primarily to syntax.

§4.5.1 Łukasiewicz' syntactical system

Consider the system of four axioms (“primitives”) devised by the Polish logician Jan Łukasiewicz in 1951:"  

P1. \(aAa\)
P2. \(aA\)
P3. \((cAb \land aAc) \rightarrow aAb\)
P4. \((cAb \land cAa) \rightarrow aAb\)

To these primitives we may add two purely notational definitions:

D1. \(aEb =_{def} \neg aAb\)
D2. \(aOb =_{def} \neg aAb\)

Now, embed these primitives and definitions within our sentential calculus. To effect deductions we may make use of any theorem in the sentential calculus, along with *modus ponens* and Substitution. With attention solely to the syntax, we may prove the following theorems:

\[
\begin{align*}
T1: & \quad aEb \rightarrow \neg aAb \\
T2: & \quad \neg aAb \rightarrow aEb \\
T3: & \quad aOb \rightarrow \neg aAb \\
T4: & \quad \neg aAb \rightarrow aOb \\
T5: & \quad aOb \rightarrow aOb \\
T6: & \quad \neg aAb \rightarrow aOb \\
T7: & \quad aAb \rightarrow \neg aOb \\
T8: & \quad \neg aOb \rightarrow aAb \\
T9: & \quad aAb \rightarrow bMa \\
T10: & \quad aAb \rightarrow aAb \\
T11: & \quad (cEb \land aAc) \rightarrow aOb \\
T12: & \quad (cAb \land aAc) \rightarrow aAb \\
T13: & \quad (cAb \land cAa) \rightarrow aOb \\
T14: & \quad (cEb \land aAc) \rightarrow aAb \\
T15: & \quad bMa \rightarrow aEb \\
T16: & \quad bMa \rightarrow bMa \\
T17: & \quad (bEc \land aAc) \rightarrow aEb
\end{align*}
\]

Now, precisely because the primitives need not *mean* anything, we can assign various meanings to them — *interpret* the operators. Looking ahead, we can see that, for any such interpretation to be *true*, it is necessary that the two operators be *reflexive* (P1, P2), that the first be *transitive* (P3) and have P2 as a consequent (T10), that the second be *symmetrical* (T9) and in this way transitive with the first (P4, T12).

Two such interpretations are:

\[
\begin{align*}
\text{xy}: & \quad x = y & [D: \text{numbers}] \\
\text{xy}: & \quad |x| = |y| & [\text{for “absolute value of } n]\end{align*}
\]

or:

\[
\begin{align*}
\text{xy}: & \quad x = y & [D: \text{quantum mechanical operators}] \\
\text{xy}: & \quad xy = yx & [“x commutes with y”: contingent here!]
\end{align*}
\]

Recall that, without some interpretation, some instillation of meaning, we could never decide whether a syntactical form is true or not.

Still, although Łukasiewicz's syntactic system allows for a variety of semantic interpretations, the most “natural” interpretation is:

\[
\begin{align*}
\text{xy}: & \quad \text{all } x \text{ are } y & [D: \text{Aristotelian terms/beings}] \\
\text{xy}: & \quad \text{some } x \text{ are } y & [D: \text{Aristotelian terms/beings}]
\end{align*}
\]

Whereupon a, b, and c recall the traditional S, P, M. Again, the four primitives are all true (on the modern interpretation, the truth of “some \( x \) are \( x' \)) requires that the domain D not be empty).

Hitherto in our logical study we have built upon the logical principle that variables take instances, so that we can work out our logical system independently of the instantiations. And claim to know, in advance, that our determinations hold for an infinite number of actual arguments. Knowledge by form precedes knowledge of instances.

However, we now have yet another order of instantiation: meanings for symbols. In this case we instantiate not by drawing upon *individuals* in a domain outside the system. Rather, we draw upon the formal meanings within our intellectual intuition: meanings generated by earlier considerations of the space-time grid or by our own logical work.

Now consider four more interpretations:

I. \(xAy: x \text{ is finitely } (> 0) \text{ to the right of } y\)

\[\text{xy}: x \text{ is finitely } (\geq 0) \text{ distant from } y.\]  
\[D: \text{points on a line}\]

II. \(xAy: x = y\)

\[\text{xy}: x \neq y.\]  
\[D: \text{integers}\]

III. \( xAy: x \) is at distance \( d \) (\( 0 \leq d \leq 1 \)) from \( y \)
\( xly: x \) is finitely (\( \geq 0 \)) distant from \( y \). \([D: \text{points on a plane}]\)

IV. \( xAy: x \leftarrow y \)
\( xly: x \rightarrow y \)  \([D: \text{sentential forms}]\)

When interpreting the four primitives in any of these four ways, we notice that each set of meanings generates only three necessarily true propositions. In each set, one is recognizably either false or contingent:

I. \( aAa \) is always false: no point on a line can be to the right of itself.
II. \( aIa \) is always false: no integer can be unequal to itself.
III. The third primitive is contingent: \( b \) can be as much as two units from \( a \).
IV. The fourth primitive is contingent: \((c \leftarrow b) \land (c \rightarrow a)\) → \((a \rightarrow b)\) is not a necessary truth.

If we wished to devise formal systems reflecting the semantics (consistent and sound systems in which we could affirm every theorem properly derived from the primitives), we would in each case assert only three of the four primitives. And, in each case, we would then discount those theorems requiring the employment of the false or contingent primitive.

And in this way we have perhaps proved the independence of each of the four from its three peers.

For if our sentential theorems and rules of inference can never generate non-necessary conclusions from necessarily true premisses, and if we are satisfied that, in each set, three primitives are necessarily true while each remaining one is not, then we could never, in all eternity, derive the remaining one from the other three alone.

§4.5.2 Bernays' syntactical system

Recall now the four primitives necessary for Russell's system:

\[
\begin{align*}
\neg(p \lor q) & \lor p & \text{Tautology} \\
\neg(q \lor (p \lor q)) & & \text{Addition} \\
\neg(p \lor q) \lor (q \lor p) & & \text{Permutation} \\
\neg(q \lor r) \lor \neg(q \lor q) \lor (p \lor r) & & \text{Summation}
\end{align*}
\]

The question is whether we can devise interpretations of the logical operators assuring us that each set of three has a life of its own to the exclusion of the fourth. In Aristotelian fashion, we would like to provide a counter-example showing that, for instance, the following is invalid:

1. \( \neg q \lor (p \lor q) \)
2. \( \neg(p \lor q) \lor (q \lor p) \)
3. \( \neg(q \lor r) \lor (p \lor q) \lor (p \lor r) \)
4. \( \neg(p \lor p) \lor p \)

However, unlike Aristotelian logic, our modern logic provides us with two rules that allow us to generate a potentially infinite series of lines. How can we rest assured that the "conclusion" will never appear at some point in the series?

As apparently first suggested by Paul Bernays\(^*\) in his habilitation of 1918, we may interpret the two logical operators, the curl and the wedge, with more than the two values semantically at issue in logical analysis (namely, true-false). For instance, consider the following three-value interpretation (also called "model"):

\[
\begin{array}{ccccc}
p & \neg p & p & q & p \lor q \\
0 & 2 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
2 & 1 & 0 & 2 & 0 \\
1 & 0 & 0 & & \\
1 & 1 & 0 & & \\
1 & 2 & 1 & & \\
2 & 0 & 0 & & \\
2 & 1 & 1 & & \\
2 & 2 & 1 & & \\
\end{array}
\]

We can see from the table on the following page that, on this interpretation of the operators, the three "premisses" all have the same property throughout their "worlds":

\[
\forall v (v \in W \rightarrow v = 0),
\]

while the "conclusion," namely Tautology, does not have this property,

\[
\exists v (v \in W \& v \neq 0).
\]

If we can show that every line derived from the three premisses has the

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property of those first three, we may conclude that no line with the second property could ever rightly appear—assuming that the three “premises” are consistent with one another (and this we have already proved). We must then prove the Induction step.

Our sentential calculus has only two Game Rules: Substitution and *modus ponens*. We must now consider carefully how each application of either of these rules bequeaths the property \( \forall v(v \in W \rightarrow v = 0) \) to the next line—assuming all previous lines have it.

Substitution always presupposes that formulas function as *schemata*: that the variables serve only as *place-holders* for propositions and propositional forms having *any* value. Thus, if a formula is a tautology in the classical (two-value) sense, the array of “worlds” accounts for all the possibilities, and Substitution in any line will produce a formula displaying exactly those possibilities already displayed in the original.

In contrast, *modus ponens* might in fact not bequeath the property enjoyed by any two previous lines. For instance, on the following interpretation:

\[
\begin{array}{c|ccc|c}
\text{right} & 0 & 1 & 2 & \sim \\
\text{t} & 0 & 0 & 0 & 2 \\
\text{f} & 0 & 0 & 1 & 0 \\
\text{e} & 1 & 0 & 0 & 1 \\
\text{i} & 2 & 0 & 0 & 1 \\
\end{array}
\]

all four primitives have the property \( \forall v(v \in W \rightarrow v = 0) \). Yet Excluded Middle, so easily derived from Tautology, Addition and Summation by Substitution and *modus ponens*, does not have this property (rather, it has values of both 0 and 1). Thus we must ask what condition must be met if *modus ponens* is to bequeath the value of its predecessors.

The operation of *modus ponens* takes the following form:

1. A \text{ Antecedent} \\
2. \sim A \lor C \text{ not-Antecedent or Consequent} \\
\therefore C

The induction hypothesis is that the two premisses both have the property in question. Under what conditions does the Consequent have that property? The generalized table expressing the conditions for any three-valued interpretation is:
where we can complete the table according to the definitions of the operators and assure ourselves that whenever \( A \) and \( \neg A \lor C \) have the preferred value of 0, so too does \( C \). That last (untenable) interpretation looks like this:

\[
\begin{array}{c|cccc|c}
A & \neg A & \neg A \lor C & C \\
0 & 1 & 0 & 1\\
0 & 0 & 1 & 0\\
0 & 0 & 1 & 0\\
\end{array}
\]

and we can see (in the second row) that modus ponens does not bequeath the preferred value.

On the other hand, if we complete the table for one that will in fact prove Tautology independent, namely:

\[
\begin{array}{c|cccc|c}
A & \neg A & \neg A \lor C & C \\
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
\end{array}
\]

we will discover that modus ponens does in fact bequeath the preferred value. Therefore, if every line preceding the application of modus ponens has the value 0, so too will the line resulting from the application.

With the Base Cases and now the Induction Step, we may conclude that every line ever generated from the three premisses and the two Game Rules will have the property of those first three lines. The fourth formula, Tautology, has a property contradicting this original property. Therefore, if the first three premisses are consistent on the normal interpretation (two values), the fourth will never show up in the system generated from the three.

Meta-theorems: Independence of Primitives

An exhaustive examination of all possible interpretations reveals that there are exactly six models meeting both the conditions and proving the independence of Tautology with 0 as the “preferred” value in three-value interpretations. Similarly, there are exactly eight interpretations proving that Addition is independent, and exactly four showing that Permutation is independent.

But there is none for proving Summation independent in the same manner. Bernays himself resorts to a four-value interpretation, namely:

\[
\begin{array}{c|cccc|c}
\lor & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
\end{array}
\]

Even with a computer program, the discovery of suitable four-value interpretations is very tedious. I here offer three more:

\[
\begin{array}{c|cccc|c}
\lor & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
\end{array}
\]

Yet Bernays offers another way of introducing interpretations (models): the use of two preferred (or distinguished) values. This he does (unnecessarily, as it turns out) to prove Addition independent in a four-value interpretation with two “preferred values, namely 0 and 2:

\[
\begin{array}{c|cccc|c}
\lor & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
\end{array}
\]

* Using James. B. Gerrie’s software program The Logical Theorist, one can discover all these interpretations (plus the one’s to follow) within several hours of dog labor. This program, which is mainly designed to facilitate the construction of systems of sentential and predicate logic of the sort illustrated by Principia Mathematica, may be downloaded from Dr. Gerrie’s website: http://www3.ns.sympatico.ca/jimgerrie/LOGIC53.ZIP
Constructing an entire table for each of the primitives, we would discover that the property bequeathed by the other three “premises” is:

\[ \forall v \in W \rightarrow (v = 0 \lor v = 2) \]

whereas the fourth formula, the unobtainable “conclusion” includes 1 as a value, and therefore has the contradictory property:

\[ \exists v \in W \land \neg(v = 0 \lor v = 2) \]

And the Induction Step requires that we consider eight rather than three conditions:

<table>
<thead>
<tr>
<th>A</th>
<th>~A</th>
<th>~A ∨ C</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>0</td>
<td>1</td>
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<td>2</td>
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<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The three “worlds” in which both the “premises” of *modus ponens* have one of the preferred values also yield “conclusions” having one of the two preferred values.

Now, we may apply the same principle of two preferred values to three-value interpretations, in which case we may discover that there are exactly six more proving Permutation independent. And an exhaustive survey of all the possibilities reveals that there are none of this type for the other primitives.

So all four primitives are independent. The proofs depend on the discovery of suitable properties along with suitable contradictions and proofs of bequeathal. There are a total of twenty-four such proofs for three-value interpretations, and who knows how many with four-value interpretations.

It should be noticed that the introduction of two preferred values requires us to abandon the last vestige of truth-functionality. While with only one preferred value, a three-value interpretation might appear to permit two forms of falsehood, with two we introduce systems in which decidable units (formulas, lines) have a life of their own, possibly enjoying one value and possibly another — a condition we associate with contingent propositions. And yet this dynamic feature in no way interferes with the necessity required of any system.

§4.5.3 *The recourse to syntax*

We can now give a stronger significance to the notion of “primitive”: not only do we posit primitives (Łukasiewicz’ four, Bernays’s four), we also need each of them if the system is to be complete.

However, a greater significance of the technique lies precisely in the peculiar power obtained by abandoning absolute reliance on the meaning of operators: in our examination of Russell’s four primitives, we might just as well have “marked on paper” ⊗ in place of the curl, and ⊙ in place of the wedge — and, as Bernays himself did, designate values with Greek letters (α, β, γ . . .) instead of numerals (0, 1, 2 . . .). Then, just as (following Łukasiewicz) we can assign entirely different meanings to the (meaningless) “marks” xAy and xLy, so too (following Bernays) we can assign entirely different meanings to the wedge and the curl. The choice of symbols/meanings is ours. Such neutral marks as ⊗ and ⊙ provide yet another domain of instantiation.

The peculiar power lies in the style of reasoning: reasoning with a view exclusively to the syntax. Classical logic, whether Aristotelian, Stoic, or modern, directs reasoning to material already available, namely to matters of concern already imbued with the contrast between truth and falsehood (political and scientific discourse). In contrast, the technique we have employed to prove primitives independent engages a reasoning that manufactures its own materials. Yet the reasoning itself, our own performance as distinct from the materials on which the performance focuses, remains classical in structure. For any proof of independence presupposes that the primitive formulas are tautologous on the classical model. Moreover, the bequeathed properties are formulated on the
classical model, as are their contradictories. And, of course, both Substitution and modus ponens represent classical inference.

Thus we are led to distinguish two discourses: the material discourse with purely syntactical definitions of the operators and sequence of lines, and the reflexive discourse with the classical (two-value) understanding of universality, contradiction, and inference.

This distinction between material and reflexive discourse allows us to decide questions regarding infinite sequences. Instead of trying to reason directly about truth values in an infinite sequence of logical operations, we manufacture a syntactically parallel sequence, one having nothing to do with truth, employ RMI, and conclude from the parallel proof back to the original infinite sequence. Paul Bernays does this not only to establish the independence of primitives but also to decide in advance whether (some) quantified formulas can be proved within a system (see below, §4.7).

In his famous essay of 1925, David Hilbert argues more elaborately for what I have illustrated in regard to the relatively simple question of independence:

Our principle result is that the infinite nowhere becomes real. Neither is it available in nature nor is it admissible as a legitimate basis for rational thought—a remarkable harmony between being and thinking. In contrast to the earlier efforts of Frege and Dedekind, we are convinced that certain intuitive representations and insights are necessary conditions for scientific knowledge, and that logic alone is not sufficient. Operating with the infinite can be made certain only by way of the finite.*

By “logic” Hilbert means the classical logic of universality, contradiction, and inference as constrained by direct focus on reality. By “certain intuitive representations and insights” Hilbert means ones that we can...

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Meta-theorems: The Dispensability of RCP

§4.6 Dispensability of RCP

Essential to Natural Deduction (ND) is the Rule of Conditional Proof. With this rule, we may not only derive theorems rather easily, e.g. Principia Mathematica’s Syllogism:

1. \( q \rightarrow r \) 
2. \( p \rightarrow q \) 
3. \( p \) 
4. \( q \) 
5. \( r \) 
6. \( p \rightarrow r \) 
7. \( (p \rightarrow q) \rightarrow (p \rightarrow r) \) 
8. \( (1) \rightarrow (7) \)

We may also comfortably apply the rules of ND to sets of empirical premisses to derive empirical conclusions—without bothering to close off the assumptions into one long conditional.

Axiomatic Deduction, on the other hand, does not recognize the technique of supposing an antecedent, deriving a consequent, and discharging the assumption to obtain the needed theorem as a conditional.

The question then arises: Under what conditions might we dispense with RCP without forfeiting the ultimate advantage of obtaining desired conditionals? From the standpoint of ND: Could we conceivably renounce, at some point, further use of RCP, without impairing our ability to prove conditionals? From the standpoint of AD: Could we conceivably work out on scratch paper a conditional proof of a desired theorem using RCP, and then translate the sequence back into our axiomatic system to obtain the same result without RCP?

---

* This retreat into reason marks much of the music and the painting typical of the twentieth century. Musicians and painters produce their own syntax—non-classical in the sense that it does not articulate the indication of anything already at issue, does not “mean” anything, point to anything—and yet work this syntax out all the more rigorously with the logic basic to classical composition. See Book Five, §3.1 Origin: structure vs. form, pp. 800 ff.

* “On the Infinite” (1925), differently translated in Philosophy of Mathematics, edited by Benacerraf and Putman (1983, p. 201). See also Hermann Weyl’s Philosophy of Mathematics and Natural Science (1949), especially pp. 50–66: “It cannot be denied that a theoretical desire, incomprehensible from the merely phenomenal point of view, is alive in us which urges us toward totality. Mathematics shows that with particular clarity; but it teaches us that that desire can be fulfilled on one condition only, namely, that we are satisfied with the symbol [representation] and renounce the mystical error of expecting the transcendent ever to fall within the lighted circle of our intuition.”
Formally, the question reads: Can we, from
\[ \Gamma \text{ plus } A \vdash B \]
est assured, by way of a meta-logical proof, that we could also obtain
\[ \Gamma \text{ minus } A \vdash A \rightarrow B, \]
where the gamma represents the background of accepted formulas, whether primitives, derived, or even contingent. The first formulation above represents the procedure of ND, where we then introduce, by way of RCP, what the second formulation pretends to derive without RCP.

To consider the question indifferently in regard to AD and ND (for present purposes, their sentential parts only), we may derive, with the rules of the latter, formulas representing seven of the eight ND-inference rules: &-intro. (both versions), &-elim. (both versions), v-intro. (both versions), v-elim. (also, to cover the introduction and elimination of \( \rightarrow \), the two formulas for double negation). Including these formulas in the \( \Gamma \) of ND, just as we include the primitives of Russell's *Principia Mathematica* in the \( \Gamma \) of AD, we reduce the number of rules to the same two for both systems: *modus ponens* and Substitution. The only real difference between AD and ND is that the one allows yet a third rule of inference, viz. RCP. The question is now: Do we really need it?

As always in the effort to settle a question about shapes that grow infinitely, we must envision a series corresponding to that of the positive integers. In the present instance, the series can be that of the number of lines in the derivation of \( B \) from \( \Gamma \text{ plus } A \). The first possible ND-derivation will be one-line long:
\[ \Gamma \]
\[ 1. \ A \]
\[ 2. \ B \]
where the antecedent formulas might be numbered as .01, .02, ...— so that the first line of any imagined ND-derivation will be numbered 1. The \( A \) stands for an assumption introduced in the spirit of ND, and the full question is now how we can obtain \( A \rightarrow B \) in AD.

Now, how might we have obtained \( B \) in ND? Given the limitation to our two rules, there are only three ways: (1) by some intermingling of formulas in \( \Gamma \) to form progeny without even using \( A \); (2) by employing \( A \) and some formula in \( \Gamma \) in a operation of *modus ponens*; and (3) by deriving (in this case of the briefest conditional proof) \( A \) itself (i.e. to produce the prototypical \( p \rightarrow p \)). What do we need to have at our disposal in order to derive, in each case and without RCP, \( A \rightarrow B \)?

Case (1): no use of \( A \).
1. \( B \) Present in \( \Gamma \)
2. \( B \rightarrow (A \rightarrow B) \) Needed Formula: call it *Redundance*
3. \( A \rightarrow B \) 1,2 *modus ponens*

What we here need in our \( \Gamma \) is the formula reading “having something entails that anything implies it.” This formula is immediately derivable in AD, and easily derivable in ND.

Case (2a): use of \( A \) with *modus ponens*, where \( A \) as the antecedent.
Trivial, since the formula in \( \Gamma \) must be precisely \( A \rightarrow B \), \( \Gamma \) minus \( A \) becomes \( \rightarrow B \), \( 1 \rightarrow (A \rightarrow B) \) Needed Formula: call it *Redundance*.

Case (2b): use of \( A \) with *modus ponens*, where \( A \) is a complex and \( B \) is detached from it by some other formula in \( \Gamma \).

Case (3): deriving \( A \rightarrow A \).
1. \( A \rightarrow [(A \rightarrow A) \rightarrow A] \) *Redundance*
2. \( A \rightarrow (A \rightarrow A) \) *Redundance*
3. \( (1) \rightarrow [(2) \rightarrow (A \rightarrow A)] \) *Ersatz-RCP*
4. \( (2) \rightarrow (A \rightarrow A) \) 1,3 *modus ponens*
5. \( A \rightarrow A \) 2,4 *modus ponens*

We have covered all possibilities of ND-derivations one-line long invoking an extraneous assumption, and shown that, with two extra formulas we can obtain the implication without employing RCP.

We may now formulate the hunch that we will soon have to test: that, with two formulas in our system, namely:
1. \( p \rightarrow (q \rightarrow p) \) *Redundance*
2. \( [p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)] \) *Ersatz-RCP*
we can get along without RCP. The two formulas will be used often in such routine sequences that we profitably formulate two derived rules:

\begin{align*}
\text{DR1: } & A \therefore B \rightarrow A \\
\text{DR2: } & A \rightarrow (B \rightarrow C), A \rightarrow B \therefore A \rightarrow C
\end{align*}

For the Induction Step we now assume that every derivation less than \(n\)-lines long may be obtained without RCP, and proceed to show that and how the next derivation can also be obtained without RCP.

The \(B\) at line \(n\) might in fact be obtained directly from lines already present in \(\Gamma\), in which case the proof-procedure employed in establishing the base cases suffices. The only problematic cases are those in which two intervening lines of the present sequence have been used in the application of \textit{modus ponens}.

The initial derivation (starting with the assumption of \(A\)) ends at some line \(n\), and may be called \(D_{n'}\). Then the two lines used to obtain the last crucial derivation (of \(B\): ignoring definitional transformations) may be called \(D_i\) and \(D_j\). Now, since our only rule is that of \textit{modus ponens}, one of these last two must take the shape of \(X \rightarrow B\), and the other must take the shape of \(X\). And both these we can, according to the induction hypothesis, obtain as consequents of \(A\). Starting, then, with these two implications (lines 1 and 2), we may proceed with dispatch:

1. \(A \rightarrow X\) \hspace{1cm} \text{Letting } D_i = X
2. \(A \rightarrow (X \rightarrow B)\) \hspace{1cm} \text{Letting } D_j = (X \rightarrow B)
3. \([A \rightarrow (X \rightarrow B)] \rightarrow [(A \rightarrow X) \rightarrow (A \rightarrow B)]\) \hspace{1cm} \text{Ersatz-RCP}
4. \((A \rightarrow X) \rightarrow (A \rightarrow B)\) \hspace{1cm} \text{2,3 modus ponens}
5. \(A \rightarrow B\) \hspace{1cm} \text{1,4 modus ponens}

Since \(B\) stands for whatever had been derived at point \(D_{n'}\), we have proved the Induction Step. Together with the base cases, we may then infer that no matter how long a derivation-by-assumption might be, we will always be able to construct an in-house proof, from \(\Gamma\) alone, of the corresponding conditional.

Once, that is, we have in our \(\Gamma\) the two special formulas. In the case of axiomatic deduction, we may either include the two as primitives (as many logicians, e.g. Łukasiewicz, have done) or derive them from the other primitives (e.g., as Russell’s *2.02 and *2.77).

As in the proof of the Rule of Replacement, proof of the dispensability of RCP elaborates a principle of \textit{construction}. We have demonstrated a reliable \textit{procedure} by which we may \textit{effect} a finite proof when needed. For an illustration, consider the formula for syllogism with which we began, eliminating one assumption at a time:

1. \(q \rightarrow r\) \hspace{1cm} A
2. \(p \rightarrow q\) \hspace{1cm} A
3. \(p \rightarrow (q \rightarrow r)\) \hspace{1cm} 1 DR1
4. \(p \rightarrow r\) \hspace{1cm} 3,2 DR2.

Now eliminating one more assumption:

1. \(q \rightarrow r\) \hspace{1cm} A
2. \(p \rightarrow (q \rightarrow r)\) \hspace{1cm} 1 DR1
3. \((p \rightarrow q) \rightarrow (p \rightarrow r)\) \hspace{1cm} 2,3 MP
4. \((p \rightarrow q) \rightarrow (p \rightarrow r)\) \hspace{1cm} 2. Ersatz-RCP

And, finally, eliminating the last remaining assumption:

1. \((q \rightarrow r) \rightarrow [p \rightarrow (q \rightarrow r)]\) \hspace{1cm} Redundance
2. \((q \rightarrow r) \rightarrow [\text{Ersatz-RCP}]\) \hspace{1cm} DR1
3. \((q \rightarrow r) \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]\) \hspace{1cm} 1,2 DR2.

The significance of the meta-theorem, called the Deduction Theorem, lies not in further asceticism in the employment of inference rules, but rather in the crossover between arguments and theorems, between the mundane and the transcendental. Since the proof of the converse of the Deduction Theorem, i.e. \(\Gamma\) \textit{minus} \(A \vdash A \rightarrow B\) implies \(\Gamma\) \textit{plus} \(A \vdash B\), is immediate, the two procedures are formally equivalent. Their formal equivalence illustrates not only the relevance of applying theorems to concrete arguments but also the possibility of recoiling from concrete arguments to prove the theorems necessary for dealing with the arguments.

A moral application: I rise in the morning with a number of principles gathered under \(\Gamma\). I come to believe that a condition \(A\) obtains in my environment: the car won’t start, or my boss has committed an outrage. From \(A\), now apparently secured in my \(\Gamma\), I derive \(B\), another proposal: I’ll call the garage, or I’ll submit my resignation. The Deduction Theorem formalizes the reminder that instead of straightaway concluding \(B\), I could conclude instead \(A \rightarrow B\). Now I may more easily suspend my judgement until such time as I have \textit{confirmed} that \(A\) in fact obtains.
§4.7 Decidability of predicate formulas

We prove a theorem by a sequence of inferences: by deduction, whether natural or axiomatic. In addition, a sentential formula we can decide, in advance of any proof, whether we should be able to prove it with our rules of inference: here, we construct a truth table and scan the possibilities to determine whether it holds in every possible world. The sentential calculus is, as we say, decidable: we have an effective, i.e. finite and mechanical procedure for deciding whether or not a formula holds no matter what the sentential variables take as instances.

A predicate formula might be negatively decidable: we might be able to devise a finite domain for which the formula fails to hold, and we might then judge the formula to be inherently contingent in form. I say “might” because our failure to devise a finite domain could stem from the inherent necessity of the form. Or it might stem from the formula's need to have an infinite universe of discourse.* And for trivial cases a predicate formula might even be positively decided, viz. if it is simply an instance of a sentential theorem, such as $\forall x Fx \rightarrow \forall x Gx$.

Because a quantified formula such as, e.g.:

$$\forall x (Fx \rightarrow P) \rightarrow (\exists xFx \rightarrow P)$$

naturally draws its arguments from an infinite domain we cannot straight off construct a truth table to decide it effectively. Instantiated for an infinite domain $a, b, \ldots \infty$, this last formula would appear as

$$[(Fa \rightarrow P) \& (Fb \rightarrow P) \& \ldots \infty] \rightarrow [(Fa \vee Fb \vee \ldots \infty) \rightarrow P],$$

and a truth table would require an infinite number of columns, and then also of rows.

As it turns out, however, there is an effective procedure for deciding formulas consisting of only monadic predications. In search of this procedure (of the hunch necessary for proving its universality), let us re-

* For instance, the following tripartite formula

$$\forall x \exists x Rxx \& \forall x \forall y \forall z [(Rx y \& Ry z) \rightarrow Rx z] \& \forall x \exists y Ry x$$

requires that in the domain there always be one more item than has been introduced so far. The formula describes the most fundamental property of the natural number sequence, with $Rxy$ reading “$x$ is greater than $y$”; it also holds for the universe of stags, with $Rxy$ reading “$x$ will vanquish $y$,” providing only that the herd perpetuates itself ad infinitum. — With these counter-examples we can see that the method we are looking for must be limited to monadic-predication formulas.

consider the formula above. Any effort to prove that it does not hold requires that we show the possibility of the consequent being false and the antecedent being true. While both parts of the formula extend over an infinitude of arguments, we can see that the consequent can only be false if $P$ is false and everything in the domain yields a true assertion when coupled with the predicate $F$; and under this condition of truth the very first conjunct of the antecedent is false, so that the formula itself is true. We have failed to devise a possible world in which the formula would not hold, we followed the only path possible for devising one, therefore the formula must hold for all possible arguments.

Intuitively, we can see in this first example that it makes no difference whether natural or axiomatic. In addition, a formula we can decide, in advance of any proof, whether we should be able to prove it with our rules of inference. Here, we construct a truth table and scan the possibilities to determine whether it holds in every possible world. The sentential calculus is, as we say, decidable: we have an effective, i.e. finite and mechanical procedure for deciding whether or not a formula holds no matter what the sentential variables take as instances.

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Meta-theorems: Decidability

This formula happens to hold for an infinite domain. But how can we be sure, without employing inferential rules? Here is where we need to think infinitude holistically:

Whatever the applicable domain, infinite or not, we know in advance that, as far as the two concepts go, each item in the domain falls into one of four classes:

- Class 1: $Fa \& Ga$
- Class 2: $Fb \& \neg Gb$
- Class 3: $\neg Fa \& Gb$
- Class 4: $\neg Fa \& \neg Gb$

where the Greek letters stand for anything in each of the respective classes; so that we could have said that, for the sub-domain of Class 1, $\forall x (Fx \& Gx)$, and so on for the other three classes.

Now, consider our formula again with a finite world of four individuals:

$$[(Fa \vee Fb \vee Fc \vee Fd) \rightarrow (Ga \& Gb \& Gc \& Gd)]$$

$$\rightarrow [(Fa \rightarrow Ga) \& (Fb \rightarrow Gb) \& (Fc \rightarrow Gc) \& (Fd \rightarrow Gd)]$$

The four individuals in this last formula allow for each of the four possibilities corresponding to the four classes into which the applicable
domain may be exhaustively sliced. We may then construct a truth table with these four, each row of which will represent one of the four classes. Each individual is then a delegate, and among the four of them all possibilities are exhausted, even for the infinite domain. So if the formula passes this table-test, it holds for the infinite domain as well; and if it fails the test it does not hold for any domain of four (possibly even less).

In this one case we seem to have tamed the infinite. And it seems clear that we might be able to generalize the decision procedure: count the number of concepts (Fx, Gx, ..., ) calculate the number of classes (subdomains) into which the (infinite) domain may be divided (the permutations being expressed in the formula $2^n$, with $n$ as the number of concepts), and instantiate the formula with a number of individuals equal to the number of classes (one individual as delegate for each class); whereupon we may employ the customary sentential techniques.

The chief shortcoming of our decision procedure so far is that we have not tried it out on an infinite series. Predicate formulas come in all sizes and shapes. To envision the infinite possibilities, we must imagine the construction of these formulas in an infinite series, the difference between any two points being defined by and confined to finite operations.

One way to formalize the procedure is to imagine five Transcendental Acts to establish the relevant Base Case, and then perform a sixth Act to establish the Induction Step.*

**Act One:**

From the chaos of possible well-formed predicate formulas, create an order. The number of quantifiers (either $\forall$ or $\exists$) will determine the point at which a formula will be defined. Thus at Point 1 we will have such formulas as $\forall x (\phi x \lor \neg \phi x)$ and $\exists x (\phi x \rightarrow \psi x)$, etc. At Point 2 we will have such schematic formulas as $\forall x \forall y \phi x y$, $\forall x \exists y \phi x y$, $\exists x \forall y \phi x y$, and $\exists x \exists y \phi x y$, where the schema $\phi x y$ might be $(\phi x \lor \psi y)$, $(\phi x \rightarrow (\psi y \land \xi x))$, etc. Evidently, at each point there will be an infinite number of forms, since the string indicated by the schema my go on without limit.

* I here follow the argument of Paul Bernays and Moses Schönfinkel, “Zum Entscheidungsproblem der mathematischen Logik.” in *Mathematische Annalen*, 99, 1928, 342-72; cf. §2 (the paper goes on to consider formulas with polyadic predications, some kinds of which are also decidable).

**Act Two:**

Create another series, exactly parallel to the first: viz., by taking for each formula in the first a corresponding Conjunctive Normal Form in the second. That each formula in the first series has an equivalent CNF in the second has been the subject of an earlier proof. For instance, the CNF of $\exists x \phi x \rightarrow \forall x \phi x$ is

$$\forall x \forall y (\neg \phi x \lor \phi y),$$

and the CNF of $\forall x \phi x \rightarrow \exists x \phi x$ is

$$\exists x \exists y (\neg \phi x \lor \phi y).$$

The purpose of assigning, imaginatively, definite meanings to the “marks on paper” is to assure ourselves that each application of a predicate to an item in the domain could result (imaginatively) in a clear yes-or-no decision. In other words, we must introduce the condition for each separate instance of $\phi x$ and the like being truth-functional.

**Act Four:**

Allowing now our predicates (not yet the formula containing them) to range over their domain, we notice mathematically that they will sort the items in the domain into piles, according to whether or not the item,
confronted with each of the distinct predicates, has or does not have the property imagined. The item may have all the properties (in which case it belongs to all the piles); it may have none (in which case it belongs to a general pile of rejects); and it may have any number in between. For a formula having three predicates, there will be eight piles. In general, the number of piles will equal \(2^n\), where \(n\) = the number of distinct predicates imagined (a number equal to the number of distinct predicate-variables syntactically evident in the formula).

The purpose of noting the power of sorting by independent predication over a domain is to create a new domain: that of the piles themselves. In technical terms, we may now consider as a domain the classes that are created by the (imaginary) sorting procedure. And, most importantly, these classes (piles) will always be finitely countable: any formula in our series is well formed, any well-formed formula has a finite number of distinct predicate-variables, the decision for any one predicate is binary, the total number of possible decisions (classes) will be \(2^n\), where \(n\) = the number of distinct predicates.

**Act Five:**

Now create another, a New Formula, one patterned on whatever formula we chose from those already created. The New Formula will have the same shape-on-paper as its original, but its domain will be the classes (piles) gerable for the original, and the predicate-variables will be ghostly counter-parts of the original: e.g., for the three above, “containing rocks each weighing less than one kilogram,” “containing rocks each with traces of silver,” and “containing rocks each inscribed with initials CW.”

Crucial to note is that the New Formula will be valid if and only if the original is valid. For the new predicates are the same as the original, except that they have been modified to apply grammatically to the classes (piles) instead of to the original individuals.

But of course there is a difference between the two sister-formulas: the New Formula ranges over a finite domain. We can therefore expand it to cover the entire domain, thereby dispensing with the quantifiers, and proceed to check its validity just as we would check the validity of a sentential formula. For instance, the formula \(\forall x \phi x \rightarrow \exists x \phi x\) in our first series reappears in our CNF series as \(\exists x \exists y (\neg \phi x \lor \phi y)\) which, expanded over \(2^1 = 2\) classes \(\alpha\) and \(\beta\), becomes

\[
[(\neg F_\alpha \lor F_\alpha) \lor (\neg F_\beta \lor F_\alpha)] \lor [(\neg F_\alpha \lor F_\beta) \lor (\neg F_\beta \lor F_\beta)],
\]

whereupon we see that the formula cannot be false. Unlike its contrary, \(\exists x \phi x \rightarrow \forall x \phi x\), the CNF of which is \(\forall x \forall y (\neg \phi x \lor \phi y)\) — which, when expanded over \(2^1 = 2\) classes \(\alpha\) and \(\beta\) becomes

\[
[(-F_\alpha \lor F_\alpha) \& (-F_\alpha \lor F_\beta)] \& [(-F_\beta \lor F_\alpha) \& (-F_\beta \lor F_\beta)],
\]

whereupon we can see that the formula can be false. In these two examples, we have determined by inspection the modality of the New Formulas. The modality in each case is the same as the modality of the corresponding original. Thus we have decided not just negatively, as in the second example: a procedure possible already in our first and second series. We have also decided positively, as in the first example: a procedure requiring that we apply our wits to the third series we created in parallel with the second.

Apart from the two examples, the imaginative (and transcendental) act can be repeated for formulas having any number of predicate-variables: no matter how many such variables, we can create a New (predicate) Formula corresponding exactly in modal status (necessary, contingent, or self-contradictory) to any original at Points 1 and 2 in the CNF series. It works for two such variables, and for every one more we double the number of classes forming the New Domain.

**Act Six:**

At any Point \(n\) there will be \(n\) quantifiers, according to the way we have imaginatively ordered the chaos of well-formed predicate formulas. The induction hypothesis is that the formulas at all points prior to and including Point \(n\) are valid if and only if their corresponding New Formulas are valid. And the question is whether this assumption warrants the inference that any formula at Point \(n + 1\) will be valid if and only if its corresponding New Formula is valid.

The inference is trivial. The difference between formulas at one point and the next is the addition of one more quantifier, either universal or existential. In the CNF series, the additional quantifier sits in front of a formula which itself is hypothesized to be decidable. For the string over which all the quantifiers range has the same number of predicate-variables and has the same configuration as some one string occurring at previous points; the major difference is that there is one more object-variable (assuming the additional quantifier has some work to do). The same procedure then applies to the string: the classes will be determined
as \(2^n\), where \(n\) is the number of distinct predicates. The other difference is that the expansion will double in size: with wedges if existential, with ampersands if universal. But if the formula is valid without this doubling, so too will be the formula with this doubling.

Most striking, perhaps, is the difference between this proof and the proofs regarding Replacement, Conjunctive Normal Form, and the Deduction Theorem. Each of these earlier proofs leaves us with a mechanical procedure for constructing what the proof claims can be constructed in all eternity. The present proof does not deliver any such procedure.

§5. The truth of induction: anticipation

The results are already astounding. The rule of inference invoked by Pascal, formulated as a rule by De Morgan, applied routinely in modern mathematics, and borrowed by logicians: this rule yields unwavering confidence in generalizations over ever-varying instances in a never-ending sequence. Such generalization answers to classical universality, and yet differs remarkably from it.

In classical logic, mathematics, philosophy and natural science, universality pertains to the sameness lurking in multiple instances. Leibniz could still speak of differences as only numerical (the number of \(\text{H}_2\text{O}\) molecules in a given cubic meter of air, the number of horses in a corral) and thus only functions of encounter (\(\alpha\iota\sigma\theta\iota\sigma\iota\zeta\); the reality itself is one and the same (each molecule, or each horse, is, in its intelligibility, identical to the other molecules, the other horses). Kant notes that this “principle of the identity indiscernibles” is “based on the presupposition that if a certain distinction is not to be found in the concept of a thing in general, then it is also not to be found in the things themselves.” Such universality ultimately implies transcendental form, i.e., the fixity of the destination of each instance encountered. Especially to us today, Socratic, Platonic, and Aristotelian induction then appears as “pedagogical”: either a means toward or a description of the learning of the universal—whether understood as the destiny of organisms (plants, animals, cities) attaining to or departing from their fulfillment, or (more in the modern vein) as the plan of the universe divinely pre-ordained.

The universality engendered by modern mathematical induction pertains to points in a series and allows for differences in predication at each point. Indeed, the chief question to be answered in such an induction is whether there is a difference from point to point and, if so, whether we can account for the difference as a function of the point itself. The Rule of Mathematical Induction generalizes over the differences, yielding a unity of form over the never-ending series. The operation itself, then, requires an intellectual engagement with points represented by the natural-number series—and thus with number itself, i.e., with progressive numbering (as when we number the days on which there’s to be a gift of grain, or the blocks in which there’s a cardinal to be inscribed, or the line of a proof at which there’s a new formula to be inferred).

Relying on the fact that the operation works, we ask in contemplation how it works. In Kantian fashion, we elicit the conditions of the possibility of engaging in the series formally structuring the days of the year, positions in a pyramid, or lines in a proof. And the formal structure is here that of number: 0, 1, 2, 3…

How are we to contemplate number? Number has always appeared to intellectuals as the prime evidence of the ubiquity of intellectual operations already in pre-intellectual (pre-philosophical, pre-academic) experience. We don’t even notice what we immediately encounter—don’t encounter anything at all—except as we reckon it as this one thing differing from those other things (differing in kind or in number) and yet either integrating itself with them into a unity or suggesting another unity: the simplest experience already contains the germ of number. Yet only intellectual engagement itself reveals the germ—reveals it already in the experience itself. In contemplation, philosophers then formulate the manner of the revelation: we encounter the thing in sensation (\(\alpha\iota\sigma\theta\iota\sigma\iota\zeta\)); we count, order, and in this sense discern things in intellecction (\(\delta\iota\nu\alpha\omicron\iota\alpha\eta\zeta, \nu\sigma\omicron\eta\iota\zeta, \nu\omicron\theta\zeta\)). Yet the question remains how, exactly, we best understand the status of number, and therewith of numbering: as reflecting transcendent or transcendental unity (roughly, the Platonic and the Kantian answers, respectively).

The classical understanding has the advantage that number ultimately (in everyday dealings: immediately) dissolves in the face of what the
number is numbering, namely a form. Thus both empirical determinations (blue, loud, sweet, . . .) and rational ones (double, odd, square) continue to refer to the “things themselves.” It follows, then, that reference to anything supposedly infinite or unlimited (τὸ ἀπειρον) — such as our modern mathematical induction seems to require — would send us to “something in itself,” thus something definite, i.e. delimited: a contradiction. However, Aristotle himself ponders this question rather than dismissing it—since “number, time, and magnitude” all require that we think the possibility of “one more.” He summarizes by referring to the fourfold of “causes” (ὕλη, κίνησις, τέλος, εἴδος):

Apparent is this: that it is as material that the infinite is a cause, i.e. that it is itself privation (στερησις) and that the subject all by itself is what’s compiled and sensory. . . . Thus it is absurd to make of it something that itself contains rather than what is contained. *Physics*, 208 a 1-3

When we speak of the infinite regarding number, time, and magnitude, we recall the ability of something to “go on,” and where there is ability there is also possibility — remarkably distinct from actuality, i.e. from anything we contemplate intellectually. The reference here is to compilations arising in ἀισθησις, “sensation,” i.e. as ὕλη, “material.” In sum: the infinite is “potential” — or is only potentially.

Modern mathematical thinking gradually shifted the reference of the study of number onto what we now call the Newtonian space-time grid, where the “always one more” refers to points in space and time (a remarkable confluence of the three “infinities”: “number, time, and magnitude”). Without the reference to either forms or sensations, and with intellectual attention directed solely to relations, this shift allows us to “contain” the infinite: most clearly, to formulate statements about what can happen throughout the Newtonian grid: to contain what can happen, and in this precarious sense “encompass it all.”

Still, such formulations as are permitted by RMI capture the “all” only as “all that can . . .”: the capturing, the containing, may be called “actual” — the actually infinite — but the “content” is precisely the ability (and therewith the inability of being otherwise) — the potentially infinite.

Mathematical induction locates the achievement of modern thinking generally: an intellectual formulation of possibility. Whereas the ancient understanding of what is commits us to what has already been and must be recollected, our modern understanding of what is commits us to what is coming and must be anticipated. Thus we might say that modern mathematical induction provides a representation of the kind of commitment essential to modern thinking. If so, the “substance” of which it is a representation might be kin to what Kierkegaard calls “repetition”:

. . . repetition is a decisive expression for what “recollection” was for the Greeks. Just as they taught that all knowing is recollection, so will modern philosophy teach that all living is repetition.

The question, at least, must be raised. For, as Aristotle says, there is no “substance” in mathematical work itself (no ὑβσία in arithmetic or geometry: *Metaphysics*, 1073 b 7)."

* Heidegger was once asked what he thought about “logistic” (another name for modern logic). He replied:

Logistic has nothing to do with philosophy. It’s pure calculation, a higher stratum of mathematics. It’s the mathematization of everything conceivable, applicable to everything possible. It holds universally and therefore gives the impression that it’s true. It plays an important role in mathematical work. The Europeanization [of the planet] entails the faith that logistic presents philosophy as it really is, that formulas can say anything at all about the essence of anything. Logistic has expanded to the point that it plays an uncanny role in mathematical research (computers). What Descartes began is here unfolding in an uncanny way. Even China will—probably in the next decades, or perhaps centuries—exist in the European manner, just as Japan: this is Europeanization, i.e. the essence of modern thinking (and not just in [computing] machinery but also in our kind of technology), a determination of humanity’s basic relation to being. And it’s my personal opinion that any change, any turn in our destiny, can only issue from here — from where this final condition of the planet today has emerged.

These are very loosely formulated comments, taken in shorthand by an auditor during a seminar in 1951 (unbeknownst to Heidegger, but later read and confirmed by him: *Gesamtausgabe* 15 [1986], pp. 437-8). Note that, although he starts by saying that “logistic has nothing to do with philosophy,” he ends by saying that “any change, any turn in our destiny, can only emerge from here.” That is, while simply “practicing” logistic will get us nowhere philosophically, contemplating it remains an essential condition for any change, any “re-brirth.” The philosophic task is to render this whole way of thinking once again transparent to its origin, first of all to the “basic relation to being” this way presupposes (roughly, the ontology of modern philosophy), and then possibly to the “matter itself” — all over again. Book Five resumes this topic.
Axiomatic Set Formation

Over and above the operators and rules of general logic, a theory of set formation requires that we work with one new primitive operator, that of membership: $M_{xy}, x$ is a member of $y$. Since both variables here take complicated substitution instances, clarity and calligraphy require that we introduce a neater linear symbol, $\in$, so that $M_{xy}$ appears graphically as $x \in y$ (just as $I_{xy}$ reads more conveniently as $x = y$).

The intuitive reading of membership is “belongs among.” Empirical instantiations lead to formulations which we may determine by inspection to be either true or false:

$S \in T$, “Smith is a member of the group of Teachers at Mount Allison” may be true, while $R \in C$, “red belongs among the colors {green, red, blue},” certainly is.

$N \in P$, “Napoleon is a member of the group enrolled in Philosophy” is false, as is $L \in V$, “laziness belongs among the virtues {honesty, courage, patience}.”

$3 \in \{3, 7, 12\}$, “3 is a member of set comprised of 3, 7, and 12” is true, as $D \in \{T_1, T_2, T_3\}$, “my desk at home is among the tables in this classroom” is false.

And we may even formulate necessary general truths:

$$\forall x \forall y(x \in \{x, y\}) \land (R \in C) \rightarrow \exists x(x \in C).$$

Expressed as $M_{xy}$, membership immediately appears as a dyadic relation. Unlike such relations as “$x$ is Identical to $y$” and “$x$ is Larger than $y$,” the membership relation does not enjoy many properties that are necessary a priori. Counter-examples show that $M_{xy}$ is not necessarily reflexive and not necessarily symmetrical, but proof that these properties are impossible for the membership relation, i.e. that $\forall x(x \notin x)$ and $\forall x \forall y(M_{xy} \rightarrow \neg M_{yx})$, will require a special axiom (the Axiom of Foundation) serving primarily this purpose. And although it is con-
Appendix III: Axiomatic Set Formation

ceivable that the relation be transitive (in a given society, being a member of a family might entail being a member of whatever the family is a member of), counter-examples show that it is not necessarily so: \( N \in C \) & \( C \in U \), “New Brunswick is a member of Canada and Canada is a member of the U.N.” is true, but \( N \in U \) is false.

Our intuition of the Membership relation depends heavily on our intuition of what counts as substitution instances of the variables \( x \) and \( y \). Certainly sets. Yet many intuitive examples of membership involve individuals: each student is a member of a class, an individual tree is a member of the collection of trees on my property. Yet it seems totally contrary to our intuition to suppose that anything, least of all a set, can be a member of an individual. It seems, then, that \( y \) must be restricted to sets.

When Ernst Zermelo introduced in 1908 the project of axiomatizing the theory of set formation, he solved the problem immediately by restricting all variables to sets. The advantage of this stipulation is that we might shorten our formalisms considerably (never having to note that certain maneuvers apply to sets only). The disadvantage is that we would leave behind a host of questions, and therefore a number of opportunities to contemplate the significance of set formation. Zermelo was primarily interested in transfinite sets (sets with infinite members: e.g. the set of integers, the set of points on a line). Now that the original project has worked itself out, we may take the broader view, posing questions that at the outset appeared incidental to the project and useless hindrances to its realization. And the core contemplative question is this: How does set formation fare at the interface, i.e. with individuals?

First, though: What is a set? A collection of items, we must say. But a collection that abstracts from considerations of how the items are related to one another. In contrast, then, an individual might be defined as whatever \( x \) has members; or:

\[ y \text{ proves itself to be a set if and only if it makes sense to ask for a decision whether there is some } x \text{ that is a member of } y. \]

Since fundamental to the theory of set formation is the axiomatization of a set having no members, i.e. of the Null Set, it is important not to define a set as what has members.

And we also say that once something counts as a set, we henceforth abstract from any consideration of interconnections among the members. This abstraction makes most sense when we confine ourselves to sets with finite membership like \( \{b,4,red,\{a,b\},honesty\} = \text{Stuff} \), which allows of five truths-by-inspection, including \( 4 \in \text{Stuff} \) and \( \{a,b\} \in \text{Stuff} \). Here we are obviously letting the membership define the set, unlike Plato’s and Aristotle’s definition of human beings as “what is animal and has λογος” (which sets the standards for membership). Paradoxically, however, we also define even finite sets in advance, e.g. the set of students at Mount Allison University under the age of 17; and also transfinite sets, e.g. the set of all numbers neatly divisible by 41. But the principle of extensionality (as it is called) requires that once a set is given (or has been constructed) its being relies solely on the elements it contains. This temporal consideration becomes essential to the resolution of the paradox engendered by the set of “impredicables.”

§1. The impetus for a theory of set formation

At the interface, we respond more or less intelligently to what we encounter. In the West the hallmark of such response is the imputation of a predicate belonging to a subject: \( P \) belongs to (or does not belong to) this-S-here, to some \( S \), or to all \( S \). We identify individuals this way, and we continue to respond to individuals according to our understanding of the form they illustrate. The shallow structure of this predication appears grammatically: there is a subject, and I impute a predicate to it. The deep structure of this predication we may state phenomenologically:
The Impetus for a Theory of Set Formation

something looms on my horizon and I address it in a way that aids or deters its presence. Philosophers from Thales onwards, but especially Plato and Aristotle, have debated the intricacies of predication at various levels, always in an effort to highlight our own responsibility at the interface. At issue in this responsibility is the intelligibility of our circumstances. And therefore also an interpretation of our own nature.

Already in late Scholastic philosophy predication of a being ("Horse is what is that under the tree!" or "Horses just don't respond well to that kind of treatment!") received an interpretation that makes of predication what we today call classification: S belongs to P ("There's yet another classification is what that is under the tree!" or "Horses just don't respond well to that kind of treatment!"). In a natural language these formulations may seem nearly equivalent in meaning, but between the original "P belongs to S" and the gradually evolved "S belongs to P" lies the difference between the beginning and the ending of Western philosophy. At the beginning, the verb "belongs" intends to take our own response back into what we encounter. At the end, the verb intends to take what we encounter back into our response. The difference between Aristotle and Kant. And a difference on the question of what we count as being primarily, secondarily, ... .

In mathematical terms we may formulate the difference as that between \( \varphi \) and \( x \), where \( \varphi \) mimics traditional predication (monadic: \( \varphi \) belongs to \( x \)). Imbued with modern thinking, we would like to believe that always, or most always, any predication in the form of \( \varphi x \) is a naïve way of formulating a classification, \( x \) belongs among the things we call \( \varphi \). The philosophical tendency authorizing this interpretation is called nominalism when the defenders of the tendency hold that \( \varphi \) is in each case “only” a name for the collection of \( x \)'s identically classified (and deny that the classification reflects a form, a reality setting the standard for its members).

We may even formulate in mathematical terms the transition from traditional predication to modern classification:

\[
\exists C \forall x (x \in C \leftrightarrow \varphi x),
\]

the generating part of which reads:

\[
\forall x (\varphi x \rightarrow \exists C (x \in C)).
\]

The bi-conditional version mathematicians often call the Principle of Abstraction, while the simpler conditional emphasizes how a set is constructed (abstracted) from \( \varphi x \). And \( \varphi x \) then reads more broadly as “\( x \) is a function of \( \varphi \)” (\( x \) takes on its values according to the recipe provided by \( \varphi \)). Inserting various functions, we form the various sets of special interest:

1. \( [(x \in A) \& (x \in B)] \rightarrow \exists C (x \in C), \)
   where the new set \( C \) is the intersection of \( A \) and \( B \), e.g. those things that are both animal and bony: \( A \cap B \).

2. \( [(x \in A) \& (x \notin B)] \rightarrow \exists C (x \in C), \)
   where the new set \( C \) is the difference of \( A \) and \( B \), e.g. those things that are animal but not bony: \( A \setminus B \) (often \( A - B \)).

3. \( [(x \in A) \lor (x \in B)] \rightarrow \exists C (x \in C), \)
   where the new set \( C \) is the union of \( A \) and \( B \), e.g. those things that are either poisonous or flammable: \( P \cup F \).

4. \( (x = x) \rightarrow \exists C (x \in C), \)
   where the new set \( C \) is the set that has every \( x \) as a member, similar to what Aristotle called a higher genus (e.g., one-ness, is-ness, \( \varphi \sigma ι \xi \)):

   a Universal Set.

5. \( (x = x) \rightarrow \exists C (x \notin C), \)
   where the new set \( C \) is the set having no members, the nothingness (le néant) Plato says belongs to everything that is \( \hat{1} \): the Null Set, \( \emptyset \).

6. \( \neg \varphi x \rightarrow \exists C (x \in C), \)
   where the new set \( C \) is the absolute complement of \( \varphi \): it will contain everything that is not \( \varphi \). For instance, there will be a set of all things that are not horses.

† Mathematicians nearly always express the broader sense of function by putting \( x \) in parentheses: \( \varphi (x) \). This notation highlights the role of \( \varphi \) as a function (at least a two-place relation) rather than a simple (one-place) predication. However, for the present purposes there is no need for the extra punctuation. Later thinkers like Soren Kierkegaard and Martin Heidegger have understood human beings as illustrating this nothingness most blatantly, if not exclusively.
Gottlob Frege is now known as the logician who, like Leibniz before him and Bertrand Russell after him, insisted that fully legitimate knowledge (modern science, in the current views of the time) required a script not only overcoming the ambiguities of natural language, but also laying out clearly the (preferably few) basic propositions and rules of inference necessary for all (but especially mathematical) reasoning. One of the hopes for such a system was that, secure at the basis, one could insert anything at all into the variables and generate logical truths without further worry. And one of Frege's basic propositions (Basic Law V) contained as a special case precisely the Principle of Abstraction from which the six formulas above are derived.

In 1902 Bertrand Russell wrote Frege a letter in which he pointed out that one function for \( \phi x \) would be: \( x \) does not have itself as one of its members, or \( x \notin x \) (unlike, for instance, the Universal Set generated above, or perhaps the set of all abstractions). Returning to the full formula of the Principle of Abstraction:

\[
\forall x(x \in C \leftrightarrow x \notin x),
\]

with the putatively existent \( C \) in place.

Now, since this formula holds for any substitution instance, we may substitute for all three occurrences of \( x \) in the formula the \( C \) putatively assured by the Principle:

\[
C \in C \leftrightarrow C \notin C,
\]

from which a contradiction immediately follows: \( C \in C \land C \notin C \).

Therefore, the Principle of Abstraction cannot serve as a reliable building tool in the construction of a system intending to formulate, with absolute precision, the innermost workings of intelligibility.

In retrospect, it is easy to say that we should have expected as much. Whereas the one half of the Principle, \( x \in C \), seems clear enough, the other half, \( \phi x \), takes as instances not only membership relations (1. through 3. above) but also identity relations (4. and 5.) and even banal monadic predications (6.) as instances — generating, in the last three cases, somewhat bizarre sets. And finally a self-contradictory set. But this is wisdom in hindsight. It is a measure of just how much intellectuals eventually valued the Fregean project that thinkers during the next decades aspired to develop systems that would serve the same or similar purpose. As Ernst Zermelo's.

§2. Zermelo's axioms

There are eight axioms basic to the theory of set formation:

- **Axiom of Extensionality:** \( \forall A \forall B [\forall x (x \in A \leftrightarrow x \in B) \to A = B] \)
- **Null Set Axiom:** \( \exists C \forall x (x \notin C) \)
- **Axiom of Separation:** \( \exists A \forall x \forall y [x \in C \leftrightarrow A \{ (y \in A) \land \phi x \}] \)
- **Pair Set Axiom:** \( \forall y \forall x \forall z [(x = y \lor x = z) \to \exists A (x \in A)] \)
- **Sum Set Axiom:** \( \forall A \forall x \forall y [\exists B (B \in A \land x \in B) \to \exists D (x \in D)] \)
- **Power Set Axiom:** \( \forall A \forall x [x \subseteq A \to \exists B (x \in B)] \)
- **Axiom of Infinity:** \( \exists A [\varnothing \in A \land \forall x (x \in A \to x^+ \in A)] \)
- **Axiom of Choice:** if \( \forall x [x \in M \to \exists z (x \in z)] \)

then \( \exists \forall x [x \in M \to f(x) \in x] \)

Each of these has a simple reading. Extensionality asserts the principle that sets are basically what they contain. Nullity allows for sets to have nothing in them. Separation (a schema) serves in place of the Principle of Abstraction, insisting only that the new set \( C \) be constructed (reversibly) out of a previously formed or formable set (without any employment of itself, of course: \( C \) must not be free in \( \phi x \)). Pairing reasserts in reverse the ancient principle that having two things means that we detect a unity against which to measure the diversity. Summing allows us to form a set out of the members of the sets of a set. Powering permits us to form a set of all the possible (unordered) combinations of the members of a set. Infinity postulates a set that will have the Null Set and then always one more set as elements. And Choice formalizes the principle that we may always distinguish one element within each set of a set of sets, and thereby form a further set comprising just these distinguished elements.

They all provide ample occasion for contemplation and, in addition, the tools sufficient for developing a vast field of logical and mathematical ideas. For our purposes, it suffices to flesh out each Axiom to show the direction it takes.

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* See the translations in the excellent volume *Conceptual Notation and Related Articles* (Oxford: Clarendon, 1972), the central work of which scholars often refer to as Frege's *Begriffschrift* (concept-scripture), first published in 1879. See also the partial translation of Frege's *Basic Laws of Arithmetic* (University of California Press, 1964); it is in this work that Frege proposed his Basic Law V, a limited case of which is the Principle of Abstraction in which Russell detected the possibility of generating a contradiction.
§2.1 Basic theorems for extensionality and nullity

The first task will be to establish that there is only one Null Set. The existential quantifier in all the axioms stipulates that there is at least one. In each case we will have to establish that there is only one.

The uniqueness of relevance in the theory of set formation is extensional, and we define it in exact parallel to the way Russell defines the uniqueness of an individual:

\[ \exists! \exists \{ \phi \text{S} \} \leftrightarrow \exists! \exists \{ \phi \text{S} \} \land \forall T(\{ \phi T \} \to T = S) . \]

The tactic in all uniqueness proofs is to invoke the condition for \( \phi \text{S} \), then to suppose the same formula with another variable (schematically: \( \phi T \)), and finally to show that the two (\( S \) and \( T \)) are extensionally identical. Thus, with \( \phi \text{S} \) as \( \forall x(x \notin C) \) we may prove:

**The Uniqueness of the Null Set:** \( \exists! \forall x(x \notin C) \)

1. \( \exists C \forall x(x \in C) \) Null Set Axiom
2. \( \forall x(x \notin A) \) Assumption: might there be another set like \( C \)?
3. \( \forall x(x \in C) \)
4. \( y \notin C \)
5. \( y \notin A \)
6. \( \lnot(y \in A) \land \lnot(y \in C) \) 4.5 Conjunction & \&-def.
7. \( \forall z(z \in A \leftrightarrow z \in C) \) 6 \( \lnot(p \land \lnot q) \to (p \leftrightarrow q) \), \&-intro.
8. \( A = C \) 7 Extensionality
9. \( \forall x(x \notin A) \to A = C \) 2 — 8 RCP
10. \( \forall x(x \notin A) \to A = C \) 9 \&-intro.
11. \( (3) \land (10) \) Conjunction
12. \( \exists C[(3) \land (10)] \) \&-intro.
13. \( \exists! \forall x(x \notin C) \) 12 \&-intro.

Having shown that there is only one such set described by the Null Set Axiom, we may introduce a special symbol for it and henceforth use it (definitionally) in place of the nondescript \( C \): \( \emptyset \). And now we may prove a working theorem:

**The Null Set Theorem:** \( \forall x(x \notin A) \leftrightarrow A = \emptyset \)

1. \( (x \notin \emptyset) \) Uniqueness of the Null Set, \&-elim.
2. \( \forall(x \notin A) \) Assumption
3. \( (x \notin A) \)
4. \( (x \in A) \to (x \notin \emptyset) \) SC: \( \lnot p \to (p \to q) \)
5. \( (x \notin \emptyset) \to (x \in A) \) SC: \( \lnot p \to (p \to q) \)

Zermelo’s Axioms

6. \( \forall x[(x \in A) \leftrightarrow (x \in \emptyset)] \) 3.5 Conj. def. of \( \leftrightarrow \), \&-intro.
7. \( A = \emptyset \) 6 Extensionality
8. \( \forall x(x \notin A) \to A = \emptyset \) 2 — 7 RCP
9. \( A = \emptyset \) Assumption
10. \( \forall x(x \notin A) \) 1.9 =-exch. \&-intro.
11. \( A = \emptyset \to \forall x(x \notin A) \) 9 — 10 RCP
12. Theorem 8, 11 Conj. def. of \( \leftrightarrow \)

And from this we obtain immediately a useful correlate:

\( A \neq \emptyset \leftrightarrow \exists x(x \in A) \)

We may now define a set freely, according to our intuition at the interface. For instance, something is a set if and only if either it has at least one member or it is identical to the Null Set:

\( y \) is a set \( \leftrightarrow \exists x(x \in y) \lor y = \emptyset \).

This definition allows for there being something that is not a set, e.g. an individual or an intensional whole.

§2.2 Basic theorems for separation

We may easily derive the existence of the intersection of two sets and prove its uniqueness:

**The Intersection of Two Sets**

\( \exists! \forall x[x \in C \leftrightarrow \exists A(x \in A) \land \exists B(x \in B)] \)

1. \( \exists C \forall x[x \in C \leftrightarrow \exists A(x \in A) \land \exists B(x \in B)] \) Separation \( \varphi x \) as \( \exists B(x \in B) \)
2. \( x \in C \leftrightarrow \exists A(x \in A) \land \exists B(x \in B) \) 1 \&-elim. \&-intro.
3. \( x \in D \leftrightarrow \exists A(x \in A) \land \exists B(x \in B) \) Assumption: another set?
4. \( x \in D \leftrightarrow x \in C \) 2.4 Replacement
5. \( D = C \) 4 Axiom of Extensionality
6. \( [x \in D \leftrightarrow \exists A(x \in A) \land \exists B(x \in B)] \to D = C \) 3 — 5 RCP
7. \( \forall D[(6)] \) 6 \&-intro.
8. \( (2) \land (7) \) Conjunction
9. \( \exists C \forall x[(2) \land (7)] \) 8 \&-intro., \&-intro.
10. Theorem 9 \&-intro.

In the language of classical logic, we might say that the set authorized by Separation and Extensionality is singular as distinct from both universal and particular. Russell’s definition of uniqueness mimics such singularity by a conjunction of particularity and universality — in keeping with the spirit introduced at the close of Scholastic philosophy, where the subject
of a singular proposition (Socrates is running) received an interpretation as a universal (All that is Socrates is running).

The logical significance of establishing the uniqueness of an existing set is that we may re-name it: just as we renamed as \( \emptyset \) the set \( A \) axiomatized as having no members. In the present case, we re-name \( C \), unique relative to \( A \) and \( B \), in a way reflecting its intuitive sense: \( C \) is itself that unique set, the membership of which is the same as the members shared by the existing sets \( A \) and \( B \). In brief: \( C = A \cap B \). To obtain a working version of Intersection, then, we may eliminate the uniquely-existential quantifier in favor of its new name and leave the universal quantifiers implicit:

\[
x \in (A \cap B) \iff [\exists A(x \in A) & \exists A(x \in B)].
\]

Now, given the conjunctive structure of the Axiom of Separation, we can easily repeat the same process for:

The Difference of Two Sets

\[
\exists ! C \forall x [x \in C \iff \exists A(x \in A) & \exists B(x \notin B)],
\]

which allows us again to re-name the unique (relatively singular) \( C \), employing the generally more workable \( A \sim B \) in preference to the visibly more accurate \( A \cap \overline{B} \), and thereby obtain a working version of Difference:

\[
x \in (A \sim B) \iff [\exists A(x \in A) & \exists B(x \notin B)].
\]

A special case of Difference occurs in the formation of a "complementary set" from a negated predicate. For example, with \( \varphi x \) as "non-black" or "non-mammal," we may proceed to form a set \( N \), but one separated out from an already-existing set (e.g., "animals"): \( x \in N \iff [\exists A(x \in A) & \sim \varphi x] \)

Our current Axioms permit no "absolute complement" (everything except...), as the Principle of Abstraction would seem to authorize. In Aristotle’s terms, we must always invoke a “higher genus” (\( \alpha νομα \ \alpha δος τον \)) when considering two items, whether comparing them (apples and oranges: fruit) or negating them (as in forming a complement).* 

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* Metaphysics, 1016 a 19. However, Aristotle calls an expression such as "non-men" an indefinite, or unrestricted name, \( \delta νομα \ \alpha δος τον \): On Interpretation, 16 a 31.

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The Axiom of Separation itself reveals properties paralleling those of Aristotle’s intricate discussions of definition by genus and differentia. Understanding anything (a set \( C \)) takes place against a background (a property, a \( \varphi \)). The background appears in Aristotle’s considerations as a source: a person stems from a family and must distinguish himself as an individual (by growing up). A human being is animal, but must distinguish his or her specific being by \( \lambda \delta γος \) (by speaking up articulately). Of course, in Aristotle’s account the task is ontological rather than merely formal: failure to distinguish things (or oneself) by some property \( \varphi \) leaves things a blur (and oneself merely an instance of one’s family, or merely animal): every individual (human or non-human) contains its genus as that back into which it falls when it fails to rise to itself, and sets the background against which the individual plays out its duration (death setting in when the individual falls away even from its generic being).

In formal terms, the Axiom of Separation simply requires that sets be formed out of a pre-existent set. But the Axiom also mimes the semantic requirement that we separate what we are talking about (our own system) out of a larger context, one that we move within and draw upon, but never completely formalize, in order to bear down on what we are talking about. Thus we will find the project of axiomatizing set formation (or any system) essentially frustrating if we aspire to formalize everything: it will be like trying to pull ourselves up by our own bootstraps (we must acknowledge our own stand, be grateful for it, rather than trying to establish it as well as what we do from it). As Alfred Tarski put it (Logic, Semantics, Metamathematics, p. 402):

> People have not been aware that the language about which we speak need by no means coincide with the language in which we speak. They have carried out the semantics of a language in that language itself and, generally speaking, they have proceeded as though there were only one language in the world. This analysis of the antinomies shows, on the contrary, ... that a language that contains its own semantics, and within which the usual logical laws hold, must inevitably be inconsistent.

* See his Metaphysics, iii, 3 (998 b 14) and v, 18 (1024 a 29), as well as the lengthy consideration (for debating) in his Topics iv, 1-2, and vi, 5-6.
For instance, if I tell you that everything I say is a lie, then you and I must understand this claim as pertaining to some set of utterances in which my present declaration does not figure as a member.

Whether interpreted ontologically, formally, or semantically, there seems to be a restriction on our ability to talk: we cannot rightly talk about everything. With two lemmas, we may easily prove that there cannot exist a set of all sets:

**The Finitude Theorem**

\[
\neg \exists U \forall x (x \in U)
\]

1. \( \exists U \forall x (x \in U) \) Assumption (for ~-intro.)
2. \( \exists U(x \in U) \) 1 \( \exists \)-elim., \( \forall \)-elim., \( \exists \)-intro.
3. \( \forall x \{x \in C \leftrightarrow [\exists U(x \in U) \& x \neq x]\} \) 2 Separation, \( \forall x \) as \( x \neq x \)
4. \( C \in C \leftrightarrow [\exists U(C \in U) \& C \neq C] \) 3 \( \forall \)-elimination
5. \( C \in C \leftrightarrow C \neq C \) 4, 2 Lemma A
6. \( C \in C \& C \neq C \) 5 Lemma B
7. Theorem 1 \( \rightarrow \) 6 RCP & \( \sim \)-intro.

Aristotle insists that we talk about things that are (so that individuals receive the honorific title of “primary”). He also insists that we cannot talk about everything: \( \tau \delta \pi \alpha \nu \) reduces to \( \pi \nu \nu \nu \alpha \) of something, an “allness” that signals full understanding only when directed to the \( \epsilon \theta \delta \alpha \zeta \) of individuals. The \( \epsilon \theta \delta \alpha \zeta \) of an individual, represents what makes it whole (\( \delta \lambda \alpha \nu \nu \nu \)), grounds the universal appearing in our responses to it: precisely as the particulars of something become clear, we address it in its wholeness: \( \tau \delta \kappa \alpha \theta \delta \lambda \alpha \nu \nu \nu \).

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* About \( \tau \delta \pi \alpha \nu \): In his *Posterior Analytics* (1, 5: 74 a 32) Aristotle argues that one who knows triangles will know what applies to each kind of them. In his *Metaphysics* (V, 26: 1024 a 1) he distinguishes “all” (\( \pi \nu \nu \nu \alpha \)) from “whole” (\( \delta \lambda \alpha \nu \nu \nu \)): any quantity has a beginning, middle, and an end, and therefore an all; but if it makes no difference what is where (an “unnurtured” set) it is only an all, whereas if it does matter what is where the quantity becomes a whole (an ordered set). Liquids are just alls, whereas things having a nature remaining the same through change (wax and a jacket) are wholes in addition to being alls. In his *Politics*, he underlines the doubleness of meaning of “all,” i.e. all (each) severally vs. all (together) commonly (the question is how people

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Plato (throughout his *Sophist*) and Aristotle (in his *Metaphysics*, XIV, 2:1089 a 15) emphasize the difficulties of talking about non-being. In contrast, we moderns simply posit the Null Set. In a sense, we start at the opposite end. It is then remarkable that so much of modern formalism mimes, or rather mirrors the logical principles of Antiquity.

**§2.3 Basic theorems for pairing**

The Pair Set Axiom, remember, reads:

\[
\forall x \forall y \forall z \forall x[(x \equiv y \& x \equiv z) \rightarrow \exists A(x \in A)]
\]

This formulation is equivalent to:

\[
\forall x \exists y \forall z \exists x[(x \equiv y \& \forall z(x \equiv z) \rightarrow \exists A(x \in A)]
\]

The Axiom mimes an essential act of intelligence: the recognition of sameness and difference, of two different things appearing as members of the same set. An empirical illustration: I claim that, whoever stole my logic book it was either Tom or Linda: I now construct a set such that whoever stole my book is a member of that set (pair). One formal illustration: we may instantiate all three variables with \( \emptyset \) to obtain \( \exists A(\emptyset \in A) \), and therefore also \( \exists x \exists A(x \in A) \), whereupon we assure ourselves that in our system there exists at least one non-empty set. Another: we may instantiate \( y \) and \( z \) with \( x \) to obtain \( \forall x \forall A(x \in A) \), whereupon we assure ourselves that in our system every set (and every individual) will be a member of some set.

We can fatten the Axiom into a bi-conditional in order to prove (routinely, now) that there is a set \( C \) that is unique relative to the variables \( y \) and \( z \):

**The Pairing Theorem**

\[
\forall x \forall y \exists C \forall x[(x \equiv y \& x \equiv z) \leftrightarrow x \in C]
\]

1. \( (x \equiv y \& x \equiv z) \rightarrow \exists A(x \in A) \) Pair Set Axiom
2. \( x \in C \leftrightarrow [\exists A(x \in A) \& (x \equiv y \& x \equiv z)] \) Separation
3. \( x \in C \rightarrow (x \equiv y \& x \equiv z) \) 2 SC: \( [p \leftrightarrow (q \& r)] \rightarrow (p \rightarrow r) \)

---

can affirm \( \sigma \delta \sigma \alpha \) as all their own—in which of the two senses this might be possible or impossible: each citizen affirming each \( \sigma \delta \sigma \alpha \) in the community, e.g. children, spouses, and dwellings, to take care of as his or her own, or everybody affirming everything collectively, therefore indifferently as public property.
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4. \((x = y \lor x = z) \rightarrow [\exists A(x \in A) \land (x = y \lor x = z)]\)

5. \((x = y \lor x = z) \rightarrow x \in C\)

6. \((x = y \lor x = z) \leftrightarrow x \in C\)

7. \((x = y \lor x = z) \leftrightarrow x \in D\)

8. \(x \in D \leftrightarrow x \in C\)

9. \(D = C\)

10. \(\forall D\left( [(x = y \lor x = z) \rightarrow x \in D] \rightarrow D = C \right)\)

11. \(\exists C(6) \land (10)\)

12. \(\forall y \forall z \exists C \forall x((x = y \lor x = z) \leftrightarrow x \in C)\)

Again, then, we may re-name the set \(C\) — conveniently, as \(\{y,z\}\) — and thereby eliminate the quantifier \(\exists! C\). This new name contains its own “quantifier” — that of singularity — and in this sense the braces bind the variables. The name recalls, by way of its constituents, its origin in the variables governed by the universal quantifiers. The working version:

\[x \in \{y,z\} \leftrightarrow [(x = y \lor x = z)].\]

The pair set is basic: it represents the fundamental act (the intellence) of making one out of two. And although we routinely talk of sets as having more than two members, we thereby bury the basic thought. Speaking more carefully, we define the unit set, \(\{x\}\), as a set of two items that are identical, a triplet set as one containing a pair and a unit set, a quadruplet set as one containing a pair of pairs, and so on:

\[
\{x\} =_{\text{def}} \{x,x\}, \\
\{x,y,z\} =_{\text{def}} \{x,y\} \cup \{z\}, \\
\{x,y,z,w\} =_{\text{def}} \{x,y\} \cup \{z,w\}.
\]

This procedure mimes the Platonic understanding of our ability to count things: numbers are (constructed, generated) from one-ness and from two-ness that is unrestricted.* Zermelo himself will represent the integers 0, 1, 2, 3, ... as \(\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \ldots\) : each of these is a pair that may be constructed by employing the Pair Set Axiom consecutively (thus we may construct an image for any given integer — not yet, however, for the set of integers, transfinite as this set will be).

The following theorem expresses formally the commutability we would associate with a simple pair:

\[
\text{The Unordered Pair Theorem} \\
\vdash [\{x,y\} = \{u,v\}] \rightarrow [(x = u \land y = v) \lor (x = v \land y = u)].
\]

1. \(\{x,y\} = \{u,v\}\)

2. \(u = u\)

3. \(u = u \lor u = v\)

4. \(u \in \{u,v\}\)

5. \(u \in \{x,y\}\)

6. \(u \lor u = y\)

7. \(v = x \lor v = y\)

8. \(x = u \lor x = v\)

9. \(y = u \lor y = v\)

10. \(x = y \lor x \neq y\)

11. \(y = y\)

12. \(x = y\)

13. \(x = x \lor u = x\)

14. \(v = v \lor v = y\)

15. \(v = y\)

16. \(x = u \lor y = v\)

17. \((x = u \land y = v) \lor (x = v \land y = u)\)

18. \((x = y) \rightarrow (17)\)

19. \((x \neq y) \rightarrow \vdash [(x = u \land y = v) \lor (x = v \land y = u)]\)

20. \(x \lor u = y\)

21. \(x = y\)

* Aristotle discusses this view in his *Metaphysics* (XIII, 7-9): δ ζητο ντριμος και της αοριστου (1081 a 15). For a summary of the ancient accounts of Plato’s lost lecture “On the Good,” in which Plato himself seems to have spoken in these terms (one-ness and unrestricted two-ness), see Konrad Gaiser’s article in *Phronesis* (XXV, 1 [1980], pp. 5 ff.). The peculiarities of Greek grammar permit two readings of δοριστου: unrestricted or unrestricted, un-terminated or in-terminate. In the philosophical discussion, the word takes on the positive meaning of open-ended-ness, an openness essential to what we now call mathematical induction (given any point \(n\), there is a successor, a point \(n + 1\). Ontologically, the thought might be illustrated by the a child learning that there is another mother, not only its own, so that mothers (anything: friends, horses, mountains) come in sets (kinds, forms: a second instance driving one toward this sometimes sad, sometimes joyous thought).
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Zermelo’s Axioms

Notice that \( \varphi \leftrightarrow D \) does not follow, namely where \( \varphi \) is false as well as \( C \): when, that is, there is nothing satisfying the requirement stipulated by \( \varphi \). Thus we can see how Separation acts as a prophylactic.

We can then re-name the unique set as \( \bigcup A \), “the union of the set \( A \),” and form the now-familiar working version:

\[ x \in \bigcup A \leftrightarrow [\exists B(B \in A \land x \in B)]. \]

And with both the Pairing and the Summing Theorem we may establish the constructibility of the union of two sets:

The Union Theorem

\[ \forall A \forall B \forall x[(x \in A \lor x \in B) \leftrightarrow \exists C(x \in C)] \]

1. \( x \in A \lor x \in B \) Assumption
2. \( x \in A \) Assumption
3. \( A = A \) Identity-intro.
4. \( A = A \lor A = B \) 3 Addition
5. \( A \in \{A,B\} \) 4 Pairing Theorem
6. \( A \in \{A,B\} \land x \in A \) 5.2 Conjunction
7. \( D[D \in \{A,B\} \land x \in D] \) 6 \( \exists \)-intro.
8. \( x \in \bigcup\{A,B\} \) 7 Summing Theorem
9. \( x \in A \rightarrow x \in \bigcup\{A,B\} \) 2—8 RCP
10. \( x \in B \rightarrow x \in \bigcup\{A,B\} \) parallel with 2—9
11. \( x \in \bigcup\{A,B\} \) 1.9,10 \( \lor \)-elim.
12. \( \exists C(x \in C) \) 11 \( \exists \)-intro.
13. Theorem 1—12 RCP, Q-intros.

As in the previous cases, we may easily fatten this theorem into a bi-conditional, prove that the generated set is unique, and then re-name it to reflect its uniqueness:

The Union of Two Sets

\[ x \in (A \cup B) \leftrightarrow [(x \in A \lor x \in B)]. \]

§2.4 Basic theorems for summing

The Sum Set Axiom, remember, reads:

\[ \forall A \forall x[\exists B(B \in A \land x \in B) \rightarrow \exists D(x \in D)]. \]

This formulation is equivalent to:

\[ \forall A \forall B \forall x[(B \in A \land x \in B) \rightarrow \exists D(x \in D)]. \]

This Axiom represents our ability to examine the subsets within a set and to gather together into a new set all the members we discover in the subsets. Empirically: the United Nations has countries as members, and each country has members (say, provinces); the Axiom authorizes us to do what we know we can do, namely to form another set comprising all the provinces of member-countries (bypassing the initial countries).

Again, we must fatten the Sum Set Axiom into a bi-conditional in order to establish the uniqueness of the new set it authorizes:

The Summing Theorem

\[ \forall A \forall !C \forall x[\exists B(B \in A \land x \in B) \leftrightarrow (x \in C)] \]

The proof follows the same pattern as the proof of the Pairing Theorem and others to come:

- \( \varphi \rightarrow D \) a uni-conditional (axiom or theorem)
- \( C \leftrightarrow (D \land \varphi) \) Separation
- \( \vdash C \leftrightarrow \varphi \) from the sentential & predicate calculi

§2.5 The basic theorem for powering

The Power Set Axiom, remember, reads:

\[ \forall A \forall x[x \subseteq A \rightarrow \exists B(x \in B)]. \]

The new symbol, \( \subseteq \), represents inclusion: \( (x \subseteq A) \) reads “whatever is a member of \( x \) is also a member of \( A \).” Such is the symbol that logicians use to express the extensional interpretation of All S are P: anything
Appendix III: Axiomatic Set Formation

included under the heading S is also included under the heading of P (so that S and P lose their original intensional meaning and now designate sets). The formal definition of inclusion reads:

\[ A \subseteq B \iff \forall x (x \in A \rightarrow x \in B). \]

The Axiom allows us to form a new set \( B \) by sorting through a set \( A \) in search of all the possible (unordered) selections of its members. Interpreted as miming an act of intelligence, the Axiom states that having a collection of items, we may choose to “take” any combination of the items from none to all; and that we may construct a new set containing as members all these possible “takes.” The number of such items is equal to \( 2^n \), where \( n \) is the number of items in the original set; thus the name of the Axiom.

Empirically: if there are three items on a dinner menu, the Power Set of the menu has eight items (from ordering nothing to ordering all three); Canada is a set of ten provinces, so there are 1,024 possible trips (including a stay-at-home trip).

Formally, it is evident from the definition of inclusion that we can already prove both \( \emptyset \) and the original set itself to be members of the Power Set of any set.

To establish a more flexible working theorem, we must prove both the bi-conditional and uniqueness:

**The Power Theorem**

\[ \forall A \exists ! C \forall x [x \subseteq A \iff x \subseteq C] \]

Then, inserting a proper name for the unique set \( C \), \( \emptyset A \), we obtain the working theorem:

\[ x \in \emptyset A \iff x \subseteq A. \]

With an understanding of \( \emptyset A \), and also the assumption that we can rightly form the set of all integers (a transfinite set), one can show that the power set of the set of integers is larger than the set of integers. Cantor then developed a theory of transfinite numbers on this basis: the next infinite number “after” the first one (which Cantor designates as \( \aleph_0 \)) is \( 2^{\aleph_0} \).

§2.6 The basic theorem for infinity

Hovering over all set formation is the presumption that sets may have infinite memberships. After all, the universal quantifiers spread over domains of any size. Yet our theory intends to account for, or at least hold in abeyance our understanding of domains.

For the purposes of mathematics, we must in any case postulate the existence of a transfinite set: that’s what numbers are all about. With the natural numbers in mind, we may postulate a set comprised of members as follows:

There exists a set having one “seed” member, and for any member in this set there is a “next” one, its successor.

This formulation certainly mimics the natural number series, with “0” (or possibly “1”) as its seed member.

Instead of employing any new notion as the seed member, we may make use of the only other existing set we have axiomatized absolutely: the Null Set. The Axiom of Infinity then reads:

\[ \exists A[\emptyset \in A \& \forall x (x \in A \rightarrow x^+ \in A)], \]

where \( x^+ \) reads “the successor of \( x \).” Any set of this sort is called “inductive.” In the case of the natural numbers, we know what “inductive” means: for any number \( n \), there is a number \( n + 1 \). In the case of our infinite set, we must define the successor as a set formation.

Zermelo proposed one definition: \( x^+ = \{x\} \). On this definition, our infinite set would begin:

\[ \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, ... \}, \]

whereupon we could define 0 as \( \emptyset \), 1 as \( \{\emptyset\} \), and each number as the set with the corresponding number of brace-pairs.

Von Neumann proposed another definition: \( x^+ = x \cup \{x\} \). On this more commonly used definition, our infinite set would begin:

\[ \{\emptyset, \{\emptyset\}, \{\emptyset,\{\emptyset\}\}, \{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\}, ... \}, \]

whereupon we would especially need to recall the definitions of the unit set and the triplet set.

Von Neumann’s definition of “successor” invites us to define the natural numbers as follows:

\[ 0 = \emptyset \]
\[ 1 = \{\emptyset\} = \{0\} \]
\[ 2 = \{\emptyset, \{\emptyset\}\} = \{0,1\} \]
\[ 3 = \{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\} = \{0,1,2\}, \]
whereupon the pattern becomes clear: each number so defined not only contains the corresponding number of elements, it also comprises each of its predecessors—which accords well with the ancient understanding of number as amount. Solely with our previous tools of set formation (no need of our new Axiom yet), we can prove the following:

\[
\begin{align*}
1 &\in 2 \\
1 &\neq 2 \\
1 &\subset 2
\end{align*}
\]

It appears, then, that we have a set of items corresponding to the natural numbers. However, this set is not formally ordered: the items are not yet ordinals. Nor can we perform the usual arithmetic operations with them: they are not yet cardinals. Nor, in fact, have we proved that we have a unique set of these motley items.

Let us backtrack. And define (“abstractly”: see below, §5) the set of natural numbers (“omega”) as the set containing those elements that are members of any inductive set \(B\) “starting” with \(\emptyset\):

\[
\omega = \{ n : \forall B([\emptyset \in B \& \forall x(x \in B \rightarrow x^+ \in B)] \rightarrow n \in B) \}
\]

where we insert a definition of the successor function. The question is now whether there is a unique set of such \(n\). What we wish to prove is:

\[
\exists! W \forall n [n \in W \leftrightarrow \forall B([\emptyset \in B \& \forall x(x \in B \rightarrow x^+ \in B)] \rightarrow n \in B)].
\]

In line 2 we draw upon our new Axiom for the first and last time in this proof. And now we prove, in the customary manner, a bi-conditional version of line 6:

\[
\begin{align*}
1. & \forall B([\emptyset \in B \& \forall x(x \in B \rightarrow x^+ \in B)] \rightarrow n \in B) \quad \text{Assumption} \\
2. & n \in A \quad \forall \text{-elim.} \\
3. & \exists D(n \in D) \quad \exists\text{-introduction} \\
4. & (1) \rightarrow \exists D(n \in D) \quad \text{RCP}
\end{align*}
\]

The proof of uniqueness also follows the customary pattern. And with uniqueness assured, we can formulate our result succinctly—with the conventional omega as the name for the set of natural numbers:

\[
n \in \omega \leftrightarrow \{ \forall B[\emptyset \in B \& \forall x(x \in B \rightarrow x^+ \in B)] \rightarrow n \in B \}.
\]

We now have a set whose membership permits the development of number theory: ordinals and cardinals, and the usual operations associated with each.

§2.7 A basic theorem for choice

Since Zermelo, and even by him, the Axiom of Choice has been formulated in several different ways. For all the differences, Zermelo summarizes its significance informally:

...it is always possible to choose a single element from each element \(M, N, R, \ldots\) of \(T\) and to combine all the chosen elements, \(m, n, r, \ldots\) into a set \(S\).\(^*\)

The Axiom posits two different claims: first, it is possible to determine a “distinguished element” in any non-empty set \(B\) and, second, these special elements can be gathered into a set of their own. This double claim may be formulated as a rule holding for any set \(T\):

\[
\forall x \in T \rightarrow (x \neq \emptyset \& \forall y[(y \in T \& y \neq x) \rightarrow \neg \exists z(z \in x \& z \in y)])
\]

\[
\therefore \exists \forall v \exists x \in T \exists a \forall v[v = a \leftrightarrow (v \in s \& v \in x)]
\]

\(^*\) From Zermelo’s 1908 “Investigations in the foundations of set theory I” (collected in From Frege to Gödel, edited by Jean van Heijenoort, Harvard [1967], p. 204). The formal statement reads:

If \(T\) is a set whose elements are all sets that are different from \(\emptyset\) and mutually disjoint, its union \(\bigcup T\) includes at least one subset \(S\) having one and only one element in common with each element of \(T\).

In his “New proof of the possibility of a well-ordering” (also 1908), Zermelo summarizes again:

...a simultaneous choice of distinguished elements is in principle always possible for an arbitrary set of sets. ibid, p. 186

Finally, Zermelo’s earlier (1904) version reads:

...even for an infinite totality of sets there are always mappings that associate with every set one of its elements; or, expressed formally, ... the product of an infinite totality of sets, each containing at least one element, itself differs from \(\emptyset\). ibid, p. 141
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The “premiss” states that every element in the set $T$ is non-empty and shares none of its elements. The “conclusion” states that for any $x$ of $T$ there exists at least one singleton set $s$ containing a unique element $a$ that is also an element of $x$ of $T$.

This version, requiring that the elements of $T$ be disjoint, serves as a principle of arithmetic multiplication: a set of 3 things and a set of 5 things permits us to choose one item at a time from the set of 3 and pair it with items, one at a time, from the set of 5, and finally to form a set of all these pairs, one having 15 items (pairs).*

Other versions do not require that the elements of $T$ share no elements; for instance, the post-Zermelo version presented in the original list:

For any set $T$ not containing $\emptyset$, there exists a choice function $f$ (i.e., a function whose domain is $T$) such that, for each member $x$ of $T$, $f(x) \in x$.

\[ \forall x [x \in T \to x \neq \emptyset] \]

\[ \exists f [\forall x [x \in T \to f(x) \in x]] \]

To derive this shorter rule from the longer one above, we must call upon the notions of function and Cartesian product, notions dependent on the Pair Set and the Sum Set Axioms.†

For our purposes, we need only a simple consequence of the Axiom of Choice, the barest version of which reads:

\[ \forall x [(x \in T \& x \neq \emptyset) \to \exists a x[a \in x \& a \in s]] \]

With this version, and where $(x \sim \{a_i\})$ is the set of the elements in $x$ less its “distinguished element,” we may prove the Choice Theorem—trivially true unless we assume $M \neq \emptyset$:

**The Choice Theorem**

\[ \forall x [x \in \emptyset \rightarrow (x \sim \{a_i\}) \in \emptyset M] \]

1. $x \in \emptyset M$ Assumption

* Russell and Whitehead already noted the need for a Multiplicative Axiom of this sort. See Russell's “On some difficulties in the Theory of Transfinite Numbers and the Order of Types” (1906), reprinted in Essays in Analysis.

† For a detailed derivation, see Foundations of Set Theory, by Fraenkel, Bar-Hillel, and Levy (North Holland, 1958, 1973), p. 56. Notice that a function, especially in mathematics, already implies order.

---

The Foundation Axiom

2. $x \subseteq M$

3. $\forall x (y \in x \rightarrow y \in M)$

4. $\exists x \exists y (a \in x \& a \in s)$

5. $a_i \in x$

6. $\exists x (z \in x \& z = a_i)$

7. $z \in \{a_i\}$

8. $\{a_i\}$ is a set

9. $y \in x \rightarrow y \in M$

10. $\forall y \left[ (y \in x \& \sim (y \in \{a_i\})) \rightarrow y \in M \right]$

11. $(x \sim \{a_i\}) \subseteq M$

12. $(x \sim \{a_i\}) \in \emptyset M$

13. theorem

1 — 12 RCP \( \forall \)-intro.

At line 9 we can see that any difference of $x$ will be an element of the power set. But the differentiating set must exist: this existence is first assured by the Axiom of Choice.

Since $M \in \emptyset M$, our Choice Theorem allows us to generate a sequence of sets, starting with $M$ and eliminating one element at a time, by recursion to the Axiom of Choice. Thus, where

$M' =_{\text{def}} M \sim \{a_i\}$,

each successive element is bereft of the distinguished element of its predecessor, and we may visualize an “inverted staircase”:

\[
\begin{array}{c}
M \\
M' \sim \{a_i\} \\
M'' \sim \{a_{i-1}\} \\
\vdots \\
\end{array}
\]

If $M$ is finite, we will reach the point where a “step” is empty, and then our recursive appeal to the Axiom of Choice no longer works. On the other hand, if $M$ is infinite, we might visualize the right-hand gap indicating the bereft-ness as halving itself at each step downwards.
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§3. The foundation axiom

If the Principle of Abstraction did not lead to a contradiction, it would legitimately provide us with nearly all our Axioms as theorems. Forced to abandon it in favor of the Axiom of Separation, we must introduce axiomatically the others. Each of these then highlights, and allows us to contemplate, basic suppositions on our part.

But what does it mean to suppose these Axioms? In a deductive system we suppose beginnings in order to march ahead toward endings. Devoting ourselves primarily to deductions, we naturally resist the temptation to retreat into a contemplation of beginnings, since the challenge is to advance. In contrast, and for whatever reason, a philosophical mind yields to the temptation.*

The Axioms formulate basic acts of intelligence. They explicitly suppose the priority of Nullity (over being) and of Extension (over form), and also suppose the centrality of creating new sets from old sets with invented differentiations, of creating a single set from the detection of a duality, of cutting through subsets to form a new set out of their members, and of sifting through a set to form a new one from all possible combinations of its members. And all these suppositions draw upon and give meaning to a single beginning, the act of membership, symbolized by $\in$.

Membership is primitive. That is, we shall not try to define it in formal terms. To do so would only introduce something else as primitive. The question is whether we can think ourselves back into it, recalling it as we move on.

One thing (individual or set) belongs among others, counts as a member of a collection, is illustrating a set. We recognize things this way, both empirically (this is a woman, this woman is my wife) and formally ($\in$ belongs to the set of our symbols, Identity belongs to the set of reflexive relations). We also understand the behavior of things this way, again both empirically and formally: we anticipate and remember things at least partly according to how they fit or fail to fit into some type (a pattern of disease, a pattern of deduction). We make sense out of situations, or fail to do so, according to whether and how we judge them to be situations of a kind. In a very formal way, membership represents the principle of synthesis, something necessary for the barest intelligibility at any moment: even to make a mistake we must have hazarded a membership judgement, and be girding ourselves for another, for a revision. For something is “given” only as we “take” it—in the taking of it. This give and take is even prior to asserting a relation.*

The Axioms then deserve the status Kant accords to “synthetic a priori propositions,” and the membership relation symbolizes the originary transcendental act. Yet if we choose to understand the Axioms and their origin in Kantian fashion, we might also remember that the Axiom of Extensionality, absolutely necessary for the development of the system, places a very serious restriction on the notion of membership, and turns it into a mirror image of the broader form-instance (one-place) relation which gave birth to philosophical concerns in Antiquity.

Now, there is something negatively special about the membership relation. The identity relation quite clearly entails the rules we devise to employ it (that we may always introduce instances of the formula $x = x$, and that from $x = y$ and $\varphi x$ we may infer $\varphi y$), and it is an easy matter to prove with these rules the other properties that belong to it, such as reflexivity (but also symmetry and transitivity). In contrast, our intuition of membership precludes reflexivity (the possibility that any set could be a member of itself). Yet the system so far developed remains quite consistent if we leave such involution in the modality of contingency.

Consistent but weird. Consider the simple example of some set $\{A\}$, about which we could assert $A = \{A\}$: since $A \in \{A\}$, we could then obtain, by identity-exchange, not only the undesired reflexivity, $A \in A$, but also a strange symmetry, $\{A\} \in A$. Most weirdly, perhaps, such a set would allow, by successive identity exchanges, the formation of an

* In Plato’s famous image, mathematicians march down from hypotheses, whereas philosophers move up from them (Republic, 510B). In his Metaphysics, iv, 3 (1005 a 25), Aristotle claims that all (all mathematicians?) make use of axioms, but non-philosophers do so only as much as suffices for their own work, i.e. for their demonstrations within their own $\sigma\nu\varepsilon\varphi\alpha\varsigma$. To turn toward the axioms is to be a philosopher.

* In his Metaphysics (1009 b 25) Aristotle reports on a report that Anaxagoras would tell his followers that things would be according to how they take them: $\epsilon\sigma\tau\alpha\tau\alpha\vartheta\alpha\alpha\rho\alpha\sigma\pi\alpha\lambda\delta\beta\mu\alpha\nu;\;\varsigma\sigma\alpha\iota\alpha\eta\alpha\rho\alpha\sigma\pi\alpha\lambda\delta\beta\mu\alpha\nu;\;\varsigma\sigma$ see also Plato’s Theaetetus 166A ff. Aristotle formulates reservations regarding these considerations, not because they are wrong, but rather because they do not adequately pave the way toward understanding how membership can appear normative (intensional) and thereby cannot justify assertions of an intellectual sort.
Appendix III: Axiomatic Set Formation

An infinitely involuted chain of formulas: \( \{\{A\}\} \in A \), then \( \{\{\{A\}\}\} \in A \), and so on (as well as a symmetrical counterpart for each of these). The conditions we would have established would replicate formally what we may experience concretely when facing two opposing mirrors placed in such a way that every image contains an image of itself.

If one contemplative purpose of a theory of axiomatic set formation is to formalize our intuitions, then we must find a way of asserting \( \forall x (x \notin x) \). Success here might also lead to a better formal understanding of why it makes no sense to talk of the set of all sets or the idea of ideas. Like perhaps Love and God, we can only have instances or moments of the thing itself, and not a super overview.

How might we preclude the reflexivity of \( \in \)? If we simply state \( \forall x (x \notin x) \) as an Axiom, we will not prevent an analogous counter-intuitive circumstance in the form of the conjunction \( x \in y \wedge y \in x \) (symmetry of the membership relation). We could preclude this axiomatically, but what would prevent longer versions of deferred involution?

In 1930 Zermelo proposed a single Axiom that solves the problem in one fell blow: every set having one or more sets as members has at least one set-member sharing nothing with the original:

**The Foundation Axiom**

\[ \forall A \{A \neq \emptyset \implies \exists x \in A \wedge \forall y (y \in x \implies y \notin A)\} \]

If we restrict all variables to sets (i.e., exclude individuals), we may formulate the Axiom more simply:

\[ A \neq \emptyset \implies \exists B \{B \in A \wedge (B \cap A) = \emptyset\}. \]

If \( A \) has only empty sets, then \( A \) itself (being non-empty) cannot figure as a member. If, on the other hand, \( A \) has one or more non-empty sets as members there must be at least one (so says the Axiom) that shares nothing with \( A \); for instance, \( \{A\} \) is not empty, so the \( A \) in this set must not share anything with it!

**The Foundation Theorem:** \( \forall x (x \notin x) \)

1. \( A \in \{A\} \) from the Pairing Axiom: \( A \) not restricted
2. \( \{A\} \neq \emptyset \) 1 Null Set Theorem
3. \( \exists y \in \{A\} \wedge \forall y (y \in x \implies y \notin \{A\}) \) 2 Foundation Axiom
4. \( z \in \{A\} \wedge \forall y (y \in z \implies y \notin \{A\}) \) 3 \( \exists \)-elim. (\( z \) restricted)
5. \( z = A \) 5 from the Pairing Th.
6. \( z = A \)

From the Foundation Theorem we can easily prove that nothing, whether set or individual, can be identical to the unit set of itself:

\[ \forall x (x \neq \{x\}). \]

This theorem parallels in formal terms a thought fundamental to the development of modern mathematical logic: the use of a term remains essentially distinct from the mention of it. Every horse is a mammal with naturally four legs, but the moment we turn to the word “horse” we begin talking about an English noun with five letters: \{horse\} is a member of the set \{\{paper\},\{tilde\},...\}; horse [used] \neq “horse” [mentioned]. Similarly, in our formal language we put the membership symbol to use (horses \( \in \) \{cows, cats, ...\}), but the moment we turn to the symbol “\( e \)” we begin talking about items in our calculus: as a two-place operator, \( \{\} \in \{\rightarrow,\forall,\leftrightarrow,\&\},\{\},\{\},\{\},\{\} \} \) each member of which is out of work at the moment. The difference between using and considering a word (a symbol, even a hammer) parallels the difference between language levels (between an object language and a meta-language). It also parallels the difference between engaging directly with something and conceiving of it at a remove: \( \emptyset \) describing a set differs from \( \{\} \), which might be interpreted as the thought of nullity. 

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**With the Foundation Theorem we may also prove that \( \notin \) (more formally: “\( e \)”) is asymmetrical:**

\[ \begin{align*}
7. & \forall y (y \in z \implies y \notin \{A\}) & \text{4 Simplification} \\
8. & \forall y (y \in A \implies y \notin \{A\}) & \text{6,7 Identity-exchange} \\
9. & A \in A \implies A \notin \{A\} & \text{8 } \forall\text{-elim.} \\
10. & \neg(A \in A) & \text{1,9 modus tollens} \\
11. & \text{Theorem} & \text{10 } \forall\text{-intro.}
\end{align*} \]

\[ \forall x (x \neq \{x\}). \]

In his *Introduction to Logic* (Minneapolis, 1966), a 13th-century textbook, William of Sherwood lists about a dozen ways we may speak of man (human being), including and properly distinguishing such ways as “man is mortal,” “Man has set foot on the moon,” “Man suffers cancer,” but also including and distinguishing such ways as “Man is a monosyllable” and “Man is a species.” From our modern standpoint, this 13th-century author is confusing use and mention. However, in pre-Cartesian literature words are modes of address in which both our responses and what looms for us figure as abstractions, elements analyzed out of a primal unity. Only with Bacon, Galileo, and Descartes did the project of establishing the drama of intelligibility primarily within our own responses replace the old Anaxagorean and Protagorean thought: ἐστὶν τὰ δόντα οἷς ὑποληφθήσειν.
In the spirit of mathematical induction, we may infer that no matter how long the string of conjunctions, once it loops back to form an involution we may demonstrate its falsehood. Such is the generality provided by the Foundation Axiom.

The Foundation Axiom brings the operation of membership into close association with our intuition on at least three different scores (reflexivity, symmetry, and however we might name $\neq$). Yet the formulation of the Axiom does not correspond to our intuition as clearly as does $x = x$ for “=". Perhaps the closest we can come to expressing the intuition in English (our meta-language, relative to the axiomatic system) is this: every non-empty set must have something else in it; if only individuals, then of course the set itself cannot be a member; and if one or more sets, then one of these must have something not shared with the original. This formulation then parallels that of phenomenology (Aristotelian, Husserlian, or Heideggerian): perception (whether noetic or aesthetic) must be of something — which does not preclude the danger that it may recoil, disintegrate, become null and void.

With these basic theorems we may now proceed to develop a system, a family of progeny.\textsuperscript{*}

§4. An elementary calculus of sets

First, recall the definition of inclusion:

$$A \subseteq B \iff \forall x (x \in A \rightarrow x \in B).$$

Theorem 1: $A \subseteq A$

Theorem 2: $(A \subseteq B \& B \subseteq A) \rightarrow A = B$

Theorem 3: $A \subseteq \emptyset \rightarrow A = \emptyset$

1. $A \subseteq \emptyset$ Assumption
2. $x \in A \rightarrow x \in \emptyset$ Definition of Inclusion
3. $x \notin A$ Null Set Axiom & modus tollens
4. $A = \emptyset$ Null Set Theorem
5. $A \subseteq \emptyset \rightarrow A = \emptyset$ 1 $\rightarrow$ 4 RCP

Theorem 4: $(A \subseteq B \& B \subseteq C) \rightarrow A \subseteq C$

\textsuperscript{*} With omissions and additions, I have followed the sequence of Theorems, 1 through 120, in Patrick Suppes’ Axiomatic Set Theory (1960, 1972).

Proper inclusion we may define as inclusion where the two sets are not identical:

$$A \subset B \iff A \subseteq B \& A \neq B$$

Theorem 5: $\neg(A \subset A)$

Theorem 6: $A \subset B \rightarrow \neg(B \subset A)$

1. $A \subset B$ Assumption
2. $A \subset B \& A \neq B$ Definition of proper inclusion
3. $B \subset A$ Assumption (for RAA)
4. $B \subset A \& B \neq A$ Definition of proper inclusion
5. $A = B$ 2, 4 Simplifications & Theorem 2
6. $A \neq B$ 3 $\rightarrow$ 6 RCP & $\neg$-intro.
7. $\neg(B \subset A)$ 1 $\rightarrow$ 7 RCP
8. $A \subset B \rightarrow \neg(B \subset A)$ 1 $\rightarrow$ 7 RCP

Theorem 7: $(A \subset B \& B \subset C) \rightarrow A \subset C$

Theorem 8: $A \subset B \rightarrow A \subseteq B$

With Intersection

Theorem 9: $A \cap B = B \cap A$ Intersection

1. $x \in (A \cap B) \leftrightarrow x \in A \& x \in B$
2. $x \in (A \cap B) \leftrightarrow x \in B \& x \in A$ Commutation
3. $x \in (A \cap B) \leftrightarrow x \in (B \cap A)$ Intersection
4. $A \cap B = B \cap A$ Extensionality

Theorem 10: $(A \cap B) \cap C = A \cap (B \cap C)$

Theorem 11: $A \cap A = A$

Theorem 12: $A \cap \emptyset = \emptyset$

Theorem 13: $A \cap B \subseteq A$

Theorem 14: $A \subseteq B \leftrightarrow A \cap B = A$

1. $A \subseteq B$ Assumption
2. $x \in A \rightarrow x \in B$ Definition of inclusion
3. $x \in A \rightarrow (x \in A \& x \in B)$ SC: $(p \rightarrow q) \rightarrow [p \rightarrow (p \& q)]$
4. $(x \in A \& x \in B) \rightarrow x \in A$ SC: $(p \& q) \rightarrow p$
5. $A \cap B = A$ 3, 4 Equiv. & Intersection
6. $(1) \rightarrow (5)$ RCP
7. $A \cap B = A$ Assumption
8. $x \in (A \cap B) \leftrightarrow x \in A$ Definition of intersection
9. $x \in A \rightarrow x \in (A \cap B)$ Equivalence & Simplification
10. $x \in A \rightarrow (x \in A \& x \in B)$ Intersection
11. $x \in A \rightarrow x \in B$ Re-Absorption
12. \( A \subseteq B \)  
13. \((7) \rightarrow (12)\)  
14. \( A \subseteq B \leftrightarrow A \cap B = A \)  

With Union

Theorem 15: \( A \cup B = B \cup A \)
Theorem 16: \( (A \cup B) \cap C = A \cup (B \cap C) \)
Theorem 17: \( A \cup A = A \)
Theorem 18: \( A \cup \emptyset = A \)
Theorem 20: \( A \subseteq B \leftrightarrow A \cup B = B \)

1. \( A \subseteq B \) Assumption  
2. \( x \in A \rightarrow x \in B \) Definition of inclusion  
3. \( x \in (A \cup B) \leftrightarrow (x \in A \lor x \in B) \) Union  
4. \( x \in (A \cup B) \leftrightarrow x \in B \) [\((p \rightarrow q) \& (p \lor q)] \leftrightarrow q \)  
5. \( A \cup B = B \) Extensionality  
6. \((1) \rightarrow (5)\) RCP  
7. \( A \cup B = B \) Assumption  
8. \((x \in A \lor x \in B) \rightarrow x \in B \) Union, Equiv. & Simpl.  
9. \( x \in A \rightarrow x \in B \) from \([p \lor q] \rightarrow r \rightarrow (p \rightarrow r) \)  
10. \( A \subseteq B \) Definition of inclusion  
11. \((7) \rightarrow (10)\) RCP  
12. \( A \subseteq B \leftrightarrow A \cup B = B \) 6.11 Conj. & Equiv.

Theorem 21: \( A \subseteq C \& B \subseteq C \rightarrow (A \cup B) \subseteq C \)
Theorem 22: \( (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \)
Theorem 23: \( (A \cap B) \cup C = (A \cup C) \cap (B \cap C) \)

With Difference

Theorem 24: \( A - A = \emptyset \)
Theorem 25: \( A - (A \cap B) = (A - B) \)
Theorem 26: \( A \cap (A - B) = (A - B) \)
1. \( x \in [A \cap (A - B)] \leftrightarrow (x \in A \& x \in A \& x \not\in B) \)  
2. \( A \cap (A - B) = (A - B) \)  
Theorem 27: \( (A - B) \cup B = A \cup B \)
Theorem 28: \( (A \cup B) - B = A - B \)
Theorem 29: \( (A \cap B) - B = \emptyset \)
1. \( x \in [(A \cap B) - B] \leftrightarrow (x \in A \& x \in B \& x \not\in B) \)

§5. Defining sets abstractly

Before proving further theorems directly based on our fundamental axioms, let us pause to consider once again the formation of sets “at will.” Especially now that \textit{kinds} no longer seem grounded in nature, we often wish to form a set containing those elements satisfying a stated (usually complicated) requirement we ourselves devise. This we may already do with the Axiom of Separation. But we may also start by referring to a set in the form \{\(x: \varphi x\)\} — the set of those items defined “abstractly” by \(\varphi\). For instance, we may wish to imagine a set \(\mathcal{H}\)
containing as elements things having the properties of being human, being male and being Canadian: \{x: Hx & Mx & Cx\}.

However, we have learned that not every dream of a set yields an extensionally distinct set, i.e. one having members uniquely its own. We might dream of a set of people 100-feet Tall to form the set \{x: Px & Tx\} and also of a set of people one millimeter Short to form the set \{x: Px & Sx\}. Yet both these sets are empty (let us assume) — and therefore identical, namely to the Null Set.

So, to make use of defining sets “by abstraction,” we must be careful to retain in our formalization of the process the possibility that the sets we describe may in fact be empty. And we do this by proving a theorem that we can understand as another, albeit now legitimate progeny of the ill-fated Principle of Abstraction:

The Abstraction Theorem

\[ \exists! C [\forall x (x \in C \leftrightarrow \phi x) \lor [C = \emptyset \& \neg \exists B \forall x (x \in B \leftrightarrow \phi x)]] \]

The proof proceeds without any use of theorems regarding sets. Assuming the contradictory of the non-unique version, one quickly obtains two formulas:

\[ \forall C [\neg \forall x (x \in C \leftrightarrow \phi x)], \text{ and} \]
\[ \forall C [C \neq \emptyset \lor \exists B \forall x (x \in B \leftrightarrow \phi x)], \]

from which one may easily derive a contradiction. Assuming another set \( D \) meeting this description, one may conjoin the two formulas and, by successive applications of Distribution, obtain a string of four disjunctions, each disjunct of which consists of a conjunction: two of these contain self-contradictions and the two others yield \( D = C \).

Now, from this our Abstraction Theorem we may very easily derive two working theorems:

Theorem 33: \([x: \phi x] \neq \emptyset \rightarrow \exists! C \forall x (x \in C \leftrightarrow \phi x)\]

The contrapositive of which reads (with no need of uniqueness):

\[ \neg \exists C \forall x (x \in C \leftrightarrow \phi x) \rightarrow \{x: \phi x\} = \emptyset. \]

Theorem 34: \( \exists B \forall x (x \in B \leftrightarrow \phi x) \rightarrow \exists! C \forall x (x \in C \leftrightarrow \phi x)\)

Although we may freely christen any unique non-empty set with a name of our own choice (consonant with inherited conventions), we also have a generalized way of defining — naming — it at its birth:

\[ \{x: \phi x\} = \{x \leftrightarrow_{\text{def}} \forall x (x \in N \leftrightarrow \phi x) \lor [N = \emptyset \& \neg \exists B \forall x (x \in B \leftrightarrow \phi x)]\].

For instance: \([x: Hx \& Mx \& Cx] = \emptyset \]

or \([x: (x \in A \& x \in B)] = A \cap B\).

The first we test to determine whether it deserves a name distinct from the Null Set. The second we fact establish as a formula in our system. Moreover, since \([x: \phi x] = [x: \phi x]\), we also have a Handy Version:

\[ \forall x (x \in \{x: \phi x\} \leftrightarrow \phi x) \lor [x: \phi x] = \emptyset \& \neg \exists B \forall x (x \in B \leftrightarrow \phi x)]. \]

The following two theorems illustrate the basic meaning of defining by abstraction:

Theorem 35: \( y \in \{x: \phi x\} \rightarrow \phi y \)

1. \( y \in \{x: \phi x\} \) \hspace{1cm} Assumption
2. \( \neg ([\{x: \phi x\} = \emptyset]) \) \hspace{1cm} 1 Null Set Theorem
3. \( \forall x (x \in \{x: \phi x\} \leftrightarrow \phi x) \) \hspace{1cm} 2 DS-maneuver on the Handy Version
4. \( y \in \{x: \phi x\} \leftrightarrow \phi y \) \hspace{1cm} 3 \&-elim.
5. \( \phi y \) \hspace{1cm} 1.4 Replacement
6. \( y \in \{x: \phi x\} \rightarrow \phi y \) \hspace{1cm} 1 — 5 RCP

Theorem 36: \([x: x \in A] = A\)

Proof of this last makes use of the fact that there does exist a set \( B \) such that any element either in it or in \( A \) is also in the other — namely, \( A \) — so that, again, we may perform a DS-maneuver on the Handy Version.

Taken together, the next two make explicit the finitude built into our axiomatic system: the results are the same whether we try to form a set of things not identical to themselves or to form a set of things that are identical to themselves.

Theorem 37: \([x: x \neq x] = \emptyset\)

Theorem 38: \([x: x = x] = \emptyset\)

1. \( \exists B \forall x (x \in B \leftrightarrow x = x) \) \hspace{1cm} Assumption (for RAA)
2. \( x \in B \leftrightarrow x = x \) \hspace{1cm} \exists\text{-elim. \& } \forall\text{-elim.}
3. \( x \in B \) \hspace{1cm} Identity \& Replacement
4. \( \exists B \forall x (x \in B) \) \hspace{1cm} \forall\text{-intro. \& } \exists\text{-intro.}
5. \( \neg (4) \) \hspace{1cm} Finitude Theorem
6. \( (4) \& (5) \) \hspace{1cm} Conjunction
7. \( (1) \rightarrow \text{contradiction} \) \hspace{1cm} 1 — 5 RCP
8. \( \neg \exists B \forall x (x \in B \leftrightarrow x = x) \) \hspace{1cm} \neg\text{-intro.}
9. \( [x: x = x] = \emptyset \) \hspace{1cm} Theorem 33 (contraposition)
The next three theorems allow us to contemplate Intersection, Union, and Difference under the rubric of Abstraction.

Theorem 39: \( A \cap B = \{x : (x \in A \land x \in B)\} \)

1. \( \forall (x \in A \cap B \leftrightarrow (x \in A \land x \in B)) \) Intersection
2. \( (1) \lor ((A \cap B) = \emptyset \lor \exists C \forall \lambda \{x : (x \in A \land x \in B)\}) \) Add.
3. \( A \cap B = \{x : (x \in A \land x \in B)\} \) Definition by Abstraction

Theorem 40: \( A \cup B = \{x : (x \in A \lor x \in B)\} \)

Theorem 41: \( A \sim B = \{x : (x \in A \land x \notin B)\} \)

The last theorem in this section formulates a thought crucial to the development of set theory: if we have two properties that are equivalent in application, then the sets defined abstractly with each of the two properties are extensionally identical.

Theorem 42: \( \forall x(\phi x \leftrightarrow \psi x) \rightarrow \{x : \phi x\} = \{x : \psi x\} \)

1. \( \forall x(\phi x \leftrightarrow \psi x) \)
2. \( \phi x \leftrightarrow \psi x \)
3. \( \{x : \phi x\} = \{x : \psi x\} \)
4. \( \forall x(x \in \{x : \phi x\} \leftrightarrow \phi x) \lor \{x : \phi x\} \equiv \emptyset \lor \exists B \forall x(x \in B \leftrightarrow \phi x) \)
5. \( \forall x(x \in \{x : \phi x\} \leftrightarrow \psi x) \lor \{x : \phi x\} \equiv \emptyset \lor \exists B \forall x(x \in B \leftrightarrow \psi x) \)
6. \( \{x : \phi x\} = \{x : \psi x\} \)
7. \( \forall x(\phi x \leftrightarrow \psi x) \rightarrow \{x : \phi x\} = \{x : \psi x\} \)

Theorems 37 and 38 provide the clue to understanding why Theorem 40 cannot take the form of a bi-conditional. In general, though, Zermelo's theory of set formation establishes a formally evident tension between the conceiving of properties possibly belonging to individuals and the conceiving of sets to which individuals might belong.

And now we return to the main developments...

§6. Single-set union and intersection

The Summing Theorem allows us to form from any set \( A \) another set, its union, comprising whatever elements are in the immediate sets of \( A \). (The union of a set containing only individuals will be empty.)

Theorem 43: \( \bigcup \emptyset = \emptyset \)

1. \( x \in \bigcup \emptyset \leftrightarrow \exists B(B \in \emptyset \land x \in B) \) Summing Theorem
2. \( B \in \emptyset \) Null Set Axiom
3. \( B \in \emptyset \lor x \in B \) Addition
4. \( \forall B(B \in \emptyset \lor x \in B) \) \( \forall \)-intro.
Appendix III: Axiomatic Set Formation

Yet on this formulation, the intersection of a set that is empty would seem to contain everything. Thus there can be no theorem that states unconditionally the existence of a (non-empty) \( \cap A \) for any \( A \). Our first theorem regarding membership will display this dependence on the existence of a member-set of the set in question:

**Theorem 52:** \( A \neq \emptyset \rightarrow \forall x(\cap A \rightarrow \forall B(x \in A \rightarrow x \in B)) \]

Actually, the unconditional,

\[
\forall x(\in A \rightarrow \forall B(x \in A \rightarrow x \in B)),
\]

does not require the condition that \( A \) be empty: with \( \cap A \neq \emptyset \) one may proceed with the Abstraction Theorem, as in Theorem 41. The reverse conditional requires a bit more attention:

1. \( A \neq \emptyset \) Assume
2. \( \forall B(x \in A \rightarrow x \in B) \) Assume
3. \( \exists D(x \in D) \) \( 1 \) Null Set Theorem
4. \( \exists D(x \in D) \) \( 3,2 \) \( \exists \)-elim., \( \forall \)-elim., MP, \( \exists \)-intro.
5. \( \exists C \forall x(x \in C \leftrightarrow [\exists D(x \in D) \& \forall B(x \in A \rightarrow x \in B)]) \) Separation
6. \( \exists C \forall x(x \in C \leftrightarrow \{\forall B(B \in A \rightarrow x \in B)\}) \) 4,5 SC
7. \( x \in \cap A \leftrightarrow [\forall B(x \in A \rightarrow x \in B)] \) 6 Definition by Abstraction
8. \( x \in \cap A \) 7.2 Replacement
9. \( \forall B(x \in A \rightarrow x \in B) \rightarrow x \in \cap A \) RCP

One may complete the proof by first inserting the reverse conditional and then closing off the assumption. But what happens when \( A = \emptyset \)?

**Theorem 53:** \( A = \emptyset \rightarrow \cap A = \emptyset \)

From the antecedent, one may construct \( \forall B(x \in A \rightarrow x \in B) \); and from the denial of the consequent one may obtain line 7 of the previous proof, wherupon one may derive a universal set.

The intersection of a single set not only figures in Zermelo’s proof of well-ordering. It also represents the fundamental act of intelligence in Baconian induction: we sift through several sets of phenomena to form a new set comprising only what those sets have in common.

Four more theorems, the third of which also figures in Zermelo’s proof of well-ordering:

**Theorem 54:** \( \cap \{A, \emptyset\} = \emptyset \)

**Theorem 55:** \( \cap \{A, B\} = A \cap B \)

**Theorem 56:** \( A \subseteq \cap B \rightarrow A \not\subseteq B \)

**Theorem 57:** \( A \neq \emptyset \rightarrow (A \subseteq B \rightarrow \cap B \subseteq \cap A) \)

§7. **Power sets**

The Power Set Axiom and its working theorem represent formally the fundamental act by which we sift through a collection to form another collection comprising all the combinations of the first. A number of theorems display the results of such sifting for simple cases. First, there are two sets that will always be members of a power set:

**Theorem 58:** \( A \in \mathcal{P}A \)

**Theorem 59:** \( \emptyset \in \mathcal{P}A \)

1. \( \neg(x \in \emptyset) \) Null Set Axiom
2. \( \neg(x \in \emptyset) \lor x \in A \) Addition
3. \( x \in \emptyset \rightarrow x \in A \) Implication
4. \( \emptyset \subseteq A \) Definition of inclusion
5. \( \emptyset \in \mathcal{P}A \) Power Theorem

**Theorem 60:** \( A \subseteq \mathcal{P}B \rightarrow \mathcal{P}A \subseteq \mathcal{P}B \)

**Theorem 61:** \( \mathcal{P}\emptyset = \{\emptyset\} \)

This theorem formalizes the difficulty we experience when we try to unify a diversity of interest groups: activities in a marriage or provinces in a nation. If two lovers conceive of their relationship as based on the **Intersection** of their various interests, then this basis very likely diminishes as the interests of each increases. Politically: if we consider Canada as comprised of subsets of interests, i.e. of Provinces each with its own, we might search for a basis of unity in the shared interests of commerce, communication, and common defense; and the greater the number of particular interests in each of these three sub-subsets, the less the likelihood of achieving consensus in policy making. With a view to Theorem 57 we can imagine four remedies: (1) assure that no subset has any interests at all, (2) assure that each province has interests extensionally identical to that of every other, therefore to that of Canada (so that \( A = B \) and \( \cap B = \cap A \)), (3) assure that there are intense common interests (that commerce, communication, and common defense remain uppermost in policy considerations for all Provinces), (4) seek unity in the **Union** of interests, or (5) insist on unity in the acknowledgement of a transcendent interest. Each remedy has found its proponents: the fifth has been that of many theocracies, the first two introduced the tragic dialectic of the totalitarian regimes in the 20th century, and the third and fourth are our own (oscillatingly) in North America.
The notion of a power set allows us to show that we can form a set containing the unordered pairs of elements in a previous set:

**Theorem 62:** \((x \in A \land y \in A) \rightarrow \{x,y\} \in \emptyset A\)

1. \(x \in A \land y \in A\) \hspace{1cm} Assume
2. \(z = x \lor z = y\) \hspace{1cm} Positive Dilemma maneuver
3. \(z \in \{x,y\} \leftrightarrow (z = x \lor z = y)\) \hspace{1cm} Pairing

etc.

**Theorem 63:** \((x \in A \land y \in B) \rightarrow \{x,y\} \in \emptyset (A \cup B)\)

**Theorem 64:** \(\emptyset \emptyset = \emptyset, \{\emptyset\}\)

The proof of this theorem is greatly facilitated by the lemma:

\[ A \subseteq \{\emptyset\} \rightarrow (A = \emptyset \lor A = \{\emptyset\}) \]

1. \(A \subseteq \{\emptyset\}\) \hspace{1cm} Assumption
2. \(x \in A \rightarrow x \in \{\emptyset\}\) \hspace{1cm} Definition of inclusion
3. \(A \neq \emptyset\) \hspace{1cm} Assumption (to prove \(A = \{\emptyset\}\))
4. \(a \in A\) \hspace{1cm} Null Set Theorem (a restricted)
5. \(a \in \{\emptyset\}\) \hspace{1cm} 4.2 \hspace{1cm} \& \hspace{1cm} modus ponens
6. \(a = \emptyset\) \hspace{1cm} Pairing Theorem, etc.
7. \(\emptyset \in A\) \hspace{1cm} 6.4 \hspace{1cm} =-exch.
8. \(x \in \{\emptyset\}\) \hspace{1cm} Assumption (to prove \(x \in A\))
9. \(x = \emptyset\) \hspace{1cm} Pairing Theorem, etc.
10. \(x \in A\) \hspace{1cm} 9.7 \hspace{1cm} =-exch.
11. \(x \in \{\emptyset\} \rightarrow x \in A\) \hspace{1cm} 8—10 \hspace{1cm} RCP
12. \(A = \{\emptyset\}\) \hspace{1cm} 2,11 \hspace{1cm} Extensionality, etc.

Now we proceed with the proof of the theorem itself:

1. \(x \in \emptyset \emptyset\) \hspace{1cm} Assume
2. \(x \in \emptyset \emptyset\) \hspace{1cm} 1 \hspace{1cm} Theorem 61 \& \hspace{1cm} =-exch.
3. \(x \subseteq \{\emptyset\}\) \hspace{1cm} 2 \hspace{1cm} Powering
4. \(x = \emptyset \lor x = \{\emptyset\}\) \hspace{1cm} 3 \hspace{1cm} Lemma
5. \(x = \{\emptyset,\{\emptyset\}\}\) \hspace{1cm} 4 \hspace{1cm} Pairing, etc.

**Theorem 65:** \(A \subseteq B \leftrightarrow \emptyset A \subseteq \emptyset B\)

**Theorem 66:** \((\emptyset A \cup \emptyset B) \subseteq \emptyset (A \cup B)\)

**Theorem 67:** \((\emptyset A \cap \emptyset B) = \emptyset (A \cap B)\)

**Theorem 68:** \(\emptyset (A \cap B) \subseteq [(\emptyset A \cap \emptyset B) \cup \{\emptyset\}]\)

§8. **Ordered pairs**

Pairing itself already represents a crucial act on our part: the conceiving into a single set of two discrete elements. From this fundamental principle we may derive the crucial act of forming a set comprising two discrete elements with a difference between the two, a distinction that formalizes the conception of order — visually (e.g., written on paper), an irreversible designation of what is first and what is second. Expressing an ordered pair as \(<x,y>\), we must be able to show that, \(x \neq y \rightarrow <x,y> \neq <y,x>\).

Much of what we consider to be knowledge requires insight into ordered pairs. We detect in two events that one essentially precedes the other (factually in past time or causally in development); in two places, that we must pass through one before we may pass into the other; in two teams, that one plays better than the other; in two wrestlers, that one is heavier than the other. Generally, as Plato liked to insist, knowledge requires of us that we take the measure of things (number them), and that as we measure things we rank them; knowledge of something engages us in both cardinality and ordinality. On the other hand, we in the West have experimented with the principle that we can best construct our political condition on the principle that there is no way of ranking people at birth: that there are no created ordered pairs of people, that all people are procreated with equal political standing. Nowadays we are experimenting with extensions of this Social Unordered Pair Theorem into the domains of parents and children, males and females.

But how may we define an ordered pair, drawing only on what we have already determined? There are several ways we can proceed, each of which allows us to obtain the most important result as do the others. Let us settle for one:

\(<x,y> =_{\text{def}} \{\{x\}, \{x,y\}\}\)

The definiens expresses the principle of difference assured by coupling: each of the two basic elements, \(x\) and \(y\), is reconstrued, the one with itself, \(\{x,x\}\), and the other with the first, \(\{x,y\}\). Apart from the special case when \(x = y\) (Theorem 69), we may then prove that the \(x\) and \(y\) of \(<x,y>\) are not commutable. Formally, incommutability suffices. And such ordering by difference in coupling does corresponds in part to our intuition of ranking: George and John may form a pair of wrestlers, and to order them we must couple each with something. The “something” may, as in our definition, be George himself (e.g. he is coupled with thought of himself, while John is coupled with thought of George).

Concretely, however, we likely rank two wrestlers in reference to something else, e.g. their weight, to form a pair like \(\{\{George, scale
reading s). Two alternative definitions emphasize the otherness of the extra element:

Alternative 1: $\langle x, y \rangle = \{ \{ x, \emptyset \}, \{ y, \emptyset \} \}$

Alternative 2: $\langle x, y \rangle = \{ \{ x, z \}, \{ y, z \} \}$

In the first of these, $\emptyset$ serves as the third element, and the needed difference appears as $\{ \emptyset \}$. In the second, we postulate an element $z$ obtaining throughout the considerations (e.g., scale-readings).

Formally (i.e., extensionally), the “truth” of whichever definition we adopt depends solely on whether we can employ it to prove the non-commutability of the Ordered Pair $\langle x, y \rangle$, i.e., Theorem 70.

It may seem that Alternative 1 is merely an instance of Alternative 2, so that we would be wise to prove the later first. Yet the employment of Alternative 1 in the proof of Theorem 70 requires only that we invoke $\emptyset \neq \{ \emptyset \}$, which in fact follows directly from the Null Set Theorem, while the employment of Alternative 2 in the proof of that same Theorem requires that we invoke $\forall z (z \neq \{ z \})$, the proof of which requires the Axiom of Foundation.

In the concrete, it seems odd that we should try to rank a single individual happening to have two different names. What sense might it have to rank George “all by himself”? Yet it is characteristic of our formalism, here and elsewhere, that it allows us, even requires us, to try it out in such odd cases. In this case:

Theorem 69: $(x = y) \rightarrow [\langle x, y \rangle = \{ \{ x \} \}]$

1. $x = y$ Assumption
2. $\langle x, y \rangle = \langle x, y \rangle$ $\leadsto$-introduction
3. $\langle x, y \rangle = \{ \{ x, x \}, \{ x, y \} \}$ Definitions of ordered pairs and unit set
4. $\langle x, y \rangle = \{ \{ x, x \}, \{ x, y \} \}$ 1,3 Lemma below and $\leadsto$-exch.
5. $\langle x, y \rangle = \{ \{ x \} \}$ Definition of unit set (twice)
6. $x = y \rightarrow \langle x, y \rangle = \{ \{ x \} \}$ 1–5 RCP

Note that $\leadsto$-exch does not work directly on line 3 to yield line 4 (the braces do not embrace variables). Thus one must prove the lemma:

$x = z \rightarrow [\langle x, y \rangle = \{ z, y \}]$

Essential to our intuition of ordered pairs is the next theorem, which contrasts with the Unordered Pair Theorem we use to prove it:

Theorem 70: $\langle x, y \rangle = \langle u, v \rangle \rightarrow (x = u \land y = v)$

1. $\langle x, y \rangle = \langle u, v \rangle$ Assumption $\therefore x = u \land y = v$

Each of the following four theorems displays something of what happens when we take the union or the intersection of ordered pairs:

Theorem 71: $\bigcup \langle x, y \rangle = \{ x, y \}$

Theorem 72: $\bigcup \{ x, A, B \} = A \cup B$

Theorem 73: $\bigcap \langle x, y \rangle = \{ x \}$

Theorem 74: $\bigcap \{ x, A, B \} = A$

And we may also show how, from the elements of one (or two . . . ) initial sets we may form sets having as elements the ordered pairs of those initial elements (compare Theorems 62 and 63):

Theorem 75: $(x \in A \land y \in A) \rightarrow \langle x, y \rangle \in \emptyset \emptyset A$
Theorem 76: \((x \in A & y \in B) \rightarrow <x,y> \in \emptyset \mathcal{P}(A \cup B)\)

1. \(x \in A & y \in B\) Assumption
2. \([y,x] \in \emptyset \mathcal{P}(A \cup B)\) Commutation & Theorem 63
3. \(x \in A \lor x \in B\) 1 Simplification & Addition
4. \(x \in (A \cup B)\) Union Theorem
5. \(x \in (A \cup B) \& x \in (A \cup B)\) Indempotence
6. \([x] \in \emptyset \mathcal{P}(A \cup B)\) Theorem 62, definition of unit set
7. \([x],[y,x] \in \emptyset \mathcal{P}(A \cup B)\) 6.2 Theorem 62
8. \(<x,y> \in \emptyset \mathcal{P}(A \cup B)\) Definition of \(<x,y>\)
9. Theorem RCP

**Cartesian Products**

Given two sets \(A\) and \(B\), we may form a new set \(C\) containing all the possible (ordered) pairs from \(A\) to \(B\). Each member, each \(x\), of this new set will be composed as \(<y,z>\), where the \(y\) is a member of \(A\) and the \(z\) a member of \(B\).

This new set is called the Cartesian product of \(A\) and \(B\). For example, from a set of (three) Boys and a set of (five) Girls, you would like form a set \(D\) comprising all the possible Dance partnerships—on the assumption that each boy will ask each girl to dance with him. Or, jumping to infinite sets, the set \(H\) may be comprised of all the points on a horizontal axis, and the set \(V\) of all the points on a vertical axis: the new set \(C\) will then be the set of ordered pairs of points \(<x,y>\).

Intuitively, we can see that the number of possible pairs of a finite set \(C\) will be the arithmetical product of the number of items in \(A\) and the number of items in \(B\). Yet a Cartesian product is not usually commutative, unlike the arithmetic product (see Theorem 82).

The definition of the Cartesian Product of two sets reads:

\[A \times B \overset{\text{def}}{=} \{x : \exists y \exists z (x = <y,z> \& y \in A \& z \in B)\}.\]

But we must suspend use of this definition until we prove that there exists a possibly non-empty set fitting the description offered by this definition (whether empty or not may depend on \(A\) and \(B\)). We do this by showing that any \(x\) taken into the formula of the abstract definition is also a member of a set (\(\not= \emptyset\)) that has already been authorized. In the present case, we can show that the \(x\) belongs to \(\emptyset \mathcal{P}(A \cup B)\)—as proved by Theorem 76—whereupon we have the needed set \(D\):

Theorem 77: \(\exists y \exists z (y \in A \& z \in B \& x = <y,z>) \rightarrow \exists x (x \in D)\)

Ordered Pairs

Again, recall that an existential definition requires for its employment that it be “synthetically” fecund: that, from it, we can derive a set of which it is a subset (an “analytic” derivation does not count). Whereas the Axioms of Pairing, Summing, and Powering posit the required set \(D\), the Union Theorem, the Intersection of a Single Set, and the Cartesian Product require us to construct it. Only then may we employ Separation:

Theorem 78: \(\exists C \forall x [x \in C \leftrightarrow \exists y \exists z (y \in A \& z \in B \& x = <y,z>)]\)

With this last we may now employ the Definition of the Cartesian Product, together with Abstraction, to prove:

Theorem 79: \(x \in A \times B \leftrightarrow \exists y \exists z (x = <y,z> \& y \in A \& z \in B)\)

Whereupon we may instantiate with \(<y,z>\) to obtain:

Theorem 80: \(\langle x,y \rangle \in A \times B \leftrightarrow x \in A \& y \in B\)

Various theorems display the interaction of Cartesian products with the Null Set. First, the necessary and sufficient condition under which a Cartesian product will have no members:

Theorem 81: \(A \times B = \emptyset \leftrightarrow (A = \emptyset \lor B = \emptyset)\)

Then we may show the necessary and sufficient condition for a Cartesian product to be commutable:

Theorem 82: \((A \times B = B \times A) \leftrightarrow (A = \emptyset \lor B = \emptyset \lor A = B)\)

The next two theorems display how inclusions work together with Cartesian products:

Theorem 83: \(B \subseteq C \rightarrow (A \times B \subseteq A \times C)\)

Theorem 84: \(A \neq \emptyset \rightarrow [(A \times B \subseteq A \times C) \rightarrow B \subseteq C]\)

Proofs displaying distribution over intersection, union, and difference for Cartesian products proceed routinely:

Theorem 85: \(A \times (B \cap C) = (A \times B) \cap (A \times C)\)

Theorem 86: \(A \times (B \cup C) = (A \times B) \cup (A \times C)\)

Theorem 87: \(A \times (B - C) = (A \times B) - (A \times C)\)

An Oddity

Here is a theorem the proof of which again raises the question whether we need to assume the Foundation Axiom:

Theorem 88: \((A \subseteq A \times A) \rightarrow A = \emptyset\)

1. \(A \subseteq A \times A\) Assumption
2. \(\forall x (x \in A \rightarrow x \in A \times A)\) 1 Def. of \(\subseteq\)
Null Set Axiom

∀x(x ∈ A × A ↔ (x = \{\{b\},\{c\}\} & b ∈ A & c ∈ A))

And from 2. and 3. we obtain a peculiar result:

4. ∀x(x ∈ A → (x = \{\{b\},\{c\}\} & b ∈ A & c ∈ A))

This is peculiar because if we now assume that A ≠ ∅, i.e.

5. a ∈ A  Assumption

we discern a "loop": the a in A allows us to install b and c in A, each of which allows us to assert two more elements in A (and in A × A). And so on. We then have a strange set indeed! Much as what can happen if we suppose a set that is a member of itself, e.g. \{x\} ∈ \{x\}: a strange trans-finitude. Assuming we wish to put a stop to this looping, we must once again call on the Foundation Axiom. As it turns out, we apply it not to A but to A ∪ ∪A (see the development from line 11):

6. a ∈ (A ∪ ∪A) 5 Add., Union; thus (A ∪ ∪A) ≠ ∅, so:

7. C ∈ (A ∪ ∪A) 6 this ½ of Foundation Axiom; other half:

8. ∀w[w ∈ C → w ∉ (A ∪ ∪A)]

9. C ∈ A ⊃ C ∉ ∪A 7 Union

And now we may derive a contradiction from each disjunct:

10. C ∈ A  Assumption

11. C ⊆ ∪A  Theorem 49

From here we can easily obtain C = ∅ (lines 8 and 11 yield a contradiction), and yet we can also obtain C ≠ ∅ (from lines 10 and 4).

So then we turn to the other disjunct:

12. C ∈ ∪A  Assumption

13. B ∈ A & C ∈ B  Summing Theorem (for some B)

14. B = \{\{a\},\{a,b\}\} 4, 13 Simpl. and \&-elim.

15. C ∈ \{\{a\},\{a,b\}\} 13, 14 Simpl. and =-exch.

16. C = \{a\} ∨ C = \{a,b\}  Pairing Theorem

And each one of these leads to a contradiction; e.g.:

17. C = \{a\}  Assumption

18. a ∈ \{a\} → a ∉ (A ∪ ∪A) 4, 17 \&-elim. and =-exch.

19. a ∈ \{a\}  Pairing Theorem, etc.

20. a ∉ (A ∪ ∪A) 18, 19 MP

Which contradicts line 5. Similarly with the other disjunct. Thus we infer that A = ∅.

Only by way of the Foundation Axiom. Should we dispense with Theorem 88? Instantiations of (A ⊆ A × A) show it to be true when A = ∅, false otherwise. We could then settle for mere contingency.

§9. Relations: domains, ranges, fields, & converses

Having devised ways of ordering the members of a set, and also forming sets of ordered pairs (Cartesian products), we position our thinking to devise ways of reconstruing the category of relation as a kind of ordering. In turn, this kind of ordering will allow us to form sets expressing the crucial notion of a function.

We shall confine ourselves to the consideration of dyadic relations. Among these, some are quantifiable: "x is taller than y," "x is prior (in time, or on a line) to y" and "x is contiguous (or simultaneous) with y." Others are human: "x is the husband of y" and "x reads y." Still others are causal: "x destroys y" and "x enhances the growth of y." Each of these examples involves an intensional meaning.

We may construe a set A as a relation, in abstraction from all examples, as a set whose members are ordered relations — a purely extensional construal:

A is a relation ↔_def ∀x[x ∈ A → ∃y∃z(x = ⟨y,z⟩)].

This is a verbal, not an existential definition: it does not posit a set, it rather posits (and names as a whole) nothing more than what has already been posited. Yet our Axioms all along axiomatize Mₓy (or x ∈ y): the (logical) archetype of relations. Thus the definition of relations is not trivially verbal: it recoups an intuitive understanding, a meta-concept, already at work, and formally integrates this understanding into the system we are forming.

One result of this process of definition: we cannot go back and define as a sets such originary relations as "x is a Member of y" or "x is Identical to y." For we would produce grossly circular definitions.

Theorem 89: ∅ is a relation

1. x ∈ ∅  Null Set Axiom

2. x ∈ ∅ ∨ ∃y∃z(x = ⟨x,y⟩)  Addition

3. x ∈ ∅ → ∃y∃z(x = ⟨x,y⟩)  Implication

4. ∀x[x ∈ ∅ → ∃y∃z(x = ⟨x,y⟩)]  ∀-introduction

5. ∅ is a relation  Definition of relation
The proof that \( \emptyset \) is a relation follows almost immediately from the definition and from the familiar paradox of implication. The contemplative question is: What sense can we give to \( \emptyset \)-as-a-relation in such formulas as \( (A \sim A) = \emptyset \)? As so often happens, our very abstract definition brings strange cases into its purview.

**Theorem 90:** \( R \) is a relation & \( S \subseteq R \rightarrow S \) is a relation

1. \( R \) is a relation & \( S \subseteq R \) \hspace{1cm} Assumption
2. \( x \in S \rightarrow x \in R \) \hspace{1cm} Simpl., Definition of inclusion
3. \( x \in R \rightarrow \exists y \exists z (x = <y,z>) \) \hspace{1cm} 1 Simplification & \( \forall \)-elim.
4. \( x \in S \rightarrow \exists y \exists z (x = <y,z>) \) \hspace{1cm} 2,3 Hypothetical Syllogism
5. \( S \) is a relation \hspace{1cm} \( \forall \)-intro. & Definition of relation
6. \( R \) is a relation & \( S \subseteq R \rightarrow S \) is a relation \hspace{1cm} RCP

**Theorem 91:** \( R \& S \) are relations \( \rightarrow (R \cap S) \& (R \cup S) \& (R \sim S) \) are relations

The **domain** of a relation \( A \), which we may symbolize as \( \bar{D}A \), is the set of every \( x \) having a \( y \) forming a pair \( <x,y> \) that is a member of the set \( A \). For instance, the domain of the relation Husband is the set of all married men. Formally:

\[ \bar{D}A = \{ x : \exists y (<x,y> \in A) \} \]

Similarly, the **range** of a relation, which we may symbolize as \( \bar{R}A \) is the set of every \( y \) having an \( x \) forming a pair \( <x,y> \) in \( A \). For instance, the range of the relation Husband is the set of all married women. Formally:

\[ \bar{R}A = \{ y : \exists x (<x,y> \in A) \} \]

Then, too, we may define the **field** of a relation as the set comprising as members each and every item of a relation, jumbled together; for instance, the field of the relation Husband comprises all married men and all married women. Thus the formal definition of a field is strictly verbal, drawing upon that of domain and that of range:

The **field of relation** \( A = \bar{D}A \cup \bar{R}A \).

And finally we may define the **converse** of a relation \( A \), symbolized as \( \bar{C}A \): it is, as the name suggests, the converse of the original; the converse of Husband is Wife, the converse of “... to the left of...” is “...to the right of (if not equal to)...”. Formally:

\[ \bar{C}A = \{ z : \exists x \exists y (z = <y,x> \& <x,y> \in A) \} \]

**Theorem 92:** \( x \in \bar{D}A \leftrightarrow \exists y (<x,y> \in A) \)

Since this theorem introduces the existence of a set, namely \( \bar{D}A \), we must proceed as in the proof-sequence of Theorems 76-79. The strategy is to choose as the pre-existent set one that is implied by the definients of \( \bar{D}A \): here, \( \bigcup A \).

1. \( \exists y (<x,y> \in A) \) \hspace{1cm} Assume (\( \phi x \) of \( \bar{D}A \)-definition)
2. \( <x,y> \in A \) \hspace{1cm} \( \exists \)-elim.
3. \( \{ x \}, \{ y,x \} \in A \) \hspace{1cm} Definition of ordered pairs
4. \( \{ x \} \in \{ \{ x \}, \{ y,x \} \} \) \hspace{1cm} Pairing Theorem, etc.
5. \( \exists B \{ B \in A \& \{ x \} \in B \} \) \hspace{1cm} 3,4 Conjunction & \( \exists \)-intro.
6. \( \{ x \} \in \bigcup A \) \hspace{1cm} Summing Theorem
7. \( x \in \{ x \} \) \hspace{1cm} Pairing Theorem, etc.
8. \( \exists B \{ B \in \bigcup A \& x \in B \} \) \hspace{1cm} 6,7 Conjunction & \( \exists \)-intro.
9. \( x \in \bigcup A \) \hspace{1cm} Summing Theorem
10. \( \exists y (<x,y> \in A) \rightarrow x \in \bigcup A \) \hspace{1cm} 1 — 9 RCP

One may then complete the proof with Separation and the sentential rule:

\[ q \rightarrow r \rightarrow p \leftrightarrow \overline{(r \& q)} \rightarrow p \leftrightarrow q \] and with Definition by Abstraction.

**Theorem 93:** \( \bar{D}(A \cup B) = \bar{D}A \cup \bar{D}B \)

**Theorem 94:** \( \bar{D}(A \cap B) \subseteq \bar{D}A \cap \bar{D}B \)

**Theorem 95:** \( \bar{D}(A \sim B) \subseteq \bar{D}A \sim \bar{D}B \)

Looking back to the proof of Theorem 92, we can devise an almost identical pattern of reasoning to establish the unique existence of the **range** of a relation:

**Theorem 96:** \( y \in \bar{R}A \leftrightarrow \exists x (<x,y> \in A) \)

**Theorem 97:** \( \bar{R}(A \cup B) = \bar{R}A \cup \bar{R}B \)

**Theorem 98:** \( \bar{R}(A \cap B) \subseteq \bar{R}A \cap \bar{R}B \)

**Theorem 99:** \( \bar{R}(A \sim B) \subseteq \bar{R}A \sim \bar{R}B \)

The next theorem will claim that if there is a set of ordered pairs \( A \) comprising, say, the sets \( <x \text{ Loves } y> \), then there is also a set (the converse of \( A \), symbolized as \( \bar{C}A \)) comprising the sets \( <y \text{ Loves } x> \). Generally, the theorem claims that for any set comprising as members \( <x,y> \) there is another set comprising as members \( <y,x> \). Again, we only prove the “license”: that any element in the definients of \( \bar{C}A \) is also in a set already proved to exist (that, in other words, \( \bar{C}A \) is a subset of an existing set and therefore itself exists).
Theorem 100: \( \langle y, z \rangle \in \tilde{C}A \leftrightarrow \langle x, y \rangle \in A \)

1. \( \exists x y (\langle y, z \rangle \in A \land z = \langle y, x \rangle) \)
   Assume (def of \( \tilde{C}A \))
2. \( \langle x, y \rangle \in A \land z = \langle y, x \rangle \)
   \( \exists \)-eliminations
3. \( x \in \tilde{C}A \)
   Simplification
4. \( y \in \tilde{D}A \land x \in \tilde{D}A \)
   Theorems 92 & 96
5. \( \langle x, y \rangle \in \tilde{D}A \times \tilde{D}A \)
   Theorem 80
6. \( z = \langle y, x \rangle \)
   2 Simplification
7. \( z \in (\tilde{D}A \times \tilde{D}A) \)
   5, 6 \(-\operatorname{exch}\)
8. \( \exists x y (\langle y, z \rangle \in A \land z = \langle y, x \rangle) \rightarrow z \in (\tilde{D}A \times \tilde{D}A) \) 1—7 RCP

Theorem 101: \( \tilde{C}A \) is a relation

Theorem 102: \( \tilde{C}C \subseteq A \)

Theorem 103: \( R \) is a relation \( \rightarrow \tilde{C}CR = R \)

These last two theorems display the condition for symmetry between a set and its converse: it is possible that a set contain more members than the converse of its converse (when, namely, the set contains items that are not ordered pairs). Once, however, all members of a set are ordered pairs (once, that is, the set is a relation), the converse of a converse is extensionally identical to the original.

Theorem 104: \( \tilde{C}(A \cap B) = \tilde{C}A \cap \tilde{C}B \)

Theorem 105: \( \tilde{C}(A \cup B) = \tilde{C}A \cup \tilde{C}B \)

Theorem 106: \( \tilde{C}(A - B) = \tilde{C}A - \tilde{C}B \)

Relative Products

An important relation is that of the relative product of two sets, or \( A/B \): if we have two sets containing ordered pairs, we may form a third set comprising ordered pairs created in each case as follows: from every \( \langle x, z \rangle \) in \( A \) and \( \langle z, y \rangle \) in \( B \), the newly formed set \( C \) will have as a member \( \langle x, y \rangle \). Thus there must be common elements in the range of \( A \) and the domain of \( B \). For instance, a club might have paired its members by wealth; John has less money than Richard and Richard has less money than Henry: from \( \langle J, R \rangle \) in the first set and \( \langle R, H \rangle \) in the second set, we may start forming a third set with member \( \langle J, H \rangle \).

The set-theoretical notion of a relative product captures the heart of transitive relations: field mice are smaller than dodo birds, and dodo birds are smaller than grizzly bears, therefore field mice are smaller than grizzly bears. Both in this argument and in the notion of a relative product, nothing follows if there is no example of the mediating term.

Relations: Domains, Ranges, Fields, Converse

Also, since \( A/B \) is not commutable, if \( A = \langle R, H \rangle \) and \( B = \langle J, R \rangle \), we are left with \( A/B = \emptyset \).

\( A/B =_{\text{def}} \{ w : \exists x y [z = \langle x, y \rangle \land x \in A \land \langle z, y \rangle \in B] \} \)

To employ this definition constructively, we must first establish the existence, in our formative system, of the set so defined. In this case, we issue a license to the definition by showing that any member described in the definiens is also a member of the Cartesian product of the domain of \( A \) and the range of \( B \).

Theorem 107: \( \langle x, y \rangle \in A/B \leftrightarrow \exists z (\langle x, z \rangle \in A \land \langle z, y \rangle \in B) \)

1. \( \forall w \) of \( A/B \)-definition Assume & thrice \( \exists \)-elim.
2. \( \langle x, z \rangle \in A \)
   Simplification
3. \( x \in \tilde{D}A \)
   Theorem 92
4. \( \langle z, y \rangle \in B \)
   2 Simplification
5. \( y \in \tilde{R}B \)
   Theorem 96
6. \( \langle x, y \rangle \in \tilde{D}A \times \tilde{R}B \)
   3, 5 Conj. & Theorem 80
7. \( w = \langle x, y \rangle \)
   1 Simplification
8. \( w \in \tilde{D}A \times \tilde{R}B \)
   7, 6 \(-\operatorname{exchange}\)
9. \( (1) \rightarrow w \in \tilde{D}A \times \tilde{R}B \)
   RCP

For the rest, see the proof-sequence of Theorems 77 through 80.

Theorem 108: \( A/B \) is a relation

Theorem 109: \( \emptyset / A = \emptyset \)

Theorem 110: \( \tilde{D}(A/B) \subseteq \tilde{D}A \)

Theorem 111: \( A \subseteq B \land C \subseteq D \rightarrow A/C \subseteq B/D \)

Theorem 112: \( (A/B)/C = (A/C)/(B/C) \)

Restricted Relations

Very often we wish to specify that a certain relation \( A \) holds, but only when the \( x \) in each of its members \( \langle x, y \rangle \) is a member of a preliminary set \( R \). In other words, we wish to place a restriction \( R \) on the domain of the relation \( A \). For example, a tax to be levied might be conceptually determined to consist of the ordered pairs \( \langle \text{income}, 30\% \rangle \), but the income applies only to what is earned (excluding pennies from heaven, gifts from Santa Claus, and the like). We want, then, to generate a set of the form \( \langle x, y \rangle \in A \land x \in R \). Defined abstractly:

\( A[R] =_{\text{def}} \{ z : \exists x y [z = \langle x, y \rangle \land x \in A \land x \in R] \} \).
Theorem 113: \(<x,y> \in A|R \iff (<x,y> \in A \land x \in R)\)

Let us contemplate the complete proof:

1. \(\varphi\) of \(A|R\)-definition Assume twice \(\exists\)-elim.
2. \(x,y> \in A\) Simplification, =-exch.
3. \(y \in \bar{\bar{R}}A\) Theorem 96
4. \(x \in R\) 1 Simplification
5. \(x \in R \land y \in \bar{\bar{R}}A\) 4,3 Conjunction
6. \(<x,y> \in (R \times \bar{\bar{R}}A)\) Theorem 80
7. \(z = <x,y>\) 1 Simplification
8. \(z \in (R \times \bar{\bar{R}}A)\) 7,6 =-exchange
9. \((1) \rightarrow z \in (R \times \bar{\bar{R}}A)\) RCP
10. \(\exists B \forall (z \in B \iff [z \in (R \times \bar{\bar{R}}A) \land (1)])\) Separation
11. \(\exists y \forall z [z \in B \iff (z = <x,y> \land <x,y> \in A \land x \in R)]\)

This line follows from lines 9 and 10: \(q \rightarrow p \iff (r \land q)\), \(\therefore p \iff q\).

12. \(-[A|R = \emptyset \land -\neg(1)\] Addition, De Morgan's
13. \(\forall z [z \in A \land R \iff (z = <x,y> \land <x,y> \in A \land x \in R)]\)

This line follows from line 12, Definition by Abstraction and D.S.

14. \(<x,y> \in A \iff (x,y> \in A \land x \in R)\)

From \(-\forall\)-elimination, \(z\) as \(<x,y>\), and \(p \iff (r \land q)\), \(r\), \(\therefore p \iff q\).

Theorem 114: \(A \subseteq B \rightarrow R[A \subseteq R\bar{B}]\)

Theorem 115: \(R[A \land B] = (R[A]) \cap R[B]\)

Theorem 116: \((R/S)\mid A = (R\mid A)/S\)

**Functions**

With the various notions associated with the set-theoretical understanding of relations, we may begin to contemplate how the notion of function has contributed to the evolution of mathematical science out of natural philosophy. For a function is a kind of relation: a set, a set of ordered pairs, a set in which to any element in the domain of the relation there corresponds only one element in its range.

Penalties for speeding are often conceptualized as functions, e.g. a flat fee of fifty dollars, plus ten dollars for every kilometer/hour over the limit: \$50 + [(x - L) \times 10] \text{, where } x \text{ is the variable for which the clocked speed in any given case must be substituted, whereupon we may calculate a definite fine. In the set Penalties, there will be a history of ordered pairs } <x,y>, \text{ with } x \text{ as a Speed and } y \text{ as the Fine.}

Mathematical formulas often take the form of functions. For instance: \(4x^2 - 4x^2\) which conveniently graphs as a parabola with \(x\) as a position on the horizontal axis and \(y\) as a position on the vertical axis, so that the formula names a set of ordered pairs of points: for \(x = 0, y = 0;\) for \(x = .5, y = 1;\) for \(x = 1, y = 0;\) for \(x = 2, y = -8;\) so far, then, the ordered pairs are \(<0,0>, <.5,1>, <1,0>, <2,-8>\). While \(y\) is a function of \(x\) (given a value for \(x\), we know the value of \(y\), we can determine that \(x\) is not a function of \(y\) (given at least some values of \(y\), e.g. \(0\), there is more than one value possible for \(x\), e.g. \(0\) and \(1\)).

The notion of a function has allowed modern mathematicians to define different orders of infinity. Two sets (e.g., positive integers and odd positive integers) have the same “size” if and only if, by definition, we can devise a function pairing each member of the one with a member of the other set. Conversely, two infinite sets differ in size if there is no such function (consider, e.g., the set positive integers and the set of real numbers).

While we usually illustrate functions with mathematical examples, the notion of a function has a wider field. For instance, in our society the concept of Husband (\(x\) is the Husband of \(y\)) might be interpreted as a function: given any item in the domain, there is a definite item in the range; in a village it would be easy to list the ordered pairs: \(H = \{<Joe, Mary>, <Tom, Joan>, <Dick, Sue>, \ldots\}\). By contrast, in a society allowing more than one wife per husband, the concept of Husband is not a function.

The definition of a function as a set has crucial applications in physics. While the classical (Aristotelian) notion of causality requires that the human intellect become transparent to transcendental nature, modern physics has led in the direction of understanding causality as transparency to transcendental conditions. The notion of a function serves to capture this formal aspect of causality: an elaborately formulated relation between sets of things, such that to every element in the domain there corresponds only one element in the range. Here, causality is understood extensionally.

The set-theoretical definition of a function reads:

\(f \text{ is a function } \iff f = \{(x,y) \in f \land <x,y> \in f \rightarrow (y = z)\}\)

Since by definition the item in the range of the function is determined for every instantiation in the domain, we will want to express this determination in the way familiar to mathematicians: \(f(x) = y\), given
a Speed, we want to be able to calculate the Fine; given the Husband we want to be able to find the Wife. This formulation may be defined as follows:

\[ f(x) = y \iff \exists z (x, z) \in f \land (y = \emptyset \lor \exists z (x, z) \in f) \]

Thus for \( f = \text{Wife} \) we have:

\[ \text{Wife}(x) = y \iff \exists z (x, z) \in \text{Wife} \]

Concretely, of course, the problem is that Wife (x is Wife of y) does not immediately provide a formula for finding the second member of the ordered pair, and not every x (domain: women) will correspond to a y. On the other hand, if \( f \) is a straightforward mathematical formula such as \( (4x - 4x^2) \), any given x allows us to calculate the (one) value of the second member; thus we may here drop the second disjunct:

\[ 4x - 4x^2 = y \iff \exists z (x, z) \in \{4x - 4x^2\} \land \exists z (x, z) \in \text{Wife}\} \]

Theorem 117: \( f \) and \( g \) are functions \( f \cap g \) is a function

A new notion, now: that of combining two functions, the symbol for which is \( \circ \) (the “combiner”):

\[ f \circ g \equiv_{\text{def}} g/f \]

The definition introduces no new set. It is another symbolic formulation of the relative product. Thus the next theorem states in effect that

\[ f \text{ and } g \text{ are functions } \rightarrow g/f \text{ is a function.} \]

The purpose of the new symbol is to allow us to read the function variables in a convenient order: when combining two functions, we hook the second mentioned back onto the first (on the reading of a relative product we would have to reverse the order).

Theorem 118: \( f \) and \( g \) are functions \( f \circ g \) is a function

1. \( f \) is a function & \( g \) is a function Assumption
2. \( g/f \) is a relation Theorem 108
3. \( f \circ g \) is a relation Definition of combiner
4. \( f \circ g \in (f \circ g) \land (f \circ g) \in (f \circ g) \) Assumption: show \( y = z \).

The penultimate theorem equates extensionally the combination of two functions, \( f \) and \( g \), with a formula in which the first, the \( f \), takes as its variable the values first delivered by the second, the \( g \). The important notion here is that we can, in effect, have functions depending on two (or more) variables. For instance, to determine the possible line of points in three-dimensional space, we must first determine where we are on two axes: the set of points \( z \) is a function of both the points along axis \( x \) and the points along axis \( y \). On a qualitative plane, Aristotle argues that Happiness is a function of Activity (of the soul in its logos-nature), \( f(\text{Activity}) = \text{Happiness} \), while he also considers the possibility that Activity itself is a function of Opportunity (luck, fortune) — i.e., \( g(\text{Opportunity}) = \text{Activity} \). Accordingly, Happiness would be identical to a double function: \( H = f(g(x)) \).

A quantitative example: the Fine the judge must impose is “somehow” a function of the Clocked-speed. Here is another factor to consider, namely the Limit placed on speed in the zone where the clocking took place. The formula for the judge might be: \( $50 + (|x - L| \times 10) = \text{Fine} \), with \( L \) as the limit in the relevant zone and \( x \) as the clocked speed. But now we can separate out the two functions: Speeding is indeed a function of Clocked-speed (a function defined by the Limit), or \( g(\text{Clocked-speed}) = S \), and the Fine is a function of Speeding, \( f(\text{Speeding}) = F \). Combing these two to allow for the variable \( x \) to cover Clocked-speed, we have \( \text{Fine} = f(g(\text{Clocked-speed})) \). To determine the Fine, one must first determine the relation between Clocked-speed and Limit, and then proceed to calculate according to the over-reaching formula. The theorem now states that for any two functions, so long as the domain of their combination is not empty, we may equate the combination-function of \( x \) with the function-of-a-function of \( x \): \( (f \circ g)(x) = f(g(x)) \).

Theorem 119: \( f \) and \( g \) are functions \( f \circ g \in \emptyset \) \( \rightarrow (f \circ g)(x) = f(g(x)) \)

Theorem 120: \( (f \circ g)[A = f \circ (g[A)] \)

§10. The truth of set formation: choice

It appears, then, that we have formally derived several notions essential to mathematical work: the notion of number (the Axiom of Infinity leads to a transfinite set of inductive elements) and also the notion of relation and thereby of function (the Axiom of Pairing leads to sets of ordered pairs). All this from our logical calculi and a handful of Axioms.

Now, these achievements suggest that set theory might have a privileged status among the rigorous sciences. That it might serve as the
motherlode of mathematics, and thereby of all the sciences relying on mathematical work.

Whatever one might make of this claim, one should not miss the *circularity* in the argument. Set theory does allow us to produce a set we might call the set of natural numbers, one we could eventually *order* into the natural sequence; and it does allow us to deduce the notions of relation and function so crucial to all mathematical science. Yet such productions and deductions have already made use of these notions. Any deductive system *presupposes* the conceptual possibility of an ordered set of theorems. All our Axioms *presuppose* the notion of $\in$, or $Mxy$—the mother of all relations. And the Axiom of Separation, the father of all theorems except those following immediately from the Null Set Axiom, *presupposes* the notion of a function — the $\forall x$ left indeterminate.

Indeed, set theory illustrates quintessentially a principle that careful thinkers in all domains must learn to acknowledge: any fundamental line of thought, whether Zermelo’s regarding sets, Plato’s regarding form vs. instance or Kant’s regarding conception vs. perception, must presuppose what it eventually shows. To uninformed ears, this sounds like cheating. But the chief problem of any fundamental line of thinking — its life-long task — is precisely to show how showing works, including its own. The achievement here is perhaps akin to what Aristotle called “thought thinking itself.”

Instead, then, of laying claim to the motherlode, we might ask where we can detect the point at which set formation opens out onto its own activity. I suggest we look to yet another principle that we have been assuming (performing) all along: the principle of *choice*.

Now, to anyone who actually performs the proofs, it is clear that the performance generally requires one to imagine *picking out* one element in a proposed set and tracing its fate: carefully following where this choice leads (or rather leading it carefully through multiple transformations), we conclude that any element in the proposed set will share the same fate. That is, we generalize from $a \in S \rightarrow a \in T$ to “given any element in $S$ it will also be in $T$.”

This picking, this selecting, takes on highly constructive dimensions in the formation of sets. For instance, from two non-empty sets $A$ and $B$,

we imagine picking, one at a time, elements $a$ and $b$ (respectively) to form an unordered pair that will itself be an element in a power set:

$$(a \in A & b \in B) \rightarrow \{a,b\} \in \mathcal{P}(A \cup B) \quad \text{Theorem 63}$$

This imagined act is essential, if only implicit, in the arithmetic computation of combinations. Or we imagine assembling $a$ and $b$ into an ordered pair that will itself be an element of a Cartesian product:

$$(a \in A & b \in B) \leftrightarrow \langle a,b \rangle \in A \times B \quad \text{Theorem 80}$$

This imagined act is essential in the mathematical understanding of multiplication: from two sets we pick out all the ordered pairs from one to the other — we imagine choosing elements from $A$ (one at a time and each only once) and pairing them with elements in $B$ (again, one at a time and each only once with any one element in $A$), and placing all these ordered pairs in yet another set, the cardinality of which will correspond to the cardinality of $A$ “times” the cardinality of $B$.

The operation of choosing — whether imagined or imaginary — lies so much at the basis of both set formation and arithmetic computations that we easily ignore it — i.e. fail even to notice it, let alone ponder it.

Zermelo, remember, drives the operation to the fore in his Axiom of Choice:

...it is always possible to *choose* a single element from each element $M, N, R, \ldots$ of $T$ and to combine all the chosen elements, $m, n, r, \ldots$ into a set $S_i$.

This principle Zermelo introduces in order to prove that every set can be well ordered, including such higher-order infinite sets as the set of rational numbers. I.e., that we may, with the Axiom of Choice, prove the existence of a set that itself is ordered and that maps the original into an order. However, precisely the fascinating question how the Axiom permits the proof of the Well-Ordering Principle detracts from the purport of the Axiom itself, which formulates a principle that governs not only modern mathematics, but also modern thinking generally.

What's new about Zermelo's formulation does not lie in the formulation given above ("it is always possible to choose..."). Rather, it lies in the application Zermelo wishes to make of it. It is not a question of choosing *one at a time* in infinite succession. It is a question of *completing* the infinite choices. The proof of well ordering (and of other theorems) requires that we conceive of it as happening *all at once*.

*Metaphysics*, 1075 a 5: here, thinking (ἡ νοησίς) is not “other than” but rather “the same as” or “one with” what gets thought (τὸ νοοῦσθεν).
...a simultaneous choice of distinguished elements is in principle always possible for an arbitrary set of sets.

One part of the Axiom states that there exists a “distinguished element” in each set — i.e., that there is a “choice function” that assures a unique selection. Another part of the Axiom is that these “distinguished elements” can be extracted all (infinitely) at once. Both parts raise serious questions for those who understand mathematics as a constructive line of reasoning, and especially for those who understand proofs as legitimate only when they keep in touch with intuition (the intuition of time, Brouwer says).

About the “distinguished element”: the Axiom states the existence of a unique element in each of $M, N, R, \ldots$ of $T$. And so it must. For the new set, $S_1$, must itself be conceived as unique; non-uniqueness would wreak havoc with the uniqueness assured by the Abstraction Theorem (as in “the Senate will be composed of one member from each Department”). Choice must be assured by a function. But where will we obtain such a “choice function”? Sometimes there is one. For instance, we may state that each Senator will be the oldest member of the Department. But there are others.

For example if $A$ is a well-ordered set, we may define the “representative” of each nonempty subset $B \subseteq A$ to be the least element of $B$; because $A$ is well ordered, we have a law at our disposal which provides us with a representative of each nonempty set of $A$.

Yet this set begs the question for which Zermelo introduced his Axiom.*

But apart from the more esoteric questions, we should not miss the fact that Zermelo has, for the first time, enunciated the principle of choice in general: we in fact do review sets to select items for the construction of other sets, and we do this quite freely; indeed, with today's computers we can devise software that will do this recursively across data collections of any size we happen to... construct.

But why the enunciation? What intellectual interest ever gave rise, first, to the prominence of selecting items from collections and, second, to enunciating this selection as a principle? These are modern concerns, and so we may ask how modernity itself requires the principle. What's at issue is the understanding of truth.

Roughly formulated, Plato and Aristotle understood truth as the double event of cognizing the universal within the singular; this event itself becomes clear only in the kind of intimacy that enhances the singular as it participates in its own fulfillment.

Just as roughly, modernity, having dispensed with the primacy of fulfillment in the event of cognition, understands truth as the double ability to cognize elements in collections, and thereby to construct something else from them. Extensionally, we create a new collection, but this extensional delimitation paves the way toward intentional reconstructions — whether a new cake or a new bomb, in either case a new way of understanding how elements become ingredients in fabrications obedient to human volition rather than beholden to natural fulfillment.

Whatever the subtleties of the modern interpretation of rationality, it introduced truth as an affair — one of reconstruction. This double interpretation stems from the new understanding of knowledge as illustrated first of all in intellectual work (rather than in artisanal work). Henceforth, knowledge-at-its-best commits us to what Kant calls “productive imagination,” i.e. incessant re-construction within conceptual frameworks that retain their legitimacy only as they constantly open out onto perceptual givens (or to the givenness, in intuition, of the space-time grid). Indeed, our entire system runs in close parallel to the modern passion for the conceptual knowledge inaugurated by modern rationalists and empiricists alike, prior even to Kant's monumental critiques of this passion. Set theory is a kind of Doppelgänger of modern science, distinguished from it chiefly by its powerful Axiom of Extensionality (the deferral of intensality).

Selections, whether imagined to happen one at a time in a finite sequence, or fantasized to happen all at once over an infinite set, are possible only because cognition itself, and therewith our historical interpretation of rationality, underwent a radical shift. Although “workers in the field” may have no need to reflect on this shift, and will sometimes deny that things have ever been or could ever be otherwise, one task of philosophical work is to think it through and detect its underpinnings — the “transcendental acts” presupposed by (i.e. performed in) rigorous intellectual work.

* Charles C. Pinter, Set Theory (Addison-Wesley, 1971). pp. 112-13. Also p. 114: “The essence of the Axiom of Choice is that it is an existential statement rather than a constructive one.”
Appendix IV: Probability

Prelude

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Probability

Early on, logicians commit themselves to the assumption that any proposition \( p \) is either true or false — or, more generally, has one and only one of two “values,” designated numerically as 1 and 0. Such is the “truth-functionality” that undergirds many logical operations. Such, too, is the basis of the principles of non-contradiction and excluded middle.

Yet propositions rarely “come” to us with values already assured. We may hear or read, sometimes even propound a proposition as if its value were assured, but we learn that there is a remarkable difference between assumption and assurance. Reflection entails a skepticism of sorts: the ability to doubt the value assigned — in order to seek assurance, or even to withhold judgement entirely.

No propositions about past and future happenings (these understood as “empirical”) are fully assured. Jurors and judges must decide propositions that often turn out to be wrong. Soldiers and generals must endorse propositions without knowing for sure whether they are true or not. And many propositions regarding what is present — immediately available to perception — refer us back to what happened before, or forward to what will happen eventually: these, too, then fall short of complete certainty regarding the value to be assigned.

Our ancient intellectual traditions insist on the uncertainty of propositions regarding empirical events, whether past, future, or present. Apart from the usual chorus of detractors and their minority reports, the main thinkers of our Greek and Roman tradition drew a hard line between the empirical and the rational. Rational were those aspects of “things” accessible only through the mediation of careful response on our part — those determinations hovering above or behind or beyond or before events immediately and often urgently crowding in upon us. The favorite examples of such aspects were drawn from geometry: our empirical measurements of the earth tilled or owned or traversed may be called into
question, but it is possible to learn to calculate formations that are certain in themselves (the most favorite example being the Pythagorean theorem) and that allow us with augmented certainty to measure out a piece of land available to perception.

It is a mark of modernity that we aspire to cross the line between the empirical and the rational — to bridge the difference. Modern thinkers like Leibniz, fully aware of the uncertainty of propositions regarding “earthly affairs,” proposed methods for calculating degrees of certainty — degrees of truth — regarding events as they just happen (did happen, will happen). A hallmark of such calculation is that it allows us to be certain about things uncertain: that it provides a kind of meta-certainty of practical use in dealing with uncertainties.

Games of chance provide clear-cut examples of meta-certainty. While it is not certain that I will draw a spade from a standard deck of cards (or did draw one, not having yet looked), we would like to say that it is certain that “I draw (did draw, will draw) a spade” has a 25% chance of being true.

Once we shift back (again, into the rational), we can develop elaborate calculi of probability, ones applicable to very complicated games of chance — and then also to predicting election-results (from public opinion polls) and turns of the weather (from meteorological observations); furthermore, to establishing appropriate pension contributions (from statistics regarding births and deaths in a given population) and to ascertaining accuracy of measurements in space and time (as is necessary in much technology). Such meta-certainty may take a form such as “our determination that the Liberals will get 52% of the vote in the up-coming election is accurate within 2% nineteen times out of twenty.”

But what does it mean to ascribe to a proposition \( p \) a “percentage”? This question touches upon the formulation, the reference, and our own understanding.

How may we best formulate this ascription? Is “I draw a spade” 25% “true”? Or should we say: There is a 25% chance that this proposition will prove to be true? Or perhaps: this proposition is true 25% of the time? In the latter two cases, time must eventually be considered in all its complexity.

To what does the formulation refer? It often describes what is inherent in the situation (52 cards, 13 of which are spades). But we may not stop there: we would have to add that there is an equal chance of drawing each of the 52 cards in the deck. Such equality, or randomness, must eventually be considered in all its complexity.

And how are we to understand the formulation? What can we make of the probability of drawing a certain card, or kind of card, from a standard deck of cards? A formula often serves to justify a practical confidence. Skilled card players recall what cards have already been played, and take their chances accordingly; some roulette players even have greater confidence in a black turning up after several whites, or lesser in a black right after a black. Furthermore, insurance companies seem to be quite confident in their calculations of the probabilities regarding different kinds of costly accidents, and of life expectancies.

From the beginning (with Aristotle), logicians have insisted that we recognize three modalities of propositions: necessary (the Pythagorean theorem), factual (here’s a spade!) and possible (yet to be decided). Whereas most philosophical work seems to concentrate on the question of the difference between fully necessary (eternally true) and merely contingent (phenomenally true) propositions, the development of probability theory concentrates on the difference between fully factual (even if still “contingent”) and the possibly factual (with falsity being one limit-case of possibility).

Probability seems to fit in neatly with the modality of possibility. Yet for nearly two millennia there was no thought of quantifying either one.

As for possibility: In both forensic and deliberative discourse one considers a proposition — considers it as yet to be decided. Kant defines this modality with an eye to scientific discourse: “Whatever satisfies the formal conditions of experience (regarding intuition and concepts) is possible.” In either case, the proposition is possible or not: there is no question of degrees.

As for probability: Since “earthly affairs” (thus throughout factual considerations: especially those at issue in forensic and deliberative oratory) must be decided and yet never allow of unconditional certainty, one must fall back on propositions that are commonly accepted and
The Aristotelian understanding of probability leans heavily on Platonic-Aristotelian ontology.† Our dealings with circumstances has twofolds: “given instance” and “transcendent idea.” Each instance arises in time at a place, full of variabilities escaping the notice of our senses. Each idea is either a generality commonly accepted among those jointly concerned to deal with the instances effectively, or it is an holistic vision worked out carefully by those few concerned to detect what is really and wholesomely at issue for the instances themselves (nature: φυσις and in joint dealings with them (community: πολις). For one accustomed to contemplate matters holistically (and to remember the variability of instances), a merely accepted generality (opinion: δοξα) is unreliable and must be overcome. Yet pre-modern thinkers understood the unreliability of opinion, and the possibility of overcoming it, much differently than we today might think. Two key differences are these: first, the life of the sensation (αισθησις) is inevitably uncertain (both with regard to the past and with regard to the future) and, second, the only way anyone can effectively bond others into a phalanx is to recall the opinions they share. On both scores, the wise will accept their finitude in earthly dealings — in order to devote their finest hours to the trans-finitude of pure intellec tion (νοησις), a form of life in which the issues (fulfillments) of instances do become clear and certain, but not their application to instances.

Such an ontology precludes the effort to know what’s probable (as distinct from knowing about it). The interface of intellec tion and sensation (as it continued to be called well into modernity) did not, in antiquity, allow the kind of cross-over familiar to us today: we “run into” the sensible dimension of things, we “know” the intelligible dimension of things. That these two dimensions somehow “come together” signals our own commitment to stand at the interface, but never allows us to dissolve one into the other (as, notoriously, Hume tried to do in one direction, and Hegel in the other). To face what is “given” (τυχη: chance, fortune, luck — good or bad), one can only gird oneself by learning a craft (τεχνη: the human ability to participate in nature’s ability). About our own craft (knowledge), Aristotle famously remarks that it “in part completes what nature is unable to finish, and in part follows nature.” Such girding may take the form of martial arts, leadership, horse-training, house-building, or even contemplation: in each case, the τεχνη directs us toward what is at issue at the interface, allows us to face what offers itself to us at this intersection, but neither dictates the course of sensation directly, nor even lets us know it qua sensation. Knowledge is confined to the completion looming over (or in) what one faces. Opinion is confined to the generalities serving in the place of knowledge. These latter are, in application, only probable: they are worthy of consideration only because we must, in forensic or deliberative contexts, act rather than only contemplate. Wisdom, if not experience, teaches that we may always be wrong.

Our present-day understanding of probability may have some kinship with the earlier understanding, but the contrasts are striking. Most obviously, modernity has developed ways of quantifying probability — both purely mathematical calculi and, in conjunction with empirical data, statistical calculi. Although we still speak loosely of a proposition being “more or less” probable, in serious affairs we must often establish a figure between 0% and 100% — or between 0 and 1. Much empirical research in the sciences, and many policies in our institutions, depend immensely on arithmetically probabilistic considerations. Far from excluding probabilities, modern knowledge of how things are, were, and will be includes at its very heart the calculation of what at one time appeared to be unknowable very nearly “by definition.”

What happened — continues, “underground,” to happen? Most clearly, knowledge has taken on another meaning — one associated with what we now call technology. It now means the ability to steer the course of affairs — this ability among other things. Yet knowledge can only shift in meaning to the extent that what is knowable shifts in tandem. The aspiration to know circumstances already sets its sights on what is to be known: each aspiration entails its own ontology. For modern thinkers, this ontology might be crudely stated in the words of Francis Bacon:

Although in nature nothing exists besides bodies performing purely individual acts according to a law, in
Appendix IV: Probability

For a sustained philosophical consideration of the historical developments of probability in modern times, see Ian Hacking’s *The Emergence of Probability*, 1975; then also his more technical *The Logic of Statistical Inference*, 1965; and his more widely ranging *The Taming of Chance*, 1990 (all published by Cambridge University Press).

On analogy with the laws established by rulers of a community, laws of nature henceforth appear as governing “bodies performing purely individual acts”: form no longer means the fulfillment of such bodies, each according to its kind, but rather the law itself. Once “what is” splits into “bodies acting” and “laws governing,” knowledge becomes an affair of formulating laws and manipulating bodies. And from here it is only a few steps—those taken by Pascal and others in the second half of the 17th century—to formulating laws probabilistically.

If at the root of our understanding of knowledge (our epistemology) there lies an often unconsidered understanding of “what is” (ontology), both these understandings presuppose an understanding of *truth*. Indeed, the question what it means to ascribe a probability to a proposition—\( \Pr(p) = n \), where \( 0 \leq n \leq 1 \)—raises all over again the question what it means to ascribe truth or falsity to it.

Before pursuing these questions, let us review the formalisms that have developed over the last few centuries, and proved themselves in the mid-20th century.

§1. An intuitive calculus of probability

Having flipped a coin and holding it concealed, we assume that the probability of the proposition “It’s heads” is 50%. Letting \( H \) stand for “It’s heads,” we may formalize this claim:

\[
\Pr(H) = .5
\]

(whereby we include the two limiting cases within the continuum—as though there were no qualitative difference between these two and what lies between them). Intuitively, these two formulas hold for *any* proposition \( p \), including substitution instances such as:

\[
(p \lor \neg p) : \Pr(p \lor \neg p) = 1
\]
\[
\neg(p \land \neg p) : \Pr(p \land \neg p) = 0
\]

The need for an “art of combinations” (as Leibniz called it) becomes evident when we must calculate the probability of a proposition referring to more than one element. For instance, what’s the probability of \( T = \) “There’s at least one tails in two flips of a coin”? Considering that there are now four equally possible outcomes, three of which satisfy the truth-condition of \( T \), we know:

\[
\Pr(T) = .75
\]

Intuitively, too, we can see that the probability of “There’s no tails in the two flips of the coin” is calculable by substraction:

\[
\Pr(\neg T) = 1 - \Pr(T) = 1 - .75 = .25
\]

Another example of a combination: What’s the probability of “I draw a king” or “I draw an ace” = \( (K \lor A) \)? Assuming a standard deck of cards,

\[
\Pr(K) = 1/13
\]

and \( \Pr(A) = 1/13 \)

The “or” obviously increases the probability of the combination. Intuitively this relation is expressed by addition:

\[
\Pr(K \lor A) = \Pr(K) + \Pr(A) = 1/13 + 1/13 = .1538\ldots
\]

The calculation becomes more complicated when we consider propositions referring to two different events. What’s the probability of “I draw a queen the first time” and “I draw a queen the second time” = \( (Q_1 \land Q_2) \)? Given a full deck,

\[
\Pr(Q_1) = 4/52
\]

And assuming that I would then keep the first queen,

\[
\Pr(Q_2) = 3/51
\]

Now, the second (imagined) event is embedded in the first, diminishing the probability of the combination. Intuitively, this relation is expressed by multiplication:

\[
\Pr(Q_1 \land Q_2) = \Pr(Q_1) \times \Pr(Q_2) = 4/52 \times 3/51 = .0045\ldots
\]
From these examples we can abstract, provisionally, a number of principles — ones which serve as occasions for further reflection and refinement:

1. \( \Pr(p \lor \neg p) = 1 \)
2. \( \Pr(p \land \neg p) = 0 \)
3. \( \Pr(p \lor q) = \Pr(p) + \Pr(q) \)
4. \( \Pr(p \land q) = \Pr(p) \times \Pr(q) \)

The first two reiterate the principles of Excluded Middle and Non-Contradiction. From the first and the third we can derive a fifth:

5. \( \Pr(\neg p) = 1 - \Pr(p) \)

Yet there is a story behind each example from which we have extracted these principles, and some of these stories, when examined more carefully, reveal the need for refinements in the formulas themselves.

Consider the flip of a coin: it is assumed, first, that there are only two possibilities (that a the coin cannot land on its edge, for instance) and, second, that each is equi-possible (that the coin is not rigged in favor of one or the other). Formally stated, \( \Pr(H) = .5 \) depends on two other formulas: \( H \lor T \) (there are no other possibilities, so that \( \neg H \leftrightarrow T \)), and \( \Pr(H) = \Pr(T) \) (based on the assumption that the two events are equi-possible). Both these propositions are contingent, yet they employ concepts of special interest for intellectual work — and ones that are perhaps even a priori in a Kantian sense (especially the notion of equi-possibility).

Consider the disjunction “I draw a king or I draw an ace.” The familiar facts of a standard deck makes it clear that the two propositions are mutually exclusive (they cannot both be true), so that the probability of the compound proposition is 8/52. The simplicity of this calculation contrasts with the complication of calculating the probability of “I draw a spade or I draw an even-numbered card.” To be sure:

\[
\begin{align*}
\Pr(S) &= 13/52 \\
\Pr(E) &= 20/52
\end{align*}
\]

Is the probability of the disjunction 33/52? Many of the even-numbered cards are spades as well, and the duplication does not add to the probability in question. A simple counter-example proves the point:

Imagine a deck of four cards, two black ones, one lettered A and one lettered B, and two white ones, one with A and one with B. What’s the probability of drawing a black or an A? Each separately is 50% so that adding the probabilities would make it a sure draw! Yet if we count the possibilities we come up with a probability of only 75%.

What should we subtract from the sum of the two initially calculable probabilities? We can imagine counting — first, the number of cards satisfying the proposition “I draw a spade” (13) and, second, the number of cards satisfying the proposition “I draw an even-numbered card” (20); finally, we may discount the number of cards that have been counted twice (the 2, 4, 6, 8, and 10 of spades: 5): we would then have counted 28 distinct draws satisfying the disjunction \( S \lor E \), the probability of which would then be 28/52. This procedure of counting and discounting (one imaginable for finite sets) is equivalent to adding the separate probabilities and subtracting the probability of duplication. Formalizing this improvement on (3) above, we have:

\[
(3') \Pr(p \lor q) = \Pr(p) + \Pr(q) - \Pr(p \land q)
\]

which will serve as the generalized formula for calculating the probability of a disjunction over one event (in our example, one draw). Of course, when \( \Pr(p \land q) = 0 \), as there is no such thing as drawing a card that is both a spade and a heart, the coda serves no purpose.

When there are successive trials (events) the coda plays a decisive role. What is the probability of “I flip at least one heads in three flips of a coin”? \( \Pr(F_1 \lor F_2 \lor F_3) = ? \). The probability of “I flip a heads” is in each case 50% but surely there is not a 150% chance of obtaining a heads! The correct calculation relies on a principle we easily forget when we drop the punctuation (parentheses): logical operations like disjunction are binary. We must first calculate:

\[
\Pr(F_1 \lor F_2) = [.5 + .5 - (5 \times .5)] = .75
\]

And then calculate the rest: .75 + \( \Pr(F_3) - \Pr(.75 \times .5) = .875 \). If the same question arises in regard to greater numbers of flips, this style of calculation becomes tedious. It is then easier to calculate the probability of not obtaining any heads for \( n \) flips, and subtract this figure from 1, since:

\[
\Pr(F_1 \lor F_2 \lor F_3 \ldots) = 1 - \Pr(\neg F_1 \& \neg F_2 \& \neg F_3 \ldots)
\]

and the multiplications may be effected in simply sequence.
The difference between \( \Pr(p \lor q \lor \ldots) \) for a single trial and the same formula for a number of trials is neatly accounted for by the Stoic distinction between definite and indefinite propositions. When the disjunction covers a single trial, the variables take instances reading “This…” — each instance having the same “this”: “This card is either ….” When the disjunction covers multiple trials, the variables take instances reading “There is…” — each instance having its own “subject”: “This time there’s heads.” The story on which our formalisms depend tacitly requires that we distinguish differing kinds of propositions. One kind that is excluded is universals: there is no sense, in probability theory, of talking about the probability of “All S are P” or “No S are P”; probability pertains to “particulars” (whether definite, indefinite, or intermediate) and relegates “universals” to the domain of formulation (devising, then, a kind of meta-universality).

Recall, too, the probability of conjunction — the example “I draw a queen first and I draw a queen second.” If the first queen were replaced and the deck reshuffled, the formulation of principle (4) above would hold. But if the first queen is kept out of the deck the formulation no longer holds. For we are then considering two successive draws, the first of which affects the condition of the second: the second draw dips into a deck of 51 rather than 52 cards, and, if the first draw extracted a queen, there would be only 3 queens left in this reduced deck. Our intuitive calculation accounted for this difference. How may we best account for it in the formulation itself? The second draw must be hypothetically formulated: if a queen is first drawn, the probability of “I draw a queen second” equals \( n \). Formally and generally:

\[
p \rightarrow \Pr(q) = n
\]

Yet in the imagined sequence, \( p \) is also given, so that the probability of \( q \) has an (imaginary) value. We may formulate this (imagined) condition of modus ponens as follows: “The probability of \( q \) given \( p \) is \( n \)” — or, formally:

\[
\Pr(q|p) = n
\]

Similarly, we could formulate our two a priori true propositions with this new symbol \( | \):

\[
\Pr(p|p) = 1
\]

\[
\Pr(-p|p) = 0
\]

Formalizing this improvement on (4) above, we have:

\[
(4') \quad \Pr(p \land q) = \Pr(p) \times \Pr(q|p)
\]

which will prove to be the formulation we need in the axiomatic version of probability theory.

These improvements on the disjunctive and conjunctive principles of probability draw upon further elements in the story. For one, both principles presuppose a finite ordered set. It is no coincidence that our modern mathematical version of probability grew out of the consideration of well-defined games of chance such as the casting of two dice each having 6 numbers, or the dealing of cards from a fixed deck of 52. We may then count the duplications in the case of disjunction, count the second condition in the case of conjunction. In more complicated applications, we can only count samples of ill defined sets.

Moreover, even though we speak loosely of the cast of two dice or the draw of two cards as yielding a simultaneous result, our formulations require that we imagine the determinations unfolding one after the other, the second being conditioned by the first. The principle here is the same as that of mathematical induction: the “next” may be a function of what “came before.” And, according to the same principle, we may extend the series of conjunctions or disjunctions indefinitely.

One of the challenges of modernity has been to apply the mechanistic principles of celestial mechanics to terrestrial events — so much more complicated and apparently irregular as these are. In exact parallel, modernity has been challenged to apply the probabilistic principles of games of chance to the much more complicated affairs of “earthly” events — such as rates of disease, mortality, criminality and the like in populations (for political, juristic and actuarial purposes), and even fortuity in nature (for insurance and even technological purposes). Both challenges presuppose what used to be called the theory of mechanism: that, namely, the laws of nature completely determine the course of events, while our knowledge of these laws, and therefore of this course, is only partial — and yet can sometimes be formulated, with a kind of mathematical exactitude, in probabilistic ways.

Thus, just as we are ignorant of some, often most of the factors involved in an actual course of affairs, so even our calculations regarding well defined games of chance presupposes our ignorance. Thus it may be that we calculate the probability of \( D = \) “I draw a spade” (from a standard deck) as 25%, and that of “I draw a spade within two draws” as...
Pr(D₁ ∨ D₂) = .25 + .25 - (.25 × .25) = 40.375%, and that of “I draw a spade within three draws as 1 - (.75)³ = 57.8125%. Of course, the probability will go on increasing as the number of trials increases. Yet such probability pertains to the compound proposition (D₁ ∨ D₂ ∨ . . .). It does not apply to any one draw: even after no spade has been drawn after 100 draws, Pr(D₁₀₀) = 25%. For our knowledge that each of Pr(Dₙ), 1 ≤ n ≤ 100, = 0 renders all those cases irrelevant to Pr(D₁₀₀). While amateur gamblers often lose their shirts, mathematicians should know that the predication of a probability to a compound proposition pertains to a set, not to members of the set. And, here, any set is composed of unknown outcomes. Ignorance is essential to the story.

Of the many developments of probability theory from these few basic ideas, there is one that deserves special remark: inverse probability. The natural sequence seems to point forward: we know the proportion between possible outcomes of “I pick a ball out of an urn” and “I pick a red ball out of the urn”—assuming equi-possibility (random picking)—and we predict accordingly (with a figure representing the probability). But consider a set-up in which “I pick a red ball” has been established, so that Pr(R) = 1, and yet there were five urns: one white urn, containing eight red balls and two green balls; and four black urns, each containing three red balls and seven green balls. We then ask: What is the probability of “The source of the ball is the white urn,” given that “The ball drawn is red”? Pr(W|R) = ?

What we know is the “forward” version: the probability of “The ball drawn is red” given that “The source of the ball is the white urn.” Pr(R|W) = 80% What formula would allow us to calculate the “backward” version, i.e. Pr(W|R)?

Consider the original situation: I approach these urns blind (ignorant) and choose a ball at random. In the forward version, I do know this: Pr(W & R) = Pr(W) × Pr(R|W); and each of these latter I also know: Pr(W) = 20% and Pr(R|W) = 80%. However, (W & R) ↔ (R & W), so I also know this: Pr(R) × Pr(W|R) = Pr(W) × Pr(R|W).

And I know Pr(R) = 40% (20 of the 50 balls are red). From these figures I can calculate the desired probability:

\[ Pr(W|R) = \frac{Pr(W) \times Pr(R|W)}{Pr(R)} = \frac{.2 \times .8}{.4} = .4 \]

And from this one example we can detect, intuitively, the formula for calculating inverse probabilities:

\[ Pr(q|p) = \frac{Pr(q \text{ vert-2}) \times Pr(p|q)}{Pr(p)} \]

For all its simplicity, this formula will supply the basis of Bayes’ Theorem. And it also serves to remind us that forensic discourse often asks us to reconstruct the probability of the source of something happening that has already in fact happened.

§2. An axiomatic calculus of probability

So far, we have formulated probabilities of various propositions by relying on our intuitive understanding of frequencies. And while we have been able to generalize some principles from our chosen examples, we may well wonder whether we can organize these principles in order to place some as axioms and others as derivable theorems.

The contemplative intent of an axiomatic revision of an intuitive calculus is three-fold: first, to drive to the surface the informal presuppositions of the original calculus; second to show that and how the formal developments of the calculus may be deduced from a very few formulas; and finally to pave the way for reopening the questions the presuppositions and the formulations otherwise close off.

The most prominent presupposition of any formula expressing probability is that the formula holds for a defined set-up, one allowing us to imagine an initial set of equally possible outcomes. Such a set implies that its members consist of random occurrence in nature, or of random choices on someone’s part. Formally, we can express this presupposition of a defined set-up as a reliance on an hypothesis. For any p having a probability n, we may write:

\[ Pr(p|h) = n \]

Or: given hypothesis h regarding the set-up, Pr(p) = n.

Very much like the traditional contention that the definition of a species relies on an intuitively evident but undefined genus—and very much like the mathematical admission of the need for a nameable and finite (but otherwise unformalized) “universal set”—the recognition of the need for a background hypothesis paves the way for contemplation of the embedded—and indebted—nature of the formalisms.
In addition to the notion of equi-possibility, the background hypothesis must include an account of how any one intervention in a sequence of occurrences or choices will affect the set-up itself, and therefore the calculation of subsequent probabilities.

The probabilistic calculations then share the same fate as geometrical calculations: the tension between the ideal and the real. In both cases, we work out beautiful theorems, only to discover that our formalisms enjoy a certainty in the study that does not carry over entirely to complexities out on the fields. In both cases, the set-up required by the calculations is engendered by the exigencies of the calculations themselves, and we only “find” the set-up in a field because we have projected it into the field. Thus the best illustrations are those in which we ourselves invent the set-up: games of chance. If, for instance, we discover that a pair of dice is “loaded” we simply rectify it—or procure another pair, or measure, as best we can, the imbalance, thereby formalizing the difference and compensating for it, creating once again an (imaginary) equi-possibility.

The presupposition of an $h$ may be integrated into our formalization of the two axioms necessary for deriving all the theorems we have formulated intuitively. As it turns out, we need only two such axioms, one regarding conjunction and one regarding negation:

$$
\Pr(p \land q | h) = \Pr(p | h) \times \Pr(q | h \land p) \quad \text{Axiom 1}
$$

$$
\Pr(\neg p | h) = 1 - \Pr(p | h) \quad \text{Axiom 2}
$$

The idea behind Axiom 1 is that the probability of two conjoined propositions is a function of each taken separately. And the idea behind Axiom 2 is that the converse probability of a proposition is still a function of that proposition.

Besides Substitution and Algebra, the rules of inference are:

- **RE**, i.e. replacement by logical equivalence
- **RI**, i.e. replacement by arithmetical identity

**Theorem 1:**

$\vdash \Pr((p \land \neg p) | h) = 1$

1. $\Pr((p \land \neg p) | h) = \Pr((p \land \neg p) | h) \times \Pr(q | h \land (p \land \neg p))$ Ax. 1
2. $\Pr(q | h) = \Pr((p \land \neg p) | h) \times \Pr(q | h) \quad 1 \text{ RE } \{(p \land \neg p) \land r \leftrightarrow r$
3. Theorem 1 \quad 2 Algebra


With Axiom 1 and the equivalence $p \leftrightarrow (p \land p)$, one may prove:

**Theorem 2:**

$\vdash \Pr(p | (h \land p)) = 1$

Then, with Axiom 2 and judicious substitution, one may prove:

**Theorem 3:**

$\vdash \Pr(p | (h \land \neg p)) = 0$

The next theorem easily follows from the algebraic principle that “equals added to equals yields equals”:

**Theorem 4:**

$\vdash \Pr(p \land q | h) + \Pr((p \land \neg q) | h) = \Pr(p | h)$

This last theorem serves well to eliminate negations, since a probability with “$p$ and not $q$” can be expressed as a difference between the probability of $p$ and the probability of $p$ and $q$. It also facilitates:

**Theorem 5:**

$\vdash \Pr(p \land \neg p | h) = 0$

Next, the much-needed theorem for the addition of probabilities:

**Theorem 6:**

$\vdash \Pr(p \land q | h) = \Pr(p | h) + \Pr(q | h) - \Pr((p \land q) | h)$

1. $\Pr((p \land q) | h) = 1 - \Pr((p \land \neg q) | h)$ Axiom 2, DN
2. $\Pr((p \land q) | h) = 1 - \Pr((p \land \neg q) | h)$ 2 RE, DeM
3. $\Pr((p \land q) | h) + \Pr((p \land \neg q) | h) = \Pr((p \land h) $ Theorem 4
4. $\Pr((p \land \neg q) | h) = \Pr(p | h) - \Pr((p \land \neg q) | h)$ 3 Algebra
5. $\Pr((p \land \neg q) | h) = 1 - \Pr(p | h) $ Axiom 2
6. $\Pr((p \land \neg q) | h) = 1 - \Pr(p | h) - \Pr((p \land \neg q) | h) $ 5.4 RI
7. $\Pr((p \land q) | h) = 1 - \Pr((p \land \neg q) | h)$ 6.2 RI
8. $\Pr((p \land q) | h) = \Pr(p | h) + \Pr((p \land \neg q) | h)$ 7 Algebra
9. $\Pr((p \land \neg q) | h) = \Pr(q | h) - \Pr((q \land \neg p) | h)$ Theorem 4
10. $\Pr((p \land \neg q) | h) = \Pr(q | h) - \Pr((q \land \neg p) | h)$ $8 \text{ RE Comm}$
11. $\Pr((p \land q) | h) = \Pr(p | h) + \Pr(q | h)$ 10.9 RI
12. Theorem 6 \quad 11 RE Comm

With Theorem 3 and Theorem 6, one may easily prove:

**Theorem 7:**

$\vdash \neg p \land q \rightarrow \Pr(p \land q | h) = \Pr(p | h) + \Pr(q | h)$

Here, as well as in the formulations of Theorems 2 and 3, we may interpret the conditional “outside” to be equivalent to its inclusion with the hypothesis “inside”—but *throughout*. Roughly:

\[ p \rightarrow \Pr(q | h) = 1 = \text{def} \{ \Pr(q | h \land p) = 1 \} \]
The following is a simplified version of Bayes’ Theorem — so named because first worked out by Thomas Bayes (1702-1761), although first published in 1763.

**Theorem 8:** \( \Pr(q|p) = \frac{\Pr(q) \times \Pr(p|q)}{[\Pr(q) \times \Pr(p|q)] + [\Pr(\neg q) \times \Pr(p|\neg q)]} \)

We may leave out the mention of \( h \) and abbreviate the proof:

1. \( \Pr(q|p) = \frac{\Pr(q) \times \Pr(p|q)}{\Pr(p)} \) Ax. 1 (twice), Comm & Algebra
2. \( \Pr(p) = \Pr(p) \) Identity
3. \( \Pr(p) = \Pr((q \land p) \lor (p \land \neg q)) \) \( p \leftrightarrow [p \land (q \lor \neg q)] \)
4. \( \Pr(p) = \Pr((q \lor \neg q) \land p) \) 3 Th. 6, \( \Pr((q \lor \neg q) \land p) = 0 \)
5. \( \Pr(p) = \Pr([\Pr(q) \land \Pr(p|q)] + [\Pr(\neg q) \land \Pr(p|\neg q)]) \) 4 Ax. 1, RI
6. \( \Pr(p) \) 1.5 RI

The foregoing eight theorems make no use of any extra-logical hypothesis; they remain purely within the logical development of the two axioms. The final two theorems show what can happen once we introduce very context-specific hypotheses.

What is the probability of “I draw exactly three spades” **given that** “I draw five times (replacing the card after each draw)” **and** that \( \Pr(\text{I draw a spade}) = 1/13 \)? The question can be generalized to read: What is \( \Pr(k) \text{ given that } \text{there are } n \text{ trials, } \Pr(F) = p \text{ (where } F = \text{the one trial has the favored result)} \), and \( k \) the number of favored outcomes? The answer is formulated in what is known as the binomial theorem:

**Theorem 9:** \( \Pr(k|n \land p) = \begin{pmatrix} n \end{pmatrix}^k \times (1 - p)^{n-k} \times \frac{n!}{k!(n-k)!} \)

Note that the variables \( n, k, \) and \( p \) are employed equivocally. In the \( \text{Pr}-\) statement at the left, each variable represents a complete proposition: “There are \( n \) trials,” “There are \( k \) spades drawn,” and “The chance of drawing a spade is one in thirteen.” This equivocation shortens the formulation but it blurs the logical distinction we are investigating: namely the very effort to assess the probability (truth) of a proposition by means of numbers.

We may prove this theorem in two stages. First we calculate one, the “star” probability: that I draw a spade three times in a row and then draw non-spades two times in a row. Second, we calculate the number of combinations in which the same favorable result occurs in merely different orders. These two calculations we may generalize from the start:
1. \( \Pr(\text{“I draw a spade”}) \text{ in each of the first } k \text{ draws} = p^k \) Axiom 1
2. \( \Pr(\text{“I draw a non-spade”}) \text{ in each of the remaining draws} = [\Pr(\neg F)]^{n-k} = (1 - p)^{n-k} \) Axioms 2 and 1
3. \( \Pr(\text{“I draw a spade” } k \text{ times } \& \text{ “I draw a non-spade” } (n-k) \text{ times}) = p^k \times (1 - p)^{n-k} \) Axiom 1

This last gives us the probability of one, the “star” outcome. Now, how many ways are there in which I could obtain 3 spades and 2 non-spades in five successive draws? This is a question of combinations. The number of “possible combinations of \( n \) items taken \( k \) at a time” is formulated as \( nCk \), which has an algebraic equivalent:
4. Number of possible favored outcomes = \( \frac{n!}{k!(n-k)!} \) Algebra

Now, we calculate the probability of the favored outcome over all these by adding them together. But, since (or rather: assuming) they all have the same probability, we need only multiply the “star” probability by the number of combinations.

5. \( \text{Theorem 9} \) 3.4 Multiplication

Finally, what theorem would allow us to compute probabilities on a complex hypothesis much like the previous one but differing in that the cards drawn are not replaced? A simpler example would be: there’s a sack of seven balls, four black and three white. What’s the probability of extracting exactly three black balls over five random extractions? In generalized form: What’s \( \Pr(\text{“there are } k \text{ black balls extracted”}) \)—where \( b \) is the number of blacks and \( w \text{ is the number of whites, } n \text{ the total number of extractions, } k \text{ the favored number of black balls, and, finally, } \Pr(\text{I extract a black ball}) \text{ varies from one extraction to the next (so there is no fixed } p, \text{ as there was in Theorem 9)\} \). The answer is formulated in what is known as the hypergeometric theorem:

**Theorem 10:** \( \Pr(k | (n \land b \land w)) = \frac{\text{bP}k \times \text{wP}n}{(b+w)\text{P}n} \times \text{nC}k \)

The formulation of the theorem introduces the concept of permutations, i.e. the number of possible ordered selections. The permutations of \( n \) items taken \( k \) at a time is expressed as \( nPk \).
The proof proceeds much as did that of Theorem 9. First, we calculate the “star” probability (call it \( SP \)), i.e. the probability of “There are three blacks extracted in a row, and then two whites”:

\[
\Pr[SP] = \frac{4 \times 3 \times 2}{7 \times 6 \times 5} \times \frac{3 \times 2}{4 \times 3} = \frac{bP_k \times wP_{n-k}}{(b+w)P_n}
\]

The generalization on the right side of the formulation answers the question: What did we do to construct the one example? Here we have an instance of what mathematicians and philosophers have called “intuitive” reason, namely a transcendent act on our part, one consisting of recalling the principles of one’s own (discursive) construction. While one empirical (a posteriori) observation does not justify generalization, one well formulated (well constructed and well contemplated) formal example may indeed lead to what Kant calls an a priori truth.

Second, then, we add together all the possible combinations of the numerator, each of these being a “lucky” extraction (again, assuming that every “lucky” extraction will have the same set of elements as does the “star” extraction, just in a different order); the denominator, of course, will remain the same throughout, representing as it does the sequence of \( n \) extractions. This “addition” we may effect by the multiplication indicated in the Theorem. Q.E.D.

Theorems 9 and 10 take respective shapes according to their respective hypotheses; the \( h \) in the one case refers to a situation in which each successive trial has the same chance set-up (the same deck of 52), while the \( h \) in the second case refers to one in which each successive trial takes place with a chance set-up that has been affected by the previous trial (one less ball). Furthermore, the proofs make use of algebraic truths regarding permutations and commutations. And with these last we are invoking the notion of sets, a notion developed in set theory.

In short, theorems regarding probability quickly embed themselves in various domains of mathematics. Indeed, the mathematician to which we nowadays refer as the originator (in 1933) of the modern axiomatization of probability, A. N. Kolmogorov, formulates the axioms in set-theoretical language:

\[
\text{The Truth of Probability}
\]

Let \( E \) be a collection of elements \( \xi, \eta, \zeta, \ldots \), which we shall call elementary events, and \( F \) a set of subsets of \( E \); the elements of the set \( F \) will be called random events.

I. \( F \) is a field of sets [a field = a system containing the sum, product and difference of any two sets in it...].

II. \( F \) contains the set \( E \) [i.e., \( E \in F \)].

III. To each set \( A \) in \( F \) is assigned a non-negative real number \( P(A) \). This number \( P(A) \) is called the probability of the event \( A \).

IV. \( P(E) = 1 \).

V. If \( A \) and \( B \) have no element in common, then

\[ P(A + B) = P(A) + P(B) \]

A system of sets, \( F \), together with a definite assignment of numbers \( P(A) \), satisfying Axioms I-V, is called a field of probability.

§3. The truth of probability: equi-possibility

Every calculus of probability, whether intuitive or axiomatic, presupposes the notion of random trials: that we can map out the possibilities in advance, according them equal chances. For example, we assume equi-possibility for heads and tails for each flip of the coin; we assume that, in the casting of a die, each of the six sides has an equal chance of landing face up; we assume that each card in a standard deck of fifty-two has the same probability of being extracted. We calculate compound probabilities in reference to such equi-possibility — on the basis, notice, of a perceived lack of any initial bias.

So it seems that we might define probability itself as what results from calculations based on the equi-possibility of certain initial events, a result expressed numerically in a percentage, or in some number \( 0 \leq n \leq 1 \). The truth, then, of any proposed probability “\( k \) is/was/will be the case” would lead us back, through some hypothesis (e.g., an \( X \)-sided block being tossed \( n \) times) to the notion of each side being equi-possible at each trial.

Yet such a definition raises a number of questions for those who examine its conditions carefully.

First, the definition seems to be circular: Is not the notion of equi-possible equivalent to the notion of equally probable? The probability of casting a standard die and obtaining two aces in a row refers back to the hypothesis that the probability of casting an ace on any one trial is one

---

in six. Correct as this formulation may be, we are still defining compound probabilities, not probability itself.

Second, the concern to formalize probability arises from the hope to know something about the flux of things that, one at a time, escape our certain grasp. At issue in the flux is the relative frequency of some one type of event. These events are facts that we record and collect into sets. And if, after many casts of a die we discover that a six-sided black lands on one of the six faces half the time, we overrule the arithmetic calculation (with “the eye of the soul”) and look (with our other eyes) to the geometric construction of the block and, if we discern no measurable differences, to the distribution of the weight inside the block — all the while convinced that the chance of the one side landing face down is not equal to the chance of each of the others doing so. Similarly in the actuarial computations of insurance companies: one looks first of all to the facts to determine probability — understood as relative frequency over a set of cases.

But the problem is familiar: such empirically assessed probability works only over “large” sets; thus we speak of relative frequency in the long run. And there is no escape from the thought that the “best” run will be longer than any stated one, i.e. transfinitely “long.” It is only for some practical purpose, or from some high meta-probability of accuracy within an acceptable range of error, that we stop after some finite number $n$ of cases and conclude that the probability of the next case $n + 1$ turning out in some stipulated way $S$ is proportional to the way these cases have turned out before:

$$\Pr(S_{n+1}) = \frac{S}{n}$$

But this reduction of probability to proportions in recorded events, coupled with an assumption that this proportion somehow applies to the next case, not only sneaks around the need for an infinite set: it already employs the notion of probability (the proportion does not definitely apply to the next case, but only probably!) and it furthermore invites us into an ontogeny according to which the future is like the past (a moot point for those engaged in any creative endeavor).

The modern notion of probability intends precisely to bring the two senses of ratio together: the sense of it as a priori calculation and the sense of it as a posteriori proportion. And this ambition raises a third question: How are we to understand our own role, our own “interest” in assessments of probability? That we do have a role and an interest is evident when we consider that, e.g., if we construct a chance set-up (say, a mechanism for a slot machine) to determine the probabilities mathematically, and then discover, after a “long enough” run, that the empirical result diverges wildly from the calculated result, we suspect that there’s something “wrong” with the mechanism (it is not properly lubricated, or whatever). Similarly, if we have calculated mortality rates for insurance premiums only to discover that the premiums don’t cover the indemnities, we assume that “times have changed.” In short, we have a stake in probabilities: a probability (whether a priori calculated or a posteriori determined) must engage us in the next case satisfactorily — where “satisfaction” entails confidence on our part (again, over the long run). To consider probability without reference to confidence would simply mean we were retiring into the mathematics. Yet surely the question of probability does not reduce to the question of confidence; it is supposed to engender it — a certain way of looking to, i.e. girding ourselves for, the “unknown next.”

In one important sense, these three questions are unanswerable: they simply formulate the conundrum that the modern development of probability theory highlights. The concern to formalize probabilities remains a concern to deal rationally with the unknown — to extract something knowable about our dealings with the unknowable. It is a question not of knowing case $n + 1$ but rather of assigning a probability to it. And what this means — that we do it, and that it makes some sense — is the question.

So we have to analyze the structure of the conundrum itself — with an eye to contemplating the “conditions of the possibility of experience” (Kant) or the “origins of beings” (Aristotle). The structure sends us into one of two opposing directions, according to whether we move within the modern or the ancient tradition.

Modern philosophy assumes the analyzability of knowledge into rational and empirical components. While a similar-sounding distinction lies at the basis of all western philosophy since Plato, the direction of the modern analysis commits us to a triadic semantics of name-concept-thing (see Book Five, §3.2, then also Book One, §7, Book Two, §4.2, and Book Three, §6.2). Bertrand Russell succinctly states this triad in his
1919 lectures on “The Philosophy of Logical Atomism”  (*Logic and Knowledge*, pp. 204-205):

I should like to say about understanding, that that phrase is often used mistakenly. People speak of ‘understanding the universe’ and so on. But, of course, the only thing you can really understand (in the strict sense of the word) is a symbol, and to understand a symbol is to know what it stands for.

This triad, established already by the Scholastics, is foreign to Plato and Aristotle. Triadic semantics seems necessary once intellectual work consists in working out the stance of “Here I stand in thought, and out there stand the objects I intend to bring into conformity with the standards of my thought”—a kind of dualism of subject and object.

On this ontology, the truth of probability lies in the exercise of our power to calculate the run of the events and thereby to guess at the upcoming ones (our condition of falsehood, on Plato’s account in *Republic*, 516C-D). Inside this unitary phenomenon we may distinguish two “sides”: just as we are somehow “both” agents and patients, so too “what is” can be analyzed into two domains: “what’s out there as yet unformalized, even unformalizable” and “what we have already formalized.” Our vocabulary will initially waver in meaning as words pertain now to one domain and now to the other. Ultimately, though, we must distinguish two concepts, and assign a distinct name to each. For instance, one sense of “possible” is “possible for what’s out there” and another sense of the word refers to the “degree of certainty” characterizing our own linguistic stance. Thus, for instance, “It’s possible for my computer to catch fire” says something about my computer, so that I could just as well have said that my computer is capable of igniting itself; and “It’s possible that my computer is what started the fire” says something about the proposition, so that I could just as well list “My computer caused the fire” along with other propositions to investigate (i.e., not yet asserted). If we wish to retain the same word “possible” in both instances, we could extend the name of each to grasp the two concepts in their difference and kinship: the first saying states a possibility in re and the second a possibility in dicto—to borrow a Scholastic distinction without prejudicing its original sense.

Finally, then, we might define “equi-possible” in a mix: the “possible” refers to what’s out there (in re), and the “equi-” refers to an acknowledged lack on our part (in dicto): we know that, for the time being, we don’t know of anything that differentiates the possibilities. In this way, it appears, we can define probability without employing the term to be defined.

Yet this solution depends entirely on the epistemology and ontology of modernity, and on the triadic semantics essential to both. Without these three, we would have no mathematical theory of probability. Nor, possibly, would we consider so important the virtue of openness with regard to the future, i.e. the spirit of objectivity so essential to modern investigation, whether of a scientific, political, or forensic sort: a certain reserve when it comes to making judgements.

Justice requires, however, that we give a hearing to the tradition out of which—away from which—the modern account of probability grew: to the Aristotelian account of knowledge, one that while taking possibility very much into account, leaves probability outside the ken of arithmetic and geometric calculation.

On the Aristotelian account, we understand beings “directly,” not by way of concepts. Or we misunderstand them, namely by relying on some shared view more or less obstructing the view (on δόξα: opinion). The direct route obviates all reliance on anything other than what we are summoned to understand. Of course, Kant set out to demonstrate that this is neither possible nor necessary: that there is no “intellectual intuition” of things in their existence, no νοῦς (pure intellection) of anything but the conditions of the possibility of experience. Given our often unacknowledged if not unconsidered endorsement of triadic semantics, we moderns find it difficult even to imagine a knowledge based on a dyadic semantics of name-thing.

Yet the dyadic ontology of Plato and Aristotle, and of their long and variable tradition, poses its own internal difficulties. Although the doctor, the captain, the flutist, the shepherd understand the beings of their respective crafts directly, these beings themselves are twofold: what they happen to be (at the moment in αἰσθησις) and what they can be (what they need to be). So there is a threesome after all—but not that of modernity. Starting with the twosome naming-being, Plato introduced the next “division” on the side of being rather than on the side of naming. Each being we deal with competently in a craft is double. In contrast, modernity detects in each judgement a doubleness of name and concept.

As any reader easily discerns in Aristotle’s *Metaphysics*, Books Five (Chapter 12) and Nine (Chapter 1), the keyword of Aristotelian ontology
Appendix IV: Probability

The Truth of Probability

is the noun “power” (δύναμις), along with the corresponding verbal form “to be able” (δύνασθαι; or simply “can,” δύναται), and the adjectival form “capable” (δυνατόν). English, unlike Greek, French, Italian, and German, does not so easily invite readers to contemplate the question in the even flow of these words. Yet noun, verb and adjectival all suggest the need to acknowledge constructive power in what we encounter, so that we might then understand our own commitment to its fulfillment “in act” or “in function” — since “what is potentially and what is functionally are in a way one” (1045 b 20), i.e. knowledge commits us to the dyad.

And one keyword in Aristotelian logic is “necessity” as opposed to “possibility”: “every proposition is either [simply] of belonging, of belonging from necessity, or of possibly belonging” (Prior Analytics, 24 a 25). There is no logic that fails to highlight the difference between considering and asserting, and between asserting and demonstrating a proposal. Kant, basing his categories on the analysis of judgement rather than beings, considered the three modalities as themselves categories.

Yet there exists an essential relation between wise detections of power in things and wise imputations of possibility to proposals. Both arise in discourse (λόγος), i.e. in our efforts to give or receive an account. While the relation between them is ultimately our own relation to beings (precisely because it belongs to our being to give or receive an account of them), we may begin by noting the relation as one of difference in meaning of the words as they apply to things. The correlate of power (ability) is ἐνέργεια, traditionally translated as “actuality,” this term to be taken in the nearly French sense of “currently in function”; another correlate is ἐντελεχεία, “completion” in the sense of “currently held within its purpose.” When Aristotle speaks of what's possible, the adjectival form is τὸ ἕνδεχον, literally “what's allowed in,” “what's admissible.” What's capable (τὸ δύνατον) and what's admissible (τὸ ἕνδεχον) differ sharply in their meanings. The power of something, its power, is its striving toward fulfillment, and it requires of us an engagement of craft, whereas our recognition of something as possible requires of us only that we “register” it, “list” it, as in public discourse where we list the various propositions an audience may have to affirm or deny in the course of debate.

Still, power and possibility are often related in logical considerations: knowing the ability of a horse, we infer possible outcomes (past or future), and if we affirm an outcome as possible we are imputing an ability to it (cf. On Interpretation, 22 a 15). Every proposition of belonging (πρόκειται) having the modality of possibility refers us to what “usually or naturally” happens: it is still beholden to “nature” in some sense, or rather holds us to it. And, of course, anything we recognize as potential we acknowledge as possibly not actualizing itself (or as possibly not getting actualized by us): thus everything simply potential both “possibly is” and “possibly is not” (cf. Metaphysics, 1050 b 11).

The ontology presupposed in (or brought into relief by) the original account of the potential and the possible deserves a study in itself; one itinerary of such study is sketched out in Books One and Two and worked out in Books Three and Five. The crucial thought required by this ontology includes the acknowledgement of the primacy of the ontological twosome as prior to, and definitive of our own linguistic responsibility. This crucial thought leads to, or perhaps follows from, an understanding of knowledge (ἐπιστήμη) quite contrary to that familiar to us in modern intellectual work of any kind, and especially in modern probability theory.

The difference may be spotted in one sentence from Aristotle’s Metaphysics (1064 a 30): “It is not possible for knowledge to be of what's accidental.” Knowledge (as distinct from simple familiarity, opinion, and the like) bears primarily on the one half of the ontological dyad that “stays the same”: on the fulfillments defining abilities. By understanding what a horse or a house necessarily is, i.e. its necessity, its fulfillment (all these being available only in intellection), I may understand “its other half,” i.e. how it happens to present itself at the moment and in sensation: I can recognize what's wrong with it and try to realign its temporal being toward its eternal being. Taken by themselves, just how they happen to be, horses and houses present to me only their accidents: what I can indeed register, over time, as a number of ungrounded possibilities for horses or houses generally. But such registration, no matter how extensive, contrasts with knowledge: it hammers a wedge between me and the ground of those possibilities, namely the other half of the dyad. As Aristotle says (1065 a 4-8), there is not even any sense in talking about “origins or causes” of what is accidentally (τὸ κατὰ συμβεβηκός δὲν); we can only have knowledge (relate to origins and causes) of what is “in-itself-ly” (τὸ καθ’ αὑτὸ δὲν).
Aristotle’s *reductio ad absurdum* for this judgement runs: if there were grounds for what happens incidentally, there would be no room for chance (τυχη), so that “to be possible, i.e. to come into being or not, would be entirely banished from happenings” — a silly opinion for all those who exercise a craft of production (ποιησις), committed as they are to “finishing what nature is unable to complete.”

And even sillier for those engaged in action (πραξις), i.e. public performance in a leadership capacity. Again, precisely in action we deal with matters so constituted that they are possibly other (ἐνδεχεσθαι ὤλας ἔχειν: cf. Aristotle’s *Nicomachean Ethics*, 1139 a 5 through 1143 b 6). Here we engage in a kind of discourse (λογος) that contrasts with that of knowledge: today we might call this kind of discourse “street smarts” (the Greek is φρονησις). Every craft, whether one of production or one of action, deals with becoming (γενεσις), requires precisely the ability to contemplate how to bring into being what “possibly is” and “possibly is not,” inasmuch as the origin of such becoming lies in the craftsman and not in the thing itself. Precisely here, in matters of genesis, we must contemplate matters fully, i.e. the dyad; Aristotle then concludes that “intellectual intuition” (νους) relates us both to the ultimate origin (what remains the same: formulated in the major premiss) and to what’s possible at the moment (formulated in the minor premiss).

Looking back on the ancient account, we moderns might discern an alternative manner of understanding the truth of what we call probability: turning away from what Aristotle calls power... some sort of possibility in the things themselves — so that we can then distinguish between this sense of possibility (in re) and the other sense required for computing probabilities of propositions. In all this, we have abandoned the ancient supposition binding knowledge, craft, intuition, and street smarts to the affair of “partly completing what nature is unable to finish, and partly keeping up with her” (Physicis, 199 a 16).

In his *Rhetoric* (Book Two, 19) Aristotle discusses at length the kind of public argument (forensic or legislative) based on potentialities (clearly in re). These are all probable arguments in the ancient sense: they are based on commonly held opinions (each both correct and incorrect, depending on circumstance), and we invoke them in an effort to incite others to choose in unison some course of action — precisely in these two domains, where only intelligent performance is possible and full knowledge is impossible. Possibility receives no mention in this discussion, but toward the end we do read two sentences regarding probability: first, in public we argue that something will happen in the future (or would have happened in the past) if another thing is happening (or was happening) that naturally happens before it (“for example, if it is clouding over it will probably rain”); and, second, in public we argue (this time regarding artifacts) that, if something has happened for the sake of something else, the “something else” will probably happen as well (“for instance, if a foundation has been laid, there will be a house”).

From the beginning, the issue of public speech was confidence. The task of public speaking was to engender in the audience a disposition to act in joint decision regarding matters possibly otherwise: whether other than they happen to be at the moment, other than generally thought to have been, or other than we project them to be. “It is about actions that we deliberate, and all actions regard what is possibly otherwise” (cf. Aristotle’s *Rhetoric*, 1357 a 2 ff.). Action requires shared confidence. The Greek for “confidence” is πιστις: we might translate it as “trust” or even “belief,” but we should then constantly recall that such trust or belief is in re, not in dicto (cf. Plato’s use of the term in his *Republic* at 511D: it names our immediate relation to plants, animals and artifacts).

While Aristotle assumes that we have the ability to engender genuine confidence in ourselves and in others, with the result that we generally translate πιστις in his works as *persuasion*, the early Christians deny this ability, at least in matters concerning life and death as a whole: for Paul and Augustine, confidence is a gift from God, and consequently we translate the πιστις at issue in sacred and theological texts as *faith* — where it likely had, at the beginning, an in re sense, even though it obviously evolved, in subsequent theology, into an in dicto sense.

If there is any favored modern meaning of confidence in action, or in life surrounded by death, it will likely be something like “acceptance”: the public is led to accept policies laid down by public-relations experts (whether in politics or in commerce); individuals must learn to accept the trials and tribulations of life as a whole (often with the help of counselling or drugs). Within these larger domains of confidence and its
lack, we embed more quotidian hopes for immediate or long-term gain: we have degrees of confidence in strategies based on the calculation of benefits weighed against risks. For example, if I have a 1% chance of winning a million dollars by betting only ten dollars, I have some confidence in the wisdom of the bet; whereas if I have any chance less than 100% for winning ten dollars by betting a million dollars, I have no confidence in the wisdom of the bet.

It is perhaps no coincidence that it was Blaise Pascal, the founder of modern probability theory, who first proposed the wager-argument for faith in God. He simply translated the question of such confidence into a quasi-numerical question of probability: an “infinite number of lives” might be gained at a “risk” of only a few modifications in one’s present life. Here, at least, the analysis of “risk and possible benefit” supersedes, or simply replaces, the question of truth. Pascal’s argument relocates the question into a utility argument, just as John Stuart Mill, a century and a half later, will relocate the questions of nobility and justice into questions of utility (cf. above, Book Two, §2).

* See §233 of Léon Brunschvicg’s 1897 collation of Pascal’s Pensées, or §451 of Jacques Chevalier’s 1925 collation.
Axiomatics
The Conditions of Truth

That there be no self-contradiction: this battle-cry has appeared from the beginning as axiomatic. We may envision three forms of such self-contradiction: that a predicate both belong and not belong to a subject, that a subject both be and not be, and that we both affirm and deny a clearly formulated proposal.

Aristotle claims that the Axiom of Non-Contradiction lies subliminally within every intelligent effort, whether productive or practical. Moreover, he understands philosophy as especially committed to this Axiom: in this new art of contemplation it serves to help us focus on origins, the knowledge of which deserves the title “wisdom” more than do the established arts of production and action.

Kant, on the other hand, claims that the Axiom of Non-Contradiction serves only to keep our intellectual household consistent: the law, as he calls it, pertains only to formal logic, a principle of analytic knowledge that provides no access at all to the way things are. To account for the possibility of wisdom, he invokes another, a transcendental principle.

The conflict between the Aristotelian and the Kantian accreditation of the Axiom reflects the difference between what I have called the Second and the Fourth Interpretations. The difference ranges over all human activities insofar as we reflect upon their origins and ambitions. It especially and profoundly affects how we understand the study of
logic: whether we construe our present efforts as ontological or as epistemological in origin and in purport.

In ancient Greek, “axiom” means “worthy.” How can Non-self-contradiction be worthy? Aristotle will tell us that it reflects our commitment to origins in what we face. Kant will tell us that it bears only on our own holdings. Despite this conflict, we can detect a number of contexts in which the Axiom routinely proves itself worthy of employment.

First, in formal proofs: Aristotle demonstrates the validity of Baroco and Bocardo by the procedure now called *per impossibile*: we suppose the contradictory of the conclusion (in each case, All S are P) and derive from this, together with one of the premises, the contradictory of the other premise. To affirm the validity of the originals concluding Some S are not P, Aristotle employs two corollaries of the Axiom: (1) a proposal introduced and leading to a contradiction must itself be negated, and (2) our denial of All S are P requires us to affirm Some S are not P.

Gentzen’s mathematical deduction formalizes the Axiom in two rules. On the one hand, we may reject any propositional form (call it \( p \)) which, introduced conditionally into our system, allows us to derive a twosome in the form \( q \& \sim q \): from success in this experiment, we may conclude \( \sim p \). On the other hand, we may remove the negation of any negative form (call it \( \sim p \)) which, conditionally introduced, allows us to derive the repugnant twosome: from success in this experiment we may conclude \( p \). The two rules differ radically: the first permits the exclusion of a culprit, the other permits its inclusion.

Second, in court trials and other investigative hearings: contradictory testimonies require judges to decide which witness to believe, if any. Responsible decision cannot endorse any two accounts that have been clarified as contradicting one another. Similarly, responsible judges will discount the entirety of any one testimony that contains self-contradictions: the witness becomes untrustworthy. In these two cases, contradiction forces us, as judges, to distinguish sharply between speech and truth. However, the speech here is second-hand, and the original event serving as a standard of truth antedates the speech: we assume that there is a true account of what is absent, but so long as accounts contradict one another, and especially so long as one account contains contradictions of its own, we must seek beyond them.

However, discourse in parliaments and other policy-making bodies differs from that in court rooms. Here, we debate the establishment of frameworks for action on matters *yet to emerge*, and our policies help to *create* circumstances. Unless we believe future events follow mechanically from present circumstances, we must acknowledge our essential fallibility: all our present accounts may fail us in the future. We may even devise several mutually incompatible accounts serving equally well to fashion coming events. Or we all might fundamentally misunderstand what would be good for us: “When the gods wish to punish us they answer our prayers” (Oscar Wilde). Outright refusal to *entertain* such forms of contradiction would falsify our position in the face of things to come.

Third, in pure research: an intellectual seeks what is universal in the subject under study. An entomologist aims to account for a certain species of insect, and proceeds to record static and dynamic properties of the specimens collected. Any property that belongs to some and not to others appears as accidental rather than essential: it does not figure in the account of the species itself (although it may very well serve further research into genetic and environmental effects on the organism). On the surface, Aristotle and Bacon both agree on this principle of elimination: we must discount properties that may or may not belong to specimens.

In modern research, intellectuals typically formulate theories generalizing over relations among events. In fields such as modern...
genetics and sub-atomic physics we find examples of research that illustrate this style in thematic ways: an interest in processes rather than in kinds. But most other academic work also follows in the footsteps of the “new science” developed during the Enlightenment. For example, histories of nations do not likely intend to reveal a nationhood to which all nations aspire, they rather intend to represent the complexity of the phenomena themselves, particular complexes that essentially vary throughout time. Important for a modern theory is that it retain legitimacy in the face of two formidable competitors. First, the phenomena themselves: the theory must keep up with them, account for them in flux. Second, the theory must cover more events than does any rival theory (and there will always be rivals, much as in a capitalistic economy). In routine academic research a theory obviously stagers under the weight of counter-examples, and will eventually collapse in favor of any competing theory that can integrate the examples into its own story.

Yet in all such routine employments of the Axiom we simply rely on it. Let us now contemplate it three times over: in the ancient, in the modern, and finally in its presently looming form.

§1. Ancient axiomatics

What we now call the Axiom of Non-Contradiction Plato first formulated as an heuristic principle:

It is clear that the same thing will not admit to doing or undergoing opposites at the same time, in the same aspect of itself and in regard to the same thing, so that if we should ever discover this happening in ourselves we will know that it is not the same but plural. Republic, 436B.

Thus we may discover something about spinning tops: what moves is any spot on the circumference, while what rests is the axis. To decide firmly whether the top is moving or resting, we distinguish axis and circumference: we discover something. Then, too, we may discover our own nature. For instance, I desire something (drink), but I also foresee what, where, when, and how to drink, so I exert pressure on myself to refrain from drinking what immediately offers itself (water in a puddle), and if I nonetheless yield to the initial desire I feel disgusted with myself: such civil war in the soul leads to the conclusion that the soul is in three parts (and that justice consists in a unification in which each part does its own thing).

As a principle of discovery, the Axiom has its proper home in discussions among those willing to learn: in dialectic. It summarizes the tensions evident in learning any art, and inaugurates the tension essential to the new art of contemplation: the learning of what stays the same despite the many variations with which we have to deal when performing any art. The discovery at issue is ontological; it is inseparable from devotion to origins (ἀρχαι) that initially escape us.

§1.1 Concrete tensions

In the study of Greek logic we all along enter into three essential tensions of escape and recovery: that between a being and its predicates, that between the subject's initial and ultimate being, and that between language and what it might bring to light. If we are to maintain the ontological purport of the Axiom, we must exercise care to think these three tensions concretely.

Sitting in the bleachers and having placed my bet, I anxiously perceive my preferred horse: how it starts, whether it races ahead or falls behind the others, when it finishes. I here concentrate on predicates, on what I and others can correctly say about the horse. Taken up into sayings, the predicates formulate what befalls the horse, its affections, its accidents. These latter vary over time, so that a saying that is correct at one time may be incorrect at another: a contradiction may arise. The Axiom states that such contradiction should drive us beyond the accidents: should alert us to the possibility of discovering the underlying being of the horse, τὸ ὑποκείμενον, the “subject” of those predicates. While today we assume the biologist, or hippologist, would devote himself to this discovery, Plato and Aristotle understand the horse trainer as undertaking this task. For one who works with horses, sayings about them waver over a solid ground (or cause: αἰτία). Having learned to concentrate on this ground, on the subject, the trainer enters into it — recovers it — as the source (or origin: ἀρχή) of those qualities, quantities, relations, etc. that I in the bleachers only perceive as results.

The trainer stands in a certain light revealing the source, whereas I waver in a twilight revealing only results. The trainer's vision is fully enabled by truth, the light of being, where the healthiness of the horse
arises as the issue. My vision in the bleachers is trapped in the shadows, where the relative performance of the horse remains the issue. The trainer can help the horse be itself and therefore possibly win the race, I can only hope for the outcome. The trainer’s condition is one of knowledge mine is that of opinion, a shared but levelled out viewing.

Knowledge, however, does not relieve one of the tension between predicates and their subject. Rather, it is a condition in which one is able to begin, in time, with a predicate (with the recognition of an attribute) and stretch it back to the recovered subject as the true beginning. This first tension recurs at every moment of actual knowledge. It leads directly into the second.

As a horse trainer myself, I am now able to face a given horse as a horse. Yet this one horse before my eyes, in my hands as I lead, feed, brush it, assist at its delivery, train it, and ... its fate — this one horse is one among many that I shall handle. The oneness of the horse lies largely in what it means for any of these horses to be a horse. The meaning of even my one horse is its potentiality for being itself, and this potentiality lies in its purpose, the actuality which is its destiny. This destiny constitutes its identity, an identity it shares with the next horse that comes into my care. Paradoxically, the unity of the horse at the moment, its identity (what we can rightly, poignantly call it) and its individuality (its difference from other horses), lies in its universality: in what allows me, as horse trainer, to recognize, treat, and evaluate the next horse. My one horse’s particular condition in need of development (natural growth, artful training) distinguishes it from others, but the actuality at issue for it, and for me as a trainer, is the same as that of the others.

Knowledge allows us to participate in a tension between individuality and universality. This tension takes explicit shape as a contradiction: this horse both is and not itself. Any subject given over to our knowledgeable care is as a mixture of being and non-being (Plato’s Sophist). We moderns may wish to say that its being consists in its momentary presence and its non-being consists in what it needs to be; on this account, its non-being reflects only our understanding at the moment; on this account, knowledge requires us to desire the fulfillment of the being. This second tension is one we acutely feel as a transformation of desire. It leads naturally into the third tension.

Whether in the bleachers betting on them, or out in the field working with them, we talk about this or that horse, and about horses generally. At its best, talk focuses us on something: on a subject, on the subject at issue in the circumstances. At its worst, talk rehearses numerous predicates devoid of origin: such talk constantly “changes the subject” because it has none of its own, embedded in the circumstance, but rather echoes talk of bygone hours. In mixed company, ourselves are caught in the middle: one of us knows (the master), the other (apprentice) only opines, and the focus must be recovered. Our talk can help us recover the focus; it can also detract from it. Thus the tension.

Our medium is language: a tongue (English, say), a shared material for λόγος, the actual event of detecting and unifying the multiplicity of attributes, together with the complexity of our own engagement at the focus (with the horse, say). As Aristotle often remarks, we may syllogize either with regard to names (προς τα ονόματα) or with regard to the matter (προς το πράγμα). In time (as children, as apprentices), we start with names: with the patterns of the tongue. We must, we might then learn to start again, at the origin, with the horse, in a σεβή, where I am following nature, helping her complete what she is unable to finish. In mixed company, the same words vary in meaning as the conditions of the participants vary. For the apprentice, as for the unregenerate spectator in the bleachers, the word “horse” has a twilight meaning allowing one to hover at a distance from horses: the word represents an empty generality. Similarly with words referring to the multiple items regarding horses and to the complex gestures required in the handling of them. For the horse trainer all these words engage one in an intimacy with these matters. Under such conflicting conditions, talk is fraught with tensions: already here, in ποιησίας. In public speaking, in πραξις, company is essentially mixed in a tension between syllogism and paralogism. And for philosophic education, for θεωρία, Plato and Aristotle devised a style to embody the tension: dialectic.

Concretely understood, each of these tensions describes our involvement in contradictions. A subject both is and is not its attributes. Any real subject both is and is not itself. And well formulated talk both focuses and does not focus on its subject.
As concrete tensions, each of these contradictions marks an instability in which we participate, often painfully. The Axiom asks us to focus contemplatively on this instability: it recalls the human (productive and practical) task to break through the contradiction, resolve the tension at the moment of working, of speaking, of discussing.

§1.2 Aristotle’s formulation of the axiom

In his *Metaphysics*, (1005 b 7-20), Aristotle formulates the Axiom very much as Plato had:

It is clear that it is for the philosopher, i.e. one who contemplates all beings as they are by nature, to examine also the origins of syllogizing. ... The firmest origin of all is one about which there can be no mistake. ... What this origin is, let us state: the inability of the same at once to belong and not to belong to the same in regard to the same.

But whereas Plato introduces the Axiom as a means of calling attention to distinctions in reality, Aristotle introduces it to define and defend the vocation of the philosopher.

Most immediately, Aristotle states the Axiom in answer to the question whether there is one kind of knowledge that bears both on “what are called axioms in the mathematical disciplines” and on οὐσία (“substance”). He answers that the two bearings converge in the vocation of the philosopher. That is, every discipline has its own axioms: the musician might hold time/beat as basic, as worthy beyond doubt and therefore beyond investigation; the arithmetician might hold one-ness and one-more-ness as basic; and nowadays the economist might hold production, the politician might hold human rights as basic. These axioms bear on what is, therefore ultimately on οὐσία (a substance from which any genuine “is” must stem). Thus the two directions of concern go together. And Aristotle then asks whether there is not a basis (an ἄρχη that undergirds philosophical work itself, one that also reflects the contemplative concern for οὐσία. Such a basis must be the firmest of all (firmest than time/beat for music), and serve across all disciplines as the foundation for syllogizing (for reasoning concretely within every discipline). This basis he then states: “the inability of the same at once to belong and not to belong to the same in regard to the same.”

The formulation deserves careful attention. It asks us to accept a limitation: many things may happen, but one thing cannot happen. The inability refers to the integrity of the subject and each of its properties. In the course of time, and in the variable ways we approach subjects and understand properties, things can indeed take on various configurations, various associations. But at any one time and in any one approach, the same property cannot both belong and not belong to the same subject. Here, in any one real situation, the effort on our part to affirm both at once lands us back into the confusions of time and approach, and away from what we (in production, action, or contemplation) must come to know. Genuine knowledge constrains us, binds us, forces us to sift through initial confusions of time and approach, to focus (for a moment, at least) on the integrity of the subject under consideration. It will be the philosopher’s task to coach others into accepting this limitation in each inquiry.

The first sentence of Book G broaches the broader question in which the question of axioms subsequently unfolds: “There is a knowledge that contemplates τὸ ὑδὴ ὁ ὑδῶν (whatever is as it is) and whatever belongs to it καθ’ αὑτὴν (as it is in itself).” This claim Aristotle proceeds to defend against the obvious objection that human beings must know things discipline by discipline only, and that no one can rightly aspire to a super-knowledge of the sort Aristotle claims (as when he says: “it is for the philosopher to be able to contemplate everything”).

We today could easily read the opening pages of Book G as an effort to establish philosophy as the queen of university studies. Aristotle would then be saying of each department that it takes some part of reality and contemplates what befalls this part, whereas we in the philosophy department contemplate reality universally: things not as what befalls them but as they are καθ’ αὑτὴν (in themselves).

Along with the question of the vocation of philosophy, Aristotle here raises what we now call the question of being. It seems to be our fate as humans to understand, well or poorly, what things are: a friend, say. And yet we can be puzzled about whether we should identify our friend with what he or she is “accidentally” (κατὰ σὺν συμβεβηκός). Aristotle remarks: if not the philosopher, who would examine whether Socrates and Socrates-seated are the same? More concretely, and more urgently, we today must examine whether democracy can be identified with such features as voting, recourse, and equal opportunity, whether a human
being can be identified with such features as a heart-beat, choice, and private rights, whether various animals and plants can be identified with the services they render among themselves or to us. The question of being arises as a suspicion that there must be, that we must acknowledge obligation to, an “in-itself-ness” of each thing we more easily recognize, and incorporate into our own lives, by way of what befalls it.

Aristotle structures the suspicion. As human beings in any line of work, we incessantly judge things to be this or that, this way or that way. Every such judgement contains an “is” (even about the past and about the future). But our engagement with the “is” takes various forms: (1) some things we understand to be central; these Aristotle calls οὐσίαι, “substances,” drawing on the common Greek usage of the word, meaning those things comprising our “estate,” what we can own and must come to understand if we are to care for them and therefore continue to have them (horses, friends, cities, houses, ships); (2) but substances also are the things they undergo (παθη); the city is under siege, my friend is being acclaimed; (3) and whatever literally or figuratively serves as a path (ὁδος) toward a substance also is; a roadway toward a city, a barn door toward the cows, perhaps a book; (4) then too events: destructions, privations, or qualities of the making or becoming of a substance, even the (logical) terms bearing on (relating to) substance, count as being; and (5) finally, even outright (logical) denials (there are no centaurs, my house is not well insulated) involve us in judgements about what is.

Each kind of “is” has its own distinct power, its own distinct relevance in our dealings. Aristotle claims, however, that the first (η οὐσία) is the one nature (φυσις), the one origin (ἀρχη), that ultimately gives sense to the others in our accounts of, in our judgements about our circumstances. It may help to remember that το ον is adjectival and that η ουσια is substantive. That is, the initial statement of the philosophic vocation (that we contemplate το ον η ον, “being as being”) sets us the task of wondering what all is involved in the act of determining how things are; in this phrase, “being” does not intend to recall the basis, the reality, of such determination (“nature” or, as later thinkers say, “the cosmos”). On Aristotle’s formulation, the philosophic consideration of the determinations we make opens out onto their basis, their “substance”: η ουσια, the “being” from which the determinations will receive their justifications, and to which any wise judge of circumstance will return.

In these pages of Book Γ the question of being may appear more as an assertion, or series of assertions, intending to justify the vocation of philosophy. In Book Ζ, another but related collection of notes, Aristotle broaches the question in a way suggesting its roots, as a question, in all human endeavors:

... both anciently and now and always, what is searched and is always obstructed, namely what something is (τι το δι ν), is this: what the substance is (τις η ουσια).

Again, it is crucial for a full appreciation of the Axiom to understand this version of the question very concretely — taking cues from its context (a discussion of the categories).

As engaged human beings, we always try to find something, and we ever again run up against resistance. In this first order of questioning we ask how things are, and the answers fall under one or another of the usual categories: I wonder about the condition of my horse, or what conditions are possible for horses in general, about the sizes and colors, about what it or they can do or undergo, about where and when and in what relation to other things, and about postures. Moreover, we strain to identify things, to discover and name the animal that has been raiding our vegetable garden, the bird that sings so sweetly at dawn, the mineral that glistens in the rockcliff, the goop that is clogging the fuel lines of my car. This tenth way of asking what something is seems to rank equally among the other nine. Yet occasionally the identity of something alters the status of the other determinations — as when we discover that not a mouse, but rather a hinge is the origin of the squeaking we hear.

Yet all these first-order determinations can fail to add up. Indeed, incompetence often takes the form of an abundance of determinations having no focus and leading to no effective dealings with circumstances. In contrast, knowledge takes the form of the ability to focus predications and, often more importantly, to suspend judgement on proposed determinations so long as they fail to focus. Focus on what? Formally, on the subject, what underlies, το δινεμενον: a metaphor suggesting that there is something yet to be discovered, something that will ground our determinations, something to which the squeaking can refer us. But what counts as a genuine subject in these senses? What this underlying something is: this is a second-order question, the philosophical question of being, τις η ουσια.
A phenomenological consideration of vocational involvements can lend support to Aristotle's contention: an apprentice on a farm, in a machine shop, or in a leadership endeavor (e.g., as a parent) first learns an array of determinations, and how to deal with circumstances according to the array, but must eventually, to become a master, learn to leap through these at any moment and to relate directly to the internal workings of cows and corn, lathes and metals, or (in πραξις) of people. Determinations themselves do not assure this leap, neither those given in advance of the moment nor even those given at the moment.

Aristotle detects in the need for the leap (from the question τι το ον to the question τις η ουσια) the justification for the vocation of “primary philosophy.” Yet the point is that the fulfillment of any vocation requires the leap. Whether as musicians or mathematicians, farmers or poets, parents or university presidents, we can be philosophers as well—must so become, if we are to consummate our endeavors. But only in the sense that we naturally distinguish between our perception and calculation of how things are predicatively and our insertion into how they are substantively, and that we are concerned to make the leap.

Aristotle himself illustrates “primary philosophy” in his dialectical consideration of possible answers to the question τις η ουσια. What counts as a substance as against what we determine about it? In Book Z, Aristotle considers at least six answers: (1) the whatness of something right here, (2) its material, (3) what it was all along to be, i.e. its τελος, (4) its wholeness, i.e. universality, (5) its familial origin, i.e. το γενος, and (6) its shape, διαμορφη. Elsewhere, e.g. in Book Θ, Aristotle discusses another candidate, one distinctive of his own work: (7) the power that can be set to work, δυναμις and its correlative ενεργεια. Eventually, he integrates all these answers into a unified vision of what we must leap into, participate in, when we finally know what we are facing, what we are doing, and what we are talking about. This vision eventually came to dominate intellectual work.

Now, just as every strict discipline takes as axiomatic, as most basic and most worthy, certain principles, so the discipline of primary philosophy must have at least one of its own. And just as music will require something more than just rhythm, so primary philosophy will require more than non-contradiction. Still, there is some sense in which the performance of each discipline must ever again realign itself with its axioms: a musician need not worry about understanding rhythm, but it nonetheless remains fundamental to the event; so, too, we must now learn to discover, to focus on, what supports our endeavors to discover and focus.

Given the context in which Aristotle states it, the Axiom summarizes the entire question of being, albeit in ways that a casual reading might easily miss. For it claims that there must be sameness at issue in any genuine belonging of a predicate to a subject: a sameness of subject (ultimately an ουσια) and a sameness of what we can say about the subject (derivative from the first, on Aristotle's account). Moreover, it claims that true belonging can be dissociated from time and approach: these latter reflect issues in our own vocation rather than the issues internal to any substance—they represent what is prior to us and not what is prior by nature. On top of these claims (constant sources of contention among primary philosophers) there is the more familiar (formally uncontentious) claim that a predicate cannot rightly both belong and not belong to a subject—that, if both present themselves at the door, at most one can be admitted. Consideration of this topping constantly turns us back to the ontological claims.

§ 1.3 Aristotle's demonstration of the axiom

Aristotle says that the Axiom of Non-Contradiction is the firmest of all, and an origin about which we cannot be mistaken. Yet he goes on to note both that many of his fellow-thinkers in fact deny it and that nearly all his predecessors in effect deny it. The firmness and the infallibility are not at all self-evident in the easy sense of corresponding to “common sense,” to what everyone can easily detect. Indeed, the Axiom will appear firm, and our own endorsement of it infallible, only if and when we fathom it.

Meanwhile, the fact of disagreement requires Aristotle to enter into a discussion of it. For the Axiom itself reflects the very heart of the new spirit of λογος: that we allow for free debate, especially on fundamental questions, so that we can freely and logically decide. Now, Aristotle usually advises us to demonstrate directly: by recalling first principles and showing that the conclusion follows. In the present case, however, such direct demonstration would beg the question: the Axiom intends precisely to state the principle on which all demonstration depends. What can Aristotle do? What might we do?
Aristotle begins and remains with what we today call an *ad hominem* argument: anyone who arrives at a debate demanding that everything be demonstrated has not yet grown up; for the Axiom represents the kind of concern presupposed by any debate. The Greek for “demanding” translates the verbal form of “axiom”: one who demands a demonstration of what itself demands our allegiance has not yet grown up. There is something strange and self-defeating about demanding to be shown how to begin at the beginning. For no one can supply the beginning for another: no one can begin for another, each must begin at the beginning, *be* both the beginner and the beginning.

In Zen and related traditions, the demand for a demonstration of origins meets with what would appear to the ancient Greeks as the response of a tyrant: e.g., a slap in the face. Plato and Aristotle contributed to the formation of our western ideals by devising a way of talking non-beginners into beginning: the elenctic demonstration. An elenchus is, in Anglo-Saxon, a *shaming*; it aims to bring disgrace on one who upholds a thesis, here the thesis that the same may both belong and not belong to the same. It is not a merely formal proof in the style *per impossibile*, or of Gentzen’s negation-introduction (assuming the ability and deriving a contradiction): for this style presupposes the Axiom and would therefore beg the question.

The remainder of Book ι reads in fact as a kind of analysis of the opponents to the Axiom, together with a series of mini-scripts allowing us, the readers already agreeing with Aristotle, to enter into the fray with each of two different kinds:

*First*, there are those who argue superciliously, for the sake of λόγος rather than through it (with regard to names, and in voice, rather than with regard to the affair itself, and in thought). These we must learn to counter by force, refuting them at their own level of words.

*Secondly*, there are those who argue seriously, from a concern for the truth of what we actually experience: the variability of events, the unreliability of generalizations and even of singular determinations. These we must learn to counter by persuasion, refuting them at the level of the realities both they and we would like to preserve.

In responding to either set of opponents, our aim is to cure a disease: the disease preventing one from growing up. The Axiom represents health, maturity. Throughout the cure, Aristotle draws upon a then-current understanding of maturity that we today might not share: the ability to address others significantly in public, i.e. to draw the attention of others onto circumstances about which free decisions must be made, individually and jointly. Silence in the face of opposition will not suffice; thus Aristotle must coach us to respond even to the most supercilious opponents. As he says in his *Rhetoric* (1355 a 35):

> It would be absurd, if [as is the case] it is disgraceful to be unable to defend oneself in body but not disgraceful to be unable to defend oneself in λόγος — the enactment of λόγος being more proper to human being than the enactment of the body.

But neither will a flow of mere words suffice: the words must focus on something, as judged by those listening. Among the many strange principles emerging on these pages (axioms, but not named as such) we today might count the following: “the one responsible is not the one who sets up the argument, but the one who abides it” (not the demonstrator, but the listener: 1006 a 26). The speaker must bring something into view, something *else*, but the completion of the speech rests with *others*. This understanding of “meaning something” (τὸ δεῖμαντα) differs radically from all modern theories, despite their many differences — for they all assume that responsibility for significant speech lies primarily with the speaker, i.e. in a self-contained construction.*

### §1.3.1 Against those stuck in words

Characteristic of the first set of opponents is their insistence that the Axiom be demonstrated. They are then willing to assume the contrary view: the *ability* of the same to belong and not to belong to the same (at the same time and in the same way). To uphold this view in the abstract (meta-theoretically, we would say today) they must concretely *say*, testify, that a certain subject both is and is not a certain predicate. Once our opponent says something, we may rightly (as listeners ourselves, and on behalf of those listening in) ask what is brought into view, into focus. Aristotle coaches us to get listeners to answer: nothing. And all those listening will conclude that the opponent has disqualified his own testimony. Not because he contradicts himself, but because he has failed

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* For a thoughtful review of the restless changes in the modern understanding of language, and the gradual emergence of the concern for meaning (as distinct from thinking and from referring), I can recommend Ian Hacking's *Why does Language Matter to Philosophy?*, 1975.
to mean anything, point anything out: he “disowns λόγος,” is “not discussing,” “is not talking,” is “like a plant.”

Embedded in our Axiom is evidently another: that we must be able, in speaking with ourselves or others, to engender what I have called a focus, what Aristotle calls τι ἃρτοφαίνον, “something definite” (1006 a 25). Prior even to the decision to affirm or deny a proposal, it must allow us to look to something: speaking and looking, λόγος and σκέψις, are in this sense inseparable (1008 a 31).

What sort of things might our opponent have to name as engendering a focus? My wife is ... faithful, women are ... child-bearers. Aristotle’s examples: Socrates is ... white, human beings are ... two-footed. We may easily be mistaken about the predications. Indeed, our own fallibility supplies the most plausible, the most experiential reason for defending the ability of the same both to belong and not to belong to the same. My wife may be faithful at one time and unfaithful at another, might be faithful in regard to flesh but not in regard to thought. And who is my wife, anyway? And what does “faithful” mean, anyway? Who’s to say — on any of these questions? Even when the grammatical subject is singular, all things are in fact rather mixed up, confused and confusing. And when the subject is “women” — why, some become mothers and others don’t, even assuming we can agree on a single meaning for “mothering.”

Yet, Aristotle argues, our opponent must, to enter the debate, take upon himself the task of saying something, getting us to look at something definite, encircled for our observation. Otherwise our opponent says ... nothing, he babbles a name with no reference, an ονομα without any πραγμα. And can only slink away in disgrace.

“What would be the use of a speaker if matters revealed themselves without his speaking?” (Poetics, 1456 b 7). We have to work to bring things into focus, to bring them out. Truth is an affair of nearness, coming in close to things, making them near (1009 a 1; cf. Politics, 1308 a 30).

Perhaps people at all times have hoped for a doctrine that would supplant the effort of speaking and listening freshly, on the spot. And modern “theories of meaning” may have encouraged this disposition: largely, by developing (out of Scholastic thought) a triadic understanding of meaning where utterances refer to concepts which in turn (like sets) cover items. And once we believe that our intellectual task is to work out concepts, we can imagine studying doctrines in complete silence: public speaking then becomes ornament.

Plato and Aristotle have what we might call, in hindsight, a dyadic understanding of meaning: speaking and revealing. The question then remains what can be revealed, and how, in what kinds of speech. In struggling with these questions, the Scholastics first developed what became our familiar (although slippery) usage of the word “concept,” whereupon speech (that of intellectuals at least) no longer has the responsibility to establish a focus on things themselves. In sometimes astounding contrast, the original Greek thinkers accept this responsibility as built into our condition of λόγος.

The mini-scripts on these pages turn on what later became the question of whether existence is a predicate. Given the modern triadic understanding of meaning, existence names the antecedent basis, supplied by experience, of predicates, and predications names the act of subsuming things (e.g., existences) under concepts; thus Kant argues very cogently that existence is not a predicate-concept, not something we might determine by working out our conceptual framework (mathematical theories of set formation illustrate this). But Plato and Aristotle treat εστιν (“is, exists”) as a primary question, the settling of which lies precisely with our linguistic condition. However, here it is not a question of deducing, from concepts, the existence at issue, an existence elsewhere. It is a question of creating or eliciting a presence, a nearness, in the course of discussion — for listeners as well as speakers, here and now.

Aristotle’s arguments against those upholding the ability denied by the Axiom do not respond to what we might understand to be the issue, i.e. whether we can both subsume and refuse to subsume the same item under the same concept; his arguments raise questions about the availability of the item itself: something for which any opponent must
assume responsibility. Insistence on the ability denied by the Axiom leads to non-availability. This non-availability takes two contradictory forms (1008 a 23): (1) either “nothing is,” nothing comes forward to allow for discussion, or (2) “everything will be one,” every predicate will belong to any named thing (since if being a bed, or being in Athens, does not belong to Socrates, or to human being, then it also does belong), so that we lose our own ability to make distinctions, the whole issue of focus-seeking, presence-making discourse. In either case, our thinking (διανοια) loses its focus (1009 a 4)—disgraceful indeed.

Yet the very need for a focus, for the elicitation of an ουσια to which listeners and speakers alike can respond, raises more questions than it answers. The need indirectly evident in the consideration of supercilious opposition to the Axiom becomes directly evident in serious doubts about it.

§1.3.2 With those attentive to events

To the genuinely perplexed, the opinion that “the same can both belong and not belong to the same” comes ἐκ τῶν ουσιῶν—a “from encounters,” might be the most accurate translation (although translators usually prefer “from perceptions”). And Aristotle adds, significantly, that those who uphold the ability denied by his Axiom “speak correctly” even though they are ignoring the full import of the Axiom (1009 a 32). To appreciate the intent of the Axiom we ourselves must carefully discern how these people, how we ourselves, may speak correctly when upholding the otherwise denied ability.

In actual encounters with my wife, with students in a class, with horses, with the corn in the fields or the water in the sea, I in fact must learn to face these ουσιαι as able to be contraries: healthy and sick, rational and irrational, tame and wild, nourishing and poisonous, cold and warm. And in some correct sense, such ability is built into what I encounter: thus the constant vacillation, both in a single individual and in the varying individuals of a kind.

We might object that these contrary properties belong to any one ουσια at different times, or to different individuals of the species at the same time. But Aristotle’s formulation of the Axiom refers to (and denies) an ability, and the rejection of the Axiom affirms that ability. And, in a way, correctly. For I must in fact acknowledge that my students have in themselves the ability to be rational and irrational; that each of my horses has in itself both tameness and wildness, that any living thing has both health and sickness in it, that the sea is both life-giving and death-bringing (Heracleitus). And to be true to these ουσιαι I must encounter them precisely as they are this strange duality: when asked, I may very rightly say that my wife is (carries within her the ability to be) both healthy and sick, a good cook and a bad cook.

Must acknowledge, I say — our serious opponents say, genuinely perplexed. Why? There is no reason external to the event of encounter. To encounter something non-prejudicially requires us to accept its embedded contrarieties. In such encounter, things are in movement, κινησις: my horse teeters between being tame and being wild, is both at the moment. The evidence? Not passing encounter, even though here we especially notice people contradicting themselves with contrary determinations. Rather, craftsmanship: the horseman, the one able to participate in the teetering, knows that contraries belong to the horse; so, too, sailors and fishermen on the high seas know that the sea is both life-giving and death-dealing, wise teachers and parents know that those in their care are both rational and irrational, competent doctors know that each healthy patient also contains all the elements of disease.

We encounter an ουσια knowledgeably only if we can approach it in its contrariety: only if we ourselves are open to it as it is ... undetermined, ὀριστον, Aristotle crucially adds. The requirement that a debater bring something into focus, τι ὀριστον, surfaces in actual encounter as a requirement that we work this definiteness (of λογος) out of the indeterminateness (of ουσια) to which we are open. Themselves grammatical contraries, ὄριστον and ὀριστον mark the concrete tension, the teetering within significant talk with one another (and within any writing that has and endows power and promise).

Aristotle proceeds to sketch out numerous mini-scripts for arguing with those who find themselves stuck in their correctness: for, on his account, the correctness of initial encounter tells only half the story. Yet the other half makes sense only as a continuation of the first half. Thus our need to appreciate the correctness. Our task is then to persuade them, that there is some φυσις, some “nature,” that does not vacillate, does not teeter, in fact lies at the basis of the movement in which any knowledgeable person obviously engages (1009 a 35).
At the heart of Aristotle's mini-scripts lies the distinction between presentation and completion. Each thing initially presents itself full of contraries, and these “all at once.” But this condition of contrariety is one of ability, δύναμις: “power,” we can translate. There is, however, a goal, a τέλος, for each such power-pack: a healthy version of the thing (person, horse, corn), an inner intent (of water, of rock, of blood, of sap). Power is correlative to such “holding out within a goal” (ἐντελεχεία: perfectedness, completedness). The Axiom refers us to this correlate of power, elsewhere called the ἐνέργεια, the “at-work-ness” of the thing: the completion of a thing arrives as a smooth functioning, as a peacefulness — an overcoming of the restlessness associated with destruction and generation (a smooth running, not a stasis).

But how can we show this correlate, persuade those who, in their seeking and loving the truth, have trapped themselves in their truncated stories? We cannot demonstrate the correlate, since all demonstration depends on an acceptance of the Axiom, and the Axiom precipitates the correlate as its point of reference.

The showing of an ἀρχή, the development of persuasion (πίστις: confidence, faith) in it, depends on a joint venture of contemplation: a thoughtful observation of the full event, what actually happens in human dealings with circumstance, successful and unsuccessful. Above all, perhaps, such showing and its eventual persuasiveness depend on “searching and loving the truth,” i.e. on an openness in regard to what counts as the full event, and especially to what counts as success and failure.

The central observation in Aristotle's version of the venture (indeed: in the entire tradition stemming from Plato) is that encounters beg us to fathom them. “Not every phenomenon (φαινόμενον) is true,” Aristotle says: not every thing that falls within the lighted circle of our encounters is for that reason fully revealed. The encounter itself, the unjudged perception (αἰσθησις) is never false: it brings its own correlate to light, albeit partially, i.e. as a phenomenon. If then we remain passive in the encounter, we are left with our own version of the phenomenon, our own φαντασία: an imaging of what has come to light. This imaging easily loses its original ground.

In idle talk and curious involvements, this loss of ground makes no demands on us. We then cheerfully notice that the size and the color of something varies according to our position in regard to the thing, that things taste differently according to whether we are healthy or diseased, that what seems heavy to a tired man seems light to a vigorous young athlete, that things appear one way when we are asleep (the time of pure leftovers, pure φαντασία) and another way when we are awake.

But in serious engagements things make demands on us: children make demands on parents, cows make demands on cow-keepers, corn makes demands on farmers, water makes demands on fishermen and sailors. We meet these demands by fathomimg the things we first encounter. Encounters are of something: an encounter bears on something, bears on us (1010b 35). We stand at a kind of crossroads: we may either retreat into our on fantasies or we can move out toward the “something” at issue in the encounter: toward what Aristotle then calls τὸ ὑποκείμενον, the immediate source of all those leftovers. Serious people, those fully engaged in a τεχνη, have already chosen which road to travel. Aristotle's proposed venture in contemplation takes us, takes the serious opponents of the Axiom, into careful observation of what happens for those who have accepted the demands evident in the performance of an art. His version of the venture, that of western intellectual development, promotes artful, later technological engagement with circumstances.

When Parmenides tells his pupil Zeno that the filling of the limbs is what gets thought, when Anaxagoras tells his followers that τὰ οντα will be just as they receive them, these thinkers plunge us into encounters, away from rootless imaging. They say succinctly and abstractly what Pindar repeats over and over again in an exhortive and moral manner. As in his 3rd Pythian Ode:

If any mortal keeps in mind the way of truth, it behooves him to receive gladly the things given over to him from the blessed powers beyond. One way, then another, do the winds blow on high. Not for long does the prosperity of men last whenever it falls to them in full measure. Small in what is small, great in what is great, shall I be. The divine (δαιμον) ever pressing in upon me I shall care for with the devices (μαχαιρία) that are mine.

The way of truth is that of reception, of accepting vacillating circumstances within whatever task happens to have enveloped one. This
acceptance of presence, this aiding of presence by way of acceptance, poses a wise alternative to human distraction.

Neither Plato nor Aristotle ever deny the wisdom of reception. On the contrary, one can find passages in their works that exhort us in much the same say: e.g., toward the end of the Republic, in the story of Er, where any present condition is envisaged as a product of how we received our δεξιωμα.

Nor do they deny the practical wisdom (ϕρονησις) embedded in Protagoras' teaching, so sympathetically rendered by Socrates in Plato's Theaetetus (166A-168C):

... For each of us is a measure of things that are and are not. Now each [person] myriadly differs from every other in just this, that to one some things appear [come to light] and to another other things. Still, I do not say that there is no wisdom, no wise man. Rather, I say that, as bad things appear and are to anyone of us, he is wise who can change them to appear and to be good. Moreover, do not lay stress on the words of my account, but become clear about what I say. Remember now I was saying: food tastes bitter to the sick, but appears and is the contrary condition. Thus in helping children grow up, we effect a change from one condition to a better. But while a medic changes with drugs, a wise man changes with λογοι. Yet nobody has ever made anyone with false views to have true views. For one is unable to view [opine, believe] things that are not—or anything other than the things one undergoes. And these are always true.

A Protagorean physician, parent, teacher, or parliamentarian accepts that things are for patients, children, students, or citizens as these people take them to be. The Protagorean's task is not to contest their measures, to show their judgements to be wrong; rather, he assumes responsibility for changing things, for doing something that will get these people to experience as good what they otherwise experience as bad.

The serious opponents to the Axiom rely exclusively on the first half of the story: a half-truth. Aristotle argues, but nevertheless a truth upon which we must draw in responding to them. Certainly a physician would shirk his duty if he insisted that the patient was wrong (that, for instance, the orange juice is "really" sweet); the physician's task is to get the orange juice to appear sweet and therefore be sweet. Similarly, a public speaker must get something, e.g. a new power dam, to appear good to the audience, and not just argue (to whom?) that the audience errs in thinking it bad.

But even more: How unjust and ineffective it is to assume that the perceptions, the encounters, of others are untrue! Doing this as a teacher, I discredit what lies closest to my students, that to which they can themselves testify: I rob them of their autonomy. But I also disregard the only ground on which they themselves might come to perceive what I am talking about. Obviously, speakers and writers must allow others to hear in the account, in the λόγος, what they themselves can acknowledge as true of their own encounters; only then can a change for the better occur—a change in what appears to them, is for them.

Again on behalf of our opponents, we must acknowledge that αισθησις is not always easy to come by. Anybody who has made a photographic study of a landscape will testify how perceptions only emerge slowly, unexpectedly, and barely emerge at all for one who simply wanders through the area. Anybody who prepares delicacies or tests wine will taste more nuances, encounter more subtleties than do others. Indeed, one of the issues for us in the performance of an art is precisely whether we will see or taste, smell or feel or hear things as they simply present themselves: whether things will appear in αισθησις so that we can register them as being even in this rudimentary sense of το δν (categorizable).

Thus our opponents “speak plausibly,” provide a “probable” account (1010 a 5). Their talk tunes in with our actual encounters, and is therefore credible. But, Aristotle adds, they do not speak truly, their account is not true. Why? Because “truth” is here receiving a new interpretation: not simply the hard-to-come-by and even harder-to-sustain encounter with things, but the fathoming of these, the penetration of phenomena, a penetration already buried within the artful dealings with things as recounted in the first half of the story. Thus Aristotle urges us to argue ad hominem in the earlier sense: to re-articulate the arguments of our (serious) opponents so that these themselves open out on the truth.

Over and over again, Aristotle's basic ad hominem (intending to reveal and to persuade) is this: in order to enhance effectively what you,
our opponents (and we also) want, viz. full and sustained encounter (αἰσθησις), you must learn an art (τεχνη) allowing you to see through the multiplicity, variability, and confusion of “first appearances” so that you can respond to the nature (φυσις) emitting these phenomena — and can then appreciate their multiplicity, variability, and confusion. The Axiom signals this needed and desired vision: it reminds us all of our destiny to transcend phenomena for the sake of sameness; thus it does not fully apply to the phenomena themselves.

Plato and Aristotle intend to root the Axiom in the human condition per se — in the life of anyone who develops a craft for dealing with encounters. However, they already single out some arts as especially illustrating the Axiom at work. Astronomy provides a pure example: an art that quickly passes beyond the variable to the stable. Arithmetic and geometry provide pure examples of arts based on formulating samenesses (even if they keep us aloof from all ουσια). These arts Aristotle calls “sciences” and notes a difference: engaging in them, we do not aspire to alter anything, we do not help nature complete anything left unfinished. He then reserves the name science (επιστηµη) for any such art that focuses attention purely on sameness. And he includes the then-new vocation (or version) of philosophy among these arts.

Eventually, however, the sciences we now call “hard,” i.e. those whose subject matter can be articulated mathematically, came to incarnate the Axiom: modern physics first of all, then the rest in imitation. With a difference, however: while formulating underlying samenesses modern science also invites us to alter appearances. Proof of a formulation lies in the ability it bestows on us to change and to produce phenomena. Modern science then sheds its ancient loyalty to contemplation and becomes what Aristotle calls a productive art. This development requires a transplantation, or at least a re-rooting of the Axiom in the course of modern philosophy.

§1.4 Identity

Almost in passing, Aristotle remarks (1011 a17) that, “...if not everything is in relation (προς τι), but some are also themselves by themselves (αυτα καθ αυτα), then it is not so that every phenomenon is true.” The “if” here is dialectical. Already at the opening of Book Γ we read the bald claim: “There is a knowledge that contemplates being as being and whatever belongs to it as it is in itself (καθ αυτο).” This knowledge comprises the domain of philosophy proper: of philosophy as a vocation unto itself. While all other vocations, committed as they are to production and action, daily bring aspects of ουσια to light, our vocation of contemplation brings their “in-itself-ness” to light.

Lying slightly buried in the Axiom of Non-Contradiction is a supposition, a sub-axiom: In each being (δει) lingers, perhaps even looms, a sameness. Aristotle names this lingering or looming sameness ουσια: an ordinary Greek word that also names what presents itself to our care, our “estate.”

Whereas the easy understanding of the primary Axiom finds routine employment in acts of denial (proposals leading to internal contradiction are to be negated, self-contradictory testimony deserves rejection), the Axiom of sameness expresses an affirmation: there is sameness, in-itself-ness. Despite related-ness.

Most things make sense to us προς τι: in the category of relation. We approach a river, something that evidently is (ον), as presenting a formidable crossing, as protecting us from the enemy, as a source for

* In Aristotle’s Politics we find ουσια summing up the question of private vs. public possession. Possessions should be private but be made public in the using of them with friends (1263 a 36, 1330 a 1). Why? Because each individual only takes care of those things that he or she individually owns. Here and elsewhere, everything depends on understanding ourselves as taking care of things. Complementing this principle of artisanship is another: the only proper way of relating to ουσια, what one possesses, is “temperance” combined with “liberality,” i.e. restraint in one’s own consumption combined with generosity in regard to others (1265 a 32). Only because the propriety of our bearing on such everyday, concrete, and tangible “substances” eludes us, and precisely because our own destiny still appears to Aristotle as bound to them in their elusiveness, does ουσια begin take on esoteric, abstract, and ethereal airs. See also Nicomachean Ethics, 1119 b 18 to 1121 a 18, and 1165 b 20, where we might well translate ουσια as our “holdings.”
irrigating our fields, as an opportunity to catch fish, as a danger of inundation. Where is the river as simply itself? So far, its is-ness appears variously on the periphery of human ways of getting on within circumstances: as a phenomenon, the river relates to someone, and this relation brings other relations in its train (crossing from one bank to the other, raising crops, catching fish, ...). Always something else looms in our productive and practical affairs, and we pass quickly on to it, something other than the river itself.

How can a river reveal itself as a unit “in itself” (καθ' αυτο), as a sameness holding fast despite the multiple and ever different relations into which our own responses draw it? Today we might pinpoint the sameness of the river in the formula H₂O: the modern chemical name allowing us to discount the “accidental” elements in the river (sediments, pollutants, even fish). And to account for the motion of what is named, we might add some considerations of slope of the bed and the force of gravity: properties, perhaps (as distinct from accidents). Yet this modern effort to pinpoint sameness takes us away from actual, i.e. artisanal dealings with the river: our scientific discourse of sameness, unlike the Aristotelian version, does not address the claims of sameness arising in the ποιησις of the farmer and the fisherman or the πραξις of the town guardian and the ship captain. Indeed, the drive to pinpoint sameness has led historically to forms of θεωρια having their own criteria of productivity and social organization competing with the artisanal ones, and differing from these latter precisely in their effort to keep formulations clear of the demands things make on us: to develop ways of retaliating, of nullifying these claims.

§1.4.1 Routine employments: questions

Identity is the Latin-based word for sameness or in-itself-ness. Whatever the difficulties we raise in contemplation, we employ the Axiom of Identity in a number of routine ways. Reviewing some of these ways, especially in an artisanal context, we prepare ourselves to contemplate this sub-axiom in the classical manner.

For instance, we identify individuals as representative of kinds: this here is a river, this here is algae in the river, this here is a mudsucker eating the algae in the river, this here is a bear fishing for the mudsucker. We here assume that our effective dealing with individuals requires that we learn to recognize their kinds. Thus we might, if we wish, employ individuals only as specimens and research their species, the sameness that will keep asserting itself despite the different and differing specimens.

Then, too, we identify individuals as being the same ones despite intervals of absence: this here is the knife that the suspect had in his pocket the night of the stabbing, this here is my long absent husband Amphytrion, this here is the house in which I was born, this here is my child even though she is now a mother with her own vocation.

In modern logic identity appears as a special relation in the axiom ∀xIx, usually written ∀x(x = x): given anything, it is identical with itself. Here we might suppose that the quantifier ranges over individuals: this (house) may have many debatable predicates and relations with other individuals, but at any given time and place it—how shall we say?—identifies itself with itself. Since individuals change according to their time and place, this notion of identity raises questions about how we ourselves can identify individuals.

In more recent theories of set formation, the axiom of extensionality posits the identity of any two sets S and T once we have decided that whatever is a member of S is a member of T, and vice versa. Since much different concepts (meanings) can cover the same individuals (references), this axiom raises questions about identifying concepts.

Throughout modern mathematical studies, from elementary arithmetic to the highest reaches of modern physics, we routinely employ the sign for equality, the same sign used in logic for identity. Children already recite 2 + 3 = 1 + 4. While the result seems undeniable, not so the weight of the =-sign. There are at least three interpretations. With a view to correctness, we might interpret the numbers 2, 3, and 4 as shorthand for sums (2 for 1 + 1, and so on), whereupon the equality sign comes to represent the axiom of identity, x = x, or, most formally, simply our ability to re-iterate the string 1 + 1 + 1 + 1 + 1. Then, with a Platonic and Aristotelian concern for truth, we might interpret the equality sign as representing our previous commitment to kinds (since counting different items makes sense only against a background of sameness), a commitment consonant with different groupings (temporal contingencies of counting). And, finally, in set-theoretic fashion, we might abstract from the substantial kinds, and consider the groupings as merely incidental, whereupon we interpret the equality sign to represent our ability to construct quantities transcendentally: S (the union
In each of these routine employments, identity is rightly assumed as we pursue the affair (and whatever we must have in order to learn is an axiom: Posterior Analytics, 72 a 17). Yet there is in each case something of a wonder, something to wonder both at and about.

When identifying individuals by their kinds, we assume that there is a constancy of kind under which the individual is subsumed: this assumption comes into explicit play when we turn to study the kind. Yet is there a sameness of kind (of εἴδος, of species)? Supposed representatives of a supposedly single kind differ so much from one another! Artisanal tools differ in construction from place to place, and even more from age to age. Biologists accept the principle that species likewise differ, i.e. change, so that any sameness we might experience lies in our own conventions of naming, not in the nature of things.

When identifying an individual as the same from one place back to another, and from one moment back to another, we assume that there is a uniqueness that survives from one situation to another: this assumption comes into explicit play in questions of ownership and of responsibility, in ordinary situations as well as in legal disputes. But is there a sameness (uniqueness) of individuals? The question bears both on the thing and on the person: in what sense is the house of my youth the same house I visit fifty years later? In what sense is the person who committed a crime at age twenty the same as the person much wiser at age seventy—for others, and even for him- or herself? Indeed, we have enough troubles becoming clear about individuals (things, persons, ourselves) at any one time and place! And perhaps we wisely look to, respond to, each presentation at the moment as new, as different from its previous incarnations; the sun might very well be new each day.

In the mathematical employments of the =-sign, the question of identity hovers ghost-like under the formulations — regarding both the sameness of individuals and the sameness of sets. ... or discover, each side of this sign somehow refers to the same thing. And here we have the philosophical problem so much discussed during the last several centuries of mathematical philosophy: reference. Precisely, perhaps only, when we aspire to develop self-contained forms of thinking (systems under which we subsume situations-on-the-wing) can we, perhaps must we puzzle over whether and how our talk bears on something “outside” our talk. Having abandoned the Platonic and Aristotelian interpretation of numbers as reflecting our engagement within the tension between individual and εἴδος, between this-here and its fulfillment, modern thinkers have devised a variety of references, e.g. activity of the human intellect, or of creative imagination (thus the acts left and right of the sign are the same, despite the differences in their articulation). Or we may abandon the notion of reference altogether, whether transcendent or transcendental, claiming that the only “reference” is that of the marks on paper or on the screen (combined with mechanical or electronic procedures for exfoliating the marks).

Identity remains elusive. Formulating the Axiom of Identity, we contemplate our reliance on it. We lose our grip on identity precisely when we try to grasp it directly. How else could it be? How can we, how dare we place before us, as an object of our own judgement, what grounds us and gives sense to every legitimate judgement? We contemplate not directly the sameness of things, but our involvement with things as this involvement draws its life-blood from sameness. In Aristotle's formulation, we cannot demonstrate origins, yet these origins come out, become clear, get revealed to the attentive mind, during the course of any demonstration. For instance, a botanist concludes that there must be water in the desert, since plants are growing there, and plants require water: whether or not the conclusion and the minor premiss are correct, the whole argument draws upon sameness, occasions the revelation of identities (of individual and of kinds).

§1.4.2 The elusiveness of beings: ranking

What more can we do? Other than combating supercilious versions of philosophy (sophistic) or engaging the young in the problems and paradoxes of philosophy (dialectic: 1004 b 23), we can contemplate the human engagement with οὐσία as this engagement throbs with the tension of identity.

The fact is that we generally make sense of things πρός τι. A hinge makes sense as allowing the door to open and close, and the door makes
sense as allowing access to the room, and the room makes sense as part of a home or some other sort of building: metal workers, carpenters, and designers alike work on each element as it ties in with others. Similarly, a gathering of students or professors makes sense as resulting from and preparing for another assembly, and assemblies in general makes sense in the general enterprise of education and research: leaders at every level of a university guide each event as it relates to others. Then, too, blood and veins and kidneys and lungs make sense only in their interconnectedness: doctors and athletes understand each of these things as parts of a whole. Items such as hinges and seminars and blood samples may demand careful attention for a moment, but lead our attention elsewhere: in Plato's and Aristotle's sense, they do not themselves present sameness, do not have in themselves any commanding identity, any strong “in-itself-ness.” Sub-artisans may remain with these items, but master-artisans integrate attention to them with attention to the units, the one-nesses that are αυτα καθ αυτα. In these three examples we can at least name the units along with their respective master-artisans: house and architect, body and physician, university and academic administrator.

What counts as a unit, as a being? Philosophers raise this question explicitly. In Aristotelian fashion, we raise it because it already, albeit implicitly permeates every fully mature human engagement as a tension and a search, and with regard to some domain of things already making sense προς τι. Naming the unit may help to highlight the question; we do not thereby answer it. As the architect, the physician, and the academic leader already know, their respective units elude us all. A house only is a house (a home, we may say in English) as a dwelling, a place that becomes a unit only under special conditions — and otherwise degenerates into a place to hang our hats and coats, making sense προς τι as a rack. A body only is a body as an embodiment of a whole human being, and otherwise degenerates into a cadaver or a instrument at someone else's behest, items having sense only προς τι. A university only is a university as a theater of human focus, and otherwise degenerates into a collection of commodities and distractions, like a shopping mall.

Precisely because the in-itself-ness, the identity of things eludes us already in our pre-philosophical vocations, we incessantly search and research for it. Plato and Aristotle inaugurated a definite style of searching, a style consonant with and lending support to the already-current Athenian disposition to debate questions of a practical and theoretical sort. They inaugurated the intellectual tradition whereby we understand the purpose of debate to be the elucidation of what things are destined to be, each as it is in itself (house, body, university, ...). What do we argue (syllogize) about, if not the identity of things? If we argue pointedly about the intricacies of a house, are we not distinguishing how a house really is from how it might factually be, in name only? Similarly, what sense can it have for surgeons to argue with one another about the best way to operate on a patient, for academics to argue about the best policies for their institution, except as they struggle to become clear about what these things are meant to be?

With identity as the criterion of beings in their elusiveness, the Platonic and Aristotelian style of intellectual searching, the new style of θεωρησις, leads to a ranking of beings. A door has more being than do its hinges, a heart has more being than does the blood flowing through it, but a house has more being than do its doors, the whole body has more being than do its various organs, and a university has more being than do its committees and departments.

In each case we decide that one thing has more being than another on the grounds of relative self-containedness of human attention: to understand hinges or fingers we must turn toward a larger being — ever-more-real ones (door, room, house, estate “in act”; hand, arm, body, human being “in act”). The necessity of this turning lies embedded already in the exigencies of ποιησις and πραξις, the “evidence” to which we appeal in our own stance of θεωρησις.

The difficulty of ranking reflects the difficulty of becoming clear about the subject of a syllogism. We reason about fingernails and cuticles, about spoons and saucepans, but — as in the case of a river — on what can we fully focus our attention? Especially in these cases, predications draw other subjects, apparently more real ones, into view. Or, rather, they draw upon such subjects as human beings or kitchens, each in their wholeness (of function, ἔργον, both Plato and Aristotle add). And these elude us as well. Yet we reason about these things anyway, i.e. prior to concerning ourselves “theoretically” about the elusiveness. Indeed, the Scholastic thinkers often remark that we reason about these things precisely because we cannot, as human beings, simply gaze upon the subject of our discourse; we differ from God in this crucial respect.
Reasoning is essentially a pursuit of subjects that elude us. Not, as in much routine academic work, a pursuit of predicates: these serve rather as avenues along which we travel during the pursuit. On Aristotle’s account, we pursue a subject by enquiring after, embedding ourselves in its “causes,” especially its τέλος and its εἴδος but also its κινησις and even its υλή. In this sense syllogism depends on identity—not, Aristotle insists, as a mere supposition, an “hypothesis,” but as a sine qua non of any λόγος, of its having any issue.

Against the background awareness of the elusiveness of subjects we may, in Scholastic fashion, devise hierarchies of subjects, and note theoretically the varying difficulties of pursuit:

Earth, air, fire and water: How can we focus on these, come to understand them? Not one of these elements comes in a natural unit. Yet a farmer must till this soil, a sailor must provide this water. In general, the understandings here indicate dependencies: the farmer understands the source of his own products as soil, wind, sun, and rain. Owing to the lack of units onto which we can focus, these subjects are the most elusive; we more easily understand them as the material for other subjects.

Blood and sap, gasoline and graphite: complexes embedded within functional units and generally measured in quantity and quality. These, too, we must variously understand, depending on our vocational involvements. Again, however, we more easily approach these in their relations to animals, plants, and machines directly under our care. Yet we recognize that they have definite characteristics of their own, above all definite functions in regard to nameable units. Aristotle and his successors would also understand them as derived from the four basic elements, and sharing elementary properties of hot and cold, wet and dry.

Organs and instruments, e.g. the leaves on a tree and the fingers of a hand, tools such as hammers and all the measuring devices in a shop. Here, now, we may speak of identity not only of a kind but also of the subject: while we may point to some soil or some blood, and even store some in containers, a leaf or a finger, a hammer or a measuring device comes into view already contained. And we may enhance an organ or an instrument by cleaning, spraying, oiling or adjusting it, by exercising or bandaging it. Yet we recognize (as a kind) and treat (as an individual) each such subject as serviceable parts of a host subject: detached from their hosts (stem or hand, shop or machine), organs and instruments lose their function, degenerate into identities in name only.

Internally functional units having lives of their own: plants and animals. In artisanal vocations, the identities of carrots and trees, cows and horses, focus our attention: we must cultivate and shelter, plant, raise, harvest, and employ them—know them as individuals and as kinds. These are the middle-range beings having a nature of their own, a nature that the artisan learns to follow and to complete. Aristotle looks to these as prime examples of ὀφθαλμα, and extends the list beyond domestic plants and animals.

Internally functional units having no life of their own: human products such as houses and ships. The identity of such things seems to depend heavily on the use to which we put them, yet even our own use is constrained by the nature of other things: ships must conform especially to the nature of wind and water, houses to the nature of wood and stone, fire for cooking and light for mobility within. Having no generative powers of their own, these middle-range beings disintegrate unless we maintain them: in this sense they are only under our care.

Communal units, challenging mixes of the natural and the human: communities first of all (from families to teams and beyond), but perhaps also churches, cultivated parks, theater productions and other art works. Such things we in some sense make, but in another sense find already making claims upon us as natural—the nature here being the human spirit rising in unison. In any case, they especially elude us, leaving only their skeletons behind, whenever we neglect them, take them for granted apart from our preservation of them.

Finally, celestial units: the moon, the sun, what we now call planets, stars, and galaxies. These super-beings lie outside our domain of care. Yet precisely by their regularity of movement, their uniformity over time, they represent a stability contrasting with the irregularity, the perishability of terrestrial individuals—as anyone may confirm on a clear moonless night far from the glow of city lights. This manifest stability Plato and Aristotle interpret as recalling the unmanifest stability of the internal destinies buried in the unstable individuals under our daily care.

The Platonic/Aristotelian tradition ranks beings according to their degree of self-containment, this being their hallmark of identity. The criterion of self-containment lies in our own ability to focus on them: the more our focus must shift to larger things, the less being; the more our
knowledge brings us into the thing’s own function, the more being. Thus, in one of Plato’s examples, the horseman, the one who puts the horse to use, moves in closer to beings, whereas the bridle-maker, the one who must follow the instructions of the horseman, is farther removed from beings — and the mere talker, the one who spins tales to please customers, merely picks up predicates by looking at horseman and instrument-makers, and misses out on subjects themselves: thus the whole horse is more real than the bridle and the parts of the horse on which it fits, and such instruments and parts are more real than the accidents noted by mere spectators and by those who pander to these for the sake of rewards.

Out of this ancient tradition grew what we today might call a “research program,” a form of θεωρία that lasted for nearly two millennia and eventually evolved into modern science: investigate each and every level of identity, i.e. study earth, air, fire and water (our chemistry), their configurations (geography, modern thermodynamics and associated mechanical technologies), the parts of animals and plants (ancient physics, modern botany, biology, psychology, and associated agricultural technologies), human configurations (literary criticism and political history, modern anthropology and sociology). Whether ancient or modern, these studies employ the Axiom of Identity, its interplay of instance and kind. And anyone who has engaged in investigation of either sort has learned how elusive these identities are.

§1.4.3 Anachronisms

A number of differences thwart our own (modern) efforts to understand and to appreciate the original (ancient) versions of θεωρία. Among these differences are crucial ones such as the shift in the understanding of form from immanence within middle-range (terrestrial) beings to transcendence of a mathematically formulable sort, and the concomitant development of “theory” as setting its own conditions of evidence and criteria of legitimacy.

Then, too, our modern theoretical education leads us to suppose that the “reality” of a middle-range being becomes clear only as we discover the elements of which it is composed, together with the mechanisms accounting for their functional composition: atoms or even sub-atomic particles have more being, represent more stability and more intelligibility than do molecules, biological drives to reproduce have more reality than do affection and loyalty, individual citizens have more reality than do communities, and so on up the chain of composition. On the other hand, the formalized mechanism by which we understand the composition of terrestrial beings stems, paradoxically, from the earlier mechanics assumed to hold only for beings at the upper end of the chain: the motions of celestial bodies. These and other anachronistic suppositions we carry with us as we attempt to understand the origins of our intellectual tradition.

Another difference deserves separate comment. We today find ourselves surrounded and engulfed by artifacts: besides sedentary things from buildings and roads to home and office furnishings and all the gadgets to ease work and facilitate play, we work and play with electric and electronically programmable telephones and fax machines, televisions, radios, and stereos, home computers and printers. Each such thing counts as a tool, an instrument, a means, an ὁδός toward something else. In the Aristotelian sense, such things have only weak identities, both as instances and as kinds: they are mainly πρός τι, we understand them differently according to what we are trying to do with them. Our overwhelming involvement with them can atrophy our sense of the in-itself-ness of what we encounter. Indeed, a world of tools can lead us to conclude that all talk of sameness, of self-contained-ness, of inherent nature, reflects an academic vacuity paralleling popular superstition.

It bears repeating, perhaps, that the “metaphysics” of identity originated in an intellectual concern to bring into discussion the issues already present to those involved artisanally with natural things. Even a king such as Odysseus could brag about his ability to plow a straight furrow; and any king had to know how to mount and ride a horse even if he acquired servants to perform the more menial tasks. But in an agricultural society everyone is surrounded by plants and animals that assert their identity in the claims they make upon one: the aristocrat may not feed his horses or herd his sheep, but he knows that each of these things has its own life, its own requirements for thriving,
its own destiny of generation and destruction. Knowledge named the ability to fit in with these given exigencies, not to redesign them according to human exigencies alone.

Knowledge as fitting in. Here we may consider the most radical challenge for those who aspire to overcome modern anachronism. On Plato's and Aristotle's account we can pursue an identity, ever elusive, only as we participate in its movement, move with it, be moved by it. The exigencies internal to ποιησις and πραξις provide the prime evidence for this account. Having learned the appropriate τέχνη, we complete and follow, we guide while miming, we are able to adjust because we share the destiny of the this-here under our care.

The factual condition of any this-here under our care — whether a horse or a house — is indeed one of movement, or rather one of oscillation between rising and declining. Plato and Aristotle, and later the Scholastics, devised a hierarchy of beings partly in order to achieve a theoretical focus on this movement: in order to develop the art of contemplation. Whereas artisans and leaders “come into their own” by participating in the movements of this-here and that-there, always different and partial, thinkers do so by participating in movement itself, the movement at work in all things. From the new standpoint of θεωρία, a standpoint insisting that there be a cause of every movement, there must be an origin of movement itself, a highest being, what moves all else but itself not moved. We know this highest being not by turning toward something besides horses and houses, but by discovering our own ability to participate in the movement of these things. This ability Aristotle especially calls νοῦς, translated variously as “mind,” “intuition,” “intellect,” “intelligence” — as distinct from, as the cause of λόγος, i.e. of our ability to gather a multiplicity together into a unity. Discovering our “own” identity is thereby a discovery of the highest identity, one that moves us and all that we might ever know. Thus the Aristotelian Axiom of Non-Contradiction opens us out not only onto ontology but also onto theology.

§1.5 Excluded middle

At first again only in passing, Aristotle remarks that for those who hold that the same can both belong and not belong to the same “there is no necessity either to say or to gainsay” (οὐκ ἐν δύνασθαι ἢ φάναι ἢ ἀποφάναι: 1008 a 4). For instance, a physician who rests with the conviction that a patient in a coma is both a human being and not a human being need not, perhaps cannot decide between the two, much less carry through with the consequences, but will remain neutral.

Insistence on neutrality appears to Aristotle as disgraceful. After all, the exercise of any art incessantly requires that one decide, at least tentatively, whether something is or is not in a certain way. If the learning and the exercising of an art belongs to our own destiny, principled neutrality, as distinct from reserved judgement, renews our own nature.

This third principle we now call Excluded Middle. Of the three, it alone aims directly at our linguistic condition. All three may be expressed linguistically: \( \neg(p \land \neg p) \), \( x = x \), and now \( p \lor \neg p \). Yet Aristotle calls only the first, Non-Contradiction, an ἀρχή. And both it and the sub-axiom we call Identity pertain to what we must learn to face. In contrast, the sub-axiom of Excluded Middle pertains to our commitment to respond to what we face. One mark of modernity will be that all three principles become linguistic in this sense: functions of our own articulation.

The agony of decision provides the basic experience of Excluded Middle. It has often been remarked that the early Greek experience of free government brought to the forefront of public attention the acts of declaring where one stands, of voting yes or no. Artisanal decisions are not public: the craftsman silently decides what is best — whether rightly or wrongly the product will eventually tell. Group decisions, however, decisions of the new type of πραξις, take shape gradually, and prove themselves not in any product (its use by others), but rather in the consolidation of the communal action itself.

In his Constitution of Athens (8), Aristotle remarks of Solon (“at the beginning, and the first leader of the people” [28]):

He saw the polis often in conflicts while some of its members preferred, through sluggishness, to let things run their own course [be auto-matic: αὐτοµάτως]. And so he established a law bearing on these people especially: whoever during conflicts in the polis did not take up arms one way or the other would be dishonored [ἐπιστήµον, disfranchised] and could no longer participate in the polis.
Three centuries later, under Roman rule, Plutarch still finds Solon's law worthy of remark and commentary (*Lives*, “Solon,” 20):

Among others, one of his laws protrudes and contravenes our usual ideas, viz. the one declaring dishonored *ἀτιμον* whoever in conflict does not take sides with either part. He likely wishes that citizens not be indifferent or insensitive toward what is shared [common], arranging their life at home and congratulating themselves on not sharing in the aches and diseases of the fatherland—but that they should promptly act on what they take to be better and juster, share its insecurities and help it along rather than waiting in security to join those who win.

Yet surely the necessity either to say or to gainsay, to affirm or deny a proposal, is not *immediate*. The difference between prejudice and genuine judgment lies precisely in whether the saying or gainsaying issues from a close examination of the matter itself: on whether we *discover* that P belongs to S. And close examination of a matter, open attention to a subject, requires us to suspend judgement, postpone the decision. Artisans often distinguish themselves from the rest of us precisely by their refusal to “take sides” precipitously in matters of their own competence: by their ability to admit ignorance, to wait for further evidence. And group decisions (those of a jury or a parliament) supposedly take shape as a result of debate. Unable to enter into a situation neutrally, at least in regard to the judgement at issue, we betray it. Contrary to what Solon's law might suggest, wisdom would seem to require that we cultivate the ability to remain neutral. For the duration of the examination, that is.

The necessity embedded within the sub-axiom of Excluded Middle hovers over our linguistic condition: over our decisions regarding our circumstances. It presupposes a genuine neutrality, an ability quite distinct both from indifference and from insensitivity. And distinct also from the opportunism that Plutarch describes and that we often find in political assemblies. Yet this neutrality contrasts with our initial and prevailing entanglements in language.

For the most part, we enter into a situation already saturated with verbal versions of what remains to be considered directly. Clever talkers spin accounts that attract us away from the matter itself. Vested interests in results prejudice the decision from the outset: for such results (e.g., payment for our work, political alliances) pertain to our own ongoing careers rather than to the subject demanding response. What we in fact need, more often than not, is a liberation from these entanglements, a liberation allowing us to look freshly at the subject *so that* we can freely decide whether a proposed predicate belongs to it or not: to discover the subject having the predicate, or to testify that the subject has revealed itself without the predicate.*

Still, we may wonder why Aristotle bothers discussing Excluded Middle as though the principle were problematic. Constructing a purely formal system, we may introduce $p \lor \neg p$ without engaging ourselves in any soul-wrenching decisions; indeed, it would seem that intellectual work presupposes a drive to decide whether or not to affirm proposals of certain kinds, and that logical work simply formalizes this drive. Similarly, artisans at every moment must decide whether or not some P belongs to the S under consideration. And judicial proceedings thematize the decision: Did X happen ... or not? Is Y guilty ... or not? In all such cases, there is no need to wonder about the status of Excluded Middle.

Exactly! In Aristotle's sense, the axioms definitive of philosophical consideration undergird not only every act of reasoning, of syllogism, but every art, whether productive, practical, or theoretical; and those involved in these (pre-philosophical) arts draw upon these axioms without any need to consider them. These axioms already hold: they sustain every art and science, every decision within knowledgeable activity and encounter. The three (Non-Contradiction, Identity, and now Excluded Middle) unquestionably deserve our endorsement.

Or almost. For we may in fact misunderstand them, misemploy them. Each is a source of illusion. For any actual individual under our
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Care both is and is not, both simply and predicatively, and we err grievously if we fail to recognize this strange duality (for we then discount its potentiality, its being-in-act, and its movement). Then, too, the identity of a thing, its specialness, its glow, its εἴδος, we grievously misconstrue as a template, a finite mould into which humans or gods may cast individuals. And especially now we may grievously misconstrue the supposedly necessary option: either to say or to gainsay, affirm or deny a given proposal.

Turning now to consider directly the sub-axiom of Excluded Middle we must elicit the special conditions under which it retains rather than falsifies the truth at issue in production, action, and contemplation.

§1.5.1 Aristotle's formulation

“But no between is admissible for contradictory sayings,” Aristotle says (1011 b 23), evidently resuming his earlier hypothetical statement of the principle. “There is no third possibility,” tertium non datur, the Scholastics say. Corresponding to this denial we read an affirmation: “but it is necessary either to say or gainsay one [predicate] of any one [subject] whatsoever.”

Aristotle's formulation of Excluded Middle recalls the original question of one-ness, singleness, unity of focus, both regarding the "point" of focus, the subject, and the "angle" of focus, the predicate. In any given case, once we achieve one-ness of both sorts, we “necessarily” either affirm or deny the one of the other.

The formulation remains ontological. Although linguistic in effect (we must say or gainsay), it refers to our ability (Aristotle says: the necessity) to decide whether to slip a “not” between any given subject (given as a genuine unit) and any proposed (genuinely single) predicate. The acts of affirmation and denial then also (derivatively) arise when denying a saying or denying a gainsaying, as when disagreeing with interlocutors: the first stems from the more basic act of denial, and the second from the more basic act of affirmation. The basic acts arise from a direct consideration of S and of P.

The requirement that the subject and the predicate each be one raises all the questions of unity we have already considered. Most prominently, perhaps, the question of what counts as a proper (logical) subject: to be one, ἕν, the subject must fall under the category of being, οὐσία; thus Aristotle need not account for what logicians now call “category mistakes” (such as demanding a stand on whether a number has a certain color, or whether a posture is rightly predicated directly of health).

Excluded Middle intends once again to remind us of the task of discourse: to talk and to listen attentively to discover what is being said, what deserves to be said—to discover the subject in what is being said about it. Excluded Middle sets the terms of this discovery: singleness of both subject and predicate forces us to take a stand, to declare ourselves on the question of whether they go together or not.

Yet the required singleness eludes us, and so long as we fail to catch up with it we in fact have the option of admitting our ignorance. Such admission, such suspension of judgement, is a sign of wisdom: the wisdom of acknowledging one's being in hot pursuit.

Especially pertinent to the study of logic is the recognition of the various ways of misconstruing the options:

The contradictory of “(All) women are devious” is “Some women are not devious” rather than simply “Women are not devious.” In this example we may detect the philosophically most dramatic distinction between the singleness of kind (destiny, εἴδος) and the singleness of case (a this-here). Yet much discourse is indeterminate in regard to what we call the quantity of a proposition: a major source of futile debates.

The contradictory of “My child is strong” is “My child is not strong” rather than “My child is weak.” In this example we may detect the difference between contradictory statements and contrary predicates. One must become clear about the exact meaning of the predicate, and then affirm or deny it of the subject, leaving contraries out of account for the moment of this one judgement. (Aristotle's example: between “Socrates is black” and “Socrates is white” there are many “betweens” of gray.)

As Bertrand Russell so well analyzed, the contradictory of “The student who entered my office last night and stole my book had a key” may be rather complex: e.g., “Either the person was not a student, there was no one stolen book of mine, I have no office, it did not happen last night, or the person had no key—or any combination of these.” Russell rightly notes that modern scientific knowledge bears largely on subjects we know only or primarily by description rather than by acquaintance (encounter), and that a description includes claims to unique existences that may require review. Yet, given the originally much different sense
of science, Aristotle would have replied that any significant debate on a predication presupposes “knowledge by acquaintance.” Only when the rather complex subject has been discovered does tertium non datur hold: either the subject had a key or did not have a key.

On Aristotle’s account, the Axiom of Excluded Middle does not itself supply a proposition of the sort \( p \lor \neg p \), even once instantiated. It would be fatuous on our part to claim that any such \( p \lor \neg p \) represented knowledge: the compound is vacuous, since it does not focus attention on anything definite, τι δρισιμενον; it precisely abandons the subject to play with the predicate in abstraction. Rather than supplying insight, this Axiom demands a decision. Our task is to consider how this demand retains its truth, its legitimacy.∗

Consider three situations: (1) a cast of dice that either brings a 7 or not, (2) a wartime challenge that either leads to a sea battle at Midway or not, and (3) a horse that either grows strong or not. There are moments in time when the options are not only clear but pressing—and as yet not decided or even decidable. The first depends, as we say, on chance, τυχη; the second on human deliberation and action; and the third on nature. In all three situations our own position (as gamblers, admirals, or horse trainers) is one of having to stand up to potentialities: in the first the outcome does not depend on us (short of loading the dice), in the second it partly does, and in the third it might engage us as helpers. Most intelligent responses on our part parallel one or the other of these scenarios. In what sense might we say that Excluded Middle governs our saying and gainsaying?

One sense: Resting assured that the actualization has taken place (the morning or lifetime after), we can return to the scene rightly convinced that either the dice rolled 7 or not, either there was a battle or not, either the horse grew strong or not. Here, our position (e.g. as historians) is indeed one of having to investigate and having to take a stand on one side or the other: there is now no between.

But the historian’s is seldom our own real position. Rather, ours is to respond to what comes, to the present that is coming. The pressing options pertain to our own decisions: whether to lay a bet on 7 (the thrill of taking chances), how to face or how to flee the enemy (the agony of deliberation and action), how to treat our horse (the task of cooperating with nature). We have no business taking a stand on the factual outcomes themselves. These are “anybody’s guess,” the concern of those not intelligently engaged (in Plato’s image: of those strapped into predicting the shadows to come).

Aristotle himself hotly disputes the application of Excluded Middle to responses bound to the moment. This sub-axiom shares the ambiguity of Non-Contradiction: it indicates our commitment to rise above the moment, to respond to the actualities at issue in the potentialities pressing in upon us. As he remarks in another work, On Interpretation (19 a 9), “in things not always actualizing themselves there is the ability both to be and not to be.” Even the Axiom of Non-Contradiction fails to hold “absolutely” in real encounters.

Yet the purpose of these reflections is precisely to goad us into thinking the things that are “always actualizing themselves:” in colloquial English, what it means to be a (good) horse, what it means to wage battle (to attack or defend, to win or lose well). Modern examples: what it means to mathematize our understanding of chance, what it means to formalize our understanding of water as \( H_2O \), what it means to formulate laws of thermodynamics. Each “it means ...” suggests that there is ultimately one meaning: one guiding potentiality, one ability (δυναμις). And that when investigating this one-ness transcending each (temporal) instance we must indeed decide between “is” and “is not”: whether or not a proposed feature belongs to the meaning of a good horse—or of water by itself, heat by itself (real water, real heat).

§ 1.5.2 Truth

Immediately after introducing the Axiom of “no between” Aristotle remarks:

This is clear once we focus on what the true and false are. For to tell as not being something that is, or to tell as being some-
thing that is not, is false, while to tell as being something that is and to tell as not being something that is not, is true — so that one who tells something being or not being will either speak truly or speak falsely. But neither [in the case of] what is nor [in the case of] what is not do we tell it as either not being or being.

Before we can rightly decide whether and how these lines clarify the Axiom of “no between” we might wonder what they mean.

On a casual reading, Aristotle's account of the true and the false seems to repeat what we have often heard from our parents and teachers, later from our colleagues and critics: we may speak truly or falsely, truly when our account squares with the facts and falsely when it does not square with the facts. Thus I speak truly when admitting that I chopped down the cherry tree or that I did not write the essay I submitted, also when giving an accurate account of what I found and did not find on an archaeological dig. I speak falsely either by mistaking the facts or by devising a fiction to disguise the facts: I mistake the facts when reporting that there was blood on the victim's shirt while closer examination reveals it was only spaghetti sauce, or when I claim that ticks leap down from trees onto their prey when in fact they don't; I misrepresent the facts when I tell my parents I was studying in the library when in fact I was conversing socially with my friends, or when I fudge experimental data in order to appear successful in my research.

Already the parental version of the question of truth might recall how much our talk does not aim for truth. Or even falsehood, we must add: How rarely we speak or listen with a view to revealing in speech how things are! Most speech serves other ends. While fabulating and pandering we might understand as betraying the apophantic intent of speech, most of our speaking and listening takes the form of questioning and commanding, inveighing and prodding. The injunction not to lie first introduces the possibility of apophantic speech into our lives.

As a sailor, I must be able to tell whether the knot is properly tied or not: the knot becomes fully present to me only if I know my knots. Yet I may err in this regard as well: the knot may appear to be well tied at one moment, but prove to be poorly tied at another — or vice versa.

As a captain, I must be able to tell whether my officers are doing their assigned work well: their engagement becomes fully present to me only if I know how to lead a ship's crew. And of course I may err in my decision: err not out of neglect (out of indifference or insensitivity), but precisely in my perception — as King Lear erred in regard to each of his daughters.

Why do we err? That we unintentionally mis-report or mis-predict should come as no surprise: when supplying others with a present account for past or future events we aspire to bridge an evident gap, and the original will always differ from a later or an earlier version. Similarly, that we intentionally mis-represent the past or the future to others: again, the gap essentially prevails, and we can play on the difference. But that we should err silently when face to face with the knot, with our officers: here there is no temporal gap to account for the discrepancy, and no one else for whom we prepare the account. When the competent sailor, the competent captain, silently focuses on the knot, on the officer, he may detect (λέγειν) as being something that is not, or as not being something that is. How so?

The most ancient answer reads: owing to the withdrawal of the being at issue (“nature loves to hide,” Heracleitus remarked). A knot, an officer, essentially eludes us. Why? because each is doubly, with its own gap: its immediate appearance and its own standard. Each side of the gap makes sense, both to us and by nature, out of the gap itself. The knot, the officer, is most immediately in its pretension to fulfillment, and we pay careful attention to the immediacy because it pulls us toward the question of its fulfillment. And the standard of this fulfillment becomes an issue, a manifest possibility, only in the immediacy, and only because the immediacy of ship-work makes demands on us for fulfillment do we
turn to consider the standards themselves: what a good knot, a good officer is.

Asking us to focus on what the true and the false are, Aristotle engages us in the gap contemplatively. His definition of the true and the false interpret truth as occurring when and only when, where and only where, we find ourselves in the gap internal to a being and successfully bridging this gap: bringing the immediacy of the knot, the officer, out toward its own measure.

Only those able to enter into the gap, able to participate in the tensions internal to each thing, can tell as being (είναι) what is (τὸ δὲν), the prime of the four possibilities. For each determination of how a thing is receives its justification, at the moment of λέγειν, from the emergence of the thing (e.g., knot) in the light of its pending fullness.

Finding ourselves in the gap we can also detect that the knot, or the officer, itself does not catch up with itself: that the immediacy falls short of its own standard. This detection of failure (privation) on the part of what we encounter depends essentially on the detection of success, i.e. of the standard. And both detections allow us to assemble the determinations of the being in its immediacy: to provide an account, a λόγος of what is.

On the other hand, we ourselves can fail to catch up with the knot, or the officer. Each eludes us in any event, leaving us with a doubleness, a difference between its immediacy and its measure, a gap in which we ourselves are caught. And we can fail to bridge the gap. This failure consists in formulating a determination in reference to one side or the other. For instance, the sailor may examine the knot and detect in it only what it is supposed to be, its standard: the positive error of failing to notice how the thing in fact is. Similarly, he may notice that the knot is not exactly the embodiment of the standard and insist that it is therefore not right: the grumpy error of failing to accept the finitude of knots. Or he may fail to recall the measure at all, detecting then only the accidental features of the knot: the error of incompetence. In each kind of case, one will detect as being something that is not, or as not being something that is.

On this account, we ourselves may fail to catch up with things as they are only because these things themselves are in a race to catch up with themselves. We ourselves are “in the truth” when, perhaps only when, we recognize that we participate in the race.

Now, when the sailor reports to the captain his findings he must say either yes or no to the query whether the knot is properly tied — say or gainsay, with no between. As must the captain when reporting on the performance of one of his officers. This predicament becomes clear. Aristotle claims, once we understand what it means to detect, tell how something is and is not. Saying and gainsaying issue from such detecting, such telling. And when we detect something we no longer waver between being and non-being — we decide, truly or falsely, one way or the other. Therefore, given that we understand our condition of detecting (λέγειν), our linguistic utterances must come down on one side or the other, either yes or no — assuming that these utterances are themselves rooted in the direct testimony and its exigency to determine how things are in their own gap.

Saying and telling, φανεῖν and λέγειν, differ from one another crucially. Excluded Middle invites contemplation of each, and of their differences. Saying, as in reporting and in predicting, issues at best from telling, from first-hand examination. Saying is a social act, one in which we take a stand regarding some proposal or question, and one that is received by another who may or may not have to take the sayer’s word for it, i.e. for the truth of the matter. In either case, however, the saying now refers to something else: refers, over yet another gap, that of time and perhaps of space, to a detection. In such reference there lies the possibility of a correspondence between saying and what has been detected. A saying can then be right or wrong, according to whether it adequately represents the detection. Starting with the saying (e.g., hearing it from another) we may check it out, verify or falsify it: but only by resorting to an original telling, detecting: an individual act—but not private, because the act of λέγειν takes one out of oneself into the gap, into the drama of things themselves. Truth is primarily an affair of detection, while saying only recalls us into, perhaps sets us up for this affair.

So far, this account of truth draws upon the exigencies of ποιησις and πράξις: upon the τεχνη, the competence essential to each encounter in which truth is already an in-built, a pressing issue. The popular wisemen of classical Greece, the sophists, had claimed to reduce questions of truth to questions of human interest: we ourselves are the measure of χρηματα, and genuine decisions had best leave aside, unraised, the question of the divine. Indeed, this reduction appears...
natural in our everyday mode of arranging things to suit ourselves and others. And there are in fact arts of arranging things in reference to human needs and the usability of things encountered: that of a quarter-master in the military, of the concierge in apartment buildings, of the merchant, of the hotelkeeper. But, Plato and Aristotle argue, such arranging, whether of a private meal or of a commercial transaction, becomes possible only after somebody has become competent in the handling of nature, e.g. in raising sheep or sailing a ship. The chrematistic arts are illusory because derivative and parasitical — both factually, in regard to the super-arrangement called the πολις, and ontologically, in regard to our nearness to things as they are in themselves. Yet the chrematistic arts appear temporally and socially as collecting and distributing the products of the basic arts, as pre-empting and predominating, so that the illusion will never dissolve absolutely.

But what about the truth at issue in θεωρία? The truth at issue for intellectuals generally, but especially for us as philosophers? How do we account for the truth of arithmetic and geometry? of music? of astronomy? of our religion? of all our modern sciences?

It is a remarkable fact that Plato and Aristotle developed and bequeathed a “theoretical” understanding of truth in reference to artisanal competence. On this understanding, truth becomes an issue only when we are engaging with circumstances while upholding their primacy over our own responses. Yet at the same time these thinkers insisted that there be a separate art, that of philosophy itself: a τεχνη of θεωρία.

The entire discussion of axioms in Book Π intends to elicit the focal points at issue in this new art: a knowledge of determinables as they are determinable in themselves rather than in relation: the ἐπιστημη of τὸ διὸ ἢ δὲν πρὸς τι. The new art of contemplation takes place primarily in live discussions: with the young and open as well as with the old and hardened (as dialectic and as eristic). And increasingly in written treatises.

It is also remarkable that, for nearly 2,000 years this art shared with the arts of the sailor and the captain an endorsement of the distinction between saying and detecting. In production and action this distinction makes immediate sense: there is something else at issue besides the having of knowledge. In contrast, the art of contemplation seems to take place essentially in language rather than on board, where knots and sailors, officers and ships find themselves daily tested, proved or disproved, by circumstances themselves. Where lies the test, the proof of a discussion, or of a treatise?

Considered as a collection, an organization of sayings, the discourse of contemplation becomes true when it gives way to detection. It seems plausible, if not clear, that the art of a discourse on geometry or music or physics (in the ancient sense) consists in engaging others in the detection: in recalling the detection, goading them into it as their own. And there is indeed something philosophical about such discourse: it requires a love of retrospective, retroactive learning — as distinct from the love of immediate interaction with circumstances. But what about the art of a primarily philosophical discourse — the sort engendered by Plato and Aristotle? On what does “primary philosophy” focus us, if not on knots and officers, right angles and flutes? What demands detecting — as distinct from saying? Knots and officers very dramatically elude those whose task it is to determine, to decide them. What eludes us, defines our task in contemplation? On what standard, on what μετρον does the truth of contemplative discourse depend?

The discourses of Plato and Aristotle in fact draw into view our own dramatic engagements in ποιησις and πραξις. These engagements are “ours” not personally but humanly: we may not be able to engender or appreciate contemplative discourse unless we ourselves have engaged in those pre-contemplative activities, but the art of contemplation focuses on these as human conditions, as issues for everyone. And as eluding not just others, but also us — much as knots do. At moments we might succeed contemplatively in catching up with these. At such moments we detect the difference between fulfilled and unfulfilled production and action. And this detection remains part of the fulfillment itself — just as the detection of how the knot really is remains within the activity of tying, untying, and retying it.

In the Platonic and Aristotelian development of contemplation as an art we may discern a double focus. On the one hand, their discourses in fact focus us on “technical” competence as the chief mode of human encounter and draw this out as the originary mode in which truth becomes a genuine issue. On the other hand, these discourses address questions regarding courage, love, nature, community — in each case with a view to the kind of fulfillment we have discussed under the name of Identity. These two foci develop in tandem, and provide an Interpretation of truth that has remained definitive of western intellectual work.
The sayings of \( \theta\epsilonωρ\varphi\alpha \) are then true or false according to whether they engage us in the pursuit of another kind of elusiveness: the illusiveness of our own condition of \( \tau\epsilonχ\nuη \), both as our destined framework of human development and as containing multiple measures of development for what we encounter.

Accustomed to engaging in “theories” designing their own sphere of evidence (their “methodology”), we moderns must strain to remember that for nearly 2,000 years of development, intellectuals continued to assume that sayings may swell up out of detectings, and may serve well or badly to engage others in the recollection of them, but can never stand in any assured correspondence with them. Within the difference, the relation remained decidedly asymmetric: \( \lambda\epsilon\gamma\epsilon\tau\nu \) gives birth to fresh \( \phi\alpha\nu\iota\eta \), but this latter (the saying) degenerates into a sham-life of its own and therefore remains forever unreliable for the return journey to the former (the detecting).

Yet we must also remember that Plato and Aristotle continued to understand our linguistic condition, speech itself, as a destiny to speak and listen, read and write out of and for detection, not as an affair of formulating sayings. The Axiom of Excluded Middle does pertain to sayings, a necessity to say yes or no, but a necessity we meet not by simply saying yes or no, much less by asserting “either the affirmative or the negative of \( p \) must be right,” but by returning to the detection, by enacting it—truly or falsely.

So far, there is no truth unique to theory itself: engaged in \( \theta\epsilonωρ\varphi\alpha \), we “borrow” the truth at issue from production and action. And our own sayings intend to self-destruct for the sake of detectings: very much as the sayings of sailors and captains hard at their own work.

§1.5.3 The shift into modernity

The birth of modernity changed the understanding of theory so that we today must wonder, investigate whether a theory itself is true. Copernicus insisted that the heliocentric theory, hitherto a familiar mathematical hypothesis playing with the observed phenomena, must also be true: must embody a collection of sayings paving a reliable path directly to the contemplation of the heavenly bodies, including the earth. Slowly, painfully, intellectual work adapted itself to the Copernican ambition. Today our universities bulge out at every corner with theories, bodies of knowledge that teachers reorganize and repeat, that beginners learn to re-say, that researchers revise and expand.

The Axiom of Excluded Middle now bears down more directly on sayings: \textit{these} are now right or wrong, and accordingly earn or lose their membership in the body of sayings inherited and bequeathed in the name of some (modern) science.

With the birth of theory as an enterprise laying claim to its own truth, the difference, the tension between saying and detecting hardly vanishes. Rather, the difference becomes exacerbated. For now the sayings must be so formulated that they stand in a symmetrical relation to detectings. To satisfy this new exigency of truth, intellectuals devise special ways of saying what they deem detectable: the special vocabularies and rules of syntax with which teachers catechize beginners in such modern sciences as physics, biology, and chemistry; more recently, economics, psychology, and sociology. The purpose of these special languages is to establish a platform of sayings that themselves can be bequeathed and inherited as \textit{assured} avenues of detection, avenues \textit{collectively} travellable, expandable, reconstructible.

On the one side: truth as a harmony of intellect within the gap, the movement of fulfillment. On the other side: truth as correspondence between sayings and detectings. Truth as individual participation in the workings of nature vs. truth as anonymous participation in a collective work of human formulation. Truth as a moment within productive and practical engagements vs. truth as a property of formulated theories.

Yet there is a common root, one likely contrasting ours with non-western civilizations: truth as an affair of special competence and therefore an affair of \( \varphi\omicron\tau\omicron\nu\iota\varsigma \): of bringing things out in their fullness, whether horses or houses, later gold or guns, and most recently human organizations. Precisely Descartes, who explicitly endorses the development of philosophy beyond the traditional contemplation of artisanal competence, aims for a philosophy with practical application, one that establishes a theoretical knowledge of “the forces and actions of fire, water, stars, heavens, and all the other bodies that surround us,” a knowledge that will serve for the “discovery of an infinitude of artifices ... but principally also for the conservation of health” (Discourse on Method, Part Six). We sometimes talk about pure research, about knowledge as an end in itself: we sense that premature insistence on application will distract from the concentration necessary for developing
the originary insights. In the end, however, our research, our modern knowledge proves itself in what Descartes calls une pratique, in the invention of artifices. As the biologist P. L. Kapitza has insisted, we can say that scientists understand life-processes (e.g., the contracting of muscle fiber) only when their analyses finally provide a recipe for artificially reproducing them. And for repairing our own.¹

At the end of the twentieth century it has become fashionable to criticize modern science as instilling the spirit of mastering nature, as encouraging if not establishing a commitment to exploit our circumstances and thereby to eat away at our own tenuous foothold on earth. And thinkers have no doubt rightly seen in the original Greek understanding of truth the theoretical beginnings of technological production: drawing our inspiration from contemplating the τεχνη evident in ποιησις, we will eventually affirm the productive mode of engagement as setting the standard of θεωρησις itself.

However, in order better to contemplate our own theoretical commitments— to question them, perhaps then to overcome them for the sake of their own truth, and to avoid the horrendous trap of simply objecting to these commitments—we might better contemplate the differences between the ancient and the modern versions.

A major logical difference lies in the understanding of truth in relation to saying and detecting. Ancient thinkers understood this relation to be one of irony. Modern thinkers understand it as one of formalism: without denying the need for sayings to give way to detections, we moderns insist that sayings re-present detections: so that others can begin with previous well formulated sayings and work back to the originary detections and forward to new ones. Excluded Middle now applies essentially to (specialized) sayings. This shift of focus reflects drastic shifts in our understanding of truth.

Paradoxically, our modern intellectual tradition now, after several centuries, implicitly supports truth as conformity to accepted doctrine. In parallel with the development of evangelism (e.g., Calvin’s), modern science starts with the incitement to individual determination but quickly transforms the results of individual assessment into a body of sayings and gainsayings to which subsequent individuals must conform. This kind of conformism has proved self-defeating in matters of religious faith. But it has proved self-confirming in matters of technological organization. Just as everyday sayings have a life of their own, so too, we have discovered, do specialized sayings: participants need only say and gainsay the right things, and the organization functions, perhaps even efficiently, safely, productively.

Plato images our helpless condition as that of mountain-folk having survived a massive flood and being left with the name or saying (δογμα) for each thing but with no understanding of the thing’s work or function (εργον). Having successfully shifted the question of truth back to (specialized, formalized) sayings, we moderns now experience our strength in the collective formation of functional sayings (bodies of knowledge) that require no individual to translate the sayings back into detections, but rather only a willingness to behave in accordance with the sayings—as one can bake a cake simply by following the recipe.²

§2. Modern axiomatics

Those for whom mathematics serves as the primary form of knowledge will rightly understand modern axiomatics as first evolving at the end of the nineteenth century with the concern to establish deductive systems. In the Appendices of Book Four we reviewed in a formal manner the beginnings of several such systems. But what renders these systems axiomatic is not the fact that they enable careful deductions from a small number of primitives, but rather the fact that they intend to occasion careful considerations of principles by which even those primitives take on significance: principles of cognition.

¹ Karl R. Popper rightly defends the primacy, in modern science, of “knowledge without a knower.” His Objective Knowledge: an Evolutionary Approach (Oxford, 1972) urges us to accept the autonomy, anonymity, and objectivity of our intellectual world, one consequence of which is currently playing itself out in our educational systems: we alternate catechize our students in established doctrines (bodies of knowledge, of sayings) and let them choose which particular doctrines to imbibe. In contrast, the pieces collected in Martin Heidegger’s The Question Concerning Technology (New York, 1977) urge us to contemplate the primacy of such knowledge.
Cognition — the taking cognizance of things — comes to special fruition in modern science. And modern science takes articulate shape in mathematical formulations — largely in reference to identities (of the form $\phi x = \psi x$, where $\phi$ and $\psi$ stand for mathematical functions). Modern axiomatic thinking must therefore include a consideration of mathematical work. As part of a the consideration of cognition. Not as the primary focus.

When we raise the question of pre-modern axiomatics we may readily agree that we must return to Aristotle and Plato, i.e. that the intellectual work of subsequent centuries remains within hearing of the principles of Non-Contradiction, Identity, and Excluded Middle as these originators formulated them. In contrast, we today do not yet have the entire story of modernity behind us. Only when we find ourselves in another story entirely will we look back in confidence on the springboards and consummations of what we presently call the modern era, that development historically located in the Renaissance, the Reformation and the Enlightenment, and leading to modern organization, modern science and popular education.

Meanwhile, I myself believe that the great contemplative works of the last few centuries will eventually reveal their greatness precisely in their embodiment of axiomatic considerations traceable to Kant's work. Not always to Kant's explicit doctrines, but rather to Kant's consistent detection of the drift of modern thinking generally: the drift toward transcendental analysis. He himself formulates this drift as a critique of reason, and his own critique as an inquiry into the conditions of the possibility of experience in general.

Since at least Socrates, all intellectual work has implied, even implored reflexivity. The motto has been: Re-consider! — and this time consider your own position within the business at hand. Since Plato, the re-consideration has highlighted our commitments to know the business at hand, to know it in addition to simply handling it. Such knowledge requires an intelligence of the in-itself-ness of the things with which we deal, an identity setting the standards of our dealings: our position is then essentially one of indebtedness to realities that transcend us — that transcend also the things we deal with, along with the determinations we make about them.

Kant recognized that the modern ambition to establish a kind of knowledge that one could have, i.e. that would allow us to retreat from the claims of things in themselves, invites a different kind of reflexivity. Once we understand knowledge to liberate us from indebtedness to the things known, we begin clearing the ground for constructing systems of our own. And questions arise about the relation between our own construction and the ground on which it occurs — or to which it refers.

The most famous of Kant's questions bears on the interface of our own knowledge and the transcendent entities traditionally at issue in contemplation: the relevance of talking about "things in themselves" and especially about the Soul, the Source, and the Cosmos. However, these macroscopic question Kant long holds at bay for the sake of a more humble question, one bearing on the interface of our own construction and the givens of the moment: What are the conditions for determining anything whatsoever to be the case? Anything in ordinary experience? A datum in carefully constructed experience (experiment)? This question requires that we back off from first-order determinations and then, instead of looking for higher-order entities and commitments, draw into contemplative view the factors already at work in those first-order determinations.

More humble, perhaps. But also very difficult. Sustained reflection of this transcendental sort has become a topic by itself in the twentieth century. Thinkers like Husserl and Heidegger have extended its mandate and renamed it in a manner already familiar to Kant: phenomenology.

In explicit contrast to the origin we call the Axiom of Non-Contradiction, Kant formulates the transcendental principle in his Critique of Pure Reason (A158, B197):

\[ \text{every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience.} \]

Kant's critical writings as a whole expand upon this short formulation — his critiques of pure theoretical and pure practical reason, and finally his critique of our judging powers. He understands this principle both as limiting pre-contemplative enterprises of knowledge and as providing the firmest foothold, the basic inspiration for the twist of thought necessary for contemplating the human enterprise as such.

\section{2.1 Synthetic vs. analytic knowledge}

Kant's formulation he himself calls "the highest principle of synthetic knowledge." The traditional Axiom of Non-Contradiction he calls "the
highest principle of analytic knowledge.” This now famous distinction between analytic and synthetic knowledge already introduces axiomatic considerations: How can we divide our cognition neatly in two, each part having its own “first principle”? What is analytic cognition? What is synthetic cognition?

However much we may try to read the distinction between analytic and synthetic knowledge back into Greek contemplation, it has no deep roots there. In his *Analytics*, Aristotle passes from apparently very formal considerations of syllogism into rather substantial considerations of grounds and origins. And in his *Metaphysics* he contemplates the origin we call Non-Contradiction precisely as bearing both “on what in mathematics are called axioms and on substance” (1005 a 20). Modern thinkers like Leibniz continued to contemplate principles in this manner, seeking a continuous thread through what we now call formal considerations on into substantial ones.

In the heat of new thoughts, we may wish to dismiss old ones as simply mistaken. However, in retrospect on both sides we can detect, not error, but rather a drastic change in modes of thinking. Of seeing, hearing, feeling, even smelling and tasting. Of understanding, evaluating, anticipating and remembering. This change we might roughly characterize as the introduction of concepts.

Plato and Aristotle understood our true position in both production and action — then, too, in contemplation — as one in which we respond directly to the things coming into view and within hearing. What emerges in such direct response turns out to be double (things as they happen to be and things as they need to be), and our own position appears as caught in the middle, whether creatively or impotently. Despite this doubleness, the semantics here remains dyadic: meaningful talk grows directly out of contact with things.

The Scholastic concern for concepts bequeathed to modernity the rudiments of triadic semantics: we respond not directly to what presents itself, but by way of concepts. In the monasteries of the twelfth and thirteenth centuries thinkers introduced a subtle shift from the earlier understanding of our participation in the duality of presentation and its completion: the completion (ἐντελεχεία) of the presentation becomes a function of our own thinking, something in which we participate independently of the strain of presentations. Like an εἴδος, a concept transcends what is immediately given. But unlike an εἴδος, a concept does not itself work to complete things like horses and communities. Presentations themselves — my house, my horse, my wife, my town — now only adumbrate, in the course of mundane life, the eternal order of things: Plato's philosophically intended stories (e.g., in the *Timaeus*) take on literal meaning, with modifications inspired from the religiously intended stories of the Old Testament.

As modernity matured, further shifts occurred: most obviously, temporal events appeared as mechanically constrained by the eternal plan of things, i.e. constrained into clock-like mechanisms. We ourselves might then become clock-makers, adjusting things according to our own understanding of perfection. And during the twentieth century we have had to learn the temporality of concepts themselves: not only that species (the natural correlate of concepts) evolve, but also that concepts emerge historically, mature tentatively, transmute into apparently self-evident routines, and finally become so out of tune with presentations that intellectuals must retire them to make room for new ones.

Despite the wide-ranging changes, conceptual thinking (triadic semantics) has so far remained in place and grown in power from late Scholasticism throughout the modern era. Most appropriately for modern science, we must understand intellectual work to take place in an established and shared conceptual network: in what Karl Popper calls a “third world,” a domain that determines what we can call objective, and that embraces and overrides both the (first) world of “physical objects or states” and the (second) world of “mental states or behavioral dispositions.”

Perhaps the most dramatic abstract effect of triadic semantics has been the admission, first promulgated by Kant, that no amount of conceptual re-working can ever rightly lead to an affirmation of existence. Thus syllogistic forms like Darapti and Felapton in the second figure, and Fesapo in the third — *any* such form concluding a particular from only universal premisses — must be deemed invalid. More dramatically, the ontological argument for the existence of God (as formulated by Descartes) will no longer hold: I may have a concept of God, i.e. a concept of perfection, but this concept cannot include (stipulate) existence

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* See Popper's *Objective Knowledge*. Thinkers such as P. Feyerabend, T. S. Kuhn, I. Lakatos, I. Hacking, M. Polanyi, C. F. von Weizsäcker emphasize that the conceptual thinking serving as the hallmark of science does not necessarily serve the understanding at issue in other kinds of involvements.
among its determinants. No concept does. For concepts have their life precisely in contrast to existence, to what happens to present itself.

"Sein ist offenbar kein reales Prädikat," Kant famously remarks: "Being is evidently not a predicate that crosses, takes us over into the domain of the real" (A598, B626). In its verbal forms ("is,"...), being is only a copula: an auxiliary that must introduce a legitimate predicate, one that "enlarges" the concept of the subject. This enlargement may take place in various ways: analytically and synthetically, as Kant has been arguing. But the question of existence can only be settled by crossing out of the conceptual domain—or by standing at its edge.

Analytic knowledge takes the shape of internal enlargement: we can unravel a concept, decide ever more elaborately what we shall mean by it. Our concept of a natural kind, e.g. "oak," may include "plant" and "tree." Our concept of an attribute, e.g. "autistic," may include various sub-concepts of behavior, some of which will likely be negative. Our concept of events such as "light" will involve relations and become very complex. However we may be inspired to enlarge our concepts, once formed they invite exposition of their components.

Gentzen's natural deduction and especially any axiomatic system illustrate the possibility of open-ended exposition. The primitive rules and propositions count as concepts only possibly referring to existence (a domain of instances, "arguments"), and each derived formula counts as a concept implicit in the original set.

Once we have developed a conceptual world having its own "life," i.e. not born of interaction with things encountered, we may speak of analytic knowledge. Anything goes in this world except outright contradiction. For once we detect a contradiction within our collection of concepts we find ourselves in a position analogous to the predicament Aristotelian outlines for those who deny the Axiom he himself formulates (either nothing is or everything is one): considered set-theoretically, a contradiction generates the Null Set (no reference, no membership, no instantiation); considered formally, a contradiction allows us to derive everything we please. Thus the Axiom of Non-Contradiction is the one principle that remains inviolable for any viable conceptual world.

But analytic knowledge is a strictly in-house affair. Both as human beings dealing with houses, horses, and communities, and as intellectuals making the understanding of things thematic and systematic, we wish to bring our conceptual apparatus into a productive bearing on what we encounter. Kant says: on what is given in experience. Indeed, we find ourselves bombarded with existence (Existenz), with what is simply there (Dasein), with effective reality (Wirklichkeit), with what is given (gegeben) in perception. And our question, already as human beings, but especially now as intellectuals lodging ourselves within conceptual networks, is whether and how our (shared) thinking bears on these bombards.

Plato and Aristotle understood the subject, the ᾨδοκεμενον of thinking to be the thing itself, already its existence (ἔχθοι) and ultimately the in-itself-ness revealed once we have penetrated the various phenomena of the thing. Kant, however, rightly notes that modernity requires us to place in the subject position the concept of the thing, a concept that we can clarify and expand prior to raising the question of whether anything exists corresponding to it: formally, whether the set is empty or not. Knowledge bearing on existence now consists of two quite independent factors, conception and perception. And any such conceptual venture into existence engenders another kind of cognition, what Kant calls synthetic knowledge.

There are easy cognitions already counting as synthetic: These stones are hot, My dog is barking, Some houses have flat roofs, Every athlete in the gym smells of rancid sweat, All apples at lunch today tasted sour. On the Fourth Interpretation, each of these cognitions involves a synthesis of concepts and percepts: concepts like stone and warm, dog and possession and barking, house and roof and flat, etc.; and perceptions of touching, hearing, seeing, tasting and smelling. Our daily experience we might possibly understand to consist of a series of such fleeting cognitions, including memories and anticipations of them. And of course in any given case we may err: the subsumption of the sensation under the concept may go wrong. But then we simply go back to the stones, to the dog, to the houses, etc., for another try.

The synthetic cognitions of interest to intellectuals are not so easy: These stones are hot because all day they absorbed the rays of the sun, transforming these into heat and retaining the heat long after the sun has gone down. This sort of cognition bears down heavily on what lies beyond, or at the edge of our conceptual network. But it also goes beyond immediate testimony. We may testify directly that the stone is now warm, much later cold; that the sun is now shining onto the stone and later drops below the horizon. But now we connect the two, stone
and sun, both in their proximity and over the period of the day: we talk of rays of the sun, the transformation of these into heat, the storage of this heat as resisting the ambient cold. Such connecting knowledge involves another level of synthesis, one overlaying simple recognition at the moment and on the spot. This synthesis spans the immediacies with a complex bridge of concepts: rays and heat and transformation and retention, some of which look like things in addition to stones and sun, others of which sound like events. And presumably such synthesis may be wrong. If it may be wrong, we should have a way of checking it out. In the one example, there may be a fireplace behind the stones, in which case we change the story: the flames in the fireplace generated heat that was transferred into the stones, through them to the other side. But in either story the knowledge at issue is universal: we have a theory about heat that covers variable factual circumstances, and we generally assume that the theory will allow us to make sense out of what is happening at the interface, at the edge where concepts and percepts meet. That is, we assume that we can know the invisible, inaudible, untouchable inards of things.

We could draw further examples of complex synthetic knowledge from Newtonian mechanics and contemporary nuclear physics, then too from the highly conceptualized versions of biology and chemistry, sociology and psychology. In the latter half of the twentieth century the incessant revision of conceptual networks (scientific revolutions, paradigm shifts) has come to puzzle both those engaged at the cutting edge of research and those who contemplate the engagement from up close. This latter-day development serves to highlight the difference between analytic and synthetic judgements. On the one hand, our purely formal systems appear to be essentially self-contained, purely imaginative albeit rigorously constructed systems constrained only by the Axiom of Non-Contradiction. On the other hand, our much-vaunted “research and development” at the interface appears to be committed essentially to devising ever-new ways (conceptual schemes) for grasping and altering events.

Now, Kant’s “highest principle of synthetic knowledge” also constrains intellectual work at the interface, otherwise so open-ended. Once again:

every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience.

Let us call this crucial principle the Axiom of Synthesis. It constrains our knowledge of things outside our own constructions by stipulating in advance that any candidate for our knowledge must meet some preliminary requirements. The formulation of these requirements introduces a style of contemplation that Kant calls “transcendental”: instead of moving out to ask about the real things we might or must know, we back up to ask about prior conditions of any such move: about our own condition as knowers at the interface reaching over the edge. Each word in this Axiom introduces a thought that contrasts with the thought of Non-Contradiction. Kant’s critical work in its entirety exfoliates these new thoughts. Our own task will be to trace out a logical line through the axiomatic part of that exfoliation. Meanwhile, some preliminary remarks:

The Axiom of Synthesis speaks of knowing objects. Since Kant, this generic word has become a permanent resident of our culture. It replaces such words as “substance,” “creature,” and even “thing.” It undoes the primacy of “estate” embedded in Aristotle’s οὐσία. When we speak of objects we can, with Kant, think of anything that puzzles us and that we would like to get a handle on: heat or lightening, protons or blackholes. Even concrete things such as horses and houses become objects whenever our relation to them becomes one of wrestling with them, pinning them down, altering them. For we are then considering them primarily as posing obstacles; they are no longer subjects we must cultivate and repair, “help into being.” We ourselves now assume the place of subjects, while all else becomes an object.

Then, too, the Axiom of Synthesis speaks of a knowledge. We may miss the fact that the knowledge here at issue is of the ordinary type, as exemplified in my first set of examples (stones being warm, athletes smelling...). The German is Erkenntnis, which does not translate the knowledge at issue in Aristotle’s Metaphysics (demonstrative knowledge: επιστήμη; closer to the German Wissenschaft). Yet Kant eventually exfoliates the Axiom to include the thought that the organized and disciplined knowledge at issue for intellectuals must have its roots in ordinary knowledge, and that will mean in our own prevailing (finite) condition.

And finally the whole phrase, “synthetic unity of the manifold of intuition in a possible experience”: What do these words mean? Unity?
Unity that is synthetic? Manifold of intuition? Possible experience? Let's start with the last, apparently the easiest.

At the interface we experience something: no longer confined to our own intellectual constructions, we open ourselves out, we take a trip out into or at least up to the Other. Anything known at this opening will require experience in some way. Not that we must have the object immediately there for inspection (touching, ...); we name all kinds of things that we are not currently present, and everything of intellectual interest at the interface escapes such presence entirely (laws of nature and kinds of things, the instincts of horses and the character of people). Yet we today generally believe, with Kant and against most Medieval Scholastics, that any putative knowledge must recall or anticipate experience of some sort (in the case of much natural science, reproducible or at least repeatable experience). An object of knowledge makes no sense unless it fits in at the interface, although not necessarily as a separable item.

At the interface we are engaged in a manifold of intuition: we find ourselves taken up into, facing a multiplicity that is also immediate. The German for “intuition,” Anschauung, does not have the ethereal ring of our English word; it names our immediate condition, as when I walkthrough my garden, feeling the uneven earth, hearing breeze in the trees, inspecting the leaves of the potato plants, smelling the flowers, perhaps tasting a raspberry. At the interface, I may concentrate my attention on one of these many sensations, but they are all pressing in upon me at once: they form a manifold. Experience starts with such a manifold—much as Aristotle already insisted.

And at the interface there must be a unity: my trip through the multiplicity of immediacy, with all its sensations, requires that this particular situation be gathered together (synthesized) into what we might today call an occasion, a situation. In the ordinary sense of knowledge, I currently know nothing in my garden unless I am currently drawing the garden together in a unity; I may in fact remain unaware of the garden, be more present with my students in class or my wife on a ski slope. The full force of the Axiom of Synthesis lies in the claim that this unity is a complex of perception and conception. And that it does not lie in the identity of the things known, e.g. in my deep knowledge of and care for cabbages and carrots. The unity is transcendental rather than transcendent.

Following the spirit of modernity, we may unpack the Axiom of Synthesis with a view to our condition at the interface of concepts and percepts. We recoil, as it were. Our task is to reveal the sub-axioms rendering the Axiom of Synthesis concretely worthy.

§2.2 Recurring to experience: space and time

The simplest act of noting something at the interface, e.g. the roughness of the soil under my feet, invokes immediately a temporality and a spatiality. The roughness (or coldness or slipperiness) appears now as distinct from before or after, and also, if registered, gets ordered in memory and anticipation. It also appears here as distinct from over there on the walk, back there in the house, and very likely gets registered in relation to these other places.

In Kant's succinct formulation: time and space are a priori forms of intuition. Aristotle called the time and the place of something “categories”: optional ways we can determine the subject of our discourse. While gathering data a modern research scientist must record the time and the place of each determination. Yet Kant prefers to call time and space “a priori forms.” Before we can even rise to the challenge of deciding when and where something at the interface is (whether absolutely or predicatively), the thing must appear in these forms: only because things appear at the interface in these forms can we categorize them. Temporality and spatiality already structure the opening onto what offers itself to us in experience. They also serve as reminders of the difference between purely conceptual cognition (e.g., the development of formal systems of deduction) and cognition at the interface: the first transcends time and space, while the second must keep recalling the time-space grid. Time and space name our grounding in any cognition that reaches out beyond the analytic.

Kant names the general human ability to stand at the interface our “faculty of receptivity.” Concretely, we can exercise and improve this faculty: effective productive, practical, and intellectual work even requires that we enhance it, develop our sensitivity to what is happening at the edge of our own formal operations. And we might also neglect and impoverish this faculty, both in ourselves and in others. In short, as human beings we have both the ability (the faculty) and the necessity (the pressure) to become and to remain alert—“as guards in the night.” For this is the beginning of all synthetic knowledge, both of daily cognition

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and of the special cognition of concern to intellectuals. We shall continue to contemplate this beginning. Given the modern ambition, everything depends on grounding our contemplation firmly in it: on beginning at this beginning rather than trying to eke synthetic knowledge out of purely conceptual developments.

§2.2.1 Appealing to experience

In contemplation we ask how we begin. The first “we” differs from the second. The contemplative “we” asks how the pre-contemplative “we” begins: in contemplation, we search out the beginnings of any human cognition. Despite the remarkable difference between contemplation and, say, production, we project an eventual isomorphism.

At first, however, the distinction is especially crucial. For when we intellectuals appeal to experience we might misleadingly recall a kind of experience that intellectual work has itself devised out of, even away from the experience within which we most primordially begin. The prototype of such devised experience we now call “experiment”—the kind of experience that Francis Bacon first outlined, illustrated, and defended in his New Organon (1620). Kant’s Critique of Pure Reason asks us to contemplate the experiential beginnings of such contrived experiment in order that we might appreciate both its power and its limitations.

Plato and Aristotle turn to one kind of experience in order to search out beginnings: the experience of craft, of τεχνη. However, now that the modern contrast is available to us, we must count the experience of craftsmanship as a privileged kind, one in which the beginning clearly transcends our factual, our temporal beginnings: human beings may end with a craft, but our question is how they can and do begin.

The birth-pangs of modernity inaugurated yet another way of appealing to experience, the way most associated nowadays with the British empiricists. Whereas the tradition of education and of contemplation had recommended that we begin, in time, by appealing to authority for an understanding of how to begin, Locke and his successors proposed that we search for beginnings by turning to the supposed beginnings of children—to our factual beginnings as we first started up in life. Experience here appears as most originally a matter of sensations: of touch, of sight, and the rest, even though we might decide to add to these outer sensations (traceable to bodily organs) those inner sensations that suggest an irreducible organ innate to the soul. But such beginnings as “rough” or “red” or “shrill” or “painful,” or even “thinking” or “hating” or “loving,” are abstractions: we intellectuals extract them from a fuller experience; they are the products of intellectual analysis and reconstrual. The path in the garden is rough and boring: this, a complex, we first receive, first intuit.

After the establishment of popular education—years of relative confinement—we especially note the difference between hearing or reading about things and actually experiencing them. In contemplation, then, we may have to appeal to “personal” experience: only one who has experienced India (poverty, love,...), has spent time in India (in poverty, in love,...) can know it. Such experience marks the real (vs. second-hand) understanding of things. The evident contrast between hearing or reading about something and then experiencing it first-hand can lead one to suppose that experience is a subjective affair, that each person has his or her own version of India (of poverty, of love,...), a private view of the beginnings. This describes, of course, the facts. Yet only those still trapped in classrooms and libraries can erect such facts into a wistful personalism. Direct experience of anything strongly engaging us may indeed lead to a profound personal transformation, but essential to this transformation is the recognition of inexorable necessitations of the things experienced.

Each of these appeals to experience—to (contrived) experiment, to (hands-on) craft, to (extracted) sensation, and to (personal) engulfment—presupposes the event under consideration. And highlights something important about it. The structure of experience, of our presence at the interface, permits the reconstruction we call experiment, it strikes us in the heightened version called τεχνη, it includes what we learn to call sensations, it becomes mine in personal crises. The contemplative task is now to step back to detect this structure—to analyze it transcendentally.

Kant’s Transcendental Analytic highlights experience only indirectly: directly, it concentrates us on the conditions of experience. And of experience not yet developed into that of experiment nor that of craft. It is that of simple cognition of the sorts described by the Stoics as eligible for what we now call sentential variables: this is a warm stone, something is squeaking upstairs, my summer cottage has a leaky roof
(definite, indefinite, intermediate). Having stepped back, we note that such cognitions (imagined as happening) already engage space and time, prior to predicking the subjects. Each definite proposition we may re-write with a preface: “HERE and NOW: this is P”; each indefinite we may re-write similarly: “HERE and NOW: there is P”; and each intermediate requires something of an apology: “THERE and THEN, a sometime presence, now only named: That named item is (was, will be) P.”

We might be able to imagine a Super Being able to “experience,” i.e. know directly all at once, no matter where or when. Tradition assigns this role to God. We, in contrast, are finite. Anything we can experience, i.e. know directly, will have to surface as a speck within the grid of space and time. However, bound to receptivity on a grid, we already transcend the sights and sounds, tastes and odors, the feel of what we receive. As puny as simple cognitions of things may seem to one expecting indications of higher-order realities, these first encounters at the interface are possible only because we order our perceptions within the grid, within the forms of intuition. The ordering begins in the simplest testimony that something is currently the case: here as distinct from to the left or to the right, up above or down below, out front or out back, and now as distinct from before and after; and then in recollection and anticipation elsewhere and elsewhen on the grid. Built into the barest reception at the interface, this order is a transcendental act of synthesizing. Both a posteriori and a priori.

Of course, the immediate content of direct cognition is a posteriori. That this here is warm and that it is a stone, that I own a cottage and that it has a leaky roof, these depend on simple cognition — somehow. For instance, academic knowledge seems committed to mature into universality, causality, necessity: every action entails an equal and opposite reaction, all animals have an instinct for survival, every art work expresses its author, its age, or whatever. Such judgements go beyond the immediacy of the interface. Perhaps because many such judgements also transcend the bounds of legitimacy, careful academics often argue that such generalities merely reflect convenient summations of one's own data-base, and have no special status, none justifying any claim to insight.

Yet already the simplest encounter requires synthetic a priori cognition: for us, there is no other way of stepping up to the interface out of formal work — as well as out of our day and night dreams. It is here, in our finitude at the interface, that Kant asks us to begin, to take stock of human beginning. And this beginning, this αρχη, as universal: as an axiom (analogous to Aristotle's) at work in any cognition, but one reflecting our own position, not the position of what appears to us. We already have our foot in the door to some sort of higher reality, albeit a transcendental rather than a transcendent one.

§2.2.2 Space as an origin

An “a priori condition of possible experience,” Kant says. And he argues that this status of space as a “form of intuition” permits the development of geometry as a science (“mathematics of extension,” A163, B204), a science whose determinations are both a priori and synthetic, therefore enjoying the modality of necessity both in themselves and in application a posteriori. Thus our confinement to space — the finitude of our knowledge—Kant converts into a very special beginning: the possibility of transcendence.

Our present question, contemplative as well as logical, reads: How does space as an origin figure as a sub-worthy within the Axiom of Synthesis? How does the Axiom draw upon this origin? How does the origin illustrate the Axiom?

I approach a house (B162). This house I have never seen before. In Stoic fashion, I think “this here is a house.” Even if I am wrong, in
order to be wrong (or right), I identify what I perceive. Apart from the question of identity (its ground: the Platonic-Aristotelian question), I can now note, in contemplation, that in order to perceive anything at all, I must receive. And what I receive from “outside” can only be “there” in relation to, and as distinct from, what is next to it, behind it, above it, and so on. Moreover, the thing directly perceived has its own internal dimensions, including its own sides, front and back, and also its own insides. Of all these sides only the front side is ever sensorially perceived. All the other sides, necessarily there even if I am sorely mistaken about how they are there, will require me to go there, to go around the house, to go into it. As I explore the house, I myself will change places, move around within a grid of a priori possibilities. As I move around, I receive (intuit) more and more: the nodes on the grid take on flesh to which I can testify more or less correctly. I both anticipate and recollect such testimony, and a priori.

All previous philosophers, and now Kant, analyze the reception into modes of sensation: touching, seeing, hearing, tasting, smelling. However, Kant now asks us to consider how the grid itself establishes the nodes at which we locate what we receive by way of the senses, whether or not correctly located, anticipated, or recollected. The grid of three-dimensional space first allows us to receive: to intuit (encounter), to perceive (focus on something definite), even to sense (taste, ...). Abstractly stated: space frames, therefore precedes, anything experience-able. Concretely formulated: the whole experienced situation takes place first, then we explore it, familiarize ourselves with its details, perhaps correct our first impressions.

Now recall the Axiom of Synthesis:

> every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience.

Space now assumes the role of one such “necessary condition”: “I perceive something other than myself” implies “I engage in space.” Similarly, as when I am engaging in purely formal work (logic or mathematics): “I am not engaging in space” implies “I am not perceiving anything other than myself.” More generally, space conditions the “synthetic unity of the manifold of intuition.” That is, the approach to the house opens out on a multiplicity, analyzable into sights and sounds, tastes and smells, and even touches (the wind on the face, the gravel underfoot, ...). Yet there is a unity to this situation, that of space itself. And this unity is assembled, is being assembled, well or badly: a painter or a prospective buyer may experience this assembly, the formal interrelations as well as the sensory details, more intently than a casual or dreamy or pre-occupied visitor. Finally, we do not live only in the present, with Stoic definites and indefinites. We represent things both in anticipation and in recollection. Most of our discourse about things refers not to current experience but to possible experience. Here, too, space is a necessary condition. When offering or considering accounts both of what is already known and of what might be known (retrospections and prospections), anything receiving predication must be projected onto the grid of space, at least roughly. Even fictional accounts: indeed, both good novelists and good liars generally take extra pains to arrange their details on a coherent grid.

Anything we actually experience appears in space, and anything we recall or anticipate experiencing we represent as appearing in space. Transcendental analysis reveals that experience is already double: the something experienced and the form of the experience. In a sense, we experience the form as well as the sensory content, even though in our routine dealings we may not become aware of this doubleness. Yet a number of familiar professions require one to develop the awareness systematically.

An architect designing a building draws it: draws its spaces. Such drawings very exactly interrelate the various spaces of the building before the building even exists. Similarly, an engineer re-draws the drawing with a view to the weights, strengths, and tolerances of the proposed building materials, the stresses and strains of gravity on their locations (the pull of the earth), and the resulting reliability of the construction (accounting, perhaps, for shifts in the earth). And any interior decorator assesses spaces not only in their proportions, but also in their interrelations: will determine, by roundabout measurements and geometrical determinations, what spaces lie on the other side of a wall—without having to drill a hole to find out.

In all such vocations, we experience space transcendentally. And we naturally assume that if the determinations we make do not hold of the spaces of a given house (or bridge, or...) we have measured them wrongly.
The intellectual exercise of geometrical demonstration allows us to experience the grid of space in its purity. Here we roam freely through our \textit{a priori} form of intuition, and determine necessary truths about spatial figures. If, then, these determinations also hold of all “possible experience,” geometry will be \textit{revelatory} rather than only an intellectual exercise. Thus Kant must argue at length that the original determinations (available in any textbook on plane or solid geometry) already enjoy synthetic status prior to applications \textit{a posteriori}: only because they enjoy this special status can we know in advance how things will appear to us on site.

Since Descartes, geometric determination has stood at the gateway of all developed knowledge of how things are “out there.” Aristotle, remember, had claimed that mathematical exactitude (\textit{ακριβολογια εκθηµατικη}) does not rightly arise in the contemplation of nature (\textit{φυσις}), and that geometry especially isolates us from full presence (\textit{ουσια}). Kant now can claim that “in every doctrine of nature only so much science \textit{proper} can be found as there is mathematics in it.” The difference signals not a disagreement on the role of mathematics in knowledge, but a total divergence in the direction of knowledge, the basis of its development.

While knowledge of the Platonic-Aristotelian sort grows out of what we encounter, taking its cues from the learning of such crafts as cabinet making and horse training, modern science grows out of the thematizing of space and time. The price we have learned to pay, and willingly, is that modern science deals with objects rather than subjects: with what appears as we bring our formalized efforts up to the interface. In Kant’s celebrated phrasing, knowledge of things outside our own formalizations, indeed these formalizations themselves, bear only on appearances and not on things “in themselves.”

In our armchairs, no longer either approaching a house or reconstruing it (as architects and the others do), we may in fact think about things apart from our facing them. There is nothing wrong with thinking them this way, as Kant will insist. We slip into error only as we begin insisting that we can settle questions while “professing to isolate the objects of the senses from the formal condition of our sensibility” (A166, B207). When a question bears on what we can receive, we must remember to account for our own condition as receivers: our finitude.

Kant has in mind the developed knowledge familiar to any educated person under the name of Newtonian science. In confirmation of the “ideality” of both space and time, Kant reminds us that everything in our cognition which belongs to intuition [feeling and volition are not cognitions] contains nothing but mere relations; namely of locations in an intuition (extension), of change of location (motion), and of laws according to which this change is determined (moving forces). What it is that is present inside a location \textit{in dem Orte}, or what it is that operates in the things themselves \textit{(in den Dingen selbst)} apart from change of location, is not thereby given. Now a thing in itself \textit{eine Sache an sich} cannot be known through mere relations; and we may therefore conclude that since outer sense [space] gives us nothing but mere relations, this sense can contain in its representation only the relation of an object to a subject \cite{Note1} and not the inner properties of the object itself. This also holds true of inner sense [time] ... because the time in which we set these representations ... contains [only] relations of succession, coexistence, and ... persistence. (B66-7)

Kant here invokes a principle central to the Platonic-Aristotelian tradition of logic: the category of relation bars us from understanding the in-itself-ness of things to which the category is exclusively applied. And he is assuming that knowledge of the Newtonian variety can serve well as the paradigm of all developed knowledge.

Space is clearly a thematic beginning, an \textit{αρχη}, of Newtonian knowledge, and of modern science generally. We can also read space back into our ordinary (pre-mathematical) knowledge. But how can we affirm that space is also a beginning, an axiom for our logical calculi?

The formal beginning of our sentential calculus is the propositional variable: \textit{p, q, r,...}. Although we formalize our system apart from any consideration of what factually serves as instances of these variables, we assume that the instances are sentences allowing for the extra-logical decision whether each is true or false (one or the other, and not both: Excluded Middle and Non-Contradiction). As formal logicians, we may withdraw from further considerations of this decision. But we stipulate,

in our farewell address, that those on site must go out to testify: go out to where they can testify face to face, or at least recall or anticipate where such testimony became or will become possible. And such testimony, actual or possible, takes one or the other of the Stoic propositional forms (direct, indirect, or intermediate), each of which requires at least imaginative recurrence to some point on the grid of space.

Withdrawn into our formal systems, we may believe that the requisite recurrence is unproblematic. On site, however, we discover that such recurrence requires an alertness, a patience, a suspension, a strain not at all easy to sustain. And that once we succeed in this supposedly simple task we can genuinely begin, get something started (an investigation, a creation). Withdrawn transcendentally, we may acknowledge both the necessity and the power of space as a beginning.

Quantifier logic formally begins with an analysis of the sentential beginning: the double variables $\varphi x$, $\psi y$, $\xi z$, ... . Once instantiated (doubly), we naturally assume that these, too, are either true or false. In Stoic fashion, we instantiate the object-variable with a deictic, or possibly deictic expression: this-here, that-there, a present but unlocated thing, or a named but absent thing. Yet in our scientific investigations we assume that the variables take nodes as instances, and all else, including proper names (Socrates, Eiffel Tower, Mount Allison University) become predicates: we predicate (rightly or wrongly) a named node with “Socrates,” “Athenian decision,” “death,” and whatever (likely with polyadic predicates). And the node is named by formal relations on the grid (and in fact changes its name as we improve our predications).

Again, as formal logicians we may withdraw from such decisions; for the most part, we may even forget that we have axiomatized space as one of their conditions. Yet the more we press our quantificational calculus over into the realm of relations and into meta-theoretical questions, the more we find ourselves nearly forced to acknowledge, transcendentally, our commitment to purify our object-variables of all predications — to understand them as points in space. And/or time.

§2.2.3 Time as an origin

Again, an “a priori condition of the possibility of experience,” Kant says. And he argues that the status of time as a “form of intuition” permits the development of arithmetic, the science of counting — a science establishing a priori necessities that are also synthetic. Time and space are twins: the one is the form of inner, the other of outer sense. Yet time also figures as a beginning of spatial organization. The twins do not share their patrimony equally: unlike space, time mediates every synthetic judgement (A155, B194).

On Kant’s transcendental account of both ordinary experience and developed knowledge, our own innermost condition of responding to what we receive is essentially temporal. Indeed, creatively so — in contrast to the earlier Interpretations, where time appears not only as a limitation, but also, by itself, a cause of destruction.* Time will also account for the primacy of the Axiom of Synthesis over the Axiom of Non-Contradiction.

Consider again the approach to a house — any fresh experience. Not only do we gradually fill out the detail of the house — put empirical flesh on its a priori spatiality. Not only do we recollect and anticipate the detail on the basis of spatial interrelations already articulated a priori and therefore formalizable scientifically. Alert to the situation, we at every single present moment assemble ... a multiplicity of moments. The very moment we see... the front, or hear... a door slam, or smell... the wood fire, we look or hear or smell again. The first moment only in its own “again” — is as its own spill-over into the next. This microscopic spilling accounts for the finitude of experience: we cannot experience the house all at once — not directly, not in sensation, not as an actual appearance. We might wish to understand this inability solely as a limitation. Yet this very limitation will on occasion reveal our own essential creativity: we must in any event construct our experience of the house.

As Kant emphasizes in his Critique of Judgement, the simplest experience is creative. The barest emergence of a house depends on its taking time — on our time, i.e. on our alertness, our willingness to engage constructively in the spill-over. For we do have the option to recoil at the first moment into old ones: rather than looking, listening, sniffing openly, we may resort to a “yet again” traceable to previous moments. That is, we may forgo the occasion of fresh experience, and fall back into

* See Aristotle’s Physics IV, 12: “It is necessary that things occurring in time ... are affected by time — as when we speak of time leading to the downfall of things, and everything growing old under the influence of time, but not our having learnt, nor anything becoming new or beautiful by time. For time by itself is rather the cause of destruction.”
routine recognition. The price we thereby pay is that we miss the house that actually appears.

Our engagement in time — our temporal condition — we normally ignore, except insofar as we order our days into schedules and calendars. Thus we easily miss both the position into which our temporality casts us and the peculiarity of the spill-over that reveals time as an origin — as crucial to beginning anything, to getting things started. However, some kinds of experiences serve especially well to recall us to our condition.

When listening intently to music we verge at each moment into the next. Yet, unlike the distracting vergings typical of routine experience, the verge in musical experience configures — incites us to configure — the previous moments, their synthesized movement, into the spill-over. The whole piece, the gathered movement, opens out onto the next moment. Similarly when reading a book or listening to a lecture: if the experience has any coherence at all, it engages us in a temporal construction not unlike that of musical experience. Paintings and sculptures similarly: they become present only as we move through their spatiality — temporally. In general, art works recall us into our temporality, create four-dimensional moments out of a multiplicity, invite us to experience moments freshly and intently. And the performance of any art, be it that of violin playing or that of cabinet building, that of chess playing or that of horse training, requires that we learn to start up and keep together a successive multiplicity in continuous movement, where each new moment configures a whole complex of moments into a convergence on this one appearance. Thus the full experience of anything takes on both spatial thickness and temporal duration.

The most basic microscopic event is the spill-over itself, out of which experience takes various forms, from distraction to concentration, from mere recognition to intense duration. The basic event is a movement apparently embracing but actually generating two moments: this-now and this-again. The retention of the duality thereby generated, the focus of this multiplicity onto the next-coming, generates a grid of distinct and abstractly separable points in time. The grid of time differs from the grid of space in a most crucial and appropriate way: the grid of space we can rightly imagine as present all at once in regard to a given appearance (e.g., of a house): we can point to the nodes in arbitrary order. The grid of time eludes us precisely when we stand back to look: the moment we cease to enact it, we lose it. As the grid of inner sense, time can only appear to us as we transform it by analogy into one-dimensional space — into a line. This we do when we draw up a list of things to do during the day, or when we write up a list of things already done. And also when we inscribe our lists (e.g., appointments) on public calendars, thereby acknowledging our communal involvement in time.

Especially since the development of forensic and scientific modes of investigation, we moderns likely admit that any legitimate determination (a Stoic definite, indefinite, or intermediate proposition) appears with a temporal tag. Most obviously, any datum recorded during an investigation must be marked temporally: the siren sounded at such-and-such time, the accused was sighted at such-and-such time, death occurred sometime within such-and-such period of time. The dialect of the inscription varies according the chronometer locally employed; the measure in a research lab may differ considerably from the measure in a barroom, and time-zones as well as calendars vary according to when and where we are. Yet we rightly assume that there are rules allowing us to establish a suitable correspondence and so to agree on a single grid of time. Only in very advanced work in modern physics can we abandon the presumption of such correspondence rules; in such work we also abandon the presumption that we can stand aloof from our own engagement, from the performative conditions of inner sense.

Now, Kant has argued that the performance of geometrical demonstrations manifests our a priori engagement in space, and that the congruence of these demonstrations with experience illustrates the synthetic status of the knowledge thereby obtained. So, too, he can argue that the performance of arithmetical calculations manifests our a priori engagement in time, and again that the congruence of these calculations with experience illustrates the synthetic status of the knowledge thereby engendered.

All familiar arithmetic operations — addition and substraction, multiplication and division — refer us back to operations of counting. Plato and Aristotle interpret counting (αριθμόν) as requiring and illustrating a transcendent movement on our part. To determine the number of horses in a field, I must distinguish horses from donkeys, and must discern the sameness shared by each horse despite its differences from others. The advanced work of determining the rank, the quality of each horse, and then of ordering them by rank, depends on my ability to fathom the essential sameness, distinguishing this from incidental
similarities (accidents): numbering-as-ranking requires advanced insight into the good of, and what is good for horses, so that I can also distinguish specimens naturally good for different human purposes (for plowing, for racing, for riding, for glue).

Yet a transcendental movement precedes any successful transcendent one. Counting the horses, I synthesize on the basis of a priori synthesis. Concretely, albeit partially stated: I must remember what I am doing, keep what I am doing together in a unity. Kant’s fuller formulation:

Were I to forget, while counting, that the units that now hover before the senses are gradually put together by me, I would not cognize the production of the group by this successive putting together of one to one, and so would not cognize number at all. For the concept [of number] consists simply in the consciousness of this unity of synthesis. (A103)

... number ... is a representation that gathers together the successive addition of one to one (of the same kind). Thus number is nothing other than the unity of the synthesis of the manifold of same-kind intuition in general—in such a way that, in the apprehension of intuition, I produce time itself. (A142-3, B182)

Several phrases here deserve careful attention. Counting, we produce the group of items counted (not the items themselves). The production unfolds one by one, and therefore requires an overall retention, a consciousness of a whole antecedent to anything given, and therefore not itself given. Moreover, counting requires a commitment to the recognition of things of the same kind (gleich-artig: “homo-geneous”), a sameness that need only be posited in some sensation (rightly or wrongly: we can always improve our discernment). The ability to count need only relate to same-kind intuition in general. And in some sense I thereby produce time itself: each response generates a determination of a time, and my own alertness to the multiplicity generates an occasion having an overall temporality.

Crucial to Kant’s transcendental argument is the creativity of human response within a complex domain of appearances (horses as appearing, not as inviting a penetration to their in-itself-ness). The unity here is not directly the unity of each item that appears, but the unity of our own field of receptivity. The forms of this receptivity, both space and time, remain wedded to intuition, but not to anything intuited. Time names our own responsiveness to what is received. And as we respond, so we create the individual determinations (concepts) of number and of time: each named number and each named moment. For “create” Kant says “produce”: erzeugen. Or “bring to stand, bring into being”: zustande bringen. In his Prolegomena to Any Future Metaphysics, we read: “By successive accumulation of units in time, arithmetic itself brings into being its concepts of number” (§10).

Other than our inevitable engagement in counting, Kant refers to the development of Newton’s “pure mechanics” as historical proof of time as a beginning of a priori synthetic knowledge. Newtonian mechanics introduces movement into numerical calculation, and we must understand movement in space temporally: locations determined at $t_1$ and then again at $t_2$. In his first Critique Kant draws exclusively on this then-new way of understanding natural things for examples of developed knowledge. In his third Critique he draws upon the judgements evident in our experience of the beautiful and the sublime, passing from the structure of these to the consideration of the holistic judgements necessary for biological investigations.

Most abstractly, a temporal grid recalls succession, the relations of before and after. Yet succession all by itself destroys all focus, prevents us from even noting the successive moments. Essential already to abstract comprehension of the nodes is simultaneity, the relation of identity and difference at each node. To these two relations we must add recurrence of the same at different nodes: persistence from one to the next. Both the counting of items and the calculation of movements depend on all three: on a thickening of time. This thickening we take for granted in mathematical science.

In ordinary experience, the thickening of time often becomes a dramatic issue: a haunting question and a hovering necessity. Here, too, we may simply take the thickening as it comes and goes, without pausing to articulate it. Art works of an especially modern cast often bring the drama of thickening to the fore. A salient example is Marcel Proust’s A la recherche du temps perdu (1913-27): this many-volume work, published in large part posthumously, recounts the thickening, embodies the congealing of succession and simultaneity, as a dramatic event revealing the persistence of place. Unlike Newtonian physics, which articulates pure intuition to reveal temporal and spatial form “without
ouσια,” Proustian narrative recalls human engagement in time and space to reveal the placedness, the supportive abode of experience.*

Logical work resembles Newtonian science more than it does Proustian narrative. Most obviously, we construct proofs in time: successively, step by step, at each of which we bring a rule together with one or two lines, simultaneously, so that they generate the next. And throughout the span of the proof we establish a constancy amid change: each symbol retains its special formal meaning, and each variable remains the same despite the difference in location. The construction of counter-examples to prove forms invalid illustrates a willingness to recognize cracks in the form: an acknowledgement that the form does not hold over all time.

Working with quantifiers, we must acknowledge the difference between “given any ...” and “there is at least one ...”: between finite openness to any, one at a time (over time, but confined to a domain) and finite presumption of existence (at some time). This acknowledgement must carry over into formulations in which the variables appear without their governors.

Working with relations, we must decide the transitivity, symmetricity, and reflexivity of relations such as larger than, prior to, farther than, next to, simultaneous with, at the same place as, and so on. Insofar as we can determine a priori the formal properties of a relation, we resort to pure intuition of space and time: to numerical designations on a space-time grid. For only the numbers prove the formal necessity of these properties. The lack of a numerical scale in relations such as “x is a better horse than y” may reveal either the foolishness of reducing these matters to relations, or the need for a Measure that transcends the individuals to which they apply. Any relation whose properties we can analyze logically are quantitative. And quantity here requires that we locate items on a scale that engages us transcendentally in time.

Finally, mathematical induction directly formalizes our engagement in time. The procedure of induction allows us to prove theorems about the whole of a system. How does it work? First, we must construct a series, a set of points temporally ordered. Then we determine what in fact happens at some beginning point, according to some formula. Thirdly, we calculate what must happen at any point on the unproven assumption that the formula has worked up to that point. And we must then grasp continuous movement through the series, as an always finite but ever-growing whole. In short, mathematical induction calls upon our intuition of the natural-number series as co-extensive with the construction of a temporal series of points.

Logical proof requires not only time, but alertness, background training, and a disposition to think abstractly. It requires that we recoil from the instances of our variables: that we think the thinking that goes into the thinking of instances. Plato and Aristotle claimed that such thinking removes us from time, allows us not only to transcend but also to abandon instances. Kant insists that it illustrates our ability to engage in time purely, to transcend instances only provisionally, in preparation to meet them in a temporality that is as much theirs as ours.

§2.2.4 The difference

Kant relegates the Axiom of Non-Contradiction to the realm of analytic knowledge. However, his relegation depends on a crucial rewrite of the Axiom. Two words must be dropped from the traditional formulation reading: “It is impossible that something be simultaneously both A and non-A.” First, the modal word “impossible”: it appears redundant. And second, the temporal qualification “simultaneously” (zugleich, “at the same time”): it seems to introduce a consideration that both limits the Axiom and betrays a misunderstanding of analysis. Kant only explains his excision of temporality from the Axiom:

The misunderstanding arises because one initially separates the predicate of a thing from the concept of that thing and afterwards attaches this predicate to its contrary; this [procedure] never occasions a contradiction with the subject, but only with a predicate synthetically connected [with the subject]—and now only when the first and the second predicates are asserted at the same time. If I say that a person who is unlearned is not learned, the condition simultaneously must be there; for a person who is at one time unlearned may very well be learned at another time. But if I say that no unlearned person is learned, the assertion is analytic, since the property (unlearnedness) now goes to make up the concept of the subject, thus the negative

assertion immediately emerges out of the Principle of Contradiction without any need for [stating] the condition simultaneously. (A153, B192)

If “John is learned (Lj) and John is unlearned (~Lj)” does contain a contradiction, we must add “at the same time.” But “no unlearned person is learned” is true because the predication (not Lx) follows immediately from the concept of the subject (~Lx). The supplementary condition of simultaneity arises only when we erroneously separate the predicate from the concept (L... from Lx) and consider it in conjunction with its contradictory (L... & ~L...). This ghostly conjunction then floats, as in a dream, over time, and we must acknowledge that at one node ~L may apply and at another node L. In short, a concept ϕ vanishes into a mere word unless it bears on a possible appearance: ϕx. Thus the Axiom reads simply: ∀x(~(ϕx & ~ϕx)), or ~(p & ~p).

Kant’s objection to the traditional formulation of the Axiom, and his relegation of it to a mainly house-cleaning role, depends entirely on an astute interpretation of concepts as of something, an x, so that it becomes only fictitiously possible to separate out the predicate as not bearing on any thing, as free-floating. Separating the predicate of a thing from the concept of a thing is a verbal trick. The two, the ϕ and the x, form the basic unit. The variable x supplies all the sameness that is necessary.

But how? For possible a posteriori values, the variable x takes appearances in space and time. Each appearance is formed already at a node on the space-time grid. Each node of intuition instantiated throughout any complex concept (Lx: x is learned), and throughout any judgement devised with such a concept, is identical with itself. The requisite sameness is that of each node in the grid conditioning every cognition. Each node at which the multiplicity of concepts converges already bears an a priori simultaneity. The stipulation of empirical simultaneity as part of the Axiom corrupts the purity of the sameness intuited a priori.

Thus Kant’s demotion of the Axiom of Non-Contradiction anticipates our understanding of time as a beginning, an αρχη: as our way of beginning with things at nodes. This understanding contrasts with the earlier understanding of time as an affecting, a πασχειν: as inviting an acknowledgement of the decline and cessation of things (Physics, 221 a 31).

The demotion also anticipates our understanding of logical modality as prior to ontological ability. Kant says that one way of formulating the Axiom starts with “It is impossible that . . .” and that this opening phrase adds nothing to the Axiom. At best, it is superfluous. At worst, the reference to the modality of the principle might shift attention back to the question of possibility vs. impossibility as a property of the inner formulation.

But how does this reference to modality ever enter into the formulation of the Axiom? Kant says: out of carelessness. Yet this concern for what is “impossible” does recall Aristotle’s original wording of his “origin of all other axioms”:

The inability of the same at once to belong and not to belong to the same in regard to the same.

The inability, the αδυνατον, bears on a series of samenesses. However we interpret this bearing, the formulation clearly intends to focus us on how beings are—ultimately in their ουσια. Because we are committed to the emergence of beings as they are, we ourselves are unable to get hold of any one-and-same thing as both being and not being. Unable to get hold of sameness: αδυνατον υπολαβανειν. Our own inability to get a hold of things except by way of their samenesses provides the basis for arguing against those “who say both that it is possible (ενδεχεσθαι) for things to be and not to be, and that it is possible to get hold of things accordingly” (1005 b 35). This one inability buried among all the abilities in the things we encounter and must come to understand—this inability accounts for the impossibility of saying and of grasping things in this one way among all others. Aristotle argues elenctically (indirectly): if things had the ability we would have the possibility.

On the Platonic-Aristotelian Interpretation, our own grasping of things remains beholden to the nature of things themselves, their beginnings. Only later, after all the labor of late Scholasticism and early Modernity, can we understand ourselves as beginning with our own grasplings: with concepts. And then all talk of power must reflect back transcendentally onto the power we ascribe to our own formulations: their modality.

We may concretize the difference between Ancient and Modern axiomatics in reference to the traditional division of human involvements into contemplation, action, and production.
On the Ancient account, our ability to begin contemplating human or divine nature depends on our attunement to the exigencies of demonstration: the developed knowledge (ἐπιστήμη) at issue in contemplation is directed to eternal things, to the actuality of things (their ἐἶδος, their ἐνεργεία). This actuality we contemplate as prior to, as giving direction to the potentiality of things (their power, δύναμις). In contrast, we deliberate (take counsel with one another) only on matters that can turn out one way or another: on formulations we acknowledge to make only possible contact with matters that elude us while still hovering over us. In our own domain, i.e. in the contemplation of eternal things, “to be possible (ἐνδεχεσθαι) and to be (ἐίναι) differ not at all” (Physics, 203 b 30).

On the Modern account, our ability to begin contemplating events (“nature,” but now in a new sense) depends on our attunement to the exigencies of objectivity: the developed knowledge at issue in our theoretical work (what we now call science) is directed to things as they change (and then to ways whereby we can change them). Whatever formulations we devise, they intend to focus us on fresh instances, freshly appearing x’s. Already the nodes at which these x’s occur are determined four-dimensionally. Thus each node, i.e. each x, differs from every other. To be sure, we may extract a kind of demonstration from the processes of scientific knowledge, but this kind is only mathematical, and essentially “without οὐσία,” an animation suspended over the next moment of application.

On the Platonic-Aristotelian account, our ability to begin acting, as leaders in communal enterprises, depends on our attunement to the desired form of the city, the πόλις. Plato’s Republic and also his Laws expound this dependence in exacting detail: to act is to develop prevailing conditions with a view to a paradigm (παραδείγμα) intellectually hovering over us.

Machiavelli’s Prince and Discourses embody the modern alternative most abruptly: we can begin acting only once we attune ourselves to what is in fact happening, to the prevailing conditions, states of affairs. For only then can we hone our insights into techniques of preserving the desired state by researching what has already happened in analogous states as a result of employing the various possible techniques. Leadership becomes statecraft, the art of the possible (Bismarck) unrelated to any science of the necessary. A markedly temporal affair, one in which we must adjust each response to the node of the moment—and devise wholly different temporal schemes depending on where we are on the globe. Reversion to the ancient model leads to our own ruin:

It remains to consider what should be the manners and rulings of a leader with his subjects and his allies. And because I know that many have written about this, I suspect that I shall be considered presumptuous as I write on it myself, departing as I do from the criteria (ordini) of others. But being intent on writing something useful for whoever tends to it, it has appeared to me more appropriate to go directly to the effective truth (verità effettuale) of the matter rather than to imaginative constructions of it—whereas many have imagined republics and principalities that have never been seen nor ever been known to exist in fact. For how one should live is so far removed from how one in fact lives that anyone who abandons what happens for what should happen engenders his own ruin rather than his preservation. (Prince, XV)

Finally, on the Ancient account we can begin to produce something—to raise sheep, to build a house—only when we have finally achieved a vision of the being for whose becoming we take care. Until then, i.e. as children or apprentices or fools, we wander helplessly among the shadows, waiting for some spark to ignite a vision of the model, the Measure against which we can really measure the condition of the sheep or the house tentatively under our care.

For an example of the Modern account we may recall Henri Bergson’s contrast of the child assembling a store-bought picture puzzle and the artist working on a painting (Creative Evolution, IV): the first has the model pictured on the box, the latter discovers the model in the temporal process of the work. Whether empirical, as in the image of the picture puzzle, or transcendent, as in ancient science, a pre-established paradigm renders time only a barrier to finishing the task, and the threat of defeat. Engaging in circumstances without any pre-established standard (certainly our own condition as creators—and Bergson understands it to be also the condition of nature herself), we must learn that time is essential to our work: not just inasmuch as our work takes time, but even more inasmuch as we gather together the process at one node in order to prepare ourselves for a spill-over into the next—so that we can simultaneously discover and let emerge the actual configuration
at that next node, and not before. We must begin with time, with this spill-over, rather than with a fixed vision of what lies ahead. Time is the nothingness of matter: Néant de matière, le temps se crée lui-même comme forme.6

Viewed from the inside, i.e. phenomenologically, each of the three kinds of active involvement only really take hold, really begin, in a transcendentally understood possibility — rather than in a potentiality embedded within the thing we deal with, its “in-itself-ness.” On the transcendental account, then, the Axiom of Non-Contradiction formalizes our cognitive condition behind-the-lines: the house-cleaning necessary for coherent engagement in the mode of possibility. Including a reference to its own modality in the formulation of the Axiom would only obscure the possibility that looms so ominously at every moment on the front line.

§2.3 Transcendental categories

Receptivity to what happens (in intuition, sensory or pure) marks a beginning. But this beginning is wedded to another, to spontaneity: what we receive abides only as we hold on to it — however briefly, however tenuously. In short we respond, we rise to grasp things. Our responses are immediately conceptual:

Our cognition springs from two basic sources of the mind: the first of these is to receive representations (the receptivity of impressions), the second the ability to cognize an object by way of these representations (the spontaneity of concepts). By the first, an object is given to us. By the second this object is thought in relation to that representation (as a mere representation of the mind). Thus intuition and concepts comprise the elements of our cognition, so that neither concepts without any intuition corresponding to them, nor intuition without concepts, can yield a cognition. (A50, B74)

Two sources, but wedded to one another in any genuine knowledge, ordinary or developed. Contemplating space and time as origins, we distinguish these two within one partner, the space-time grid on which we receive happenings. Yet in any example illustrating this one partner we must already acknowledge the other, the spontaneity of concepts: we spot a house or count the horses with “house” and “horse” as empirical concepts, and then organize such things with a priori concepts like succession, simultaneity, and duration, transitivity, symmetry, and reflexivity.

Any moment of cognition, whether ordinary or developed, is already a marriage of two origins, each enjoying a power of its own. As we now turn to contemplate the other partner we might well suspect that we are again sundering an origination we will eventually have to acknowledge in its primordial unity.

§2.3.1 Forms of response

Experiencing the simplest thing, I engage in decisions. Something appears on the space-time grid and I rise to identify it: e.g. “That there is a house.” I may be wrong, I may admit I'm not sure, but even to be wrong or uncertain I engage in four ways of reaching out, each way marking the occasion of a decision: one house, one of a recognizable kind; a house rather than something else, e.g. a silo; my decision is categorical when it might have been conditional or disjunctive; and I know the difference between asserting the fact, noting a likelihood, and proving the truth of the matter. Four decisions occur all at once, and only gradually do we learn to backtrack analytically to one or more of the four. Kant organizes them into a Table of Logical Functions.*

As I trudge along a path in the woods, “That there is a house” may appear in logical retrospection as singular and affirmative, categorical and

* Kant discusses his twelve logical functions starting at A71, B96. He claims his account remains within the ambience of traditional logic, yet in his slight modifications we can detect crucial openings onto the developments that have revolutionized intellectual work during the twentieth century.

* Following Kant, L. E. G. Brouwer succinctly formulates the centrality of time in and for mathematical work:

This neo-intuitionism sees [1] the dissolution of life-moments into qualitatively distinct parts that, separated only by time, can reunite themselves, as the basic event of the human intellect and [2] the abstracting of this dissolution from every feeling-content, right down to the intuition of two-oneness and nothing else, as the basic event of mathematical thinking.

assertoric. Yet each of these four logical determinations recalls two contraries.

<table>
<thead>
<tr>
<th>Quantity of the Judgement</th>
<th>Quality</th>
<th>Relation</th>
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<tbody>
<tr>
<td>Universal</td>
<td>Affirmative</td>
<td>Categorial</td>
</tr>
<tr>
<td>Particular</td>
<td>Negative</td>
<td>Conditional</td>
</tr>
<tr>
<td>Singular</td>
<td>Infinite</td>
<td>Disjunctive</td>
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</tbody>
</table>

Singular judgements, Kant admits, behave as universals so long as we consider only their occurrence in formal syllogisms. We must acknowledge, however, that they differ immensely from universals once we consider the cognition embodied within each form. Universals range far and wide, inclusively, requiring no landing point. Singulars, on the other hand, embody a focus on one thing, exclusively. In classical terms, we speak universals with greater ease than we do singulars: it is much easier to talk correctly about houses in general (universal or particular) than to talk incisively about one house.

Again, from a strictly formal standpoint, the predication of a complement (a “non-...”) appears affirmative, as when we obvert “No houses around here have inside plumbing” to read “All houses around here lack inside plumbing.” However, when we consider logical forms as fulfilled in actual judgements (drawing upon appearances on the space-time grid), we notice that the affirmation of a negatively defined predicate does not embody any positive insight into the subject. If we remain sensitive to Aristotelian precepts, we would restrict employment of obversion to the elimination of negatively formulated predicates. Similarly, the intuitionists of the early twentieth century ask us to distinguish radically between constructive and indirect proofs: the fact that the supposition of $\neg p$ leads to a contradiction does not entail any positive insight into the truth of $p$.

Kant’s formulation of the ways categorial, conditional, and disjunctive judgements differ from one another may appear conventional. However, it in fact reveals a major difference between ancient and modern logic. A categorial judgement brings two concepts into relation, Kant says, while a conditional judgement brings two judgements into relation and a disjunctive judgement brings several judgements into relation. The categorial provides the root of all three. And this root consists of the basic act of considering concepts in relation. Kant’s account prefigures the momentous shift away from the Aristotelian category of οὐσία into that of πρὸς τι. “All humans are animal” becomes: given any $x$, if “human” belongs to it then so too does “animal,” $Hx \rightarrow Ax$; or: $\forall x(BHx \rightarrow BAx)$, with “B” (for Belonging) as our transcendental act of inter-relating concepts at nodes on the space-time grid.

Kant’s formulation of the modalities of judgement seems to correspond exactly to the traditional three. Yet here, too, a difference protrudes. Kant explains “probable” as optional, and in a footnote remarks that the problematic is a function of the faculty of understanding: probable is any connection between two concepts we simply play with in our own thinking. Such play becomes “assertoric” only as we venture an actual truth-claim: again in the footnote he remarks that only in this second modality do we fully engage our faculty of judgement. And “apodeictic” pertains most obviously to those judgements we would call logical truths; in the footnote he remarks that such judgements reflect the faculty of reason. All three Kantian modalities bear on the status of our own conceptual play. In the Aristotelian tradition, modalities bear rather on an ontological hierarchy: the probable bears on social responses to temporal occurrences, the assertoric on isolated stabilities within our temporal condition, and the apodeictic on nature.*

Now, Kant claims that such spontaneity of thought also constitutes an origin. How so? As is well known, Kant claims that this intellectual origin remains dependent for its consummation on receptivity: on

* For a detailed account of the pains-taking intellectual development from Aristotelian to modern concern for probability, therewith also the changing senses of all the modalities, see Ian Hacking’s The Emergence of Probability (Cambridge, 1975), e.g. the summary on pp. 179 ff.
offerings, either empirical or \textit{a priori}, from the grid of space and time. The way we answer the question of the originariness of spontaneity will determine how we might understand this dependence.

For the most part, our judgement slumbers: we hardly \textit{rise} to decide what or how or why this thing (a house?) \textit{is} — let alone what such judgement requires of us, how and why the judgement might hold. Yet any woodsman, anyone \textit{lost} in the woods, will in fact \textit{rise}: moments take shape, tentatively and exactingly, according to whether and how we rise, and pressing situations press us into deciding. And as we respond, the spatial configurations and temporal sequences become sharp, perhaps out of our prior dullness: once alert to their shifting shapes, we begin to make something out of the offerings, take upon ourselves the task of identifying subjects and predicates, antecedents and consequents.

Alert, we begin. And such beginning requires that we carefully discern what is happening on the space-time grid. Discern empirical offerings, to be sure. But also the play of decision-making itself: whether we are deciding about one thing or about a generality, whether we are affirming or denying something about it, whether we are simply categorizing or setting up alternatives (a house or perhaps a silo?), and whether we are entertaining a possibility, asserting a determination, or proving with \textit{a priori} certainty (formal relations on the space-time grid on which the house appears).

Teachers can testify almost daily, or hourly, to the difference between beginning and drifting. Their pupils may or may not “catch on.” A necessary condition for catching on is that they \textit{discern for themselves} what they otherwise only hear or read about. And how does such discernment become possible? By looking to the evidence, we likely say. But it is not enough to look, one must decide. And such decision has a structure: traditionally, a fourfold one, each fold requiring a decision between general and singular, affirmation and denial, categorical or conditional, possible or settled — and in each case a variation that Kant formulates as a third.

In academia, the distinguished venue of modern logic, one earns membership by \textit{developing} what Kant calls the “spontaneity of concepts”: by learning to exercise thematically what every competent artisan does naturally — to energize one’s “critical faculty.” This critical ability aims not so much at \textit{fathoming} the offerings on the space-time grid as at questioning the spontaneous formulations of the conceptual orderings of these offerings — orderings already evident in our own perceptions as well as in those announced by our colleagues. This critical disposition appears similar to Socratic questioning, yet modern critical work induces a transcendental, not a transcendental movement of thought.

The spontaneity of concepts already dances with the receptivity of impressions: engaged presently in transcendental analysis, we distinguish these two origins, the intellectual and the intuitive, at work in any cognition. And we are contemplating this alreadyness: I do so with a view to understanding the conditions of truth presupposed in modern formalism, Kant does so with a view to determining the possible extension of reason into traditional metaphysical questions regarding the soul, God, and the cosmos as a whole.

Yet the dance of spontaneity and receptivity remains at best a kind of courtship, an engagement that may or may not lead to happy, in any case precarious marriages. Kant discusses mainly the causes and consequences of divorce as these determine the fate of philosophical efforts to elevate one or another partner into hegemony. He associates such efforts with traditional metaphysics, more exactly with the modern failures to provide a satisfactory philosophical account of developments in the sciences. Yet these failures Kant understands as reflecting a more fundamental delicacy: that of the dance itself. He acknowledges the source of this delicacy, this instability, when remarking, famously:

\begin{quote}
Without sensibility no object would be given to us, and without intellection none would be thought. Thoughts without content are empty, intuitions without concepts are blind. (A51, B75)
\end{quote}

Sensibility can be dulled: repetition of sensations allows us to forgo fresh decision-making. Each sensation in fact differs from all others by virtue of time if not of place. Yet precisely our faculty of judgement requires us to identify them, seek out a sameness. And as this search meets constantly with success our judgement may fall into penumbral sleep. One result of a dozing intellect is that we may fail to note differences as well as samenesses, differences that may \textit{make} a difference in the business at hand, even for survival. Another result is that we forget how each moment \textit{originates} in our spontaneity as well as in our receptivity: we lose, even deny the evidence required for transcendental analysis.
And spontaneity can become presumptuous. Intellection (Verstand) is especially liable to set up a life of its own. Most famously, Kant claims that traditional metaphysics stems from a fling of the intellect, an effort to construct content out of pure rationality. Indeed, the dance is asymmetrical. While we can receive only what the space-time grid happens to offer—and at worst simply miss some of it—we can conceive quite freely—overstepping the bounds of full legitimacy. We would normally call such free-ranging activity “fantasizing.” Indeed, Plato and Aristotle speak of φαντασία as our condition of operating with left-over φαίνομενα: a light-and-shadow play of what has already come to light. In Greek literature, fantasy marks the threshold of a failure to engage in λόγος, intellection. In contrast, Kant interprets the familiar failure as proving not a cessation of intellect but rather its independence—albeit a possibly vacuous one. He dwells at length on the comedy of such independence in academia (A134, B173):

A physician, a judge, a politicist can have in his head many beautiful rules, pathological, legal, or political—even to the degree that he may become a thorough teacher in these matters—and nevertheless stumble in their application: either because he lacks natural judgement-power (though lacking nothing in intellection), detecting to be sure the universal in abstracto but unable to distinguish whether a case falls in concreto under it; or because he has not been trained enough in such judgement by examples and actual affairs.

How can one lack something in natural Urteilskraft while lacking nothing in Verstand? This latter, “understanding” or “intellection,” Kant interprets as our faculty of inter-relating concepts, and notes that the results of such inter-relation may display their own integrity without having to prove themselves in any concretion. Our modern logical calculi, such as Gentzen’s, illustrate the possibility of such self-wrought intellection, and even more impressively than do the operations of traditional arithmetic and geometry. In contrast, then, our ability to judge assertorically requires something more: the ability to face up to instances “on the run.”

The question of truth now bears on the super-relation between two origins, intuition and intellection: between the domain of space-time offerings and our own power of conceptualizing. We, our individual and communal efforts, are in the truth only as these two origins co-operate. Untruth is born either as emptiness or as blindness.

§2.3.2 Justifying categories

As invoking forms of response, judging presupposes that the things judged take on corresponding forms. Better: judgement imputes forms. Examined transcendently, the dance of spontaneity and receptivity engenders corresponding categories—what Kant calls “pure concepts of intellection” (reine Verstandesbegriffe: A79, B105).

Prior to considering whether a judgement about all or some or this one house happens to be correct, the bare prospect of a decision already presupposes that we can meet with unity or plurality or totality, respectively. The moment I affirm, deny or complement, the bare prospect of such decisions presupposes that I can meet things in terms of reality or non-reality or delimitation. Whenever one talks significantly about things in categorial, conditional, or disjunctive ways, the bare prospect of deciding whether a formulation is correct presupposes that what one meets can be understood as having inherent properties, as causing or being caused, or as entering into reciprocal relations. And whenever we pause to assess the modality of our own formulations, we presuppose that the three corresponding distinctions bear on what we meet as well as on how we formulate our talk.

In sum, transcendental considerations lead us to an array of logical necessities that differs from, while strangely paralleling those of Aristotle.

Aristotle’s ten categories derive from an artisanal acknowledgement of the way things present themselves to us. Here, the things we meet are ... at a place and a time, doing something and undergoing something, relating to other things, prepared in a certain way, posing in a certain way, and so on.

Kant’s twelve categories derive from the a priori forms of spontaneity—the responses immediately at work in the barest perception. They first of all express us, our judging, our efforts to make sense out of what we face: what we receive, what we rise to meet. Kant would say that Aristotle derived his ten categories from receptivity rather than from spontaneity, from our dance partner rather than from our own response to the dance.
In his Preface to the second edition of the *Critique of Pure Reason*, he famously states the difference:

Hitherto it has been assumed that all our cognition must conform to objects. But all attempts to extend our cognition of objects by establishing something in regard to them *a priori*, by means of concepts, have, on this assumption, ended in failure. We should therefore try whether we may not have more success, in the tasks of metaphysics, if we suppose that objects must conform to our cognition. (B, xvi)

Thus Kant’s categories reveal our own condition first of all — as concerned to impute to the things we judge what corresponds to our activity of judging: *their* unity (or plurality or ...), reality (or non-reality or ...), relation (of inherence in a substance, or ...), and possibility vs. impossibility (or existence vs. non-existence, or ...).

But each imputation to what we meet remains suspended over an abyss separating our concepts and our percepts. How can we be sure that our dance partner accepts these conditions derived from our own intellecction? How can we ever justify the imputation of these categories to what we meet in intuition? Kant sets up the question graphically:

> It is clear that objects of sensible intuition must conform to the formal conditions of sensibility which lie *a priori* in the mind — clear, because otherwise they would not be objects for us. But it is not so easy to infer in addition that they must conform to the conditions which intellecction requires for the synthetic unity of thought. Appearances might very well be so constituted that our intellecction should not find them to be in accordance with the conditions of its unity; everything might be in such confusion that, for instance, in the series of appearances nothing presented itself which might yield a rule of synthesis and thus correspond to the concept of cause and effect. This concept would then be empty, null, and meaningless. (A90, B122-3)

How indeed? We can hardly peak around our concepts! Every peak we take engages us in conceptual imputation. And we do in fact discover that imputations fail to hold up in the course of our dealings. It may well be that intellectual response does engage us in beginnings — but only in false starts, in phantasms with varying half-lives.

Yet ... Can we simply discount our intellectual responses? Can we cease imputing unity, reality, causation, and the like to what we encounter? Or, failing to break the habit, at least learn not to take such imputation seriously? And thereby relegate these beginnings to the realm of convenience only — to the convenience of a routine allowing us to get through the days, fooling ourselves and others, as the Wizard of Oz does?

Kant distinguishes between our *transient* imputations — ones that bear directly on the comings and goings of things, and that we must constantly revise — and *original* (or *originating*) imputation. With what I have called his Axiom of Synthesis (“any object stands under the necessary conditions of the synthetic unity of the manifold of intuition in a possible experience”), Kant argues that *originating* imputation is a “necessary condition” for any experience at all. And that from this fundamental “theorem” we may immediately derive the “corollary” that we are justified in trying our hand at transient imputations, and in taking our results seriously (even if tentatively) as manifesting our more original commitment.

I hazard a summary of the Kant’s crucial argument. Contemplating transcendentally the first appearance of things, we discern our *a priori*
engagement in the space-time grid: space and time are conditions of the appearance, to us, of things, and therefore of experience of any concrete sort. Next: for something to arise on our horizon at all, it will already enjoy a space-time unity. That is, it must appear at a node in the grid (providing, of course, that we have indeed woken up to our circumstances as they are in this preliminary manner). Such at-one-node-ness already transcends sensation, although not intuition itself. In contemplation, then, we can testify that this initial aesthetic unity already frees us from reliance on sensation for evidence. And thereby provides the transcendental foothold for the legitimate imputation of categories. For we can discern, again in a transcendental mode, that each thing experienced (e.g., that house over there) requires a unity of experience itself: each node on the grid requires a synthesis of the manifold of nodes—a looming possibility of unified experience. In this synthesis of the manifold of intuition lurks the intellectual, the categorial operations of spontaneity. Organizing, categorizing in these initial ways, originates the unity of experience necessary for things to appear, i.e. conditions the very possibility of judging them rightly or wrongly.

Consider a simple, nearly dreamy experience—not the least artisanal: I step out my front door to inhale some cooler, perhaps fresher air. I have already grabbed the door handle, turned it in the appropriate manner, have acknowledged the way the door—doors generally—work. I now feel the cool air on my face and hands—exposed flesh. And I immediately notice the gray of the grass in some spots, and recall that there has been no rain lately—that I will have to set out the sprinkler this evening; or I hesitate, wondering whether cinch worms have not invaded once again, and whether I should seek advice on treating my lawn this spring.

My experience of any one thing—door knob, door, cool air, grass, rain, sprinkler, cinch worms—obviously engages me in the imputation of unity (vs. plurality, ...), reality (vs. non-reality, ...), substance and its attributes (causes and their effects, ...), and a modal dance. Aware of this last, I especially acknowledge the revisability of each incidental imputation: doors and grass take on added interest when I realize that I can learn something from suspending initial judgements—and learn something new about doors, or about grass. Yet here is the question: Do we have to impute? In order, that is, to have such an experience at all?

Kant argues that such imputation is a condition for the possibility of experience. Spontaneity is then no add-on feature, but an originating activation of experience. Careful consideration of the simplest experience reveals a duality of perception and apperception, as he calls it. While each sensation (cool..., gray...) is an abstraction from perception (cool air on my face, gray spot in the lawn), every perception presupposes a simultaneous collation: each is part of a whole, of a context—of an experience centered in a unity of collated parts; and these “parts” being analyzable into nodes on the space-time grid, the unity of apperception retains its a priori standing throughout both receptivity and spontaneity. Kant's *Critique of Pure Reason* calls this transcendental drama into view. Yet any philosopher, any poet, any painter, any fireside storyteller—any one who recounts an experience as an experience—will engage others in this drama as well: the drawing of a multiplicity together on the basis of a logically prior, perceptually simultaneous unity of apperception.

The transcendental argument aligning legitimate conceptual responses with factual encounters overturns the entire edifice of traditional logic, especially the “theory of judgement” inherited from the Scholastics and enshrined in the Port Royal Logic. In Kant's formulation:

I have never been satisfied by the explanation that logicians offer of how a judgement works; it is, they say, a representation of a relation between two concepts. Now, without here quarrelling with them about the one flaw of this explanation (that it fits at most categorial, and neither hypothetical nor disjunctive judgements), ... I only call attention to the fact that it does not determine wherein this relation consists.

But when I investigate how given cognitions in each judgement bear on one another, and distinguish this bearing (as belonging to intellection) from the relation stemming from the laws of reproductive imagination (a relation having only subjective validity), I find that a judgement is nothing other than the manner by which given cognitions are brought into an objective unity of apperception."

* From the second-edition version of the Transcendental Deduction of the Categories, B140-41 (similarly, his 1783 *Prolegomena*, §22). In that one qualification of the unity as objective, scholars can detect the difference between this Deduction (1787) and that of the first edition (1781). In both
Judgement is basically collation, not attribution. Unity is basically that of apperception, not that of the thing judged. Our first task at the door, a task spontaneously performed, is to hold a multiplicity together; and this we can only do by imputing unity (or ...), reality (or ...), subject-ness (or ...), and some modality to what we perceive. The second task, optionally performed, is to ready ourselves to revise such imputations: and to preserve rather the unity, reality, subject-ness, and necessity of our own field of perception — our apperception.

There is, however, a price to pay: intellection retains its integrity only in conjunction with intuition. Such conjunction is possible in two much different ways: first, while manually dealing with things, where we must respond to perceptions in their material content (the color of the grass, the length of its blades); and second, while intellectually synthesizing the space-time grid (in geometrical and arithmetical calculations). As Kant most famously argues, we have no business trying to cognize anything ungrounded in sensible or in formal intuition: “This extension of concepts beyond our sensible intuition does not help us at all. For they immediately become empty ...” (B148). In Kant's crucial appellation, concepts take on content only in regard to appearances: to what we face, never to what we can figure out abstractly (except as such figurings bear on the transcendental conditions of what we face, as in mathematics — and now, too, in contemplation).

In his first-edition formulation aiming to justify, and thereby relocate, the employment of categories, Kant sets the stage for our now-familiar first-order quantification logic:

But it is clear that, since we only have to do with the manifold [array] of our representations, and that the x (the object) to which these correspond means nothing to us, since it is supposed to be something distinct from all our representations, the unity that the object makes necessary can be nothing else than the formal unity of consciousness in the synthesis of the manifold [array] of the representations. When we have effected synthetic unity within the manifold [array] of intuition, we thereupon say: we know the object. (A105)

Such understanding of cognition fits both our ordinary and our scientific work, where we need only organize our categorial relations around phenomena. It also rules out philosophical claims to cognize such things as the soul, God, or the universe as a whole. For these things Kant lends special weight to another verb: we think the soul, the divine, the whole — in ways that alter our pretensions to know our circumstances.

§2.3.3 The kinaesthesia of categories

In the barest experience we already synthesize. Transcendental analysis of any result reveals two basic components: receptivity and spontaneity. At any given moment we synthesize elements across the gamut of these basic components. The subliminal issue, the obvious result of the synthesis is an at least tentative coherence of what might otherwise fall apart into a chaos of mini-cognitions.

Contemplating experience transcendentally, we discover the a priori necessity of synthesizing in any case. Yet on some occasions we focus reflexively on the synthesizing — as when we wonder how a number of different experiences can hang together. For instance, I may embark on a careful investigation of the gray areas I now detect in my lawn: I back off to re-consider the multiplicity of phenomena surrounding this spontaneous detection. I must now recall how grasses have been (my lawn, other lawns): I do this involuntarily, for otherwise I would not have noticed the gray as I did; now I do so reflexively, gathering data with which to compare the present. And I project how grasses might be

* In A Treatise of Human Nature (iv, vi), David Hume memorably describes this chaos as our reality:

The mind is a kind of theatre, where several perceptions successively make their appearance; pass, re-pass, glide away, and mingle in an infinite variety of postures and situations. There is properly no simplicity in it at one time, nor identity in different; whatever natural propension we may have to imagine that simplicity and identity. The comparison of the theatre must not mislead us. They are the successive perceptions only, that constitute the mind; nor have we the most distant notion of the place, where these scenes are represented, or of the materials, of which it is compos’d.
Axiomatics: The Conditions of Truth

(my lawn, lawns in general): again, I do this involuntarily when I think of setting out the sprinkler; now I do so reflexively, anticipating the variety of ways things can evolve.

My re-construction of the experience consists of distinguishing and formulating factors: rainfall and watering, sun and soil, cinch worms, insecticides and herbicides, animal and human trampling — each in measured amounts. Moreover, the re-construction requires suspension of anticipated synthesis: investigators proceed self-consciously in undecided tensions between possibility and impossibility, existence and non-existence, necessity and contingency. The two work together: suspension allows us to factor, factoring requires us to suspend.

Whether in modern investigation or in transcendental contemplation, reflexive analysis and synthesis is an art, a special talent stemming from an ability common to everyone: the ability to range freely, in suspense, over the manifold array of what presents itself — the ability to re-present both freely and constructively. There may also be a freedom of withdrawal, even of refusal. But the freedom evident in modern investigation remains in hand-to-hand combat with detected, recalled, and anticipated factors determinable only in reference to appearances — to encounterables. It is the freedom of attending carefully, unstintingly, to the complex events in a temporally and spatially extended situation. Such careful attention demands of the investigator the greatest exactitude of judgement on each detail, each datum on the case matrix. This preliminary exactitude leaves one free to construct the case matrix beyond the detail, and eventually to conceptualize the whole array: freeze it into a generalization, freeing one to engage in further research.

This ability to range freely in close attention to what actually happens while re-constructing what may happen and recording what did happen: this ability overreaches both receptivity and spontaneity. It is a third origin, the ability to activate the first two in mutual enhancement. This \( \alpha \rho \chi \xi \) deserves a name of its own, and Kant offers one: the power of imagination.

The reflexive synthesis of modern scientific investigation brings to the surface, or precipitates into nearly tangible form, the faculty of imagination apparently essential to the formation of any experience whatsoever. Simply looking at my lawn, mowing it, hearing the blade strike a rock, smelling the motor's exhaust: all this requires that I range around and, in this limited sense, beyond both the sensory givens and the conceptual determinations. For I must assume, take over, produce a construct in which sensation and conception engender moving relations. If my mind is elsewhere I may in fact not notice these things, may engender the collapse of the experience I was letting run on automatic: the blade breaks, the engine burns out — or I simply drop the affair, leave the machine lying, as children frequently do, and move off into another experience entirely.

The imagination thinkers like Descartes decry as the source of human error is what Kant calls the reproductive kind: we can in fact fall back on images left over from experience, as in passive nostalgia regarding things past and in active dreaming of things to come; and this falling back can distort both our attention to what we encounter at the moment and our conception of it. Thinkers have sometimes claimed that such imagination illustrates our non-rationality. Kant, however, detects the necessity of an original, what he calls the productive imagination — of which the reproductive is merely an offspring, admittedly handicapped until it returns to its parent. Already when introducing the Table of Categories, Kant is adamant:

As we shall soon see, synthesis in itself is the mere effect of the power of imagination, a blind but indispensable function of the soul, without which we would have no cognition at all, but of which we are hardly ever conscious.*

Our task in contemplation is now to highlight this function, this work of the soul.

We begin to function by responding spontaneously to what offers itself here and now. Failing or refusing to receive the offerings of our place and time, we dream or fantasize. Responding focally, we conceptualize: “The grass now looks gray at that spot.” Such beginning entails judging in the four ways formalized on the Table of Categories: by quantifying, qualifying, relating, and modulating. Within this marriage of receptivity and spontaneity Kant detects four schemata quietly but dramatically at work to allow the intercourse between the two — microscopically, between the receptivity of intuition and the spontaneity of

* A78, B103. In his personal copy of the text, Kant indicated he would rather have said “function of the intellect” than “function of the soul.” In general, Kant is bothered that transcendental contemplation might issue in a subjectivization of cognition. After all, we cannot cognize the soul.
conception; macroscopically, between circumstances and ourselves. Each schema provides us with an image from which it must be carefully distinguished in contemplation. And each is the product of the third origin called, now, imagination.

A schema is a procedure (Verfahren) by which at every moment the soul schematizes the marriage — and thereby produces an image (Bild, picture; Greek ειδωλον, a left-over from ειδος). Schematism is a condition for experiencing my lawn at all; it locates the heart of the Axiom of Synthesis. Kant says of it (A141, B181):

> This schematism of our intellect ... is a hidden art within the depths of the human soul, the true maneuvers of which we will most likely never extract from nature and lay unconcealed before our eyes.

Such schematism will not only account for the images by which we picture our condition (nature, the world, ourselves, society, and whatever else we try to understand without having before our eyes). It also provides us with the opportunity to formulate a number of *a priori* synthetic truths about whatever we can experience — and, therefore, whatever we can know (erkennen: cognize).

The three categories of quantity engage us in numbering. Abstractly: the schema of these categories (and of judging all, some, this one) is number. As a result, we can think of anything appearing in space with an image: spatial appearances have size (Größe) inviting numerical measurement: so wide, so high, so deep. Yet anything appearing at all, even inwardly (as when calculating the number of things, not only their spatial dimensions) engages us in sequential accounts: we keep track of them, one after and before another, in growing totalities. To apprehend what is happening inwardly or outwardly, I engage in successive addition of things of the same kind: the lawn now, the lawn then, another lawn over there. Kant says: *Ich erzeuge die Zeit*, “I engender time” — simultaneously *a priori* and *a posteriori*, i.e. at the cross-over of receptivity and spontaneity. Not up front, but in the depths of the soul: up front is my lawn, my experience of the lawn as a whole and in its details. In contemplation, we can extract this much of the depths: our evident numbering of things — a procedure (Verfahren: engagement) prior to becoming an image.

From the schematism of the categories of quantity Kant extracts an *a priori* synthetic principle: All encounters engage us in sizing, all appearances have extensive magnitudes, anything we experience has parts that we detect in the process of comprehending a whole. This principle then justifies our efforts to know our circumstances in quantitative terms: arithmetic in any case, then also geometric terms. The intellectual constructions familiar to physicists and engineers pertain to our circumstances, they are not merely intellectual: no wonder, then, that conclusions reached in the study work well out on the field. Or, rather, we can now contemplate the wonder.

The three categories of quality engage us in grading. Abstractly: the schema of these categories (reality, negation, ... — and of affirming, denying,...) is intensity: things appear in *degrees* of warmth, color, taste, pitch, odor. Gradations lie along a continuum of being down to non-being. Even more than quantitative determinations (numbering), qualitative determinations require, in advance, a “sense” of this continuum, and the possible movements of things throughout the (infinite) points on the continuum. This continuum is not “given” in experience the way my lawn is given, along with its reflected warmth, its colors, and the like. Rather, it allows my lawn to appear as warm, as green and gray, as reeking of spring. And things appear differently at different times on the continuum: the continuum is that of time itself, as becomes concretely illustrated in any effort to do something about the heat, the color, the taste: as when we record their intensities, but even more when treating the lawn with a view to the color.

From the schematism of the categories of quality Kant again extracts an *a priori* synthetic principle: All encounters engage us in evaluating degrees, all appearances have intensive magnitudes, everything we experience has a place on various continuums. Thus all those graphs so familiar to students of modern science have an *a priori* justification: even though things do not locate themselves on graphs (at any one time and place, a spot is simply gray, green,...), our intellectual efforts to locate their position on a continuum—to understand them gradationally, and therefore graphically — follows upon a “hidden art of the human soul” that *conditions*, originates, the experience of anything. That our graphings can hold for things, allow us to take hold on things, should indeed come as no surprise. Although, again, we can take the occasion to contemplate the wonder.
Each of the categories of relation has its own schema: we experience something as a substance, i.e. as a subject-appearance on which we concentrate, only because we attribute to it a relative permanence. We experience such a substance as changing (variable), and in doing so understand the event as based on a rule (something constant). And we understand different substances to effect each other reciprocally. Kant writes (A143 f., B183 f.):

1. The schema of substance is the permanence of the real in time.
2. The schema of cause consists in the succession of the manifold [array] insofar as it is subject to a rule.
3. The schema of community is the being-at-once (Zugleichsein), according to a universal rule, of the determinations of one substance with those of the others.

Strikingly, this threesome encompasses the basic categories of the traditional metaphysics of transcendence. Yet Kant intends to transform the earlier concern for transcendent grounds of transcendence into an insight recalling transcendental grounds.

1. I predicate my lawn: that spot is gray. This act presupposes a sameness against which I assess a difference: the lawn is ... and it is now gray (whereas earlier it was not gray and later it may again not be gray). What is the purport of this supposition of an abiding subject for the predications? A traditional metaphysician takes the occasion to speculate on a transcendent entity that might justify my grammatical construction and the name “lawn”—something an artisan (e.g., a gardener) may come to know, or a scientist (on the metaphysical version of science). Yet most concretely, and prior to any developed cognition (τεχνη or επιστηµη, métier or Wissenschaft), I do and must organize changing determinations, hold a temporal multiplicity together around a stable core: changing determinations make sense only in regard to an unchanging ground for the multiplicity. Still: How are we to understand, in contemplation, this original imputation of stability? Not as simply given in sensation. Nor as an inferred entity eluding sensation: such an inference has no justification in the contemplation of the necessary conditions of experiencing my lawn. Rather, Kant argues, we may understand this original imputation of stability to the sameness of time itself: time as the necessary condition of experience of any sort. As a condition conditioning experience of things, time itself abides while all things in time change. As I learn this origin, I all the more energetically

—willingly, imaginatively—organize the varying determinations around some occasion of constancy such as my lawn. Learning it or not, I synthesize the manifold array on a phenomenal substratum. My cognition consists in unifying this multiplicity of predicates transcendently. Permanence is a schema by which I enact the experience of my lawn as I step outside. The image of permanence is that of an enduring substratum, something I can talk about with my wife over dinner, or with a salesman at a store. As an image, permanence especially seduces one into thinking that things like lawns “just are”—that we can endorse this underlying reality atemporally, statically. Herein lies a source of much mischief, both in ordinary and (as Kant insists) in academic life.

2. I judge that the lawn is turning gray because ... cinch worms have invaded, there has been too much sunshine and not enough rainfall, or whatever. I suppose a temporal connection leading toward the perception at the moment; and I suppose that some intervention may lead toward a day when the gray will have given way to green. And the purport of this supposition? Again, a traditional metaphysician takes the occasion to speculate on a transcendent order of nature that might justify our daily imputation of cause and effect—a transcendent order of special interest to intellectuals who no longer root their contemplation in ποιησις and πραξις. In contrast, Kant proposes a pre-metaphysical justification. For the most rudimentary experience owes its possibility, its immediately necessary coherence, to a primordial sense of temporal connection. On the basis of this sense, I take notice of things and understand myself as taken up into their temporal fluctuations: each node of time is one presenting something both caused and causing. At each node I am affected by circumstances; and, more dramatically, at some nodes I rise to intervene. Still: How are we to understand, in contemplation, the status of this sense of causation? As David Hume most famously argued, I never perceive a cause — the way I perceive colors, sounds, tastes, smells, surfaces, shapes, and melodies. Nor can we infer causation itself: as John Stuart Mill famously illustrates—he who delineates in a logical mode myriad examples of investigating particular causes — , every inference linking cause and effect presupposes that things behave according to the laws of causation that Francis Bacon posited as the companion of bodies performing purely individual acts. We must, Kant argues, resort to the contemplation of the necessary conditions of experience in general. The first condition of relation is that of organizing
multiple predications onto a phenomenal substratum — already a temporal enactment. The second condition is now that of organizing successive determinations among themselves (each in an ordered pair) — very explicitly a temporal enactment. Cognition — from the simplest recognition to the most elaborate anticipation — depends on the unification of succession. Any elaborated cognition explicates causation in reference to rules (called laws when considered in contemplation abstracted from operation). But causation is already a schema by which we humans enact experience. The image of this schema is that of force: we in fact picture forces to ourselves in various ways — as “the hand of God” or as “the hand of man,” then too as “hormone-based instinct (for sexual intercourse)” or as “the marketplace (for prices).” Such images of causation invite us to think of ourselves as innocent bystanders, statically affected by the course of events. Herein lies another source of great mischief, especially among half-educated people.

3. I immediately judge that the gray spot in the lawn stands as one focal point in simultaneous multiplicity: there are also the sprinkler, the hose, and the fertilizer in the garage, my wife behind me in the house, the sun and a few clouds above, the lawns and houses of my neighbors, the university campus to which I must now depart. The one perception of the gray spot engages me in such being-at-once (in simultaneity — or in “co-existence,” as Zugleichsein has been translated). On the basis of such primordial engagement, we also develop highly abstract concepts regarding reciprocal influences of one thing on another at any one moment: “gravity” and “magnetism” in modern physics, ecological interdependence in modern biology. Prior to such abstractions, however, the supposition of the simultaneity of a multiplicity already conditions experience: our apprehension of the spatiality of an experience already presupposes an interaction among things. Yet my eyes and my ears select only one thing at a time in space. How are we to interpret the supposition of the simultaneity of spatial multiplicity? Again, the traditional metaphysician takes the occasion to speculate on a transcendent object of attention: the cosmos as a whole. Kant, in contrast, asks us to think transcendentally: to detect in the supposition of simultaneity a condition, an enactment generating experience kinaesthetically. This enactment on our part invites us to formulate laws of interaction that can also serve as rules of operation. Simultaneity is then a schema by which I already engender experience, and which then I might develop actively, imaginatively, as any eager gardener, scientist, painter or storyteller will do in the course of extending and re-presenting coherent experiences. The attendant image of simultaneity provides a static stimulus that can lead us to engender mappings of the earth in modern geography, and has led to cosmic astronomy with all the conundrums of space and time that Kant foresaw.

The three schemata of Relation entail their own principles: a priori synthetic knowledge of special interest to intellectuals. Everything we experience, even know scientifically, will take the form of properties of substrata, laws of causality, and reciprocity of interaction. But these principles differ from those of Quantity and Quality. The two mathematical principles regarding size are constitutive: they lead to determinations (measurements in space, time, and degree) constituting what we know. In contrast, the principles of Relation are regulative (A180, B223): we determine what happens (phenomena) by looking for properties gathered around substrata, for causes of effects and effects of causes, and for interactions; we find many things, but never substrata or causes themselves — only relations. To metaphysicians hoping for constitutive cognition of substances and causes, Kant's conclusion appears negative and disappointing. But the intended conclusion is positive and encouraging: if concepts make sense only in movement, they invite constructive and imaginative work. The employment of concepts, so easily vacuous or prejudicial, obtains and retains its legitimacy only insofar as we keep them in hot pursuit of phenomena — in forever renewed formations in open battle with new opponents. Illusion sets in the moment we close our conceptual ranks onto themselves: this will disappoint only those who hope for static conceptual formations — for a knowledge that ends rather than begins.

Kant's discussion of the categories of modality may seem conventional. But consider his statement of the three schemata (A144, B184):

* Kant offers an indirect proof at A212, B259: the assumption of appearances entirely isolated from each other would prevent us from having any sense of simultaneity of lawn-up-front and house-behind, i.e. from experience itself.

* Thus Kant strangely agrees with Aristotle: the category of πρὸς τι precludes cognition of things αὐτῷ καὶ ἄαυτῷ. Kant, of course, envisions and bolsters our modern hope for a kind of knowledge built purely on relations.
The schema of possibility is the agreement of the synthesis of different representations with the conditions of time in general, i.e. the determination of a representation at some one time (e.g., the opposite [predicate] in a thing cannot exist simultaneously, but only successively).

The schema of actuality is existence at some determined time.

The schema of necessity is existence of an object in all time.

From these three schemata, Kant derives three principles, called “postulates of empirical thought in general” (A218, B266):

1. Whatever satisfies the formal conditions of experience (regarding intuition and concepts) is possibly.
2. Whatever coheres with the material conditions of experience (with sensation) is actually.
3. That, the coherence of which with the actual is determined according to the universal conditions of experience, is (exists) necessarily.

Aristotle understood these modalities of our talk to reflect our lexical and logical commitment to fathoming each thing as it is destined to be: τὸ τι ἦν εἰναι. In accordance with the exigencies of modernity, Kant now re-locates these ways we talk into the temporal conditions of experience. Possibility, actuality, and necessity now appear as functions of time. We determine something as possible not by looking at what it permits, but at the conceptual conditions of its appearance on the space-time grid. We determine something as actual not by looking at its full functional presence, but at some node on the space-time grid. And we determine something as necessary not by fathoming its inner exigencies, but by determining the transcendental conditions of experience itself (a necessity we easily miss not because it is too far, but because it is too near).

The twelve categories function truly only in conjunction with temporal schemata productively engaged with presentations of or on the space-time grid: of the grid, as in a priori conceptualizations of points in space and time (arithmetic and geometric sciences); on the grid, as when dealing with things appearing a posteriori (empirical sciences). In active modern research, academics generally understand that their concepts function only kinaesthetically: that the images surfacing from their schemata represent intellectual work only statically, even in caricature. Still, the old illusion essentially resurfaces: for one first of all hears about intellectual research, and from hearsay one only obtains images. First-hand intellectual work, even if not first-class (paradigm-shattering), evolves as one breaks through merely reproductive into productive imagination.

The break-through, however, occurs prior to the development of intellectual competence: already as a condition of experience. In rough analogy to Plato’s famous dictum, we may say that παθέναι consists not in putting sight into eyes that were blind, let alone in supplying the mind with data, but rather in a turning of the eyes toward the conditions that already prevail.

§2.4 Thinking vs. knowing

Thinking—that's our affair, too. Not only that of those aspiring to know what's happening on the space-time grid, but now also that of those aspiring to bring into view how such knowledge works—what its conditions might be, including its limitations and consummations. Yet, as the “faculty of categories,” thinking belongs to pre-academic experience, i.e. to any understanding, whether artisanal (productive), organizational (practical), or even merely casual (imaginative, as when musing on what has happened, will happen, or is happening).

However, whereas thinking of the ordinary kinds bears “naturally” on what we encounter, on happenings found in space and time (on finite matters), thinking of our kind pushes beyond encounters toward matters that we admittedly cannot locate on the space-time grid (except allegorically, as in stories and paintings). Traditionally, these matters are three in number: ourselves in our freedom (special agency and immortality), surrounding nature as a cosmos (integrated whole), and all the evident unsettledness in and around us as having an ultimate source (divine intention serving as possible orientation for human agency and natural order). Unlike the usual matters of knowledge (whether in academic science, production, or action), these three are non-finite, infinite: we have only ideas (intellectual visions) of them. Kant formulates the difference as one of our “faculties”: our direct ability to understand phenomena as cohering and our reflexive ability to discern our understanding itself as cohering (A 644/B 672):

Thus reason really has as its object only the understanding and the intent of the latter’s engagement; and just as understanding unifies the manifold in each object by means of concepts, so reason unifies the manifold of concepts by way of ideas...
Now, Kant famously argues throughout his critical enterprise that we not only may but must think the three infinites—and that we cannot know them: that we can and must think ourselves as free and immortal agents but cannot know ourselves as such, that we can and must think of nature as ordered overall but cannot know nature in its overallness, that we can and must think the ultimate source but cannot know it. Both the ability and the destiny to think these infinites reflect the “vocation” of reason, which is precisely to assess the ordinary business of understanding phenomena in its totality, whereupon there’s a built-in temptation to shift the focus from such reflexivity (assessing the whole of our own affairs) onto the phenomena themselves, and to aspire to draw them into focus as a whole (A 643/B 671). Yet if we agree to confine knowing to the direct affair of making sense out of what happens, out of phenomena—confine it to the affairs of sensory intuition—it follows “analytically” that we have no business translating the thinking of those infinites into knowing them.

That antiquity and modernity here diverge in their vocabulary need not conceal a certain parallel. Kant, like Socrates, curtails human pretention: What business do we have making claims about Freedom, Cosmos and God? We may have to make claims about the horses we train, the houses we build, or the conduct of our children, but these are all matters of responding to what's actually happening. Our ability and destiny to think the three infinites stem rightly from letting ourselves into them.* Thus Plato and Aristotle, too, have a special name for this ability and this destiny: νοῦς. This “faculty” (δύναµις) also, as in Kant, consummates itself as “thought thinking itself.”

Yet there's also a yawning abyss between these parallel lines: starting with the “assumption” that “objects must adjust themselves to our knowing” (B xvi), Kant must eschew every suggestion, rampant in the works of both Plato and Aristotle, that “intellectual intuition” (νοῦς) (1) places us in direct contact (θιγειν) with its object, (2) is the origin of knowing (without which knowing compromises its legitimacy), (3) is directly true (not a matter of claiming anything)*, and (4) is “ours” only on loan (it's our way of partaking of the divine). In keeping with the Enlightenment, Kant locates reason (Vernunft) as precisely ours, in all our secular loneliness—essentially “out of touch” (μὴ θιγγανειν) with its own fulfillment and thereby hankering all the more for knowledge of what's in fact happening around us.

The distinction between thinking and knowing has become our most challenging inheritance from Modernity. It takes all manner of shapes that Kant himself addresses. Here, however, I would like to point out one essential consequence of the distinction, a duality of concern that already emerges in Kant's work and has since surfaced in two richly and radically different, yet strangely complementary directions of philosophical inquiry.

The one direction is that of lending support to positive science. In Kant's words (A 702/B 730): the “authentic vocation” of reason would be to “employ all methods ... solely to penetrate nature at her very core.” Within the context of educational curricula, philosophy's authentic role would be that of being a handmaiden to biology, physics, and the like—to help students in these fields get a firm grip on what they are doing, namely on the methods by which these positive disciplines achieve power over circumstances, and to get them to acknowledge the limitations justly imposed on their aspirations. Here, the central task is that of distinguishing what gets thought (Greek: noumenon) both from thinking itself and from the proper object of modern knowing, namely what actually comes to light (Greek: phenomenon). The purpose of such philosophical work is to promote, from the sidelines, the project of laying “the foundation, not of a sect or a doctrine, but of human utility and power.” One essential element of this project is the realization that the commitment to the systematic reflects only a maxim of positive science, so that any system is an idea, not a concept—thought but not known,

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* In his Critique of Judgement (§91), Kant distinguishes matters of faith (Glauben: letting oneself into . . . ) both from articles of faith (dogmas) and from “taking on faith” what others say. Fully understood, the faith basically at issue for us pertains only to the ideas of freedom, immortality, and God.

† Critique of Pure Reason, A 680/B 708: “pure reason busies itself only with itself.” Metaphysics, 1072 b 20: “thinking (νοῦς) thinks itself by partaking in what gets thought, for it becomes what gets thought as it is contacting and thinking it, so that thinking and what gets thought are the same.”

* “Saying and affirming are always either true or false—but not intuiting, inasmuch as this bears directly on what is, or what is destined to be, and there is no claim about it” (On the Soul, 430 b 27; also Metaphysics, 1051 b 25).
quite rightly regulative but never rightly constitutive (and therefore a rich source of mere pretention and unholy illusion).\(^7\)

The other direction is that of questioning human conduct. For this is now a separate question.\(^7\) Here, inquiry takes its cue from the recognition (on the part of our faculty of reason, i.e., our ability to take stock of our own responses) that, for all our judging of what lies before us, for all the determining we do of other things, we ourselves remain “this side” of what we are determining and therefore not determined (not judged, not known).\(^7\) Only in this way can we learn to acknowledge that dealing with others, as we do in socially organized effort (in πραξις), requires special consideration: respect for the autonomy embedded in the determining (for others as ends as well as means), resolve to uphold one’s commitments (duty), willingness to work for the common good, and so on. Since the first line of questioning for any institutional endeavor is “to penetrate nature at her very core,” institutions stand in need of constant reminders about the justice of what they are doing — whether what they are doing also upholds “human rights” or even “animal rights.” Within the context of educational curricula, philosophy has assumed the role of helping students recall these questions — and even that of training specialists in this field (called “ethicists”).

The distinction between thinking and knowing takes ever-larger proportions in the progression of Kant's three Critiques. After working out the ethical possibilities in his Foundations of the Metaphysics of Morals (1785), and no doubt already in his Critique of Practical Reason (1788), Kant introduces the distinction most emphatically in the second edition of his first Critique (1787: B xxvi):

> We must be able at least to think [the objects of any experience] also as in themselves, even though we cannot know them in themselves.

And to this he adds a footnote clarifying both the distinction and the opening it provides:

> To know an object it is necessary that I prove its possibility (either from its actuality as attested by experience or a priori by reason). But I can think what I want, so long as I do not contradict myself, i.e. so long as my concept is a possible thought — whether or not I can answer for there being, in the sum of all possibilities, an object corresponding to the concept. Yet something more is required to ascribe objective validity to any such concept (real possibility, for the first was only logical possibility). This something-more, however, need not be sought in theoretical sources of knowledge, it can also lie in practical sources.

Similarly, in the second-edition Deduction of the Categories (B 166), after insisting once again that “there can be no a priori knowledge except of objects of possible experience,” he adds a footnote emphasizing the danger of misunderstanding this restriction and, again, an indication of the opening it provides:

> . . . only the knowing of what we think, the determining of the object, requires intuition. Where the intuition is lacking, the thought of the object can still have its true and useful consequences for rational employment on the part of the subject. However, such employment — because it is not always directed toward the determination of an object, and thus to knowledge, but is also directed to the determinations of the subject and its volition — cannot here be displayed.

The thought here bears on Soul, Cosmos, and God — especially the freedom and immorality of the first and the providence of the third, these thoughts being essential to “practical reason,” or holistic considerations of human conduct.

\(^*\) The reference to “human utility and power” comes from Bacon’s Great Instauration (Kant chose this passage as his motto for the second edition of his first Critique). Kant addresses theme of “system” in the Appendix to the Transcendental Dialectic (cf. especially A 645/B 673 and A 680/B708).

\(^†\) It is difficult for us moderns to make sense out of the pre-modern propensity to consider questions of knowledge simultaneously as questions of human fulfillment. When human being appeared as already embedded within what is known, Aristotle could say in his Metaphysics: “It is clear that it is in natural affairs that we must somehow search out and delimit what is, and that the naturalist also contemplates the soul to the extent that it is without matter” (i.e., to the extent that the soul is actual and not merely potential: 1026 a 5).

\(^‡\) “Not consciousness of the determining self, only consciousness of the determinable self (i.e., of my inner intuition), . . . is the object [given to me to know when I reflect upon myself].” This formulation is found only in the second edition of his account of the Paralogisms (B 407).
Still, we might wonder whether thinking—again, our own special province—consummates its vocation in watching over the “employment of reason” in these two domains, scientific investigations and ethical reflexivity. Is there no version of thinking might constitute a way of life somehow independent of such watching-over (which so easily becomes a form of policing)?

Kant’s third work in the series, his Critique of Judgement (1790) opens yet a third door—one that he hesitatingly calls Kontemplation. This is one Latin translation of the Greek θεωρία, Aristotle’s word for a third way of life (after artisanal production and social action), one that beholds nature, φυσις, as well as the marvels of the first two ways of life, namely their dependence, for their efficacy, on (intellectual) visions of the (natural) fulfillments guiding the natural growth and the crafted formation, in short the γένεσις of things. Yet Kant’s account in fact confines such contemplation to the realm we today call “aesthetics,” the beholding of the beautiful: “the judgement of taste is merely contemplative” (§5: is based on no further result); “the taste for the beautiful presupposes and sustains a disposition of quiet contemplation” (§24: unlike the feeling of the sublime, which agitates). And when he remarks that there is no hope of founding the disclosure of the divine in the terrors of natural phenomena (even in the experience of the sublime) because “for marveling at divine greatness a mood of quite contemplation is necessary” (§28), this contemplation is extra-curricular, not any way of life distinctively philosophic. That our own vocation, that of thinking, might script such contemplation: at most, this might be detected hovering over Kant’s third Critique, a door barely opened, through which Kant’s successors—most notably Hegel—have boldly passed.*

As in the Fourth Interpretation generally, so still in Kant: knowing remains the primary model from which both ethics and aesthetics take their bearings. Even more than ancient and medieval philosophy, modern philosophy celebrates knowing as the first awakening, and understands this awakening not as a participation in the genesis of nature, but as a stand-off of the new subject (human agency) and the new circumstance (recalcitrant object). The new knowing (as Bacon, Vico and others called it), unfolding within this stand-off, takes the form of the re-formation of circumstance for the satisfaction of human needs. As the various enterprises of knowing, familiar to us in our academic institutions, develop into clearly effective forms of manipulation, the question arises how we are to understand the human needs themselves. With this question the social sciences spring into prominence—and, with these new disciplines, the “sentimental” component within the stand-off: all the agonies, individual and collective, plaguing those who have to face the “objective” component (for actual circumstances henceforth appear essentially hostile to human interest).

But what if knowing is only a derivative manner of our being with circumstances and among others of our own kind? What if the awakening celebrated since the Enlightenment only represents a form of adolescent rebellion that, precisely because it consists in a stand-off, actually re-enforces a thraldom to what it intends to master? What if knowing is a form of fleeing, one misconstruing liberation as escape? What if genuine awakening consists in learning to think, to contemplate, to behold?

If so, then we might wonder how. And raise the question all over again what it means to be rational—and where logic, the study of thinking, can once again obtain its bearings.

§3. Essay on a fifth interpretation

How so, a fifth? After the four, what does it mean to try for yet another? Would “another” offer another device (procedure, method) for re-learning intellectual work? Or another “view” (opinion, sect) for understanding human achievements? Does an Interpretation come down to one or another or both—a device or a view?—Not in any originary sense of Interpretation.

And that locates, perhaps, the problem: Interpretations eventually rigidify—justify themselves as devices or views. At their origination, however, Interpretations justify not themselves but what they reveal. One of the tasks of any Interpretation is to account for this shift in direction of justification, well aware that its own account, too, will suffer the same fate.

The source of the problem is that we live and work and have our being within an inheritance. Intellectual work especially evidences the heritage circumscribing it, since here we contribute precisely to understanding it or (as is more often the case) some moment within or

* I myself have orchestrated the score in this direction: cf. The Art of Art Works (Victoria, B.C., 1982), especially pp. 209-35.
some aspect of it — unlike artisanal work, where the means of production draw and delimit our attention. And among all the possible intellectual endeavors, logic most of all: for logical work intends to highlight, to analyze, to contemplate the conditions rendering intellectual work possible precisely as an inheritance always poised for bequeathal. Furthermore, as most of all evidencing our heritage, logical work most easily presumes its justification: it seems to have no truck with devices or views, therefore none either with the shifting winds of ontology.

To “try for another” then means this: to try to recover the other to which inherited Interpretations themselves have offered responses — once did respond and may again respond. For any successful Interpretation (one that is in fact bequeathed) is its response to . . . whatever, since this, too, only addresses us within our response to it and only afterwards shows itself for easy naming, albeit in a rigidified and caricatured fashion.

One can of course talk about Interpretations — multiple because foreign — as though each supplanted the next, or aspired to do so. One then construes them as indeed interesting and stimulating but also already destined for dismissal as outdated. Whereupon one's own talk is stillborn as well, no matter how interesting or stimulating at the time of gestation.

Even from a strictly historical (scholarly) standpoint we can suspect that Homer was recovering something of the Minoan, Plato something of the Homeric, Jesus something of the Judaic, Augustine something of the Evangelical, Descartes something of the Scholastic, Kant something of the Modern, and that Hegel aspired to recover something of them all.

Each recovery stays with us — becomes our inheritance — not only because it succeeds (if success were enough, every good teacher would enter into history), but also because it introduces difference as well: first of all, a way by which we can learn to tell the difference between justification of the matter under consideration and justification of the Interpretation itself. And, with this, also the difference between our own responding and what itself calls for response. And, finally, differences among the various Interpretations — the delight of expository education.

Yet any project of recovery only makes full sense to ears hearkening to a loss. Something was missing in the bustle of Athenian intellectual. Something was missing in the Judaic passion for the law. Something was missing in the achievements and aspirations of modernity. Plato, Jesus, and Kant also detected an excess, a “too much” concealing the loss. And if we fail to exercise our own discernment of the loss we may settle for reviewing their objections to the pretensions born of this excess — and thereby commit the very sin at issue, the refusal to acknowledge the loss.

What loss? This question can only be addressed fully by one who has first witnessed a loss. Historical inquiry can locate literary testimonies to particular losses — e.g., the loss of the divine within secular pretensions, or that of individuality within respect for traditions. However, from the logical point of view, the loss is very generic: loss of reference.

Hitherto — in Plato and Aristotle, in Kant and Hegel — logical work drew its inspiration from some vision of consummation. More: the exercise of the craft remained in service to the vision as providing the reference. Inspiration from and service to this vision, to its reference, so much dominate the logic of the Second and Third Interpretations that they generally go unstated. Only during the Fourth Interpretation did thinkers begin to insist upon the reference — now as a separate topic of discussion, one complementing the discussion of secular cognition. This separation is most clearly evident in (and effected by) Kant's separation of cognition (the knowing confined to events in space and time) and cogitation (the thinking of nature as a Cosmos, of the soul as Freedom, and of the ultimate source as God).

At the Dusk of the Fourth Interpretation the only reference possible for logical study is Truth in battle with its opposite. Gottlob Frege was among the first to insist on this last stronghold. In his essay “On Meaning and Reference” (1892) he distinguishes between storytelling (or epideictic speech of any sort, including political oration) and statement (or proposition: what's especially at issue in academic disciplines). In the first, we move along within meanings (or signifiers, direction-givers) and in the second we pause to make sure, to decide whether we are moving properly. What's at issue in the first (e.g., in Homer's account of Odysseus' landing on his home soil) is whether we move smoothly, whether we find ourselves moved pleasantly (or even profitably, as in the case of a politically motivated oration). What's at issue in the second is Truth vs. Falsehood:
We are thus driven to acknowledge the truth-value of a statement as its reference. I understand by the truth-value of a statement its condition as being either true or false.

Here we find ourselves on the familiar ground of school-room logic since World War II. Following Frege, we may turn to the question of how propositional forms may engender legitimate progeny, in which case we will eventually find ourselves interlocked with the problems of mathematical formulation. Or we may return to the question of truth conditions, in which case we will find ourselves interlocked with the problems of scientific verification. Formally, the difference between truth and falsehood suffices to construct the difference between validity and invalidity of theorems and of inferences in primary (sentential) logic.

Materially, too: in a Kantian spirit, modern intellectual work of all sorts accounts for phenomena (data, regarding either “culture” or “nature”) with a view to the same contrast between which propositions to doubt, which to uphold, and which to reject, always relative to a “story” (a complex γενος already in place).

But what kind of “reference” is this — truth-value? We aim for it, we devote whole research programs to it — but what is this “it”? For a while, we could deflect the question by promises of “progress.” Still today, and in the spirit of the Enlightenment, researchers and politicians assuage the doubts of a paying public by talking of improvements in medicine (pharmaceuticals, surgical techniques), in exploitation (of natural and human resources), in communication (electronic media), in national defense (technological warfare), in cultural understanding and appreciation (tolerance in a multi-ethnic world), and so on. But these “references” come down to anticipated “benefits,” i.e., promotions of human interest. This is as it should be in a liberal society. And for those content with such promotion, there is no need at all for any further Interpretation.

The logic at the heart of modern intellectual work now, at the Dusk of the Fourth Interpretation, clearly extols decision (strangely both individual and collective) as the reference: our own acceptance (affirmation, inclusion) vs. rejection (denial, exclusion) of statements. Legitimate intellectual work will ground these options in the Acceptance (vs. Rejection) of the Basic Story in which the work unfolds. In short, the reference is no longer transcendent nature and its possible divinity, nor transcendental subjectivity and its possible divinity, but volition “pure and simple” — as Nietzsche was the first to proclaim in a modern vein. But can this volition genuinely serve as a reference? Has it not all along been a deferral of reference, a procrastination concealing a void and promoting what might be called consumerism “pure and simple”?

Significantly, even the fine-sounding reference to Truth vs. Falsehood has lost ground not only in modern intellectual work (where the reason lies in fear of exercising authority) but also in logic — where many remarkable achievements depend on suspending its primacy.

* Über Sinn und Bedeutung, reprinted in Funktion, Begriff, Bedeutung, (Göttingen, 1966), p. 34 of the original pagination:

So werden wir dahin gedrängt, den Wahrheitswert eines Satzes als seine Bedeutung anzuerkennen. Ich verstehe unter dem Wahrheitswert eines Satzes den Umstand, daß er wahr oder daß er falsch ist.

Frege is arguing against a purely formalist (“syntactical”) approach. Note, too, that the two crucial terms, Sinn and Bedeutung, have been translated variously: Sense and Nominatum, or Sense and Meaning; this latter couple in Translations from the Philosophical Writings of Gottlob Frege (Oxford, 1952 & 1980), pp. 162 ff.

† For a history of the “tabular method,” see again W. and M. Kneale, The Development of Logic (1962, 1984), p. 420: it originated in Boole and Frege and “was popularized by Post and Wittgenstein in 1920.”

‡ Works in the “philosophy of science” have driven this point home in much of its subtlety: e.g., those by Karl Popper, Thomas Kuhn, Paul Feyerabend, Imre Lakatos, and Ian Hacking.

† Nietzsche’s part in this deserves careful study in itself. Thinkers at the end of the 20th century sometimes look to Nietzsche’s work as “post-modern.” This is justified only in the limited sense pertaining to any thinker who “blows the whistle” on current thought. He himself continues the modern project, only now in greater clarity. Michel Foucault aspires to discontinue it; I think, for example, of his L’ordre du Discours (his inaugural address at the Collège de France in 1970): “It is perhaps risky to consider the opposition between true and false as a third system of exclusion” (third after taboos on discourse regarding sexuality and politics, and the division between rationality and madness), yet we will overcome “logophobia” only on the condition that “we call into question our will to truth, restore to discourse its character as an event, and finally throw off the sovereignty of the signifier.” Of course, this condition, too, deserves careful study in itself.
§3.1 Origin: structure vs. form

At the origin of ancient logic lies the principle of non-contradiction — itself reflecting our commitment to form, the ειδος looming in whatever arises for encounter (its “nature,” φυσις).

At the origin of modern logic lies the principle of synthesis — itself reflecting our commitment to know what arises for encounter as pre-formed by space and time.

At the origin of the logic slowly but surely emerging since the early 20th century lies — I propose — the principle of structure as prior to formin either of the received senses. Before fully stating this principle, let us review a number of premonitions of its emergence.

Consider the proofs of independence presented by Paul Bernays in 1918. To show, for instance, that Summation cannot be derived from Tautology, Addition, and Permutation, he suspends the truth-value interpretation of propositional forms and introduces instead arbitrary, “senseless,” purely syntactical “options” defining ∨ and ~. Instead of T or F, an array of four: alpha, beta, gamma, and delta:

```
  ∨  α  β  γ  δ
 l  α  α  α  α  α δ
 e  β  α  β  α  β  α
 f  γ  α  γ  γ  β  β
 t  δ  α  β  γ  δ  α
```

With these purely syntactical values, the other three “primitive propositions” in the sentential logic of Principia Mathematica all have the value α, while Summation itself does not. And, crucially, any formula derived from the other three must have the value α. Thus Summation can never be among these latter derivations.

Bernays’ proofs are strictly syntactical. They are not, as traditional proofs of any sort had been, semantic (reliant on the Truth vs. Falsehood imagined for each propositional form, once instantiated). Positively stated, they engage us in manipulations governed by rules pertaining to

* See Appendix II on Axiomatic Induction, p. 570 ff.
It is precisely this “formalist” understanding of the logical development in *Principia Mathematica* (of any deductive system) that allows us to show that it has a “property” much more significant than the independence of primitives: its ability (our ability) to configure a formula, within the confines of the system itself, that it (we) cannot decide. In a piece published in 1931, Kurt Gödel configures, in a sequence of forty-five recursive and strictly syntactical (vs. existential, or semantic) definitions, just such a formula, and displays its undecidability. The proof is, as he says, “entirely constructive, i.e. one can prove [it] in a way that is intuitionistically unobjectionable.”

Commentaries differ on what to make of Gödel’s result—other than that it indeed dashed the hopes of creating a single and complete system of logical and mathematical deduction from a finite number of starting-points. Does the result show the finitude of human rationality (à la Kant)? Does it prove the arbitrariness of human rationality (à la Nietzsche)? Does it illustrate the need to “throw off the primacy of the signifier” (à la Michel Foucault)? Does it prove the purely “constructive” character of knowledge (à la mode)? Or anything at all?

Gödel’s result, it must be emphasized, does not take the form of a self-standing theorem that can then be put to use within other affairs. It is a strictly in-house determination, and its significance is confined to the construction of axiomatic systems. Thus its significance lies largely in the manner of its achievement. As in the case of proving the independence of primitives, so also in the case of proving the ability to devise syntactically an undecidable formula, we witness the power of the

* According to his biographer Constance Reid (Hilbert, 1970, p. 60), Hilbert told his students in his 1898-99 lectures on the “elements of geometry” that “Euclid’s definitions of point, straight line, and plane were really mathematically insignificant” (they say nothing about the physical universe):

In other words, whether they were called points, straight lines, planes or were called tables, chairs, beer mugs, they would be those objects for which the relationships expressed by the axioms were true.

And here “true” means “emerging in the syntax” (formed by the axioms), not “pointing to how things really are.”


Notice, though, that he continues to emphasize their role as a means (as reducible in the way Hilbert explicitly denies).

Nicholas Bourbaki (a collective pseudonym for the multi-volume work entitled *Eléments de mathématique*, Paris: Hermann) recalls that, while “all mathe-
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Of late, and perhaps for very good reasons, we easily talk and hear about the “structure” of almost anything: of animals, plants, and minerals, of the brain, the genome, and the atom, of consciousness, behavior and society—not to speak of the structure of buildings, dams, and other artifacts. We constantly see and draw pictures representing these structures—maps allowing us to orient ourselves from one point in the complexity to other points—“representations” or “concrete objects” of the sort Hilbert recommends. Yet these representations seem to function as forms awaiting instantiations (some one animal or brain, the one society or dam, etc.). But we also continue to instantiate the formulas of Principia Mathematica, proving that they retain their function as forms. That structure can also serve as form does not entail that the two be synonyms.

Until we have fully disclosed the role of structure at the basis of logical work, we must admit that the withdrawal into syntax recommended by Hilbert, and illustrated by his students, ... discourses of logic and mathematics — useful but artificial, and therefore exhausted in the results it produces.

Before embarking on the project of fully disclosing the role (and therefore the power) of structure, let us consider some testimonies from other disciplines.

This one from Werner Heisenberg, one of the 20th-century’s greatest physicists:

... In earlier epochs ... nature ... was a realm that lived according to its own laws, a realm into which man had somehow to fit himself and his life. In our age, though, we live in a world that has been so completely transformed by human agency that everywhere we constantly come up against humanly devised structures and in this sense only encounter ourselves.*


That is, no longer acknowledging the full force of the metaphorical expression “laws of, i.e. in nature,” modern natural science devises, works with, and presents to us laws about nature—better called structures. These structures pertain to what we might still call “nature”: they result from a definite manner of dealing with circumstances and they serve to strengthen our dealings. Indeed, proposed formulations must prove and improve themselves in such dealings. Compared with its ancient homonym, science now provides not so much contact with or insight into the inner and restful nature of circumstances as rather strategies for dealing with the actual course of events in all their restlessness. But then we really only encounter laws, structures, and strategies—not nature herself. And these laws, structures, and strategies represent ourselves, i.e. inherited and projected achievements.

Or consider the understanding of music that has taken shape since the beginning of the 20th century. J.-Claude Piguet, a Swiss philosopher versed in both music and musicology, likes to contrast the accomplishments of Debussy and Stravinsky: while the first “begins with expression (objectivity in an image) in order to renew form,” the second “begins with form to achieve a new kind of expression”:

In effect, Stravinsky inaugurates a new manner of saying things in music, one that is not tied directly, but rather indirectly to the things being said—... a manner of saying just about anything, not according to the rules that govern the things being said but only according to the laws of saying itself.

Piguet reviews a number of examples contrasting Stravinsky with Haydn, Mozart, and Strauss, all of whom composed instances within musical forms—rather than taking the form itself as the “subject.” One effect of retreating into form itself, of transmuting form into syntax, is that new work can consist of modifying languages that already prevail. Piguet quotes Ansermet: Stravinsky “is the first musician who quite openly speaks the language of others to get their language to say something different from what they themselves say and, moreover, to get them to speak solely by way of their manner of speaking.” But, as Piquet goes one to say:

At the same time, Stravinsky gets music to express what it never before was able to express, namely the fact that music is a language. His music is thus a formalism, but a formalism that...
is expressive because it is consistent with itself. That in his theoretical works Stravinsky repudiates expression in music should not deceive us: this does not mean that music lacks expression. Simply put, Stravinsky expresses nothing by means of form alone, he rather gets music to express the fact, in itself abstract, that music is always form. Between a symphony by Mozart and a symphony by Stravinsky there is thus the same difference as there is between a concrete tree and the abstract concept of a tree: both enjoy the same formal properties, but the concrete tree is an expressive form, while the concept of tree expresses a form.*

These testimonies are anecdotal, of course. It may be that only physics, or some branches of it in academia,—that only music, or some schools of recent vintage,—happen to abandon the primacy of form-instance, happen to revert to the primacy of structure-interpretation. And even happen to be deluded in doing so.

We could recite in detail further anecdotes to recall or anticipate the habit, whether good or bad. For instance, often at issue nowadays are our modern democratic institutions. Reading the U.S. Bill of Rights and the first few Amendments to the U.S. Constitution as well as the casual writings of, say, Thomas Jefferson (powerful documents of the Enlightenment), we easily fall into a kind of nostalgia: this literature clearly and beautifully formulates an understanding of public institutions construing them as forms to be instantiated by individual human interests (vs. Divine Providence); and yet, within barely fifty years, thinkers like Emerson, Thoreau, and Whitman had to counteract what they took to be the decline of the original intent — institutions were slipping into . . . structures.

One last, very anecdotal testimony: the prevalence of pocket calculators. Since Plato, the summoning and awakening of intellection has required that one examine instances to extract a form (e.g., Republic, 521C - 525C). An up-to-date example: How many combinations of four-member teams are possible, given thirteen players? Looking at the way this one instance is resolved, one can extract a form for calculating the number of possible “teams” of any size from a “pool” of any size. Yet our current world works quite well for most of us (office workers, for instance) who need only press the button marked \( n \text{C}_r \) on a portable calculator (and similarly with many other “functions,” including the ubiquitous sign for identity). Many otherwise competent students never learn the form-instance version; the calculation takes place within some situation of interest having its own . . . structure.

What might appear as a marvelous advance of intellectual culture—or, alternatively as a decline into mindless routine or, again, as a plot to rearrange circumstances to satisfy individual or collective desire—may actually reflect a basic condition that only now, after earlier Interpretations have worked themselves out, is fully surfacing. Yet, since there are also ugly versions of this “new” condition (as there are ugly versions of just about anything), we may always be tempted to try to revive some favored older Interpretation — along, no doubt, with the account of decline or perversion it offers.

But might there also be an originary and healthy version of the predominance of structure? Any such version will likely require of us not only that we account for decline and perversion all over again, but also that we learn to appreciate the original power and viability of the older Interpretations — to revive them in this sense (as I have aspired to do in the four previous Books), without inviting us into any kind of necrophilia or necromancy. Thus forewarning, I proceed…

§3.2 The bed of structure: world

Just as our hero of the Second Interpretation is clearly Aristotle (who revamped Plato, who had revived Socrates), and just as our hero of the Fourth Interpretation is, I argued, Kant (who revamped Descartes and Hume, indeed modernity in general), so I propose that Heidegger can serve well as a hero of the Fifth Interpretation. For his Being and Time raises all over again, and radically, the question where and how we basically are, and what all arises for encounter, basically and derivatively, in our various ways of being where we are. This work is the first ever to address the question of structure as distinct from form (even if there are strong adumbrations of this task in Kant's three Critiques). It in fact provides an elaborately articulated bed for those intellectual concerns that have found themselves committed to talking about structure — pre-
eminently in the discipline of logic, but now routinely in nearly every discipline. And whereas these other disciplines (including logic as a “normal science” in Thomas Kuhn’s sense) hypothesize (as Plato would say) the elements of structure, Heidegger invites us to “travel up to the primary origin” (Republic, 511).

This origin Heidegger calls World—a name we may fruitfully juxtapose with Non-contradiction and Synthesis. And, since world names where we already abide—live, move and mostly lose our being—the style relevant to any discourse embodying it is neither that of dialectic nor that of critique but rather that of “bringing near”—the style of phenomenology.

I propose, then, that we try to bring the phenomenon, the happening of our abode near, starting with Heidegger’s frank statement of it (Being and Time, p. 86):

The where-in of self-referring understanding as the upon-which of the letting-be encountered of beings in which being-bound-up has its being: this is the phenomenon of world.

This statement of origin encapsulates many previous considerations and anticipates not only the remaining pages of Being and Time, but also the entire project of Heidegger’s thought as developed in the succeeding three or four decades. Let us break it down a bit…

Our basic abode is a “where-in”: a context of dealings, often an easily nameable “work-world” (house work, office work, commercial fishing) or “leisure activity” (football, poker, sport fishing). This context is already intelligible in a “self-referring” way: things make sense, we understand what’s going on in reference to the goings-on themselves, the dealings often having a name from which we have our own names: home-maker, baseball-player, card-player. This “where-in,” this context of dealings, serves also as an “upon-which” through which beings arise for encounter, a context upon which they impinge, into which they intrude: beings worked on (at-hand beings: most obviously tools and materials) and beings worked with (others engaged in the dealings: most obviously our siblings, our parents, our teammates, our opponents, our employers, our clients). Beings arising for encounter are in the mode of “being-bound-up”: each thing, each focus of attention, fits in with other things, other foci, and the flow of such mutual fittings “precedes” any one being arising for encounter (each item is “relevant,” comes into relief, already in reference to other items, i.e. as bringing other things into relief). And this context “always already” prevails, prior to contemplation: it is the “phenomenon of world.”

What first of all is—the beginning, the δόξη rendering the discovery of other beings possible and perhaps even urgent—is world, our abode. Not, notice, myself or ourselves, not the “I think” (subjectivity) so dear to modern intellectual work. Nor nature in the ancient sense of the “self-engendering” (φύσις) governing that to which we must learn to respond effectively by both helping and following the generation (whether in horse-training, house-building, or polis-leading). The self, whether individual or communal, remains buried, more or less lost in its abode even if I formally recognize this abode as essentially “mine” or “ours.” And nature, understood more simply as what we must learn to face, remains buried and more or less lost or betrayed in our initial meanderings within our abode. Thus the Principle of World may enable both self and nature to arise as issues rather than be posited as origins: the self as individual, multiple, and communal; nature as singular, particular, and whole.

Now, with one momentous difference, the family name of our abode, of each and every one of our local abodes, is structure. The difference is this: the structure of world—its very being—essentially enwraps us (we can also say: engulfs us, overwhelms us, absorbs us, even defines us). This enwrapping distinguishes the originary discourse aspiring to bring our world near from that post-philosophical discourse proving the independence of primitives and incompleteness of deductive systems. Similarly, discourse including our own enwrapping differs remarkably from modes of discourse presupposing structures (of plants, of societies, of the brain, of the genome) as objects of research: as presupposed, they stand in no need of our telling where or how they become evident. As in the original “ontological proofs” of the existence of God proffered by theologians, every “proof” of structure presupposes it as the basis of the research in which the “proof” takes place. Still, it is evident that “there are” structures: they constitute the discourse itself and regulate the research. If the structures at issue in post-philosophical discourse are emanations of the structure originally enwrapping us in our abode, they either recall this origin or seek refuge in discourse itself, and in neither case find their proper home in the plants, animals, societies or brains supposedly studied. If, on the other hand, contemplative discourse
succeeds in bringing near the structure essential to our own abode, the structure we here learn to address will indeed come into direct evidence, namely as what enwraps us.

In order to extract the structure of world, let us bring near something of where and how we are pre-philosophically — and even post-philosophically (i.e., when pursuing our respective intellectual disciplines, each stemming from roots in some original contemplation of yesterday or yesteryear). Evident are …

Many chores, each of which belongs to a context — say, the preparations at home before going to work or just “going out.” Some major tasks, then: whether we find ourselves taking these as essentially our own or somehow another’s, the completion of them already establishes the relevance of subsidiary tasks. An abundance of things to be used, cleaned, repaired, avoided, procured, arranged — always within a complex (each thing stands with others, either movement to and from them or in storage anticipating later movement). An array of people with whom we cohort, for whom we work, around whom we pass — people we look out for, direct in their movements, pursue, avoid, take as models, reproach to their faces or behind their backs. There are accounts to be made — book-keeping, tax-declarations, inventories, reassessments of many sorts. There are appointments to be made and kept, along with places to be or not to be. Above all, there are successes and failures in all these — both remembered and anticipated, and with these a variety of moods pervading the abode (hope and despair, joy and frustration, love and hate, confidence and doubt, admiration and disgust, encouragement and discouragement).

Yet any one chore, task, thing, person, account, appointment, place, or mood “makes sense” — becomes pre-contemplatively evident — in “connection” with other chores, tasks, etc. Indeed, each chore recalls major tasks, other people, accounts to be made, etc. They all hang together (make sense, become evident) in a complex engulfing us: in a world. More: each of these items (“beings,” τα οντα) is its urgency: take from any one the tonality, the modality, the degree of its claim on us and it becomes a shadow of itself — precisely for us (they all become senseless except as possible fillers of a world now an empty routine) but even more obviously for others (who don't see the point of what we are doing). In one word: each concerns us (me, you, he, she, it), is within the circle of a world that, in its nearness or farness, assumes various nick-

names recalling an identifiable circum-world: home life, office work, skiing trip, laboratory research into HIV, or whatever.

Such urgency (concern) takes various explicit forms. We tend the fire in the stove, we weed the garden: we take care of things and of affairs (a complex of items: “Don't worry, I'll take care of that!”). We answer to others (to parents, employers, customers), answer for them (for our young children, aged grandparents, stricken friends), or with them (e.g., buddies on a field trip): we care for people in ways often quite distinct from taking care of things. People, even those we pass by on the street during rush-hour, we recognize as co-inhabitants having claims on us as co-survivors or co-governors, as beings also having to help “make the world go around” — and thereby to be reckoned with in ways often puzzling, uplifting, or enraging, but different from fires, stoves, gardens, and the lampposts, fire-hydrants, curbs, stoplights, hats, shoes, and eyeglasses also making demands on us.*

One thing is very evident about any abode and yet cannot be listed among the items arising for attention nor be appropriately reduced to an urgency attached to any one item: our circum-world must “come to something.” This “must” deserves extra-special attention now in contemplation. For this exigency of, in, or about our abode already demands attention apart from contemplation, all the more so when we attempt to skirt or skew it. In fact, the what, why and how of this “must come to something” has always received, and will always receive a variety of “causal explanations” from fields as diverse as theology and biology and economics. But all such explanations refer to something non-evident: they infer from the evident urgency to some cause (the commandments of God, the instinct to pass on genes, the desire to maximize utility). Leaving these aside, let us take stock of urgency as phenomenally evident in our respective abodes — in World.

Most evident is the urgency to keep up with things. Chores, tasks, tools, our friends and enemies, our children and spouses, our employers and our subordinates: all are in movement that engages us, movements of anticipation and recollection, projection and recognition, surprise and routine. Having to keep up with things within this movement, we may

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* My own account of the various traditions and powers of logic require me to tend carefully to many items arrayed in my study: I take care of these (my books, my writing utensils, the proofs). Yet my account is all the while addressed to others: caring for them while not at all taking care of them.
overstep or go too fast, miss a beat or fall behind, get off track, lose track, or lose touch. Very common, then, are the moments of frustration that again take various forms: we blame something, the lampposts, for being out of place or unfit or missing; or some people for failing to do their part or for taking too large a part, for opting out or taking over our abode. In compensation, we then praise things or people for the opposite: for contributing to the movement. But, preoccupation with one or another aspect of the movement distracts from the purport of the evidence: the concern pervading an abode embraces the abode as a whole. The whole itself is “at issue”—pressures us—and not only the chore, the task, the thing, the person (and least of all the mood: this results from how we are in the whole, how we answer to the pressure).

And this issue, this pressure, takes various forms: remaining true to our ancestors (not dishonoring the family name), succeeding in what we do (not botching the mission), acting with a view to future generations (not living selfishly in the consumption of resources). Yet all these imperatives emanate from the one exigency: to do justice to the whole—or to some whole (elsewhere, elsewhen).

Such are some of the fillers for the spaces left blank in the stark statement of the phenomenon of world:

The where-in of self-referring understanding as the upon-which of the letting-be encountered of beings in which being-bound-up has its being…

Like all truly philosophical Interpretations, Heidegger's provides only the bare script—precisely so that others may choreograph it on their own stages and with the instruments, movements, nuances, and colors appropriate to the stage. Although my account has not brought out the exact structure of world, we can already note that Heidegger's Script readies us for reading treatises, writing novels, leading communities, undertaking academic research, treating others, and tending our various gardens—and for doing all this in ways differing from those presented by the other great Scripts of our traditions. First of all, however, it readies us for contemplating our condition as a whole—and thereby itself answers, or rather lets us answer, the evident exigency to face up to, to account for, τὸ παν. This exigency every Script presupposes, right from the beginning down to the latest fashion (always some sort of “positivism” left over from more exacting Scripts and claiming that everything would come together much better if we all stopped worrying about the whole). With this difference, perhaps: the “all” immediately evident is strictly local. The whole is that of our own affairs, always inclusive of others and of things around us, but not the affairs of others per se and not the affairs of nature per se—a facing, an account embodying human wisdom, as Socrates called it (Apology, 20D).

Moreover, the Fifth Script allows us to appreciate the evidence regarding moods. Our abode always has a mood essential to it first of all—to the situation in which we find ourselves, to our circum-world. This in contrast to the explanatory supposition (“construction”) that I and you and others “have” moods and then “project” them onto the situation (as a movie projector does). If only we can suspend this historically conditioned reduction, the evidence speaks loud and clear: situations themselves are urgent in various ways, and their mood reveals (clearly in contemplation if often only obliquely on site) how we are in the urgency at the moment—in the situation as itself urgent. And, in revealing how we are in the essentially urgent situation, moods likewise reveal how things and others are in the situation. People are scared and things are scary because the situation is dangerous; people are uplifted and things are uplifting because the situation is uplifting: the cause is then the situation itself, not anything on the hitherside, or on this side, of it.

One advantage of this scripting of mood is that shapers of situations (parents, teachers, poets, painters, architects) can assume as part of their task the exigency to “create mood”—without manipulating the “feelings” of people by providing various “environmental stimulants”: as a logic teacher, e.g., I can appreciate the need to answer to the exigencies of proof and the like in a way that can lead others through the frustrations essential to the subject (humility) and the elations of understanding (victory).

Precisely because moods first of all pervade any abode, any circum-world, announcing at every moment how the movement is shaping up, it is seldom relevant to focus on them. Such reflexive focus may occur when we are moved to contemplate the situation as a whole—as when we marvel at how well or how poorly the movement is proceeding. It also may become relevant when we assume the role of pathologists concerned that an individual keeps responding to some other situation rather than to the exigencies arising in the situation at hand. Our phenomenological contemplation intends only to bring near where and how we are, namely in a world, and to note—in partial answer to how
we are in a world—that mood belongs first of all to world itself, and for this reason may then rightly (but derivatively) be attributed either to human responses or to what arises for response.

Meanwhile, we intellectuals, and especially we logicians, focus on structure. All these fillers, thrown together rather pell-mell as an anatomist might lay out the organs of a corpse: How do they represent structure—each a node on an integrated grid, as it were? How do these organs represent an organization—one that precedes its organs? For atomism is always an intellectual option presenting us with an ever-plausible reduction of often distressing complexities to apparently more manageable simplicities. The challenge of inductive thought is to bring the whole near so that we can analyze it into its elements while also inducing a counter-reductive movement. In the end, however, the whole can only become evident to one moved to face it in all its own evidence.

A start: the primacy of some story over the incidents we may cite by way of indicating where and how we are. Apart from the Master Scripts provided by our traditions (most clear in our own case is the literature of the Enlightenment, then for this the literature of Christianity, and for this the literature of ancient Greece and Judaism), our condition is already one of a multiplicity of mini-scripts: those of our home life, our work life, our free-time activities, these overlaid by scripts regarding proper behavior in public and sometimes by a local script derived from one of the Master Scripts and prescribing a way of holding all the mini-scripts together.

We can look directly at any mini-script we choose—so long as we refrain from subsuming it under some overall script even though (precisely because) these overall scripts permeate the mini-scripts as well (at least on the Sabbath, so to speak). What do we see? At least three “sights”:

1. Each thing (doorbell or pen, fire hydrant or curb, . . .) is as for something else—leads to something else, recalls something else.

2. Each response to each thing is as for the situation—intends to augment, mollify, or destroy it.

3. Situations prescribe what “must be done” by someone—and this someone may be anyone, so that it is possible to point to various people who should (might, must) do it or should (might, must) have done it.

These three “sights” we may extract, contemplatively, from any situational whole, a finite and configured situation “on the move.” The primacy of the situational whole, a finite and configured situation “on the move.” The primacy of the situational whole over its elements becomes evident in a variety of mini-experiences. For instance, we miss things lying before our eyes, blaring in our ears, prickling our skin . . . so long as the mini-script fully absorbs us in our world. Also, when things are going especially well it can be difficult even to say who the agent is: it seems that things work of their own accord (or by the grace of . . .). Then, too, the appropriateness of a thing or of a response in one situation changes for the next: we often make big mistakes by not asking all over again what’s appropriate for the one situation as distinct from the other.

Yet the unity of the elements, only emerges into its own light (so that we can see them all), and into its full power (of regulating things and responses), as we tell its story—as we talk about our abode with one another, write about it in a letter, read about it in a report. These ephemeral stories pervade our world already, they issue from the articulation of the situation itself—the precarious “togetherness” of all those elements. And such daily talk leads into those fuller accounts of situations in novels, stage plays and other art works—stories that might include lots of strange coloring, perhaps nowhere configured on earth, but nonetheless convince us that they reveal more “how things are” than do our local tales. Why more convincing? Because they draw upon and reveal the structure of the circum-world engulfing us a priori.

Now, if we were to pause to tell fully the story of any mini-script—that of a home, a workshop, a vacation spot, or a society—we would “naturally” fill out the structure to include places and times: but these as characterizing the structure, as provided by the structure, rather than as referring to the space-time grid familiar to us since the Enlightenment (this grid then comes into its own as a modern extraction from our being in a world). Only in such a story can the various elements I listed pell-mell appear as they are, namely in essential relations with one another. The relational whole then becomes clear, i.e. comes near—in the storytelling itself—and thereby allows us to extract a map of the places and a calendar of its events that in turn can be called the structure of the world being told, the structure belonging to any world.

We can highlight four features of this structure that figure prominently in logical study—signification, assertion, non-being, truth:
(1) Purpose pervades structure at every point. That is, one “thing” (moment, item, location, person) sends us directly to others. Whether it is something made, something learned, or something designated as a sign, each signifies. Making, learning, and designating only occur within a significiation belonging to the functioning of a world, to its structure.

(2) Every situation is filled to the brim with talk, or the urgent possibility of it: talk about what arises for encounter, talk with others in various relations to what arises for encounter, talk about situations in their possible integration or disintegration. Some—but not all—of this talk surfaces in explicit assertions (judgements: as in law courts, legislative assemblies, and contests), where decisions must be made about how things stand.

(3) Any circum-world is essentially empty — like gossamer. It presents itself to us not only as absorbing us but also as occasion for... things arising for encounter, others to show themselves. World is by being... nothing. Thus, most everything arising within a world presents itself as not... what it could be, what it was, where it should be, and so on; similarly, situations present themselves as not... solid, coming together, satisfying, not... as they were, as they should be, as they will be. The “nothingness of the world” (as Heidegger calls it) protrudes all the more when one supposes world to be “all there is”—for then various compensations follow (prescriptions for making the best out of an empty life, fanaticisms for reforming others, quick-fixes of a pharmaceutical nature).

(4) Incessantly at issue in a world is the right way to deal with things and situations: whether something is the “real thing” or the spurious article and whether the accounts we hear and give are correct or not.

Each of these topics not only beds itself in our abode but surfaces in logical study. Full blooded logicians have always accounted for them, each according to some Interpretation. Any Fifth Interpretation must do likewise. And Heidegger’s Principle of World can lead the way.

But first there is a question that must already be haunting any attentive reader...

§3.3 Precedents

Had no one noticed? If the workings of world not only predate the univocities of transcendent nature and the unities of transcendental reason—if world already engulfs us—how could the perceptive thinkers of our tradition have failed to enunciate the Principle of World themselves? Must we—must those who now commit themselves to enunciating this Principle—decreed that our predecessors had turned a blind eye to world? May we take pride in our own good eyes?

Rather than follow our forebears in the Enlightenment, let us sift through our tradition to unearth half-buried acknowledgements, even recognitions of the Principle.*

In answer to the question where we initially are, where things initially arise for encounter, our ancestors, shrugging, would all have answered: in a πόλις—a city, a community. It is when asked how we ultimately are in a city, and how things ultimately arise for encounter, that thinkers diverge in their accounts. Plato and Aristotle ask how—but always how by nature (answering to the recognition that the city firmly holds us in its grip from birth to death, as squirrels are held by the trees). Our Judaic and Christian ancestors ask how—but always how by God (answering directly to the recognition of ultimacy, here envisioned as formed by a covenant). Modern thinkers generally ask how—but always how by contract (answering to the recognition of human responsibility for arranging circumstances).

For all the differences regarding the ultimate, there is agreement on the basic, the initial phenomenon: being in a city means being under the rule of law—of “laws,” whether these are stated formally or informally, in written form or in silent gesture, and whatever their origin. Although our being under the rule of law may never grow beyond that of children,

* Cf. Being and Time, p. 66: “...leaping over the phenomenon of world... one tries to interpret world from the being... of those beings that are on hand in the world but initially not at all discovered: from nature”—“nature” in the Kantian sense, Heidegger adds in a marginalium. Moreover (p. 67):

The very fact that [our being-there in a world] does leap over it suggests the need for special precaution in order to obtain the right point of departure for gaining access to the phenomenon of worldliness, and thereby to prevent the leaping.

And such prevention requires a thorough understanding of the original efforts.
to whom law initially appears as the rule of others, and may degenerate into a tyranny even as we ourselves have grown beyond mere subject

to law, the essence of being under its rule is agreement — ἀδιάκοπος, common assent, a λόγος that perseveres over time and place, and thereby

found the possibility of the multiple temporary agreements typifying our initial condition. Thus we naturally raise children not directly into insight

but rather into ways of conducting themselves in the city: in the purview of others and in the face of things (other people and natural events

locating the demands made upon us all). In Greek literature our initial being is called ἔθος — habitation, set manners of being with others and

handling things arising for encounter. And it is out of whatever “habitation” we inhabit that we will grow into whatever we ultimately are:

into the disposition — ἡθος, the “character” — opening us up onto (or closing us off from) what's by nature ultimate for us. 

Fast-forwarding to modernity, we detect stark contrasts. Law is no

longer the source of healthy natural growth, but rather a sometimes convenient, unfortunately necessary constraint on natural growth: shared

assent and divine covenant give way to human contracts, and set manners of conduct become automated and merely conformative responses (“mere habit”). While insisting on shifting attention from the divine to the secular, on showing how we can arrange circumstances to suit ourselves rather than any ruler of cities or creator of nature, modern thinkers so much endorse the centrality of initial human interest — i.e.,

the concern for the “pay-offs” of law, education, and the “new knowledge” — that the context in which we initially find and lose ourselves appears essentially illusory: mere contrivance. So much so that 19th-century thinkers had to resume the question of how we could ever perceive of ourselves as being in there together — with one another and with what arises for encounter. Hegel constantly recurs to the theme: for any context to work, any “politics,

die Menschen müssen dabei sein können: people must be able to be in there, each with his or her own interests, and only thereby together with others as well.

* Cf. Plato's Laws, 792E, and Aristotle's Nicomachean Ethics, 1103 a 17.
† Hegel elaborates this thought in his lectures on the philosophy of history (cf. Die Vernunft in der Geschichte, edited by Hoffmeister, Felix Meiner Verlag, pp. 81-86; translated by Leo Rauch, Introduction to the Philosophy of History, Hackett Publishing, pp. 25-27). See also Hegel's Phenomenology of Spirit, the last pages of Chapter Six on spirit and morality, where the question is how we

If something like the Principle of World still governs the thought of Plato and Aristotle, it disappears altogether as later traditions began to explain the contextual aspect of our condition. For any explanation reduces what it explains to the status of a derived phenomenon. And a derived phenomenon is precisely “accidental”: we may, even must alter it (adjust it, recover it) with a view to what's essential — as Hobbes, Bacon, Locke and modern thinkers generally advise. Moreover, we can blame it for causing all kinds of problems, and especially for distracting us from what's essential.

But why the disappearance? An all-to-easy answer: later thinkers did not remain alert to the fundamental facts of our condition. Another: developing economic conditions rendered obsolete the Greek patterning of the city on the family. A third (most in line with Heidegger's) is that the original (and unavoidable) insistence that φυσις (“nature”) be the origin already concealed the Principle of World. Let us consider...

Plato's Crito envisions the laws reproaching Socrates, should he choose to escape from prison and flee Athens: he would be breaking away from all that had brought him into being, namely the laws themselves, the συνθήκη, the “being-together” that precedes individual birth and growth and establishes the condition of shared concerns. Socrates' acknowledgement here often shocks readers coming from an appreciation of the Apology, where Socrates appears to stand up to and against the demands of his fellow citizens, be they wrong or right in either action or opinion. For we are asked to distinguish between standing up for the city and standing up to the individuals who happen to be in there with us, even governing us.

Plato is the first to raise the distinction as a question — tentatively in his Crito, and then elaborately in his later works. In his Republic, for instance, Glaucot and Adeimantus set the challenge to Socrates: How can an individual choose justice in the most unjust city? Thrasymachus had already outlined the view of justice modern thinkers will enunciate millennia later: the origin of law, namely the coming-together conditioning our passing agreements with one another, is the awareness, on the part of individuals, of the advantage to be reaped should they

can “arrive at being-there” (zum Dasein gelangen). Fanatics such as Hitler, Stalin and Mussolini could only attract their respective populaces — as they so remarkably did — because they offered an alternative to the self-serving individuality that easily precludes “being in there” together.
choose the “middle road” between the best (to do, with impunity, what we happen to want) and the worst (to be exploited, without recourse, by others). On this common view (“enlightened self-interest”), justice is a compromise, a fiction, even a deceit that must be maintained for children but merely played by adults (whereupon the city would never deserve the allegiance Socrates enunciates in the Crito). Socrates must show that there is a genuine justice that individuals might uphold even as they live in a factual city governed by such apparent justice — and, Adeimantus adds, by stories that clearly enunciate the Principle of Private Advantage.

Socrates responds to this challenge by seeking a justice founded in nature: in the nature of the soul and the nature of leadership as founded in that of the soul. Such “nature” is identical to the power at issue directly in each of us separately. This power underlies (explains, sustains) the togetherness (συνθηκη) and the sharedness (δομολογη) obviously organized by the factual cities in which we happen to live, from which we happen to receive our sustenance, and to which we often raise objections. But now the city in which we factually find ourselves is understood as a derived phenomenon after all — one among many beings receiving support only tentatively from what underlies them (besides cities, also trees and horses, clocks and houses). All the more reason, it seems, to take care of such beings in order to forestall, as long as possible, their ever-pending demise.

So the πόλις — so close to World — henceforth appears as a “natural entity” that rightly commands attention. In no way is it wise to treat it as a plaything of those who happen to govern it — any more than a horse can long survive as the plaything of a feckless or willful youth who happens to have received it as a gift. Plato’s account leads directly into the Principle of Governance essential to western civilization that he contrasts starkly with that of the Persian kings who did treat their cities as playthings.*

While we must all learn to be grateful for Plato’s achievement, we should not fail to notice that it henceforth prevents our understanding of the πόλις from being congruent with our understanding of World as an a priori condition. Rather, ever since Plato, those who are wise will cultivate the polity of their separate souls as the a priori condition of their participation in any factual city. For the actual city now resides in λογος — just as does the actual horse for the horse-trainer dealing with passing horses.*

All this is as it should be — at least for us in the western world. Yet in this one case the distinction between factual instance and logical form encircles us all: we have no choice but to work out, or at least live with, “both” cities, i.e. the tension between the two. Thus this one “natural entity” differs from most others (only horse-trainers must work out, live out, the tension between instances and forms of horses). Indeed, Plato and Aristotle never cease bringing city into consideration — as though, after all, it did locate our initial condition.

Of course, the tension does not presuppose two separate items, not even two separate domains. Rather, two interlocked dimensions. We are asked to distinguish the pressure of immediacy (προθεσης), already recognized by children and slaves, from the activation of the intellect (νοησης) appropriate to adulthood and freedom — and especially to guardianship (shepherds of sheep, teachers of youth, rulers of communities). Such activation takes place, if at all, in a τεχνη. And it is here, in the exercise of competence, that the tension takes on its truest form. One who takes care of sheep does so within a city and for his master or his customers — where transactions take place by convention. But to succeed in these services, the shepherd must tend to the sheep as they are by nature. And, in general, anyone who is competent in handling affairs will be caught in this tension between convention and nature — even if the handling of the conventions of human intercourse becomes for most of us “second nature” (εθος). Thus, even a fortiori, serious intellectual work, too, will concentrate attention on “what is by nature” and only sply out into considerations of “what is by convention.”

Right from the start of our intellectual traditions, the origin (αρχη) of contemplation is nature (φυσης) as against convention (συνθηκη). This origin pertains not only to what happens around us (what arises for encounter) but also to any human enterprise that gets off the ground. Just as plants and animals “catch on” and “get going” as they are their proper nature (and they “lose out” and “fall apart” as their nature withdraws), so

* Plato’s Laws, 690E ff. Cf. also Werner Jaeger’s Paideia, Vol. III, pp. 234-7. Aristotle continues and strengthens the principle when discussing at length the question whether the best law or the best person should rule, concluding that law should rule and yet can only do so with the help of what we today call an “executive” (Politics, 1288 a 1).

* Cf. Republic, 592A. The analogy with the horse-trainer is here only implicit.
too, and in exact parallel and interconnection, we ourselves catch on and get going, in whatever we are learning, as we enter into a craft that attunes us to the nature at issue in whatever we have learned—whether that of plants and animals (for farmers and the like), that of earth, air, fire and water (for blacksmiths and the like), that of points, lines and planes (for architects and the like), that of pitch, rhythm and harmony (for orators and the like), that of movement, rest and time (for physicians and the like), or whatever.

The “true beginning” of western philosophy is φύσις—nature as looming through convention, as grounding the context in which we find and lose ourselves.8

Of special logical import for the tension between convention and nature is Plato’s Cratylus. As the dialogue opens Hermogenes tells Socrates that he sees no way in which names (δόματα) can be correct or incorrect except in reference to convention. As readers, we recognize not only that school children believe this (since for them the correctness of their behavior, speech and answers to questions is certified by what teachers happen to have decided) but also that older cynics will also believe it (since effective behavior and speech in transactions does in fact depend on one’s ability to play on convention).

Later, Cratylus, when questioned by Socrates, agrees that “primary names” (τὰ πρῶτα δόματα) can be revelatory (δηλωμένα) of matters (πράγματα), and that our task is to make them “fit” (οἶκος); he thereby disagrees with Hermogenes, who maintains that names are revelatory only to those who already know the matter, so that names themselves are only “conventional” (συνθημένα). However, Cratylus rephrases the “fit”: names are “resemblances” (δομοιόμενα) of matters (Socrates even adds that they are “mimings,” μιμμένα).

The tension between these two views defines much of the dialogue. In the end, though, Socrates intercedes with his own view: while convention in fact predominates naming (so that common assent (δημολογία) does in fact rule over the correctness of names), names must ultimately “resemble” the matters they name. It is only because the draw (ἡ διάκρισι) of resemblance is stingy (γλίτσα) that convention must serve as an expedient.9

The crux of the discussion emerges when Socrates asks Cratylus wherein the power (δύναμις) of names lies, what “good” (καλὸν) they are. Cratylus answers: their power and their good lies in instruction (διδάσκειν). For learning names and knowing matters go hand in hand—very much as we routinely assume when concerned that the youth learn a trade, a vocation, or an academic subject. Yet such learning can mean familiarizing oneself with accepted manners of behavior and of speech. In contrast, genuine learning, full knowledge (ἐπιστήμη), happens only within the ability to “search and discover” matters themselves. Thus the question comes down to how it is possible to become one who names correctly from matters themselves (from beings: τὰ οντα) rather than by accustoming oneself to the names one has received. And is that not our problem, especially as logicians? How one might become a “primary positor of names”?9

* Cf. Heidegger’s Introduction to Metaphysics, p. 10 of the original 1953 edition. On pp. 29–30 he emphasizes the purpose of the contrast:

To question how it stands with being—this means nothing less than to repeat, to fetch back, the beginning of our historical-spiritual being-there in order to transform it into the other beginning. Such is possible. It is even the measure-setting form of history, since it ... by reducing it to something past and now familiar and simply to be imitated, but by our beginning again at the beginning, more primordially, with all the strangeness, darkness, and insecurity attending a true beginning.

Especially in his posthumously published writings we find Heidegger returning ever again to the distinction between the “first beginning” (nature) and the “other beginning” (the happening of world). See Beiträge zur Philosophie (1989: Volume 65 of his Collected Works), p. 171 (any primordial appropriation of the first beginning entails obtaining a foothold in the other beginning) and 185 (any leaping into the other beginning requires a return to the first, and vice versa). Also Metaphysik und Nihilismus (1999: Volume 67), p. 135 (the other beginning of thinking examines the truth of being as event, as teeming). And Die Geschichte des Seyns (1998: Volume 69), pp. 23, 27, 30, 132, 135, 143, 154, 213.

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To question how it stands with being — this means nothing less than to repeat, to fetch back, the beginning of our historical-spiritual being-there in order to transform it into the other beginning. Such is possible. It is even the measure-setting form of history, since it starts at the basis of the happening. But a beginning does not get repeated by reducing it to something past and now familiar and simply to be imitated, but by our beginning again at the beginning, more primordially, with all the strangeness, darkness, and insecurity attending a true beginning.

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9 Of course, all along in the dialogue we are asked to consider what we might mean (naturally or conventionally) by “names,” “correctness,” “matters,” “fitting,” “resembling,” and even “miming.” The dialogue deserves much more careful reading than my sketch intimates.

† Cratylus, 436B: ὁ δὲμενὸς πρῶτος τὰ δόματα.
And, ever since, logicians presuppose the contrast—from Aristotle all the way through Frege. The discipline of logic appears to be defined by the supposition that the sense and direction obviously at work in names owe their origin to the indication and revelation of matters as they are “themselves by themselves,” i.e. of “nature” prior to the sense and direction evident in “convention”—whether the “nature of things” be called οὐσία (our estate in its wholeness) or more modestly Sachverhalt (the way circumstances are shaping up).

And the result of this leaping over the consideration of world is that we seem justified in proceeding directly to the analysis of signification, assertion, negation and truth as though we ... and withdrawing into purely syntactical considerations—whereupon one must fight to retain the title of logician.

And another result is that contemplation loses ground: topics contemplation once drew into some consideration, only to be relegated to secondary status, eventually resurface in other guises. Habit eventually appears as conditioned reflex to be studied by psychologists, convention eventually appears as societal arrangement to be studied by sociologists, anthropologists and historians, law eventually appears as behavior-control to be studied by jurists—and nature becomes a storehouse of materials to be studied by chemists, physicists, geologists, and biologists. Each study draws pictures of the structures pertaining to its topic. And even though many will insist that these structures do in fact encircle us, the encirclement is never directly evident but only presupposed. Such structures, belonging as they do to the investigation rather than to what is investigated, are regulative rather than constitutive (in Kant’s sense)—they are constructions modelling data that, taken as “evidence,” seem to “support” the pictures drawn, i.e. themselves.

But what, now, if we can “bring near” our world, our abode—something like convention—as having its own integrity, and thereby its own tensions of signification, assertion, negation, and truth—all prior to those we recognize as pertaining to nature? Prior, that is, to everything we may discern as arising for encounter in our abode, as well as to everything we may discern about our own way of rising to it.

§3.4 Signification
To be in a world is to be caught in signification: to be caught up in, to be carried along in, to be lured on through structure. I “always already” belong to this more or less overwhelming, at least overreaching signification—as do others (whose absence may make them all the more evident). What arises for encounter also belongs to the signification. Only derivatively does the signification belong to me (as when I become a leader) or to others (as when they stake out claims on a situation). Only derivatively does signification belong to the tools, the plants, the animals, the mountain peak, the starry sky, the forest fire, the raging ocean (as when we acknowledge that our own continuation in the signification depends on how these things turn out).²

The name “signification” intends to bring near the workings of world, the movement essential to the structure of world—what “holds things together”: me with my circumstance, others with me, each tool, plant, animal, mineral together with us. As “composing” all these, and not as a “composite” to be explained in reference to its elements.

Signification happens—is as an ongoing affair “always already” engaging us, demanding that we move along within its own hegemony. Thus even reflection on it appears heretical, a mode of withdrawal from the hegemony, one that we in the West have learned (gradually, painfully) to tolerate somewhat—mainly to the extent that it has led to enhancements (as Thales’ reflection on the heavens proved itself in his ability to corner the olive market: see Aristotle’s Politics, I, 4). And even more heretical the contemplation setting out to bring world near and not merely some focal point within affairs in relation to some other (e.g., how meteorological conditions in winter relate to olive crops in summer: reflection in the narrower sense).

Not only heretical but also strangely difficult. It may help to note two somewhat opposing modes (possibilities) of engagement in world: arduous play and routine dealing. In chess-playing and football-playing signification explicitly defines the happening. Not only do such games

² I think of Jack London’s “To Build a Fire”: “everything depends” on getting that fire started. —In contrast, Aristotle asks us to discern what belongs to subjects, Augustine asks us to discern what belongs to God, and Moderns from Descartes and Locke through Kant and Hegel ask us to discern what belongs to our thinking, our discerning—to mind, to spirit.
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prescribe rules regarding movement (the incessant repositionings), competition (looming outcome: victory vs. defeat) makes explicit each item as signifying and each gesture of ours and others as both following the signifiers and in turn signifying. And there's always a clear centerpiece, or set of items, around which all others gather (on the board, on the field: the arrangement at the moment, the position of the ball). Such games also make evident the centrality of orientation in space, directedness of gestures, timing and time-constraints.

Moreover, the signification evident in games happens as a whole—a whole in which the multiplicity of items signify each other interrelationally and also the whole itself:

The relational whole of this signifying we call signification. This constitutes the structure of world—the structure in which our being-there in any instance is.*

What discloses itself more clearly in games (because already explicitly at issue for the players) can disclose itself to us, in contemplation, as characterizing even the most soporific routine. Indeed, world can appear in the mode of “mere routine” only because the relational whole precedes—conditions the possibility of—all those signifiers appearing on the chessboard or the playing field. The relational whole of life at home and at work, of handling chores and attending funerals, are as routines (conventions, habits, customs). And therefore they may become “merely so”—all the more catching us up, pulling us along, luring us through.

One prominent feature of being in a world is our own ability to move within it, to sustain its movement, to move along with all those signifiers. This ability always appears at risk, and so becomes pronounced in various forms of effectiveness: we can be better or worse at those games, even in those more soporific routines. We might then worry about our own failures or take pride in our successes. We might also stand in awe or envy, pride or disgust, at the level of ability evident in the performances of others. But apart from the different levels, apart from questions of competence (τεχνη), being in a world “always already”—at every moment—engages us in a multiplicity of prospects: things can go one way or one of several other ways, I can pursue one or some other. Thus the prominence of this one feature of being in a world: multiplicity hovers over even the dullest routine, apart from any specifiable competence. And with it the question looms whether I can incorporate what arises for encounter, doing so according to the “rules” afforded by the signification defining the world in which I find myself (whether making a move on a chessboard or washing dishes after supper).

In short, the ability more or less explicitly at issue for us, in a world, pertains first of all to possibilities looming at every moment and setting the tasks of the moment. Considered as a question of my individual performance at the moment, the ability seems to be mine. Yet the world that engulfs me specifies the ability. Whatever situation we are in, there is an understanding belonging to it, and our role in this understanding “always already” pertains to possibilities in it—“always already” embraces both the recognition and the anticipation of the multiple “twists and turns” invading each moment.

World is intelligible, we understand what's going on, precisely inasmuch as it appears as a multiplicity of crossroads already locating our own “stand”—and not because it presents to us a fixed outline with pre-determined junctures and pre-determined gestures. At times, the multiplicity, the indeterminateness bothers us; at other times we achieve an overview and the road to take clearly protrudes, so that we feel very confident or even very distressed at “where things are going.” But prior to these special times, our being-in-world already consists of, hangs together with, understanding: our ability to move within looming possibilities.

Or, rather, to be moved. For our involvement in signification is not one of first reading maps and then choosing roads to take. Rather, each item arising for encounter already sends us to the next—without, however, assuring which “next”—and our choosings usually takes the form of re-readings and course-corrections (even “damage-control”). The sending is here first of all minuscule: one note played sends me to the next (whether I'm playing or listening), one word uttered sends me to the next, the slightest slip on ice sends me to another gesture, spilled milk

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* Being and Time, p. 87. On “being-there” Heidegger eventually added a marginalium: “the being-there in which human being has its being.” Readers often miss the point that Heidegger's analysis of “being-there” focuses contemplation on world, on being-in-world, on our being in there—precisely not on ourselves (as modern philosophy has encouraged us to do). In his later works (especially his posthumously published notes), Heidegger insists that the culmination of contemplation lies in achieving this focus rather than in any results that may follow from it. Or rather allowing ourselves into it.
Axiomatics: The Conditions of Truth

Heidegger succinctly formulates the remarkable collusion of what, in reflective hindsight or foresight, appears incompatible: the understanding already at work in my being in a world (and allowing for both flexibility and rigification) both holds the complex of interrelations in an a priori disclosure (holds the whole complex “up front”) and lets itself be “referred” within this complex and thus by each item arising for encounter. Thus I can “feel” both free (the complex is mine to dispose of) and necessitated (I am sent from one thing to the next). Herein lies the phenomenon (the coming to relief) of signification: the happening that allows all those minuscule “signifiers” (then, too, the majuscule ones, i.e. signs) to function, our own ability to become an issue, and the world to make sense (hang together and have direction).

Such signification precedes the distinction we must make in logical study between meaning and reference. Signification grounds (provides the necessary condition for) both. As direction-giving, as moving (us) toward the next thing, signification institutes meaning, provides “predicates.” As allowing circumstances to reveal themselves, one at a time and together, signification institutes reference, provides “subjects.” And these provisions are open at every moment to revision: correctness is always at risk — and so also, but very derivatively, “reference” in Frege’s sense (truth and falsity).

The distinction between meaning and reference (a possibly very legitimate one, and very powerful) arises in a derived mode of being in a world — namely, in the mode of cognition most obviously at issue in modern academic settings.

Already in Antiquity, knowledge (ἐπιστήμη) named a strengthened relation, on our part, to things as they are — to nature. The paradigms for this epistemic relation were drawn from “hands-on” manners of dealing with circumstances. Plato and Aristotle then inquire into the source of these manners, answering in unison that this source is intellectual contact (νοῦς as the “faculty,” νοησις as the activation): contact with nature (φύσις) as what accords each thing its being (both that it is and what it is — what it must be). Modern philosophy first worked out the kind of knowledge we today more easily recognize as the “issue” in our institutions of research and education: systematic determination of what arises for encounter (including human ways of rising to circumstances), but now divorced from the hands-on paradigms of Antiquity — and therefore in a contrived isolation from world. To know something academically (“objectively”) is to determine how it arises in isolation from our care (as Heidegger puts it). Such knowledge-at-a-distance then seeks its coherence in the conditions of the investigation itself — a mode of being in world that “takes” things not in their terms but in terms of the methodology defining the particular investigation and “reports” only what arises for determination within these pre-set conditions.

Once it became clear (as it did to the sharpest thinkers of the 19th century, and even to academics after the First World War) that science in our modern sense has no exclusively right access to “the way things are,” thinkers generally, and not just Heidegger, recommended that we return to pre-scientific being-in-world and bring this into view not only as the condition for the possibility of knowledge but also as locating the questions and urgencies addressing us all prior to academic concerns and therefore addressed by those academics working in the “humanities” at our educational institutions.

The battle is not over. As John Dewey testifies around the same time as Being and Time was being written:

Knowledge is still regarded by most thinkers as direct grasp of ultimate reality, although the practice of knowing has been assimilated to the procedure of the useful arts. . . . [It should] be seen that science is an art, that art is practice, and that the only distinction worth drawing is not between practice and theory, but between those modes of practice that are not intelligent, not inherently and immediately enjoyable, and those that are full of enjoyed meanings. When this perception dawns, it will be a commonplace that art — the mode of activity that is charged with meanings capable of immediately enjoyed possession — is the complete culmination of nature, and that “science” is properly the handmaiden that conducts events to this happy issue. *

* Experience and Nature, 1925 (pp. 357-8 of the Dover edition, 1958). Of course, Dewey has his own way of pursuing this thought of “priorities.” In his Art as Experience (1934), he shows how scientific statements lead to experiences, while discourse at its best constitutes an experience (pp. 84-5 of the Putnam and Perigree editions). — Many efforts to return thoughtfully to pre-scientific experience do so in the name of science, with the result that they...
In contrast to our first involvement in world, our scientific involvement rightly insists upon, or rather itself institutes, the difference between what is encountered and how it is encountered. An ontology, then: “what is” is two-foldedly. Plato and Aristotle rooted this twosome in what is encountered, modern philosophy roots it in how anything is encountered. Both illustrate what Heidegger calls the “ontological difference” essential to western metaphysics (to western intellectual work generally). And both “leap over” the phenomenon of world.

Of course, our immediate inheritance is that of modernity. And here the difference between meaning and reference commits us to a very remarkable version of the ontological difference, namely that of the Cartesian Grid. For here “reference” is determined four-foldedly in space and time. Such determination locates the \( x \) in regard to which a conglomerate of meanings can focus — when, that is, it returns to its reference. So much do this understanding of space and time undergird academic research and institutional operation that it appears “naturally” basic to being in world. Yet, pressed to its own fulfillment, this understanding blatantly conflicts with the understanding of space and time essential to our first being — with the understanding marking out our own being, with the intelligibility marking out our circumstances, and with the signification in which both we and our circumstances move.

What happens as we rise in the morning, handle matters during the day, and retire at night? First and foremost, this: the signification carrying us along looms as a whole, and at every moment. This whole is finite: it conditions the incessant preparation, as though everything had yet to be done. Yet it is also “long established”: it specifies fixed patterns to be repeated over and over again, as though there were nothing new in the world. Still, the task is to do each thing appropriately, as each moment requires. — Here, prior to the construction of a time-line and of coordinates in space, “future” is precisely the signification looming as a whole. And as a burden, a fixedness that we can now dissect as having a “past” and trace on a time-line extending backwards — and that, oddly, we now extend forwards in an effort to “straighten out” the signification as it looms as a fixed burden. Yet our first involvement within the looming alreadiness neither dissect nor traces: it heeds. For at every moment our ability is at risk — whether it meshes with what arises for encounter, fits it, does justice to it.

Primordial time, as Heidegger occasionally calls it, roots in the looming of the signification defining a world: this looming locates what we in contemplation must acknowledge as the future that allows us to prepare and foresee — and, optionally, to forge a time-line mapping possible interventions in the circumstances disclosed by the signification itself. But, on site, we first of all “must” — on pain of death or injury or simply failure — allow the signification to focus us on what arises for encounter: here, time appears as exigency, as “ripe” or “opportune” — as \( το καιρον \).

Similarly with what we now take for granted as “space.” We first of all encounter things in or out of their places. But places here are places of things. Each thing is its referring-us to a next-thing. And such reference “takes place” as a whole — the whole of the signification defining a world. Thus each place is within a region — the “place” of work, of play, of rest: my place, our place, the integrity of which looms at each moment. Thus signification “gives” a region: offers it, bestows it — so that it becomes possible to acknowledge it as a gift, to uphold it, even to defend it.

If primordial time pre-eminently marks out our ability (namely the ability to stand up to the exigency of possibilities already looming at every moment), primordial space pre-eminently marks out the region of things that may (or may not) become home for us. Home as what allows us to dwell and heed rather than only chase after things. As what allows us to heed what arises for encounter as itself embedded in a whole that also demands our care.

On this account, primordial time and primordial space “ground” the times and spaces we necessarily presuppose in logical study — presuppose as undergirding the distinction between meaning and reference and thereby the whole project of extensional logic and the extensional set theory growing out of such logic. But “ground” here does not have the status of something we can “get to” by reflection. Rather, it has the status of what we suppress in “where and how” we already are. The task of contemplation, then, is to “bring near” or “recall” the origin (world) as we work within its structure of signification. Or, better, and as Heidegger has formulated it, to allow ourselves back into the origin even
as we proceed from it. And such “allowance” itself becomes possible only as we acknowledge that this origin already looms ahead.

Heidegger’s account provides the basis for distinguishing between structure and construction. Structure we may call the relational whole of signification—a whole in which we find and lose ourselves, world itself. Whether as gardeners, politicians, musicians, chemists or simply idlers, we will of necessity understand circumstances structurally—because, as we can witness in contemplation, the abode through which we move, and into which things arise for encounter, is structure itself. Because our abode is structural, we can highlight some particular structure of the garden, the city, the music, or whatever, and thereby refine our understanding of the exigencies of (but now appearing in) the garden, the city, the music, or whatever. And thereby anticipate and even forecast what will arise for encounter.

But we can also opt for a sideline position: that of the spectator or critic or sportscaster. From the sidelines, the original unity of pre-casted fore-casting dissolves into a mere mixture of what has already fallen into place and what the workers or the players or the idlers are fore-casting. Now the whole affair appears “out there”—and as a “mere construction”: the rules of the game, then also the interpretation of what has happened “to date,” and finally the plans for action—all these taste strangely arbitrary and essentially alterable (especially the interpretation of what has happened “to date”). And if we ask what is compelling about the work or the play as a whole, or why relations among things appear necessary, or why our own responses take the form they do, we resort to the supposition that these conditions are essentially social (conventional) and therefore urgent only for one who chooses to move within the particular society of gardeners, politicians, musicians, chemists, of whatever.

The “constructivist” interpretation of knowledge repeats the “minority report” already lingering over Greek thinking regarding the “convention” (συνθήκη) — whether “correctness” is just a matter of custom (Εθός) and mutual agreement (δομολογία). For two millennia, the retort (the “majority report” issued by great thinkers) has insisted that the exigencies of nature (φύσις) — as evident in the competence (τέχνη) necessary for handling what arises for encounter — require that we acknowledge a “ground” precisely for convention. Even Kant’s effort to account for the evident exigencies of experience and knowledge posits a “ground” — namely the a priori conditions of the possibility of experience, but these conditions understood as reflecting human nature (prior, of course, to human convention). And, just as these traditional efforts “leap over the phenomenon of world,” so too does the “constructivist” interpretation. Contemplating world as prior to both what arises for encounter and our own agency, and distinguishing between “inside” contemplation and “outside” observation, we can assess exactly how “structure” names our abode and “constructions” extrapolate from our abode and are both correct and variable.

Heidegger’s account also provides the basis for raising once again the question of rationality. The easy answer, washed up on our shores from traditions, assures us that rationality is one property of human beings: we have the ability to doubt configurations of human response and natural circumstances, then to figure out the why and wherefore, and finally to devise new configurations. Traditionally, too, this one property parallels another: the ability to be affected by configurations, to respond to these in ways expressing our being affected by them, and finally to endorse or decry them according to the satisfaction or frustration they accord. On this scheme, rationality represents part of our nature.

We hear this account of rationality over and over again in the debates over which part of our nature deserves enhancement or repression in various situations. The account also figures strongly in modern attempts to educate and channel human behavior (whether in politics or in commerce, in school or at home). Yet this easy answer not only blocks contemplation of the basis of doubting, figuring and devising, it also suppresses the conditions from which such “rationality” has been extracted.

World itself is rational—quite apart from whether people or things within it have “rationality” as a property. For it is essentially structure, a finite relational whole wherein things arise relationally already and already send us along to next things. Only then can things appear to behave in bizarre ways. Only then can human agency take bizarre forms. Only then can we re-assess and rearrange circumstances. Only then can we find our own performance, and that of others, distressingly ineffective—and continue to deplore them or proceed to improve them.

At the beginning, however, the question of rationality was not so easy, and Heidegger’s account can help us appreciate its vigor. Plato’s and Aristotle’s works, and still Kant’s, approach rationality as a dual phenomenon: dianoetic and noetic, discursive and intuitive. Its dianoetic
dimension has its \textit{home} in the signification essential to being in a world: movement through structure. Its noetic dimension has its \textit{home} in the taking-up-oneself of one's situation as a whole — “readying resoluteness” as the carrying of the burden of the wholeness as it looms ahead and yet burdens us from behind. On this account, rationality is rooted in a threefold attendance: to the fixedness of world, to the opening of world, and to the wealth of world — to world as thrown at us (as “old”), as throwing itself forward (as “new”), and as presenting things to be taken care of and people to be cared for. Our being-in-world — in any given situation — is to be tending (moving and caring) in three directions at once. And only keeping these three together can we ourselves “exercise” rationality — namely, the rationality belonging to world itself (to our being “in there”). Otherwise we are just carried along, and rationality appears as conformity to the incidental flow of the affair — as convention only.

\section*{3.5 Assertion}

Looming, and thereby simultaneously both pre-cast and fore-casting, signification forms a \textit{discourse}. Every moment is as a multiplicity of \textit{sayings} in which we participate. Prior even to moments of “reception” or “inauguration” of any saying, we are \textit{engulfed} in sayings.

From all the many kinds of saying, Plato and Aristotle select one for special consideration: sayings that make \textit{claims} about how things are or can be (and, derivatively, about how things were, must, might, should be). Such sayings are indeed special in the discourse at issue at the birth of the West: in the discourse of legislative, forensic and epidemic assemblies where speakers compete with one another — as well as in epistemic discourse, where speakers and listeners alike attend to the nature of things (the sciences of music and astronomy, then too of arithmetic and geometry, and finally of the movement of minerals, plants, and animals).

And thus was born a child first called “analytic” and later renamed “logic”: the \textit{analysis} of claim-making \textit{λόγος}. This kind as constantly contrasted with other kinds: sayings that direct, urge, request, discourage, hearten, delight — even those that inform and dictate, sentence and promise.

A saying that makes a claim deserves a name of its own: assertion (in some dialects: judgement or proposition). The birthmark of an assertion is its possibility of being either right or wrong, and thereby our ability to affirm or deny it (a sign of its provenance in competitive and epistemic discourse). The field of study we call logic evolved as a form of questioning the conditions for this possibility of discourse, and of our own ability to engage in it.

In brief, Aristotle's supposition is that the possibility of an assertion's being right or wrong depends upon the difference between its subject and its predicate: whether and how the predication “accesses” the subject — reveals it.²

Again in brief, Kant's supposition is that this possibility depends on the difference between a manifold of encounters (cognitions) evolving into a sustainable or unsustainable “unity of apperception”: any one assertion receives its integrity from the integrity of a whole discourse.³

Both suppose a \textit{bonding}:

How do predicates bond (us) with subjects? Plato and Aristotle answer: by way of “nature,” \textit{φύσις}. For human activity of any sort appears in contemplation as beholden to the growth and decay of things — of plants and animals most obviously, but then too of soils and lakes, and then of course human artifacts such as ships, houses and clothes — and the exigencies of these (their nature) provides the bonding of human response (predication) to what arises for encounter.

How do cognitions bond (us) within a manifold? Kant answers: through our own unifying power (productive imagination, Kant says;

² While Aristotle bequeathed the vocabulary of subject and its predicates (τὸ ὑποκείμενον and \textit{αἵ κατηγορίαι}), Plato initiated the distinction as one of nouns and verbs (τὸ ὅντος and τὰ ῥήματα: cf. his \textit{Sophist}, 260d-263e).

³ \textit{Critique of Pure Reason}, B141: “a judgement is nothing other than the manner by which given cognitions are brought into an objective unity of apperception.” In Kant's sense, “cognition” is then “microscopically” \textit{prior} to judgement. Hegel first extended the thought “macroscopically” (see his \textit{Phenomenology of Spirit}, Preface: propositions are true only in a “system” and cannot be shown to be such in isolation, since each changes its meaning according to our own progress within the system). Whitehead highlights the \textit{formal} aspect of this thought when noting how mathematical logic repudiates the primacy of the subject (reducing it to an \textit{x}, a “bare objective for consideration”): see Book Four, §4.3.2.)
In his *German Ideology* (1845-46), Karl Marx, a scholar of Greek philosophy in his youth, detects the source of the agony of modernity, namely of what we usually call the “industrial revolution,” in the historical evolution from nature-based-ness (Naturwüchsigkeit) into society-based-ness. This loss of Naturwüchsigkeit we must learn to accept at several levels: artisans no longer enjoy ontological priority (the division of labor no longer pertains directly to natural exigencies), capitalistic industry now... relate directly to the material conditions of their own lives rather than to the exigencies of transcendent nature.

Critics who have failed to understand Heidegger’s works are prone to complain that he does not offer an account of nature, of society, of ethics and of the divine — a metaphysics. While the intent of some “anti-metaphysical” works is to purge us of our traditions, Heidegger’s works intend to re-open these same traditions — and thereby provide openings through which nature, society, ethics, and the divine might re-emerge from the penumbra.

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How do assertions arise out of world? What bondings does world itself provide that are then fixed up or dimmed down when we find ourselves involved in claim-making? To answer, to address this question, we must return to discourse itself — the happening of our talk before it evolves into claim-making. Again, then, everything depends on the art of “bringing near”...

First a distinction: Much talk takes the form of re-talk, and this we must also learn to consider as derivative, albeit in a way apparently opposite to the way claim-making is derivative. Re-talk occurs as a mode of saying and hearing exclusively what has already been said and heard: the sort of talk into which academic lectures easily fall, but also the sort that simply consolidates the togetherness of those present (sometimes, as in vicious gossip and political harangue, with a contrast to those not present). While such merely consolidating re-talk appears to make claims (about the weather, about colleagues, about injustice or foolishness or perversity of others), it includes no genuine effort to assess these — whereupon they forfeit the logical status of claim-making. While re-talk obviously derives from the prior talk at work in the signification structuring a world, we must turn directly to this source in order even to understand how it, too, lacks much that is essential to talk actually at work.

At-work talk happens in our own being at work: it accompanies our getting something done — whether in the kitchen, in the grocery store, in the laboratory, or anywhere “out in the field.” But the original accompaniment is integral to the work being done — unlike both re-talk and claim-making, both of which (although in remarkably different ways) idle (as an engine idles: cf. Wittgenstein’s *Philosophical Investigations*, §132).

What can we say about primordial talk? What can we “see” — hear — happening? At least four things. Such talk (1) focusses on something, (2) does so sharedly, (3) does so with some degree of urgency, and (4) takes a shape of its own. (Cf. *Being and Time*, §§33 and 34, especially pp. 162-3).

(1) Talk focusses: whether as speaking or listening, reading (e.g., instructions on a bottle) or writing (e.g., a reminder), we address something arising for encounter — or, rather, find ourselves addressed by it. What sort of things address us in our talk? Some matter of concern, one that is itself in-the-making and engulfing us — an affair (πραγματικός: a
rich word often misleadingly translated as “fact”). Within this “matter” on which talk focuses we there is also an itinerant focal point (the contents of the bottle, the item on the grocery list). Such focal points we also call things, but these things are in the flow of the affair, a multiplicity (manifold) of things arising for encounter — within a world.

In primordial talk (shop-talk, as it were), what the talk addresses, what addresses us in the talk, what the talk is about… are things in use. That is, things at hand.” Such talk can appear “shallow” — but not because it has no access to things, but rather because it accesses them only for the sake of getting on to the next things.

(2) Talk shares: when speaking or listening, reading or writing, we focus together on what addresses us. Since primordial talk focuses a world on what arises for encounter, no one else need be present to our eyes or ears. Even a note to myself signifies by virtue of a world, namely a shared affair (that of driving, shopping, paying,…). Sayings belong first of all to world itself, to its signification. Precisely when quarrelling with one another, we are “of one mind” (one world); indeed, much contentious talk, as between lovers or married couples, arises precisely because the sharedness appears clouded, oppressive, or threatening. For sharedness is tenuous not in that it might fail to happen at all, but in that it might not become strongly communal (as can happen in teamwork) and remain merely routine.

(3) Talk evinces: since every saying (uttered, heard, written, read) already brings a world into play, it already embodies a style, an attitude, a posture — already sings or stutters, endorses or hides its pre-casting and forecasting. Even a posted sign reading “No Parking” evinces a command, even a threat. A passing “Hello!” may evince an acceptance or a hesitancy of affirmation. Remarks at work may establish a mood of hope and encouragement, of cynicism and skepticism, frenzy or calm — this mood now permeating the entire situation. Talk may appear authoritarian or cowardly, conciliatory or arrogant, confident or hesitant. The mood of the talk is the mood of the situation itself. If consolidation of the enterprise counts, so does the modality of its talk — especially that of its leaders.

(4) Talk formulates: words are written on the posted sign or a scrap of paper, I utter a sequence of English words or simply say “No!” when asked whether it’s time to break for lunch. Talk takes the form of visual or audible gestures — sometimes even tactile, gustatory or olfactory ones — that manifest focal points within the focusing of the situation at each point. Rooted in the signification of an already-shared world, these gestures are themselves already shared and already carry a mood of urgency (or lack of it) while they are focusing us (our world) on what arises for encounter.

Such is talk at work: it focusses on things at hand, it belongs to a shared affair, it evinces a mood, and it takes manifest shape in gestures. Yet talk also talks when not at work. What happens in such re-talk? Various losses: not in use (idling, as it were), things appear in the talk randomly, coming in and passing out of focus — as a photograph too long exposed for something in motion. Not pulled together at focal points, the togetherness essential to discourse appears contrived — as in a lecture where the speaker fails to capture the attention of the audience and yet the hour captivates all alike. Not assumed as a task, the situation presents burdens that are only elsewhere and likely arouse indignation — as when one reviews newspaper accounts of the actions of politicians grappling with the vagaries of the economy or the threats of war in far-away lands. Not serving a shared and exacting focus on anything, words and other gestures take on their own life, so much reflecting the speakers (or writers) and so much affecting the listeners (or readers) that the gestures themselves become the “point” of attention — as in much speech-making in classrooms, cafés and legislative assemblies, as well as at banquets, funerals, and the like. Such talk echoes the primordial talk from which it is itself a kind of “leftover” that may be simply warmed up and served ever again. But it also hearkens to its origin, and may set to work again at any moment — if only we can find ourselves back in harness. We must take stock of it — bring it near in contemplation — in order to protect ourselves from the temptation to take re-talk as the measure of talk itself. (Cf. Being and Time, §35.)

Clearly, neither primordial talk nor re-talk consists of the claim-making at issue in the study we call logic. For neither one suspends sayings in order to examine them and to decide whether they are true or
false—as does the specialized talk of forensic investigation or scientific research. Just as clearly, however, assertions issue from primordial talk (and are side-tracked in re-talk). Assertions are the children of those four parents: focusing, sharing, evincing, and formulating. But how? To answer this question—so crucial for the contemplative study of logic—let us bring near a situation where one is indeed concerned to determine the status of a claim—where the claim not only appears in the modality of possibility but also demands a careful decision whether or not to affirm it. What happens here—in contrast to what happens in primordial talk?

(1) Instead of “simply” putting things to use, we here take pains to lay them out before us. The focus of attention is now something encounterable apart from its manner of sending us on to other things in the affair. Rather than smoking the cigarette, or offering it to another, I look at it to determine its properties: What materials compose it? How much has it been smoked? Are there signs of what sort of person smoked it? In short, predicates become the issue. More importantly, our understanding of the “thing itself” (the subject) has changed. What arises for encounter, no longer engaging us in its world, now appears as stock available for careful inventory: things no longer appear as at hand, but rather now as on hand—a mode of being reduced from that of at-hand-ness. What lies before us (τὸ ὑποκειµένον) must be fathomed—since now we stand aloof from it, within a contrived world of investigation, and every determination of it belongs to this world rather than to the world in which the “subject” originally arises for encounter and to which it originally belongs.*

(2) Instead of “simply” taking part in a joint enterprise, we here (in investigation) take pains to convince one another of predications one of the other.

* In Being and Time Heidegger employs the metaphor of “dimming down” to describe the effect of this historically developed mode of being: “theoretical looking on has always already dimmed down the world to the uniformity of beings on hand” (p. 138: cf. Nietzsche's Birth of Tragedy, §15, on “the theoretical man”). Similarly: positing something (e.g., a hammer) as a subject of investigation dims it down to a “this-here” (p. 155: cf. Aristotle on τὸ τοδὲ τι, e.g. Metaphysics, 1030 a 6 and 1070 a 10). Also: everydayness dims down the uncaniness evinced by dread (p. 189); the levelling essential to everydayness dims down possibilities (pp. 194-5); being-toward-death does not dim down the threat essentially embedded in our situation (p. 165).

us determined and now presents to others. At issue in a philosophic discussion (dialectic) is the insight of another (and of oneself). Yet in merely instrumental investigations the issue is a result that the group may then present to the public—to the “media,” say, or to a trial court—whereupon it serves the interests of re-talk of various sorts. To remain investigative, discourse must retain an articulated invitation to testing its claims.

(3) Instead of simply carrying the burden of the whole as it happens to weigh upon us—i.e., moving within the multiple exigencies of the situation as it presents them—we here carry it in one way: dispassionately. The mood appropriate to investigation is one of distance: disinterested, detachment, impartiality. Yet this very mood is itself a passion—an insistence on being disinterested, a desire to be detached, even a pride in being impartial. This mood, this way of rising to what arises for encounter, first allows us to establish a methodology, i.e. a world of our own pre-establishing what arises for encounter, whereupon we can then remain open to whatever may happen within this framework. The mood of objectivity consists in systematic pre-judgement precluding all other pre-judgements.

(4) And, finally, instead of saying, hearing, seeing, gesturing in consonance with the significations of world and focussing on the at-hand things “naturally” at issue, we here turn back on our various “native” responses to reformulate them as themselves the issue. Since Plato and Aristotle, the vocation of philosophy has assumed the task of contemplating this modification of discourse. As a form of reflexivity, the modification is radically historical: it precedes our contemplation of it (philosophy does not first institute it) and it defines the world we have inherited (especially our institutions of higher education, where each discipline establishes its own formalized discourse). Logic proper (as distinct from its “commercial use”: Republic, 525C) invites us to contemplate the workings of this modification in order to ascertain not only its workings, but more crucially its power and limitation.
These four transformations of primordial discourse happen \textit{together}; although one or the other may be emphasized at some one moment of investigation, each can come fully into its own only with the other three. And any assertion has this fourfold transformation as its source: as any one of the four falters, an assertion loses its status as an assertion and either returns to some ordinary business of getting things done or becomes fodder for various kinds of re-talk (including what some critics call “nonsense”).

Just as Plato and his successors in Antiquity, and Kant and his successors in Modernity, so now Heidegger and his rightful heirs invite us to contemplate the source of assertions. Now, however, the source of assertions appears neither as transcendent thinghood (nature) nor as transcendental selfhood (spirit), but as all-embracing worldhood.

And just as our Platonic tradition aspired to retrieve what the “life of the assertion” abandons (namely, \textit{ουσια}), and just as our Kantian tradition aspired to retrieve the human agency otherwise abandoned by modern science, so the Interpretation most ... worldly condition empirically and commercially (as evident in much academic psychology, sociology, biology, and economics).

Now, the structure of assertions transcends space and time — so that the best of them, the epistemic ones, hold no matter \textit{where} (as fire burns the same in Persia and in Athens: \textit{Nicomachean Ethics}, 1134 b 36) and no matter \textit{when} (they engage us in \textit{το αει ον}: what always is). How can this be? How can assertions take on such ubiquity and eternity — given that so much talk, even in academia, proves to be only local and fleeting? Plato and Aristotle answer: by reflecting (addressing) nature in her ubiquity and eternity. Kant answers: by reflecting (embodying) the \textit{a priori} conditions of the possibility of any experience at all. But how might we answer? And in a way that, while not falling back on these earlier traditions, turns to their source — and might thereby justify them?

First of all, by relocating the question to read: How do space and time already enter into \textit{primordial} talk, into discourse out of which assertions might arise?

Discourse already belongs to a situation — \textit{is} the articulation (keeping together) of a world. And this means: discourse belongs to a \textit{place} — a home or a ship, Persia or Athens. What provides this place is a world: its signification incessantly sends us to items placed within a context. But any placed item, any item \textit{in} its place or even out of it (misplaced), has its place within an area, a region, a space. It has its place because it \textit{fits}: each such place \textit{is} as one among an interwoven many. So, for instance, one has learned one’s native tongue by learning to make one’s way through the place overreaching a multiplicity of places: by participating in the signification structuring a world. In this sense, one’s native tongue remains tied to one’s native land, and learning a foreign tongue requires that one learn \textit{where} one is.* Primordial talk is so spatial that we have no need to take note of its spatiality. Indeed, we must learn it all over again if we are to bring it near — as in the often explicitly autochthonous talk of genuine literature.

Yet discourse is not “given” to us, neither in whole nor in part. As the articulation of the signification structuring a world, primordial talk \textit{hangs over} us. At each moment, whether in the silence of more or less concentrated work (in the garden or at the office, alone or with others), or in the roar of more or less dissipated debate (over dinner, during a committee meeting), or in the apparent ease of the simplest re-talk (chat over coffee, instructions for beginners), discourse \textit{looms as a whole}. In silent concentration, one may \textit{hear} the whole, while in a roaring debate one struggles even to hear and interconnect the parts, and in re-talk one draws upon parts with hardly any connection at all. Still, in \textit{any} mode, discourse looms not only as constricting (as prescribing a world), not only as a liberation (as opening windows), but also as demanding response on our part (both as speakers and as listeners): our own “agency” is at issue. Not yet one of making or assessing claims, this agency is more akin to simply \textit{expressing} itself (Here I am! There you are! Such a one is talking!). Any “convincing” regards more the affirmation of joint being-

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* Cf. Wittgenstein: “the speaking of a language is one part of an activity — of a form of life” (Philosophical Investigations, §23). Heidegger also broaches the question why so much of our vocabulary is spatially metaphorical: “Is it by chance that meanings are ... predominantly spatial?” (Being and Time, p. 166; cf. also the comments on the origin of pronouns, p. 120).
there than any decision about what is said.\footnote{In one of his mini-dialogues Wittgenstein brings out something essential about primordial talk (while also perhaps addressing the question of Plato’s \textit{Cratylus}). An interlocutor remarks: “So you say that the agreement among people \textit{[convention, συνθήκη] is what decides what’s correct and what’s incorrect?” and Wittgenstein replies: “What’s correct and what’s incorrect is what people say; and it is about language that people agree. This is no agreement regarding opinions \textit{[here: claims], it’s agreement on a form of life” (\textit{Philosophical Investigations, §241}).} Namely, “about” responding jointly to what’s looming. Discourse here is essentially temporal—in the \textit{primordial} sense of time: it engages us “\textit{from the start}” in a looming and demanding pre-established whole (and, as looming and demanding, offers many directions of movement). Relics of this temporality are evident in the complicated ways the grammar of a tongue engages us in temporal distinctions (so that foreigners inevitably betray themselves as non-natives in their failure to enunciate these distinctions as natives do). But these relics themselves are only possible because discourse is temporal already \textit{prior} to the fact that we talk coherently about what's coming, what's at hand, and what's established, marking these dimensions out with a plethora of grammatical distinctions.

Now we ask again: How can assertions transcend space and time? Or: How can the \textit{structure} of assertions (whether subject-predicate and syllogistic, or “the manner by which given cognitions are brought into an objective unity of apperception”) embody insights regarding “everywhere” and “always”?

So long as assertion appears to be the \textit{primary} mode of discourse, namely the “preferred” or “distinguished” mode of talk, we will find ourselves driven back either to Plato or to Kant, both of whom understood full discourse as committed to enhancing our grasp of what’s \textit{on hand} (of nature considered as such). If, however, we succeed in bringing discourse near as it embodies the signification structuring world, we may answer the question differently: the moment we turn discourse back on itself it appears as a claim to be assessed. \textit{We then} look to what the claim \textit{is about}. But what we are now looking at has lost its world. Having lost the world essential to primordial discourse, the \textit{structure} of the assertion rightly seems to hold \textit{regardless} of where and when we call upon it. Because it has forfeited its roots it holds \textit{anywhere} and \textit{anytime} we call upon it: it is essentially portable. But this portability first reflects a \textit{loss}, not directly an insight. Its ubiquity and eternity actually stems from its holding \textit{nowhere} and \textit{never}, i.e. only at the moment of “application.” And then here, having “content,” assertions easily forget their provenance and appear to talk about what holds everywhere and always (whether in Persia or Athens, a million years ago or a million years hence).

The power of assertion-based discourse (discourse ignoring its basis), lies precisely in its portability. But for that very reason it is powerless to provide \textit{insight} into what it is about: it holds just as much nowhere as everywhere, and just as much never as always. Goethe’s Faust may present the tragic version of this self-contradiction, but the discourse of modern economics, politics, and technological science present the everyday fact of it: \textit{global} formulations about \textit{individual} situations.

Plato and Aristotle acknowledged the problem: knowledge (\textit{ἐπιστήμη}) has its origin in direct insight (\textit{νοῦς}) and remains “potential” until “actualized” at its origin (\textit{Metaphysics}, 1087 a 15); words (\textit{τὸ διάσωμα}) must grow out of the matter itself (\textit{τὸ πρᾶγμα}) if our souls are to become healthy (\textit{Cratylus}, 435C and 440D). But, having reversed this very priority, insisting now that “objects” must conform to our cognitive faculties, Kant can only recommend restraint; this restraint stems from the acknowledgement that, wherever and whenever the \textit{conditions for assertion} fail us, we should withhold our \textit{claims}.

But once we discover both the power and the powerlessness of assertive discourse, we may well discover the power of primordial discourse to engender an intellectual discourse that recalls its origin first of all. A discourse that may even recover logic itself.

§3.6 Negation

But assertions not only assert what \textit{is}, they may also assert what is \textit{not}. Such assertoric negation takes a variety of forms:

(1) A predicate may \textit{not} apply to a singular subject (a this-here, \textit{τὸ τὸδε τὐ}) in either of two ways: it may touch something either accidental or essential about the subject, and so its negation may either not affect

\footnote{In the last sentence of his \textit{Tractatus}, Wittgenstein refines the Kantian injunction: “About what one cannot talk about one must remain silent.” Unlike his \textit{Philosophical Investigations}, this early work assumes that any powerful intellectual talk must take the form of assertion.}
the subject's own being or entail the non-being of the subject itself (temporary deprivation or eternal destruction). This form of negation allows for careful artisanal work.

(2) A predicate may not apply to a kind of being (its form, τὸ ἐιδος), again in either of two ways: either the kind does not necessarily have the property named by the predicate, or it necessarily does not have it. The form of negation allows us to distinguish one kind from others.

(3) The named subject might not be at all — again, either an individual or a kind. The non-being of a subject may reflect a fantasy on our part (the species unicorn, the individual Monster of Loch Ness), or it may reflect extinction (the species dodo bird, the individual League of Nations). This form of negation allows for the basic dramas of human enterprise, including the drama of exact knowledge (science).

(4) Negation may apply to sayings: what someone says is not the case. The denial of a saying entails an affirmation: it is the case that . . . (the contradiction of what was said). This form of negation allows us to take issue with one another, and within ourselves.

(5) Negation arises in the assertion of principles: the same being is not able both to be and not to be . . .; we are not able both to affirm and to deny an assertion; cognition is not possible without a conceptually unified manifold of space-time intuition. This form of negation lies at the heart of contemplative discourse (the consideration of what is and what is not possible).

Negation happens in casual conversation (especially in complaint), in public debate (especially in denunciations), in epistemic discourse (especially in academic pursuits), and in contemplative literature.

One of the earliest contemplative negations is that of Parmenides' didactic poem, addressed to Zeno in the 5th century, B.C.. It instructs us to acknowledge only two ways, two paths, of inquiry that we can think:

The one way, on which being is and non-being is not:
this is the path of confidence (for it follows truth).
The other way, on which non-being is and presses not being:
this road, I say to you, is utterly inscrutable;
for you neither know what is not (for such is not feasible),
nor do you say it.*

* Diels' fragment 2. Of course, everything depends on how we translate the Greek. My own translation intends simply to recall the question.

We hear, then, that we can indeed think two ways. Yet we also hear that one of these ways is “inscrutable” in itself and that along this way we neither “know” nor “say” anything. Parmenides insists that he himself says this: says, namely, that it is not for us to travel this road, even though we might try to. And he offers a reason: neither knowing (making sense of things) nor saying (pointing out, laying claim) can happen on this road.

At issue is then the interrelation between the thinking (that recognizes two roads) and knowing/saying (here: confined to what is and excluded from what is not).

Indeed, what could it mean to know something as not being — in any of the senses listed? Well, we are indeed called upon (by others, even by ourselves) to testify whether or not some predicate applies to a given subject. And we might then assert (know and say) that it does. But what might it mean to assert that it does not? Precisely in careful investigation, and in the cross-examination of witnesses, we often find it appropriate to testify negatively — in the form: I cannot say that S is, or that P belongs to S. Meaning: I did not see, hear, discover that it does. Not knowing whether or how something is, I report only that I am not able to say: I am not here saying anything. I am uttering myself, acknowledging myself, even knowing myself — how I am, namely to the side of the matter under consideration. Here, as in Parmenides' sayings, the “not” reflects back on our being — where and how we . . . are.

Namely, at a crossroads. Who can deny that our vocational condition is one constantly at a fork where we must acknowledge two directions for each of the many possibilities, but most pressingly for the one presenting itself at the moment? And not only in laboratory experiments and police investigations, but also in planting seedlings and saddling horses: this is the way, not the other. Our being at the crossroads allows for mistakes in our actions and assertions.

Yet who can deny, either, that it is possible to bluff one's way around the crossroads — to create a smoke screen of sorts, a discourse obscuring the fork in the road and leading oneself, along with others, on no path at all, but through the bushes? That is, it is possible not to engage ourselves vocationally at the crossroads — owing either to never having learned a vocation or to having abandoned its imperatives: in
either case, not to think the two ways at all, and to proceed non-intellectually, “blindly.”

Thus Parmenides exhorts Zeno that it is also imperative for him that he take stock of the “consensus of mortals,” in which there is no “true confidence”—where “consensus” (δοξα) is the “normal” way of human being (a third possibility), yet one not allowing for confidence (πιστις: fragment 1). Why imperative? Perhaps in order to avoid falling into this third possibility. Perhaps in order to recognize others as having fallen into it. Perhaps in order to distinguish clearly between the one genuine imperative and the multiplicity of incidental imperatives hounding one’s ears in daily intercourse.

The crossroads of being and non-being not only locates the central burden of vocational thinking, it also engenders an “extra” burden for contemplative discourse itself—in account for, to bring into contemplative view, the conflict itself: first of all, between being and non-being, and then between engaging forthrightly in this core-conflict and engaging in the business of disguising it (“sophistry,” as we say now, after Plato’s analysis of it in his *Sophist*).

Again, though, our traditions assuming this extra burden plunge us into the contemplation of assertion as the playing field of discourse. And they therefore address the question of negation from the standpoint of intellectual investigation. Both Aristotle’s Axiom of Non-contradiction and Kant’s Highest Principle of Synthetic Cognition lead us away from primordial λόγος into the discourse appropriate for aloof consideration of on-hand affairs: the selection of assertions for joint testing to decide their accuracy and to refine their formulation. Even though the greatest representatives of our traditions do incessantly dip back into pre-philosophical involvements for evidence or illustration, they do not form a discourse that allows us to dwell in affairs at hand. In our traditions, philosophical discourse appears to commit us to dwelling in the mode of investigation itself, and to abandon the consideration of primordial involvements to story-tellers, manipulators of public consensus, and comforters of various sorts. In short, to “leap over the phenomenon of world”—where there is no “true confidence”—or to study it from the outside, perhaps with a view to telling stories serving some outside purpose, to reform public opinion, or to anesthetize the masses.*

Thus, while Antiquity locates negation in the fragility of nature (she generally falls short of her own intent, as any artisan must learn), our Modernity locates it in the fragility of synthesis: assert what we will, the formation of the totality in which each assertion has its proper place is only as it meets correlative resistance and threatens to collapse in the face of it: time marks cognition as a flux, and nature appears as locating the threat (non-human microbes, human greed . . . ). Resistance names the hither-side of whatever organization (synthesis) has been achieved, both what is not included in synthesis and what might explode it. And yet the very threat of non-being (of refutation of an assertion, of destruction of a theory, of the collapse of the organization built on the theory) pumps blood back into the affair.†

Heidegger’s works invite us to consider an alternative—yet not so much a different contemplative account of negation as one that brings closer the fragility of world itself, one undergirding both the very “real” fragility of nature and the very “real” fragility of humanly devised logical systems, social organizations, and intellectual constructions.

* Many academic philosophers classify Thoreau’s *Walden* as only story-telling, Dewey’s *Art as Experience* as merely reconstructing practical affairs, and Heidegger’s *Being and Time* as an apology for a political agenda. Those committed to aloof analysis and synthesis will of necessity be allergic to contemplative discourse inviting us to dwell where we are.
† Front-line thinkers of resistance are: Hegel, Marx and Nietzsche; to which I add John Dewey, and Heidegger adds both Scheler and Dilthey (*Being and Time*, p. 210). Behind the front lines, in the philosophy of science, are Karl Popper and his students (most notably, perhaps, Thomas Kuhn). At “third remove from the king and the truth” is the assumption permeating our universities from the 1920s to the present: that the integrity of individuals and organizations depends on discerning enemies (within or without) and their victims (close by or far away: whether human, e.g. “minorities,” or natural, e.g. rain forests). This politicalization of Plato’s “third remove” belongs to our legacy from the Enlightenment: the discovery and enhancement of human agency, so that resistance is first experienced as constriction by others (or, more impersonally, by the conventions of society), and the first truth requires a war of liberation. Whereas debates in the media and at the universities focus on the details of such third-remove concerns, Heidegger’s works address them whole under various rubrics, most obviously “technology.”
How might world itself “involve” negation of the sort with which we are already familiar as being-in-world—and then also thematically in our capacity as logicians? What is world—such that something kin to logical negation might become evident?

Following the style inaugurated by Plato, Heidegger insists that the question what be entwined with the question how: How might we learn the basis of negation, the kinship of this basis with its logical manifestation? For a fully philosophical work (as distinct from a philosophical commentary) accompanies rather than informs us: it clears a way, leads the way, provides a map for us in our journey. What can we say about this map?

Formally speaking, a world is a where-in (of self-enclosed understanding, i.e. projection into possibility) serving as an upon-which of encounters. More casually: a context engaging us in more or less fixed manners of undertaking tasks, and thereby allows us to tend to things in our environment. More or less fixed: both entirely fixed (pre-cast) and entirely unfixed: for a world hovers over what arises for encounters, is “forever unfinished” (“time will tell” if what impends now will “behave itself” as world prescribes). Formally, again: for all its fixedness, world is also pure projection. More gently stated: world is but a dream—or, more accurately, a source of dreams, hopes, plans defining what arises for encounter.

Yet this is something we learn—contrary to what initially and mostly seems evident. And when we do learn, or rather suspect it, there are a number of easy reactions: we may laugh at our world, taking it as a joke; we may cry over it, taking it as a disappointment; we may resign ourselves to it, taking it as only the public dimension surrounding private life (much like the ancient Cynics), or we may try to correct it and thereby make our dreams, hopes, and plans more “realistic.” Yet so far we have only half learned: world has not yet revealed itself fully for what it is—namely our only home, or rather the only possibility of home. We have not yet come home, but are rather trying to sever ourselves from it: the grand illusion, even bad faith. —What, though, is then left to learn?

World that is only world (as it appears to the careerist, for instance) is indeed empty, is nothing: so Heidegger succinctly formulates what we more or less learn. Prior to any theory why it is empty, or what we might “do about it,” we experience the nothingness of world. Expressly following Kierkegaard, Heidegger reserves for this experience the term “dread” (Angst). It is in dread, the experience of world as empty, that world reveals itself as world: as the “where-in” of our primordial context, one that also forms a “where-upon” of possible encounters. Only in acknowledging its emptiness does world appear holistically—so that, for instance, we can read and write about it, i.e. engage in contemplation of the sort at issue in all great literature, and in all poignant conversation. Only the nothingness of world allows it to teach us that it is as a framework, an opening, the window of a very special opportunity: that of allowing whatever arises for encounter to be itself—rather than only fodder sustaining the on-going affairs of our world (but more on this development when we come to the question of truth).

One corollary of the nothingness of world is that world, now understood as hovering over what arises for encounter, already transcends whatever so arises:

Being near beings at hand while taking care of them, thematizing what is on hand, and objectifying the latter: these already presuppose world, i.e. they are possible only as manners of being-in-world. Grounded in the horizontal unity of ecstatic temporality, world is transcendent. It must already be ecstatically disclosed so that innerworldly beings can arise from it for encounter.¹

Every moment is one at which world is not…: most obviously (at least in our youth), not yet our own, not all that it could be, not finished; correlatively (especially in our old age) not what it used to be, not what it could have been, no longer imbued with promise. Still, these obvious negations pertain to world only, i.e. they remain relative to our worldly expectations and formations. If world is “all there is” such negations

¹ Heidegger talks about “the nothingness of the world” (das Nichts der Welt) on pp. 276-7 and 343. My own account recalls the contexts of such talk.

² Being and Time, pp. 365-6. The “unity of ecstatic temporality” has been worked out earlier: world opens out in three directions, namely resumption of its fixedness, engagement in its projection, and—crucially, i.e. most fragiley—encountering what arises. These three are unified either inauthentically (in flight) or authentically (in convergence). Yet, transcending as it does what arises for encounter while also opening out on it as possibility, unity itself is empty.
either make no sense at all or (as the pharmaceutical industry today would like us to believe) reflect a biological dysfunction — the recognition of which once again plunges us into a consideration of the function that is here violated, and so again into the question of what-is-not relative to what-really-is (here: what should be).

The contemplative questions read: How are such experiences (foiled hopes and the like) possible? How are such explanations (natural shortfall and the like) possible? Traditionally, answers have circled around the basic supposition that negation of any sort is relative to some original position (something that “really” posits itself, and that we ourselves must learn to posit in return). In contrast, Heidegger invites us to see, with a phenomenological eye, three dimensions of our being-in-world, each of which announces its own “not” (negation) prior to any being (position):

(1) Our own being there in world is pre-cast (“thrown”), a condition we fully recognize only when acknowledging that we are brought into our respective situations not of our own accord and that it is not in our power to dictate the course of affairs (p. 284: the “opposite,” the effort not to acknowledge our finitude, is delusion, not a goal to be posited, let alone achieved).

(2) Our being there is fore-casted (projective), a condition requiring us to follow up one of a pre-casted bundle of possible avenues, so that there are always roads not taken (p. 285: the “opposite,” the traveling of those other roads, remains with us as, precisely, no longer possible — even though, denying the finitude of our condition, i.e. deluded, we may aspire to trying them out).

(3) Precisely as absorbed in a world, tending carefully to affairs, we discover (in dread) that we are not able to base ourselves on any ability founded primarily in the affair, on the things we take care of (p. 343: the “opposite,” the effort to identify ourselves with things encountered, does not suggest, as it does in some traditions, that we discover something with which we might successfully identify, something above and beyond those things, or underneath them — the source of those quick fixes fashionably available in any epoch).

These three negations stem neither from failed hopes, nor from nature's shortfall, nor from resistance to our projects. They stem from world itself, but not from anything in the world. From world as but a

“window of opportunity” — of one opportunity, namely (as I will discuss in the concluding section of this essay) that the things and people arising for encounter be “set free in their own possibilities.”

Our being there in a world is, in Heidegger's language, basically one of caring: most obviously, one of taking care of things and caring for people — in all the various modalities of heeding, protecting, helping, redirecting, avoiding, loving, hating. But the imperative of such manifest caring is encircled by two others: the imperative to come back to a situation in which things press upon us, and the imperative to follow its thrust forward to where they are going. Thus to take care of things and to care for others requires that we care about the settledness of our situation (the “past” that is not at all gone: for it is our world) and the possibilities of our situation (the “future” that is already impending: for it is our world). Only when these three dimensions of care come together do we have a special moment of opening out: one in which the opening is itself the culmination rather than any goal set within our world. It is now that the nothingness of world becomes fully evident — in our care for what is both other (not world) and very much our own (to take care of, to care for).

Heidegger's account here intends to broach the question of the meaning of being, a meaning that, as it turns out, is double: on the one hand, the being of the one being that each of us is, namely being there, is as world (in the various ways Being and Time brings into view); on the other hand, the being of those beings arising for encounter through world requires liberation: these beings depend, for their being, on the modality, the maturation, of care — their being is pre-eminently one of possibility, sharing with us this one meaning: being forever at “crossroads.” It is this meaning of being, the being of beings not taking their measure from us but rather setting measures for us, that all of Heidegger's work intends to elucidate. And especially the account of “dread in the face of the nothingness of world.”

But why then all the talk of dread? Does such talk not call attention to human predicaments? How do our predicaments — or even those of world, serve contemplation intending to elucidate the meaning of being?

* See, e.g., pp. 144, 297-8, and elsewhere. In contrast, Aristotle speaks of the soul as having no “nature” of its own, being only a what I call a “window of opportunity” (cf. On the Soul, 429 a 20 ff.).
The question of being has always unfolded within another question: How do we learn the being of beings? And this question entails yet another: What arouses us to raise the question? For without a genuine arousal, questions give way to premature answers, namely those offered already by the world we happen to live in, and these answers supplant not only the question (so it is never raised) but also what the question is about (so that being itself does not become an issue, and we pass it by).

Dread, as Heidegger asks us to understand the word, as an experience of the nothingness of world, is a summoner.* Heidegger’s work speaks to the same question raised by Plato in his Republic (521C ff.) — and answered in a way that has guided the development of our entire intellectual tradition. On Plato’s account, what draws us (τὸ δικαίωμα) to the intellectual concern for the being of beings (ὁδειγμα, τὸ δν, τὸ δντα) is the clash of sensation (αἰσθησις) with intellection (νοησις). Such clashes serve as summoners of thought (παρακλητικα της διανοιας) — more so than, because conditioning the effectiveness of, the elenchus at work when one of us undoes the opinions of another. But then summoners arise when and where things arising for encounter resist our understanding of them — and the question of being is henceforth a question of how best to bring beings into our fold of understanding. This latter question vastly reduces the scope of the question of being: reduces it to the problem of “getting a handle” on what arises for encounter. And thereby increasingly precludes the possibility of a mode of thinking that would remain responsive to beings as they retain their own being.

As a summoner, dread might — depending on what we make of it — occasion an opening. A breach in the world, as it were. This opening can become very manifest in the way we talk — whether we are listening or not:

Listening to... is an existential being-open-for-others, an openness belonging to being-there as being-with. Indeed, hearing constitutes the primary authentic openness of being-there for its ownmost ability-to-be — as the hearing of the voice of that friend every being-there carries within itself.†

* Being and Time, Division Two, Chapter Two. See also his much later Was heißt Denken? (on what calls for thinking — thinking of the sort that we then rightly call thinking).
† Being and Time, p. 163. The “friend” here is the “call of conscience,” Heidegger makes clear only on p. 287. Still, such hearing also “helps others to become transparent in their care and free for it” (p. 122) — very possibly a necessary condition for genuine friendship.

§3.7 Truth

The most primordial event of truth is, Heidegger says, the disclosedness of being-there. And being-there is both “in truth” and “in untruth.” Finally, following the Greek we translate as “truth” (ἀληθεια), Heidegger asks us to contemplate the question of truth as the question of unconcealment. † — What can all this talk mean for a logician?

Talk that listens resonates with the otherness to which our talk attends — with the not-ness only possible in acknowledgement of the nothingness of world. William Faulkner recalls such talk, calling it “the best”:

He was sixteen. For six years now he had been a man’s hunter. For six years he had heard the best of all talking. It was of the wilderness, the big woods, bigger and older than any recorded document: — of white man fatuous enough to believe he had bought any fragment of it, of Indian ruthless enough to pretend that any fragment of it had been his to convey:... It was of men, not white nor black nor red, but men, hunters, with the will and the hardihood to endure and the humility and skill to survive, and the dogs and the bear and deer juxtaposed and relieved against it, ordered and compelled by and within the wilderness in the ancient and unremitting contest according to the ancient and immittigable rules which voided all regrets and brooked no quarter; — the best game of all, the best of all breathing and forever the best of all listening, the voices quiet and weighty and deliberate for retrospection and recollection and exactitude among the concrete trophies — the racked guns and the heads and skins — in the libraries of town houses or the offices of plantation houses or (best of all) in the camps themselves where the intact and still-warm meat yet hung...+
From the beginning (in the works of Plato and Aristotle), integrity of discourse appeared as depending on the “presupposition” that “there is” truth: the daily business of talking something through, the necessity of inferring conclusions from premises, the possibility of recalling counter-examples to discredit generalizations — all presuppose that discourse is beholden to truth and may therefore fall into falsehood. And at the end (our end), the question of integrity, although distinctively shifted to that of formal systems, depends on criteria for deciding whether to accept a formula into the system or not: “there is” (again, more an imperative: “there must be”) truth — although the criteria now appear in purely syntactical form, and reflect primarily the necessity for a yes-no decision.

Plato and Aristotle reveal the delicacy of truth on site — on the playing field, as it were. For instance, they invite us to distinguish between truth as accidental and truth as essential: it may be “true” that a certain man is bald or a certain woman not bald, or even that men have penises and women have vaginas, but these determinations say nothing about their ability to make shoes or govern a city — and it is in these capacities rather than in those other conditions that the essential truth about human beings will lie. Assertions are ultimately true if and only if they issue from insight into the “innerness” of the subject, its nature.

With Descartes, Kant and Hegel, the subject becomes the knowing one, the human spirit rather than what is known in circumstances (transcendental rather than transcendent), whereupon the issue shifts onto the plane of assertion: it becomes imperative to systematize our sayings, both in philosophy proper and in other intellectual disciplines. Still, though, truth remains that of an “innerness” we must “fathom” — really, one to which we must return.

During the first decades of the 20th century much intellectual work developed to the point where talk about fathoming a subject begins to lose its sense. Mathematical disciplines in fact present “systems” as testing grounds for formulations proposed for possible integration into the manifold of a “research program.” The chief criterion of inclusion becomes whether single formulations or entire systems survive “resistances” that are authenticated precisely in reference to the system itself, not directly in reference to what arises for encounter. What sense does it then make to talk of truth at all?
Christianity. Heidegger detects puzzling questions where these others refine mainstream answers.

But what is there to puzzle about in the “clean and lean” understanding of truth, namely the primacy of method and therefore, in logical study, of syntax?

First of all, its power — and not just its effectiveness, but also and more centrally its commitment to promoting power over what arises for encounter and for establishing power groups competing among themselves for such power. This commitment puzzles those who can wonder whether the commitment to power does justice to our dealings with circumstances or with our fellows. For many, this is just a matter of preference: whether one happens to enjoy power-plays as against, say, sentimental attachments to landscapes or people. But the puzzling we find in Heidegger’s works arises from a careful understanding of our Greek and Scholastic traditions in which we can find modes of discourse acknowledging rather the power of circumstances themselves as the source of any power an individual or society happens to have acquired.

How did this original account of power (the power of nature, the δύναμις of φύσις) ever evolve into the modern account (the power of human subjectivity)? The ready-made answer reads: the accounts of our distant ancestors are immature, and our own time has grown... oblivion essential to being-there hides from us the original commitment to power and thereby gives free rein to power as ours — presupposedly. Whereas, then, others might dismiss the literature of our forebears, Heidegger argues that we must recover it as our heritage and as a puzzle. But then this version of the puzzle comes home only to those who have learned to participate in the power of the original mode of discourse.

Secondly, and in tandem with the first puzzle, the “clean and lean” understanding of truth promotes the primacy of pictures — and not just the images cast by projectors in classrooms, charts in boardrooms, and TV-sets in livingrooms, but also and more importantly the development of systems as dictating the procedures of instruction, commerce, and entertainment — even the behavior of “nature” understood in Kantian fashion: “the existence of things inasmuch as it is determined by universal laws.” This development puzzles those who have learned to work with the “matters themselves” that the pictures picture; already there seems to be an unbridgeable gap between aloof understanding and hands-on understanding of situations. But it will also puzzle those who, having studied our traditions from Plato and Aristotle onward, have learned to appreciate the arguments that pictures of circumstances at best simulate and at worst distort: in the one case (of icons), they place us at a distance from where we need to be; and in the other case (of idols), they leave us with the detritus of where we need to be. And where need we be? Plato and Aristotle answer: “in contact” with circumstances as these are “in themselves” — this contact, this event, deserving the name “truth.”

What’s puzzling is that modern philosophers have developed the thought that rigorously formed pictures precisely assure whatever “contact” is needed or possible. Leibniz, in proposing the need for a formalized language, one with special characters, wrote: “No one should fear that the contemplation of characters will lead us away from things (a rubus abducat); on the contrary, it leads us into the interior of things (ad intima rerum ducet).” But the “interior of things” becomes identified with the interior of a system: learn the one and you learn the other. Such identification (of opposites, it would seem) works at the institutional level of education, government, and entertainment. As inherited, and that means already defining our institutions, this “picture-theory” of truth encircles us: we cannot refute it, and we cannot afford to dismiss it. The question is again whether we intend to promote it or to question it: whether we can rest with this inherited version of truth.

Heidegger proposes his own terminology for characterizing the “clean and lean” understanding of truth: when pursuing intellectually what we loosely formulate as “how things are,” we initially move within a formalized world, a framework that can only respond to what arises for encounter as resource (natural resources, human resources) — as on hand for encounter, investigation, and consumption. Just as what arises for encounter is thereby presupposed as having this derivative mode of being, the framework itself is presupposed as a mode of being-in-world committed to re-arranging things, putting them in place, stipulating the place for what arises for encounter, and thereby allowing us to understand

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* One of many formulations: “Natur ist das Dasein der Dinge, sofern es nach allgemeinen Gesetzen bestimmt ist.” (*Prol. to Any Future Metaphysics*, §14)

† Letter to Walter von Tschirnhaus, dated May 1678.
the latter as essentially fungible; thus the power and the picturing essential to, and at issue in, modern intellectual work. The integrated dualism of framework and resource sets the conditions for any determination we would accept into the fold of intellectual work. It locates the basic truth that is never discussed, only presupposed by (or rather for) front-line research that otherwise seems to investigate “how things are.”

Heidegger’s terminology does indeed intend to frighten us. But it also intends to lure us into raising the question of truth all over again. Already in Being and Time he asks us to look at what deserves the name “truth” in pre-intellectual modes of being-in-world — and work from here to understand how truth could eventually take on the dualism of framework and resource:

Lodged in a world, each of us is as moving along through the routine — through the signification — encountering things but passing immediately over and beyond them, working with others but taking them for granted, defined by the context but heedless of it. Thus a paradox: things, others, world are all disclosed — but they are also hidden, neglected, missed. How so? World itself absorbs one and all in its daily business, and functions well precisely as routine — so long as things at hand behave themselves. Truth is not “normally” an issue because we are already in the truth: in a world, rising to meet what arises for encounter and working along with others — all this essentially, since a world is as allowing us to take care of things, is as shared with others. But, precisely because such truth works by leaving all these matters concealed as they are in themselves, truth might become an issue.

But how does truth become an issue? This question has two senses: Under what conditions does the question of truth make sense? and What happens at moments when we find ourselves fully in truth.

Heidegger’s entire Being and Time intends to bring the conditions of truth out of the closet: first, the phenomenon of world and then the phenomena of clouding over of world, the phenomena of temporality and historicity. Yet these conditions themselves only become clear to us as we rise to carry the burden they bring to bear on us. Heidegger works out this rise under the double name I translate as “readying resoluteness.” Only at such a moment of rising may we detect the phenomenon of world itself, and then also perhaps the full extent of such phenomena as I have outlined under the rubrics of signification, assertion, and negation — the topics of logical study. Only at such moments of truth do the conditions of truth avail themselves to us.∗

And whether they do or not makes all the difference in the world when we turn to the question of what happens when we find ourselves in truth. Heidegger claims that truth is most primordially “unconcealment”: the undoing of the concealment essential to being-in-world, the full (but momentary) disclosure of that with which we are already intimate. World, essentially opaque, may become momentarily transparent. Truth is an event allowing us to make determinations (inventions of the new or confirmations of the old) as well as to form anticipations (hypotheses, hunches, theories) and to define regions of investigation (of data, evidence, problems) — in short, to concern ourselves for “what’s true” and to formulate, accumulate and bequeath “truths.” But what happens in this unconcealment, this transparency is not just the “clarity” that intellectuals from Plato onwards have demanded (at first as a result of truth, later as defining truth itself). What happens is a transfiguration of what and who arises for encounter.

In Being and Time we read a number of very pointed formulations of this transfiguration, and I cite only one:

The being toward things at hand which understands and takes care of them, and the being toward others that cares for them, are determined from their ownmost ability to-be-themselves.†

Their ability to be — their δύναμις. Beings as they are in themselves — αὐτὰ καθ’ αὑτὰ. And yet in our care — and only derivatively under the rationality we borrow from the signification and discourse “always already” embedded in our world.

∗ Cf. the last paragraph of “What is Metaphysics” (1929): “Philosophy gets going only for one whose own existence leaps authentically into the basic possibilities of being-there in its wholeness.” (Die Philosophie kommt nur in Gang durch einen eigentümlichen Einsprung der eigenen Existenz in die Grundmöglichkeiten des Daseins im Ganzen. Gesamtausgabe, Vol. 5, p. 122.)

† Being and Time, pp. 297-8. Similarly, on p. 87 he claims that things are able to declare themselves in their “in-itself-ness” precisely as we are able to respond to them as at hand in a world (our “default” condition). On p. 122 he distinguishes caring for others that robs them of their care and caring for others that frees them for their care. And on p. 264 he says that accepting our mortality first allows us to understand the ability-to-be of others and to cease confusing their possibilities with our own.
Crucial in the event of truth is a transfiguration of our “being toward” what arises for encounter. This in contrast to any determination of what is essential to what we encounter (the transcendental version of truth), and also to any determination of what is essential to the act determining (the transcendent version of truth). Determinations of either sort become renewedly possible only at moments of transfiguration. Otherwise, determinations are only inherited — as predications in search of their subject, as encasements of a nothingness to which we may awake at moments preparatory to truth.

Now, if Aristotelian logic promotes truth as the determination of things, and if Kantian logic promotes truth as the determination of our own conditions, the best of the logic of the 20th century has no truck with truth at all. Whether Barnays proves the independence of a primitive, whether Gödel proves there are undecidable propositions in any formalized arithmetic system, whether Tarski formalizes the concept of truth in formalized languages — we are not invited to engage in the question of truth in any of the traditional senses, nor in Heidegger’s.

Does the study of logic then necessarily promote oblivion? It certainly can promote it — just as any discipline can, and for the reasons that Heidegger himself discusses at length: oblivion belongs first of all to being-in-world itself, and then especially to any institutionalized discipline.

Logic, too, is inherited: it forms a large component of what’s been handed down to us, that within which academics especially work. Logic forms the backbone of what Heidegger calls the technological framework assuring that the intellectual understanding of what arises for encounter commits us to understanding it as resource to be exploited. As much as any intellectual discipline, the study of logic can simply promote the technology defining our heritage, indeed our world as inherited.

The alternative? To think through the aspirations of logic as the backbone of our heritage and as prejudicing our understanding of signification, discourse, and truth. At its best, logical study does this — or rather it can do it. It is perhaps better suited for providing the opportunity to do it than any other academic discipline. And, in doing so, it may abide by the precepts of truth Heidegger has suggested: truth as a matter of unconcealing, allowing to become transparent what otherwise remains opaque. Contrary to truth is every effort to determine which inherited accomplishments are true: for this can only mean to decide which one to accept in its opacity. We may detect in such “passion for truth” a commitment to what Plato and Aristotle called δόξα, where one only questions what view is “right.”

In the only televised interview Heidegger ever allowed, the interviewer presses Heidegger on what he means when he speaks of the danger of modern technology — a danger greater than that of weapons of mass destruction. He answers, in part, that his talk of danger makes sense only when embedded within the question about the essence of technology. He then takes the occasion to emphasize that, far from being against technology, his critical project assumes that our only salvation consists in pondering it with a view to getting it to reveal the source of its power. For only this source will reveal the power that belongs to it rather than merely to our institutions — not a power primarily ours to wield but a power to which we are all beholden.

I see in technology, namely in its essence, that man is placed under a power that challenges him, and in regard to which he is no longer free — that here something is announcing itself, namely a bearing of being on man — and that this bearing, concealed in the essence of technology, will perhaps one day come to light in its unconcealedness.

Whether that will happen I can’t say! But I do see in the essence of technology the first suggestive appearance of a far deeper mystery, what I call the “event of things coming into their own.” From this you can discern that there’s no sense in any talk of my resisting or denigrating technology.”

---


Is there any salvation at all? It first is, and only is, when danger is. Danger is when being itself has run its course to the last — and when oblivion, which stems from being itself, reverses itself.

On the margin of this passage Heidegger wrote: “The framework [das Gestell] as the most extreme oblivion — and at the same time as hint of the event of things coming into their own [das Ereignis].”
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