Math 1111 Test #1

Time: 60 minutes   Total: 60   Name________________________________

Note: In the calculation of derivatives, any method or formula may be used unless explicit instructions to the contrary are given. Show your work.

1. [5] Calculate the following limits if they exist. Show your work. Indicate if the limit is an infinite limit.

a) $\lim_{x \to 4} \dfrac{x-4}{\sqrt{x}-2}$

b) $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} x^2 - 6 & \text{if } x < 3 \\ 5 - x & \text{if } x > 3 \end{cases}$

2. [9] Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-2}$

Evaluate each of the following functions and specify their **domains**.

a) $(f \cdot g)(x) = f(x) \cdot g(x)$

b) $(f \circ g)(x) = f(g(x))$

c) $(g \circ f)(x) = g(f(x))$

3. [3] Match each type of discontinuity with the letter at right whose explanation best characterizes it.

1) Jump discontinuity ______
2) Infinite discontinuity ______
3) Removable discontinuity ______

A) $\lim_{x\to a} f(x)$ exists
B) graph of f has a vertical asymptote
C) left and right limits differ
D) None of the above

4. [6] Using only the limit definition of derivative, calculate $f'(x)$ for $f(x) = \frac{1}{x-4}$.

(No other method will receive any credit!)

5. [6] Find the **equation of the tangent line** to the curve $y = x^3 - x + 2$ at the point where $x = 1$.

6. [12] Calculate the derivative $f'(x)$ for each of the following. Show your work. You need not simplify.

a) $f(x) = x^6 - 3x^2 + \sqrt{x} + 5 - \dfrac{3}{x^2}$

b) $f(x) = (2x^4 + 5x - 2)(\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}})$

c) $f(x) = \left(\dfrac{\frac{1}{x} + 5}{x^2 + 1} \right)^3$

7. [5] Find y' at (1,1) if y is given implicitly by

$$y^3 + 5x^2y + x = 7$$

8. [5] Evaluate the second derivative of $f(x)$ at $x = 0$, that is, calculate $f^{(2)}(0)$ for $f(x) = \frac{1}{2x+1}$.

9. [6] Below are the graphs of two functions $y = f(x)$ and $y = g(x)$. On the same set of axes as f and g, draw the derivative curves $y = f'(x)$ and $y = g'(x)$.

10. [3] Carefully state the **Intermediate Value Theorem**. Draw a picture illustrating this result.